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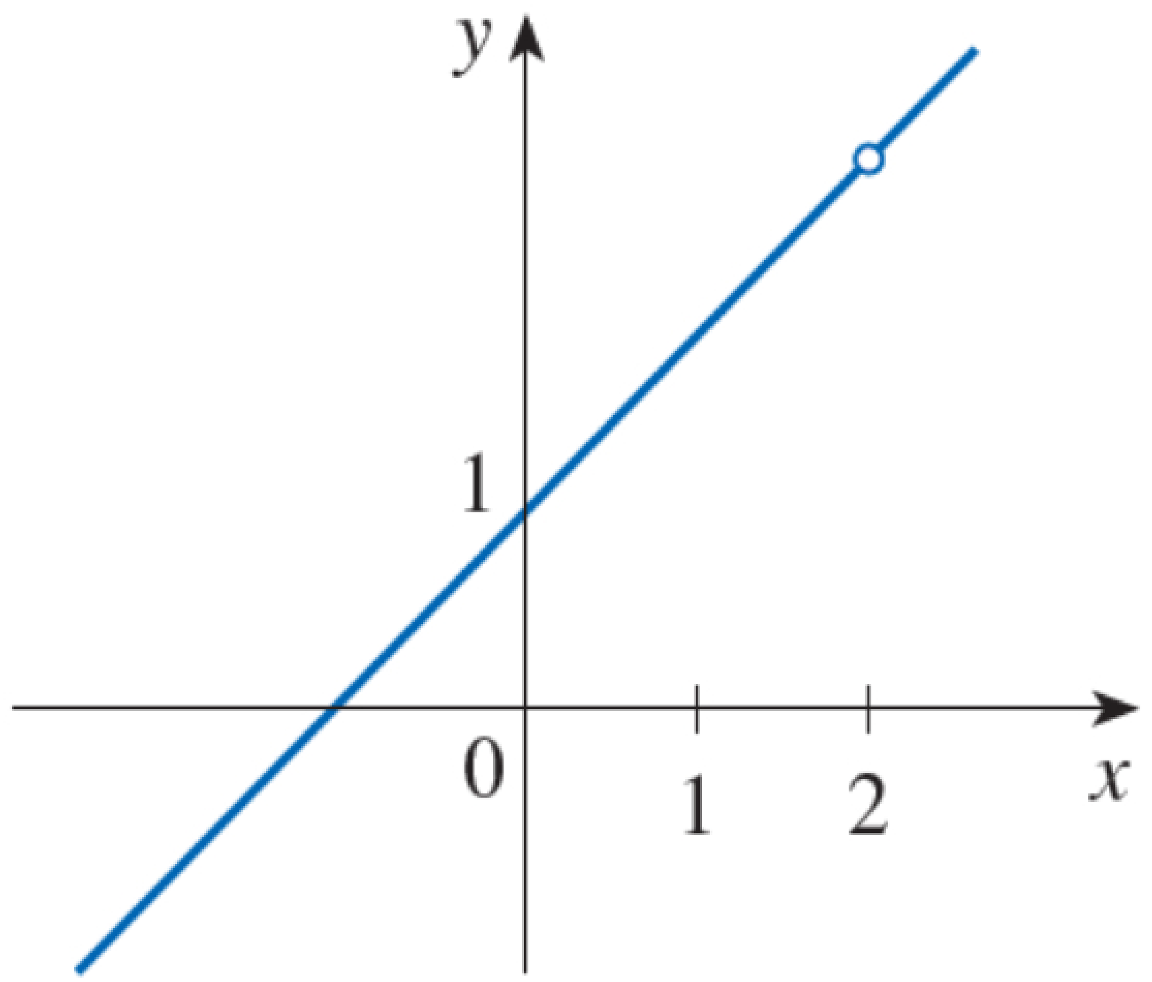
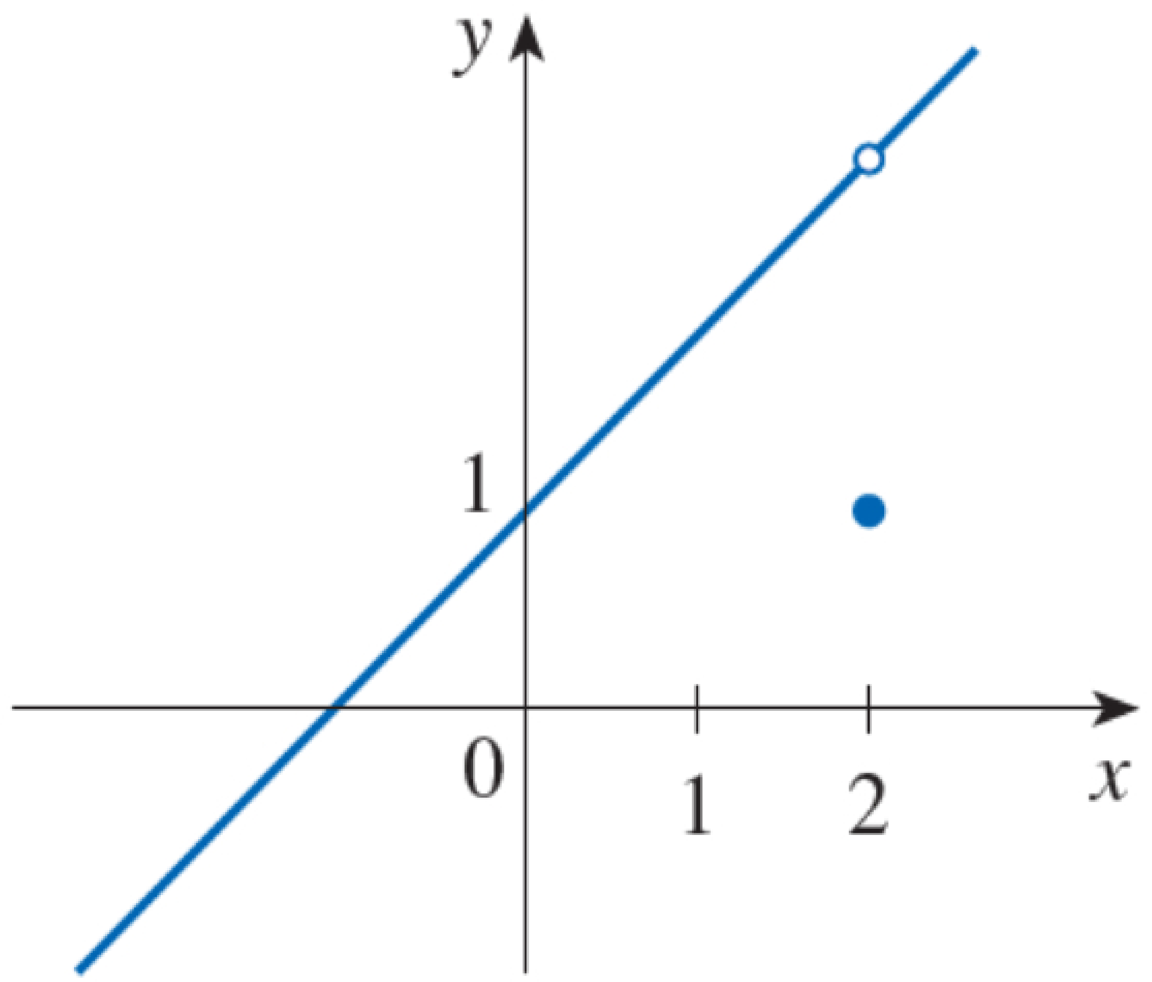
$\boxed{1}$ **Definition** A function f is **continuous at a number *a*** if

