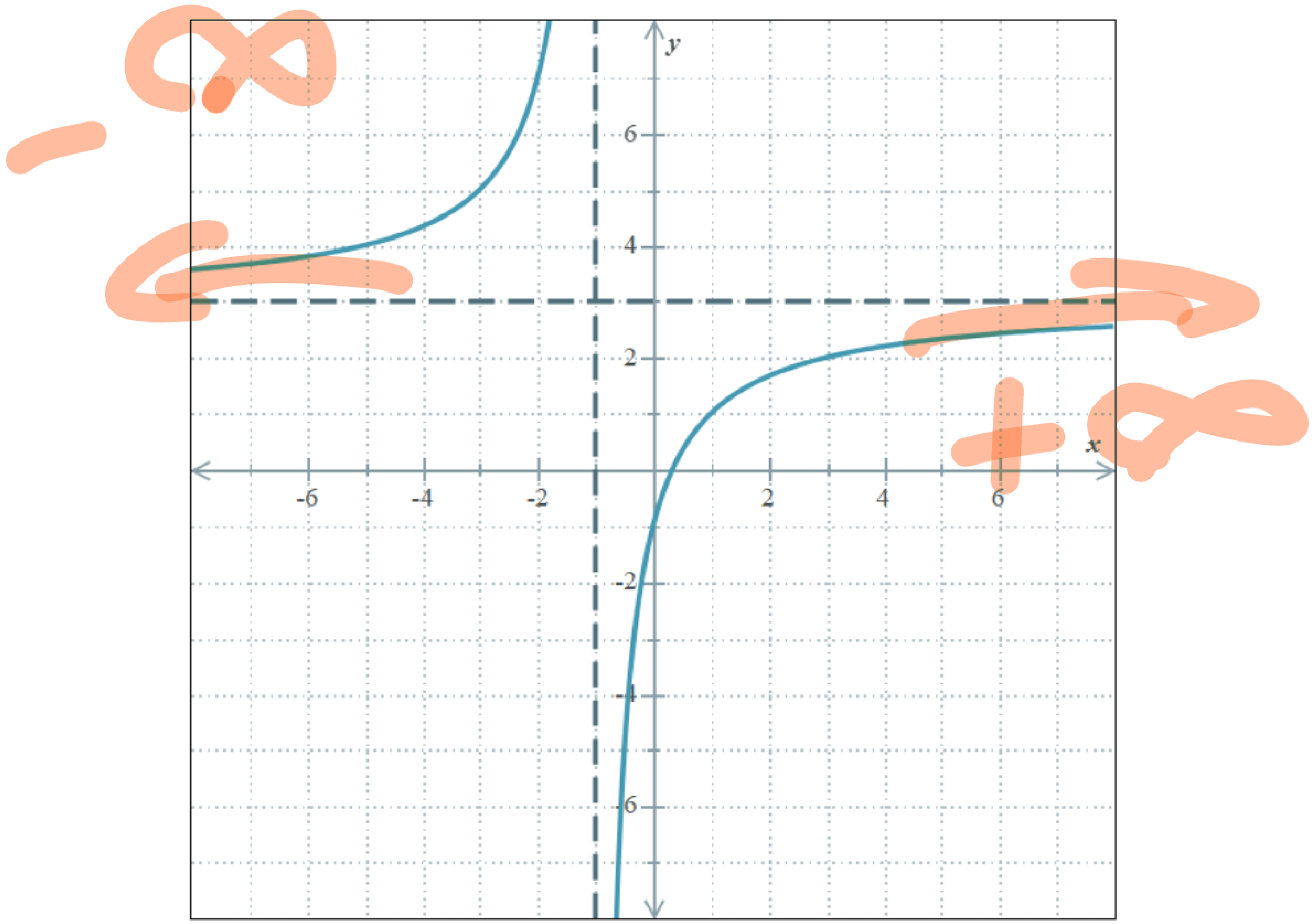
2.6 Limits at Infinity; Horizontal Asymptotes

3 Definition The line $y = L$ is called a **horizontal asymptote** of the curve $y = f(x)$ if either

$$\lim_{x \to \infty} f(x) = L \qquad \text{or} \qquad \lim_{x \to -\infty} f(x) = L$$

The graph of $g(x) = \dfrac{3x-1}{x+1}$ is shown below, along with its asymptotes.



Use the graph to find the following limits.