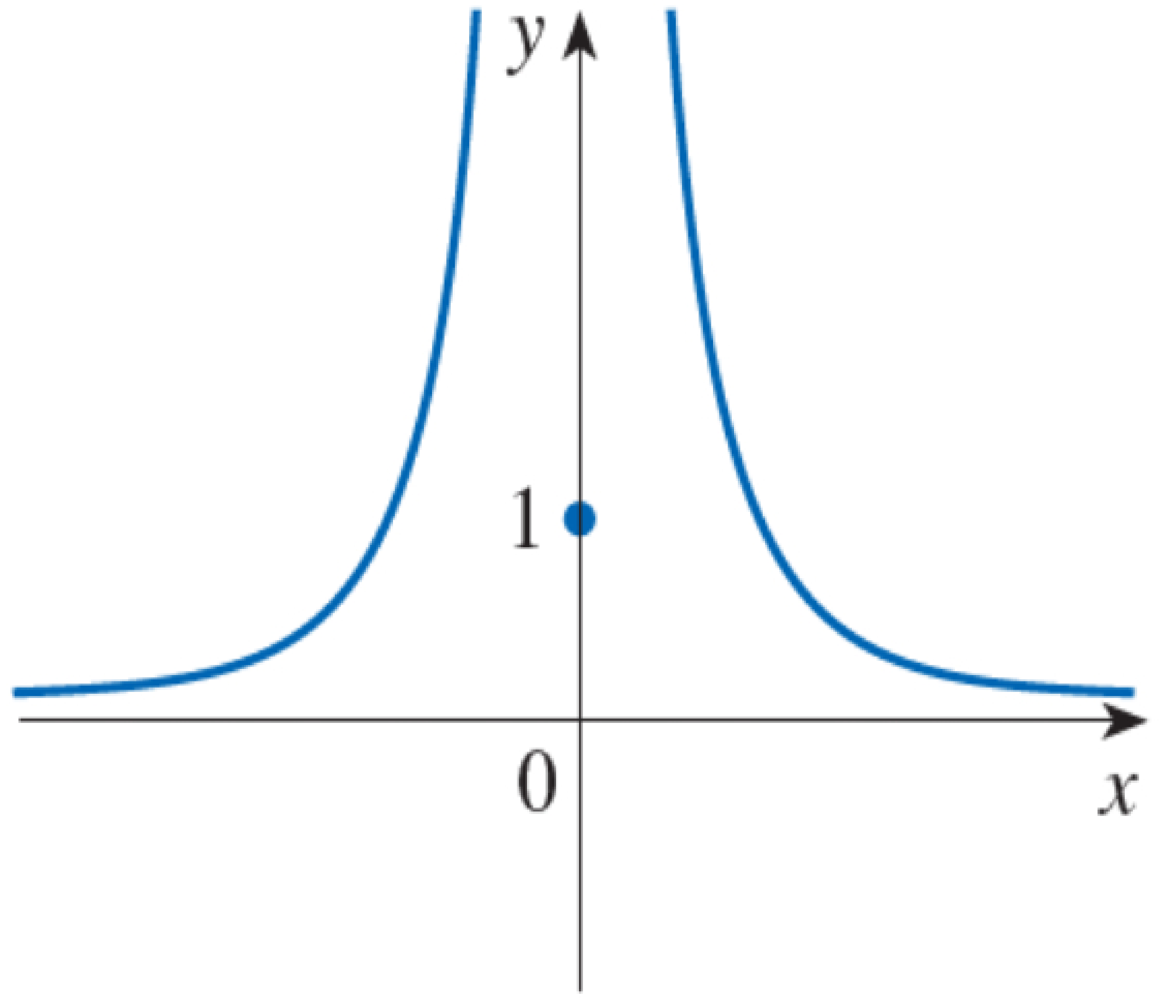
$$\lim_{x \to a} f(x) = f(a)$$



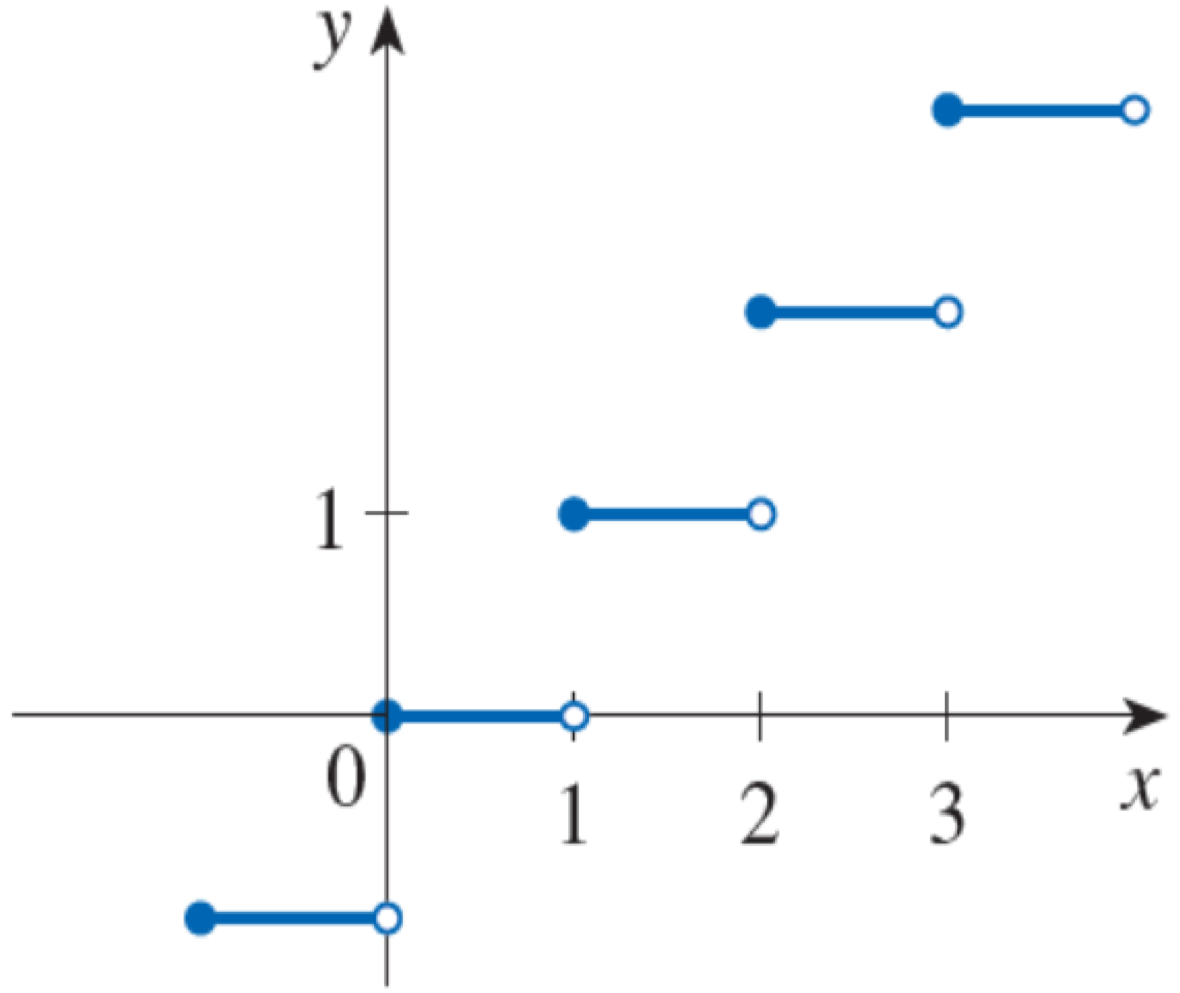
(a) A removable discontinuity



(b) A removable discontinuity



(c) An infinite discontinuity



(d) Jump discontinuities

$f(x) = [\![x]\!]$

Determining points of discontinuity from a graph

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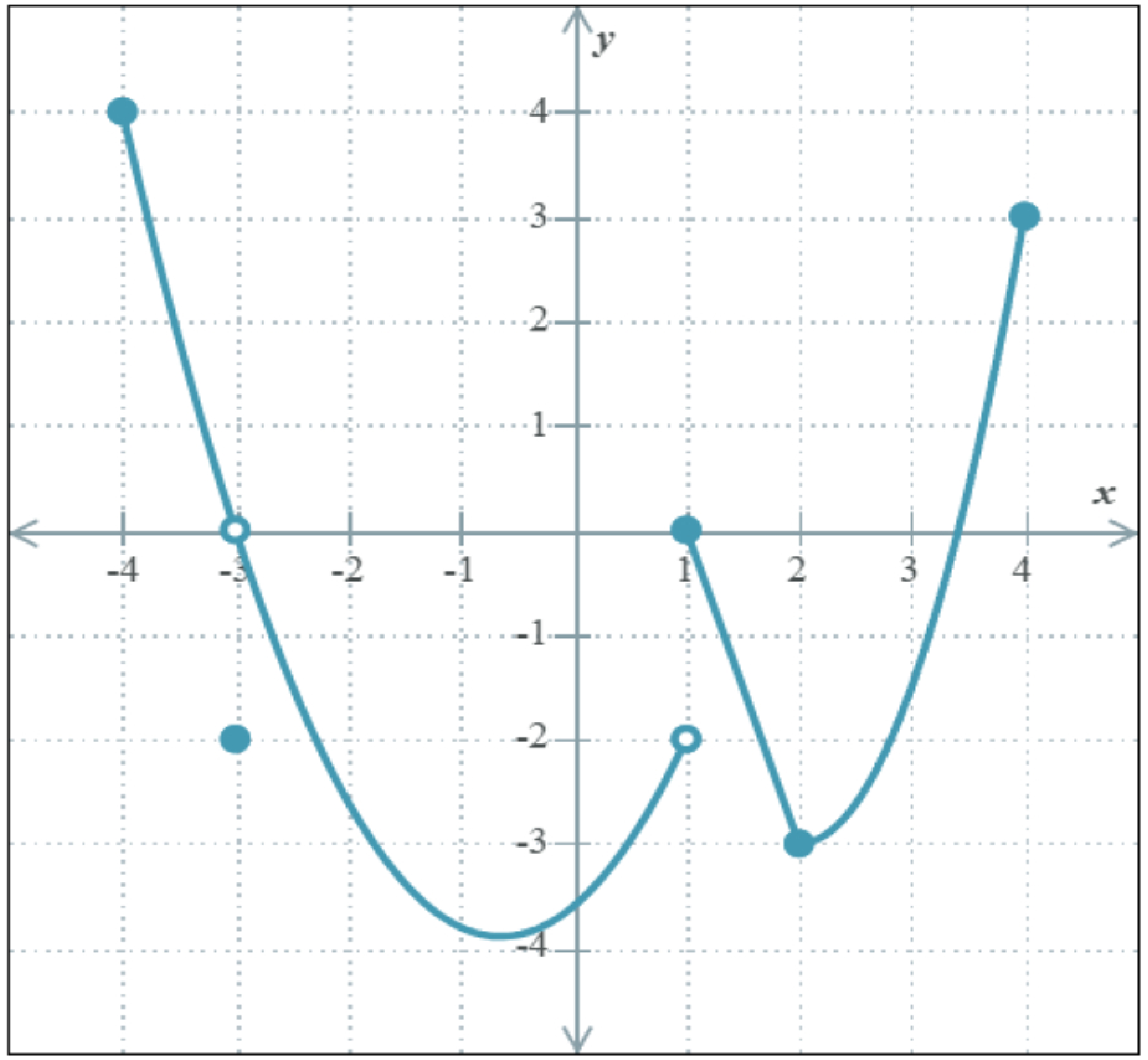
The function g is graphed below.



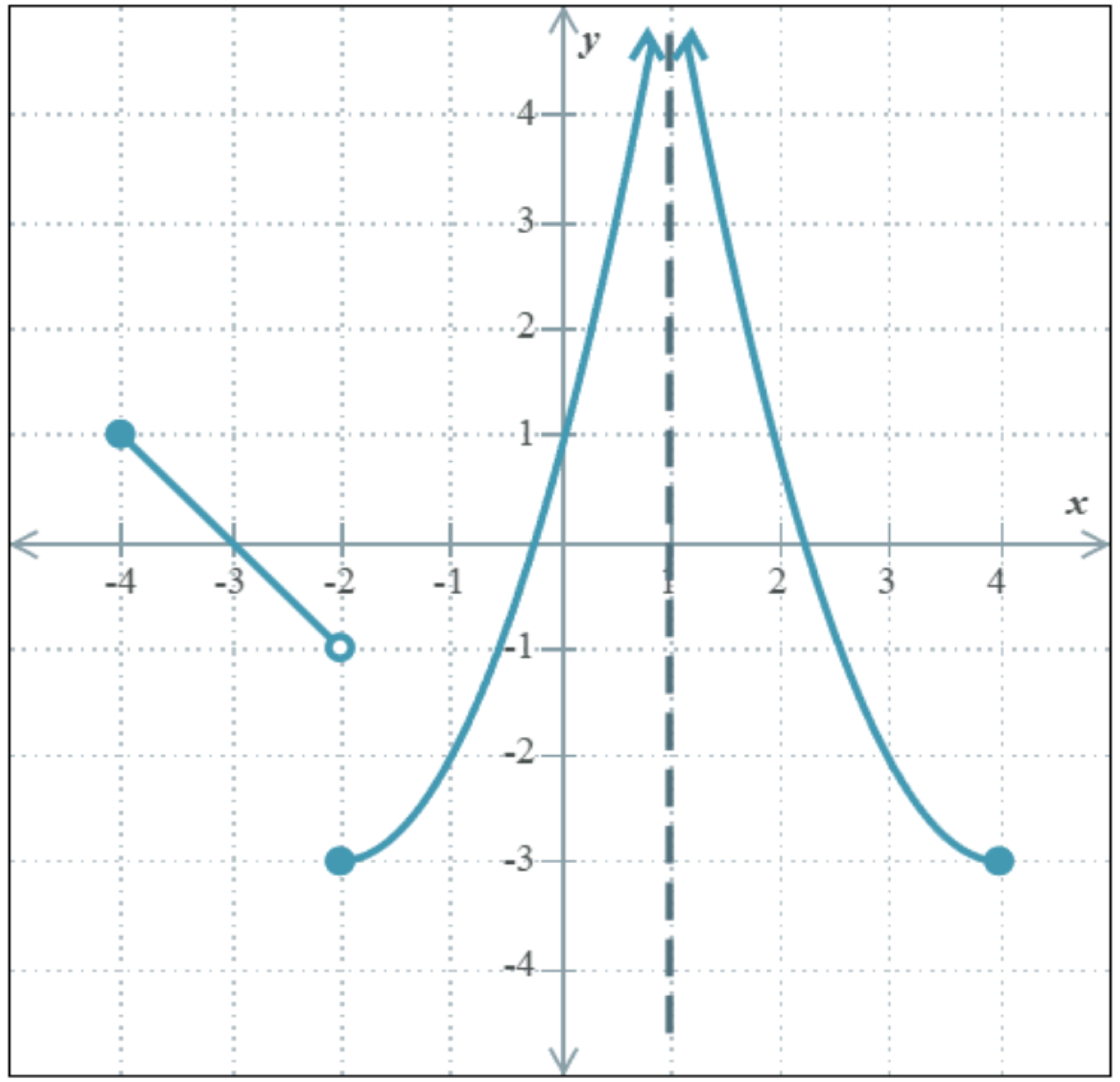
$x = -3$ Removable

$x = 1$ Jump

At what numbers in the interval $(-4, 4)$ is g discontinuous?

If there is more than one number, separate them with commas.
If there are no discontinuities, click on "None".

The function f is graphed below, along with its asymptote.



$X = -2$ Jump

$X = 1$ Infinate

At what numbers in the interval $(-4, 4)$ is f discontinuous?

Suppose f is a function that is continuous at $x=-15$, $x=-4$, and $x=1$ and the following holds.

$$\boxed{f(-15)=5, \qquad f(-4)=-6, \qquad f(1)=3}$$

Also, suppose that $g(x)=\dfrac{x+6}{2}$.

Use the above information to find the following limit. If the limit does not exist, click on "Does not exist."

$$\lim_{x\to -4} -2f(g(x)) = \square$$

$$\left[f\left(\lim_{x\to -4} g(x)\right)\right]$$

$$\lim_{x\to -4} \frac{x+6}{2} = 1 \qquad f(1)=3$$

$$-2(f(g(x)) = -2(3) = \boxed{-6}$$

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Suppose f is a function that is continuous at $x = -3$, $x = 5$, and $x = 6$ and the following holds.

$$f(-3) = 3, \qquad f(5) = 1, \qquad f(6) = -7$$

Also, suppose that $g(x) = \dfrac{8x}{x-5}$

Use the above information to find the following limit. If the limit does not exist, click on "Does not exist."

$$\lim_{x \to 5} 2f(g(x)) = \square$$

Does not exist

$$\lim_{x \to 5} \frac{8x}{x-5} = \frac{40}{0}$$

DNE

Suppose f is a function that is continuous at $x = -2$, $x = 14$, and $x = 26$ and the following holds.

$$f(-2) = -4, \qquad f(14) = 2, \qquad f(26) = 9$$

Also, suppose that $g(x) = 5x^2 - 6$.

Use the above information to find the following limit. If the limit does not exist, click on "Does not exist."

$$\lim_{x \to -2} f(g(x)) - 3 = \square$$

Does not exist

$$\lim_{x \to -2} g(x) = 14$$

$$f(14) - 3$$

$$2 - 3 = \boxed{-1}$$

The function $f(x) = \dfrac{4x+12}{x+3}$ has a removable discontinuity at some value $x = a$. Suppose we define a function g as follows, where b is a constant.

$$g(x) = \begin{cases} f(x), & x \neq a \\ b, & x = a \end{cases}$$

Find the values of a and b such that g is continuous at $x = a$. Write the exact values. Do not write decimal approximations.

$a =$ -3

$b =$ 4

$$\lim_{x \to -3} \frac{4x+12}{x+3}$$

$$\frac{4\cancel{(x+3)}}{\cancel{x+3}} = 4$$

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The function $f(x) = \dfrac{3x^2 + 16x + 5}{3x + 1}$ has a removable discontinuity at some value $x = a$. Suppose we define a function g as follows, where b is a constant.

$$g(x) = \begin{cases} f(x), & x \neq a \\ b, & x = a \end{cases}$$

Find the values of a and b such that g is continuous at $x = a$. Write the exact values. Do not write decimal approximations.

$a = -1/3$

$b = 14/3$

$$3x + 1 = 0$$
$$3x = -1$$
$$x = -1/3$$

$$\lim_{x \to -1/3} \frac{3x^2 + 16x + 5}{3x + 1} = x + 5$$

$$3x^2 + 15x + 1x + 5$$
$$3x(x + 5) + 1(x + 5)$$
$$(3x + 1)(x + 5)$$

$$\begin{array}{r|rrr} -1/3 & 3 & 16 & 5 \\ & \downarrow & -1 & -5 \\ \hline & 3 & 15 & 0 \end{array}$$

$$3x + 15$$
$$\boxed{3(x + 5)}$$

The function $f(x) = \dfrac{5x^3 - 20x^2 - 25x}{x-5}$ has a removable discontinuity at some value $x = a$. Suppose we define a function g as follows, where b is a constant.

$x - 5 = 0$

$$g(x) = \begin{cases} f(x), & x \neq a \\ b, & x = a \end{cases}$$

$x = 5$

Find the values of a and b such that g is continuous at $x = a$. Write the exact values. Do not write decimal approximations.

$a = $ 5

$b = $ 150

$$\lim_{x \to 5} \frac{5x^3 - 20x - 25x}{x-5} = 5x(x+1)$$

$5x^3 - 20x - 25x = 5x(x^2 - 4x - 5)$

$5x(x-5)(x+1)$

Determining a parameter to make a function continuous

Find a value of k such that the following function is continuous at all real numbers.

$$g(x) = \begin{cases} \frac{4}{x} & \text{if } x < 2 \\ kx + 3 & \text{if } x \geq 2 \end{cases}$$

$x \neq 0$

If there is no such value k, click on "None."

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Find a value of k such that the following function is continuous at all real numbers.

$$f(x) = \begin{cases} \dfrac{2}{(x-4)^2} & \text{if } x \le 3 \\ 3x+k & \text{if } x > 3 \end{cases}$$

$$\lim_{x \to 3^-} \frac{2}{(x-4)^2} = 2$$

$$\lim_{x \to 3^+} 3x+k = 9+k$$

If there is no such value k, click on "None."

$$\frac{2}{(x-4)^2}\{x<3\}$$

$$3x-7\{x \ge 3\}$$

$$9+k=2$$

$$\boxed{k=-7}$$

Find a value of k such that the following function is continuous at all real numbers.

$$g(x) = \begin{cases} \dfrac{3}{x} & \text{if } x \le 4 \\ kx^2 - 2 & \text{if } x > 4 \end{cases}$$

$x \neq 0$

If there is no such value k, click on "None."

Find a value of k such that the following function is continuous at all real numbers.

$$g(x) = \begin{cases} (x-2)^2 & \text{if } x < 1 \\ \dfrac{kx}{x+2} & \text{if } x \geq 1 \end{cases}$$

If there is no such value k, click on "None."

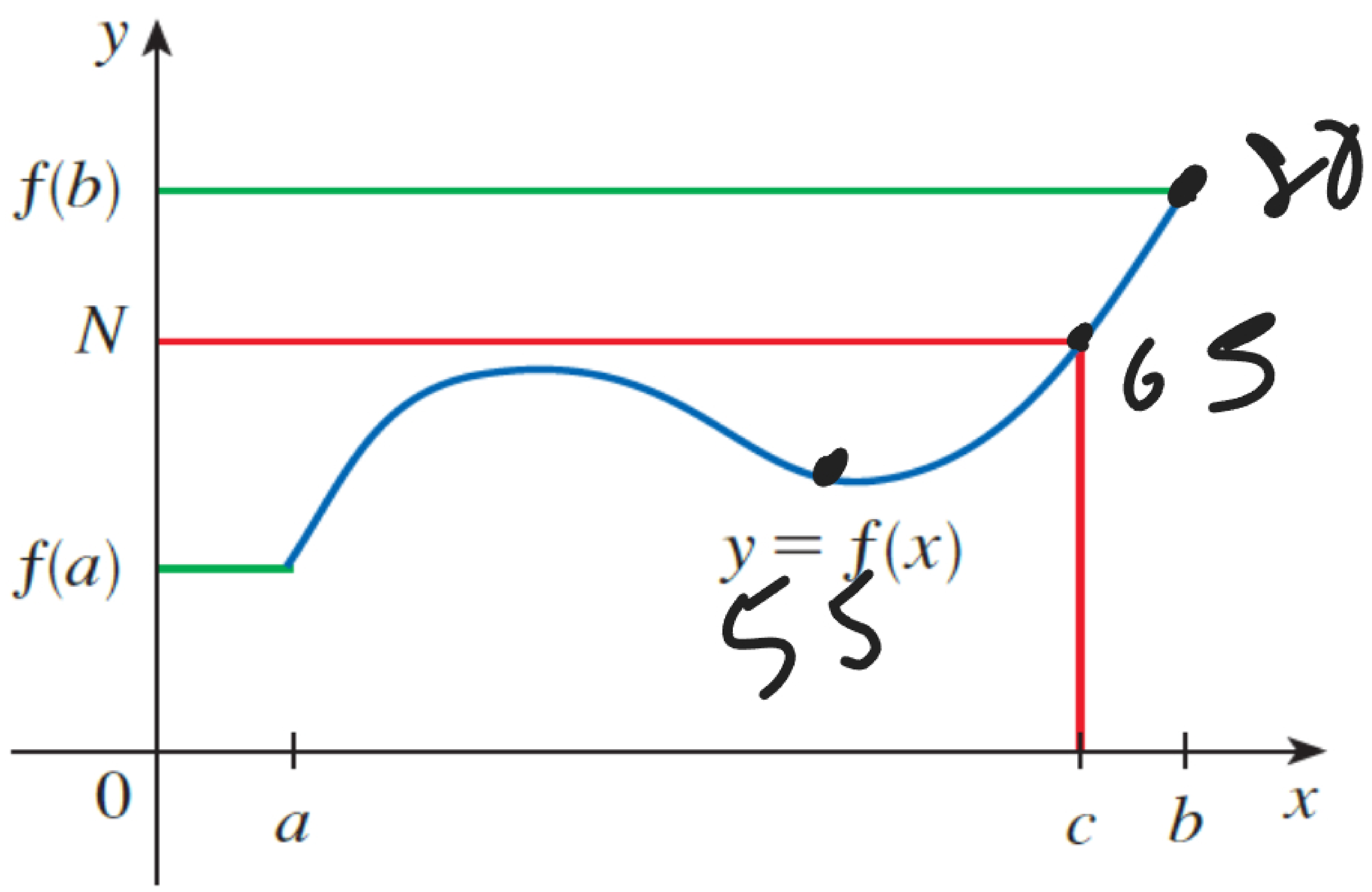
$$\lim_{x \to 1^-} (x-2)^2 = 1$$

$$\lim_{x \to 1^+} \frac{kx}{x+2} = \frac{k}{3}$$

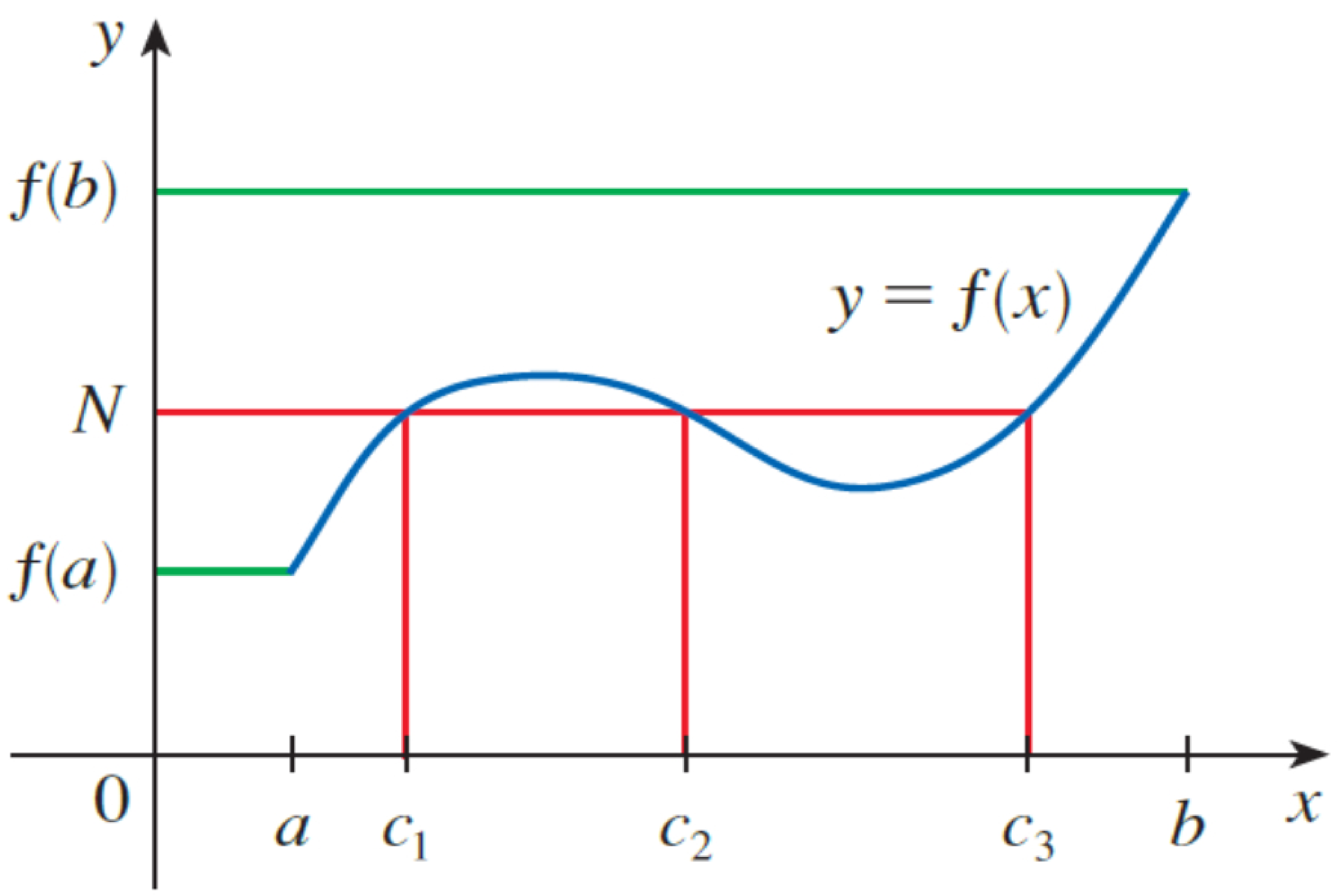
$$\frac{k}{3} = 1 \rightarrow \boxed{k = 3}$$

10 The Intermediate Value Theorem Suppose that f is continuous on the closed interval $[a, b]$ and let N be any number between $f(a)$ and $f(b)$, where $f(a) \neq f(b)$. Then there exists a number c in (a, b) such that $f(c) = N$.

The Intermediate Value Theorem states that a continuous function takes on every intermediate value between the function values $f(a)$ and $f(b)$. It is illustrated by Figure 8. Note that the value N can be taken on once [as in part (a)] or more than once [as in part (b)].



(a)



(b)

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Consider the following equation.

$$2x^3 - 15x^2 + 14 = 0$$

Answer the following to determine if we can apply the Intermediate Value Theorem (IVT) to show that the equation has a solution in the interval $(-1, 9)$.

(a) Let $f(x) = 2x^3 - 15x^2 + 14$. Choose the statement that best describes the continuity of f.

- (selected) f is continuous on $[-1, 9]$. That is because f is a polynomial function.
- f is continuous on $[-1, 9]$. That is because f is a rational function, and while there are values of x where f is not defined, those values do not lie in $[-1, 9]$.
- f is not continuous. That is because f is a rational function that is not defined for at least one value of x in $[-1, 9]$.

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(b) Find $f(-1)$ and $f(9)$. Write each answer as an exact value (not a decimal approximation). Then use $=$ or $\neq$ to compare $f(-1)$ and $f(9)$.

$f(-1) = -3$

$f(9) = 257$

$f(-1) \neq f(9)$

(c) Use $<$, $>$, or $=$ to complete each statement.

$f(-1) < 0$

$f(9) > 0$

(d) Can we apply the Intermediate Value Theorem to show that the equation $2x^3 - 15x^2 + 14 = 0$ has a solution in the interval $(-1, 9)$? If yes, then fill in the blank to explain why.

- [x] Yes, we can apply the IVT to show that the equation has a solution, c, in the interval $(-1, 9)$. In particular, the IVT guarantees that there is a real number c in $(-1, 9)$ such that $f(c) = 0$.
- [ ] No, we cannot apply the IVT to show that the equation has a solution in the interval $(-1, 9)$.

Note: The Solve feature is not available tc

Consider the following equation.

$$\frac{5}{x-4}+2=0$$

Answer the following to determine if we can apply the Intermediate Value Theorem (IVT) to show that the equation has a solution in the interval $(0, 2)$.

(a) Let $f(x)=\dfrac{5}{x-4}+2$. Choose the statement that best describes the continuity of f.

- ○ f is continuous on $[0, 2]$. That is because f is a polynomial function.
- ○ f is continuous on $[0, 2]$. That is because f is a rational function, and while there are values of x where f is not defined, those values do not lie in $[0, 2]$.
- ○ f is not continuous. That is because f is a rational function that is not defined for at least one value of x in $[0, 2]$.

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(b) Find $f(0)$ and $f(2)$. Write each answer as an exact value (not a decimal approximation). Then use $=$ or $\neq$ to compare $f(0)$ and $f(2)$.

$f(0) = \frac{3}{4}$

$f(2) = -\frac{1}{2}$

$f(0) \neq f(2)$

Equals

(c) Use $<$, $>$, or $=$ to complete each statement.

$f(0) > 0$

$f(2) < 0$

(d) Can we apply the Intermediate Value Theorem to show that the equation $\frac{5}{x-4} + 2 = 0$ has a solution in the interval $(0, 2)$? If yes, then fill in the blank to explain why.

- (selected) Yes, we can apply the IVT to show that the equation has a solution, c, in the interval $(0, 2)$. In particular, the IVT guarantees that there is a real number c in $(0, 2)$ such that $f(c) = 0$.
- No, we cannot apply the IVT to show that the equation has a solution in the interval $(0, 2)$.

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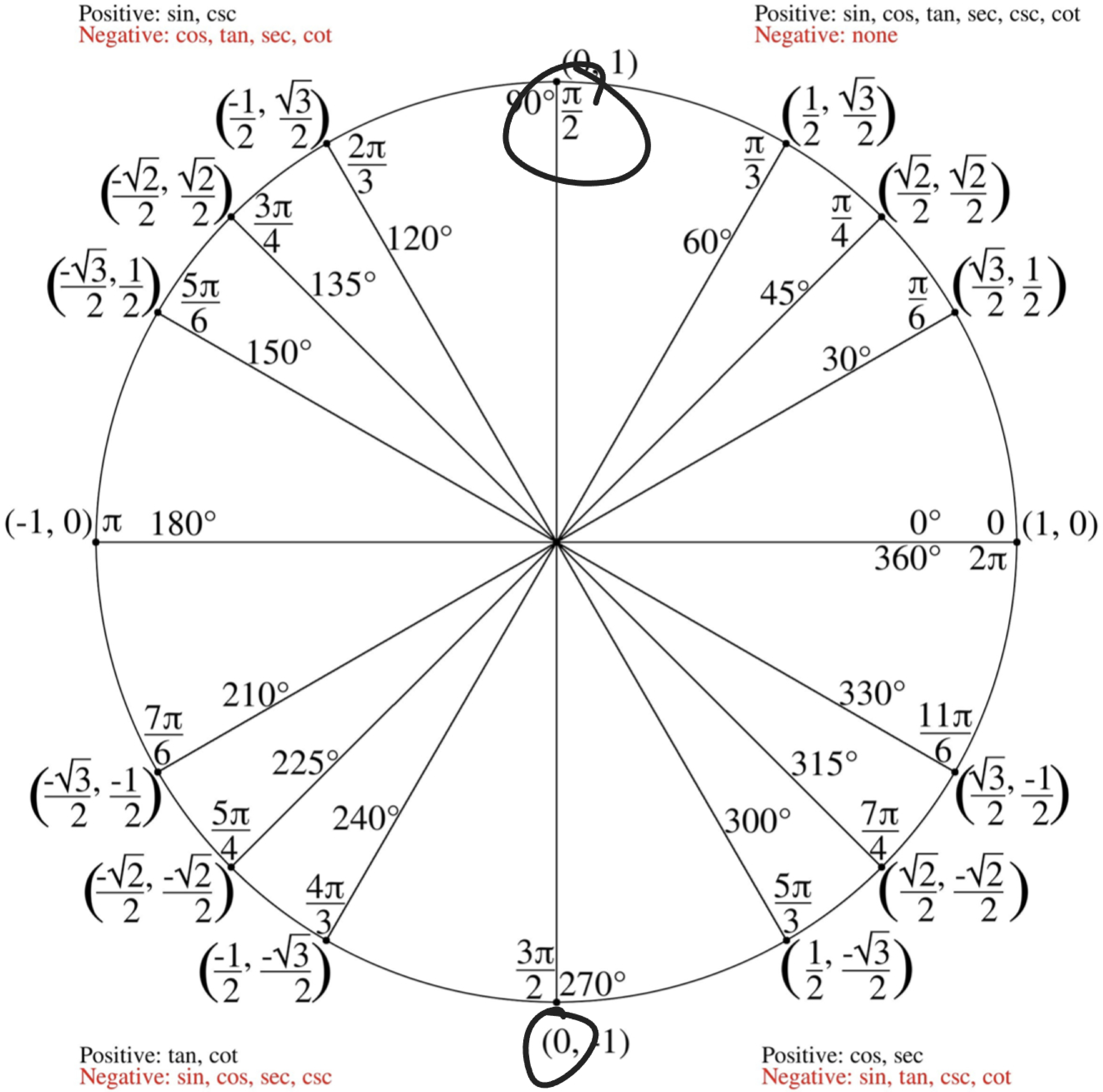

$y = \sin$

$x = \cos$

Find the following limit.
If the limit does not exist, click on "Does Not Exist."

$$-\frac{\pi}{2} \to \frac{3\pi}{2}$$

$$\lim_{t \to -\frac{\pi}{2}} \frac{5 + 3\cos t}{\sin 3t} = \frac{5 + 3\cos\left(\frac{3\pi}{2}\right)}{\sin\left(\frac{9\pi}{2}\right)} = \frac{5 + 3(0)}{1}$$

$$= 5$$

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$$\lim_{x \to \frac{3\pi}{2}} \frac{2x + \frac{3\pi}{2}}{2\sin x} = \frac{2\left(\frac{3\pi}{2}\right) + \frac{3\pi}{2}}{2\sin\left(\frac{3\pi}{2}\right)}$$

$$\frac{6\pi}{2} + \frac{3\pi}{2}$$

$$\sin\left(\frac{3\pi}{2}\right) = (-1)$$

$$\frac{\frac{9\pi}{2}}{-2}$$

$$\frac{9\pi}{2} \cdot \frac{1}{-2} = \boxed{\frac{-9\pi}{4}}$$

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$$\lim_{x \to \pi} \frac{\frac{1}{2}(x - \pi) - \cos 4x}{\sin\left(\frac{1}{2}x + \frac{\pi}{2}\right) + x} = \frac{\frac{1}{2}(\pi - \pi) - \cos 4\pi}{\sin\left(\frac{1}{2}\pi + \frac{\pi}{2}\right) + \pi}$$

$$\sin(\pi)$$

$$= \frac{0 - 1}{0 + \pi} = -\frac{1}{\pi}$$

$$\lim_{t \to -\frac{\pi}{2}} \frac{\cos 2t}{3t + \sin(\cos t)} = \frac{\overset{\cos(-\pi)}{\cos\left(2\left(-\pi/2\right)\right)}}{3\left(-\frac{\pi}{2}\right) + \sin\left(\underset{0}{\cos\left(-\pi/2\right)}\right)} = \frac{-1}{-\frac{3\pi}{2}}$$

$$= \frac{-1}{-\frac{3\pi}{2}} = \frac{2}{3\pi}$$