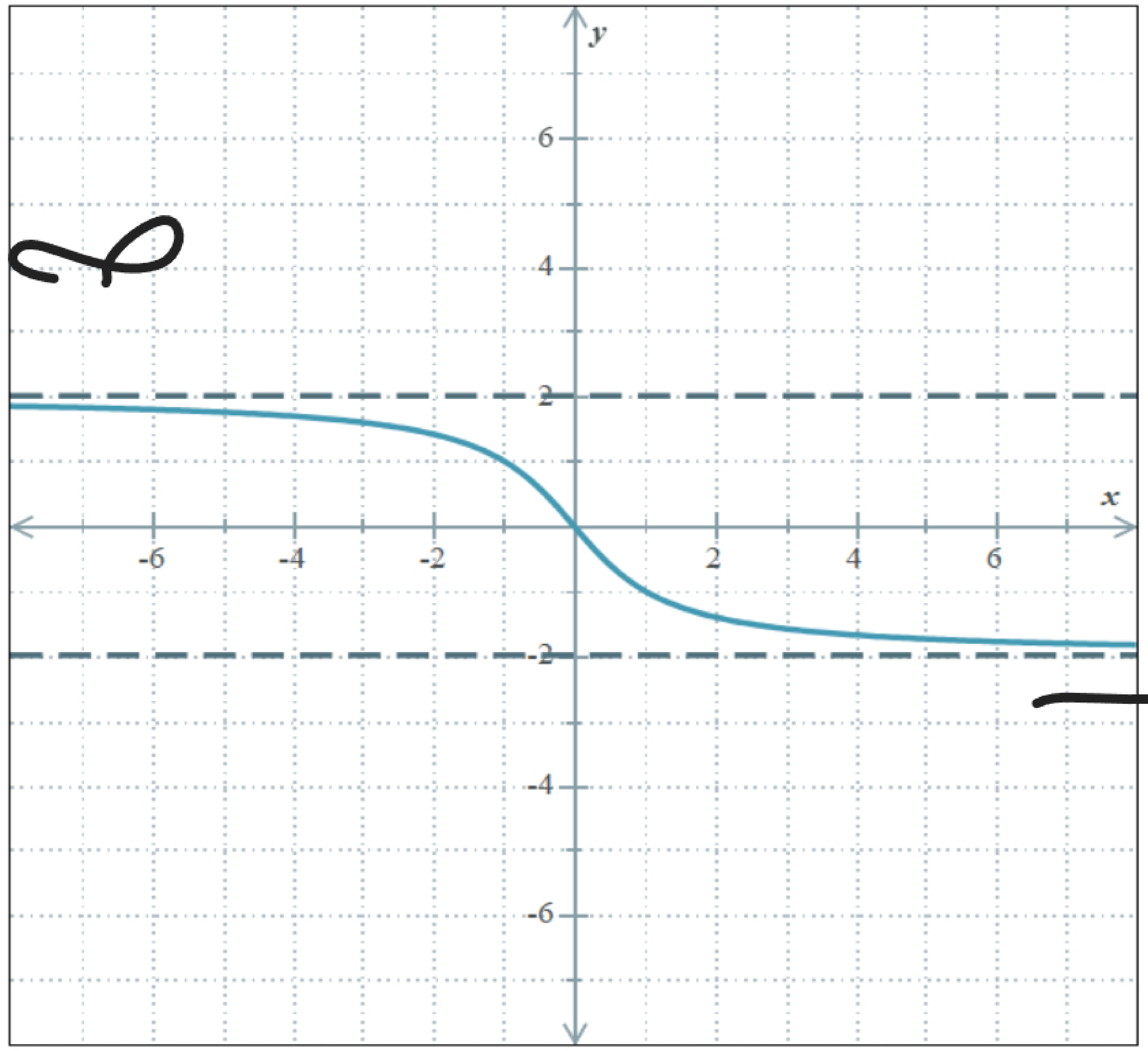
If necessary, choose the most informative answer from ∞, $-\infty$, and "Does Not Exist".

$$\lim_{x \longrightarrow -\infty} g(x) = \square \quad 3$$

$$\lim_{x \longrightarrow \infty} g(x) = \square \quad 3$$

The graph of $g(x) = \dfrac{-4\arctan x}{\pi}$ is shown below, along with its asymptotes.

$-\infty$



$+\infty$

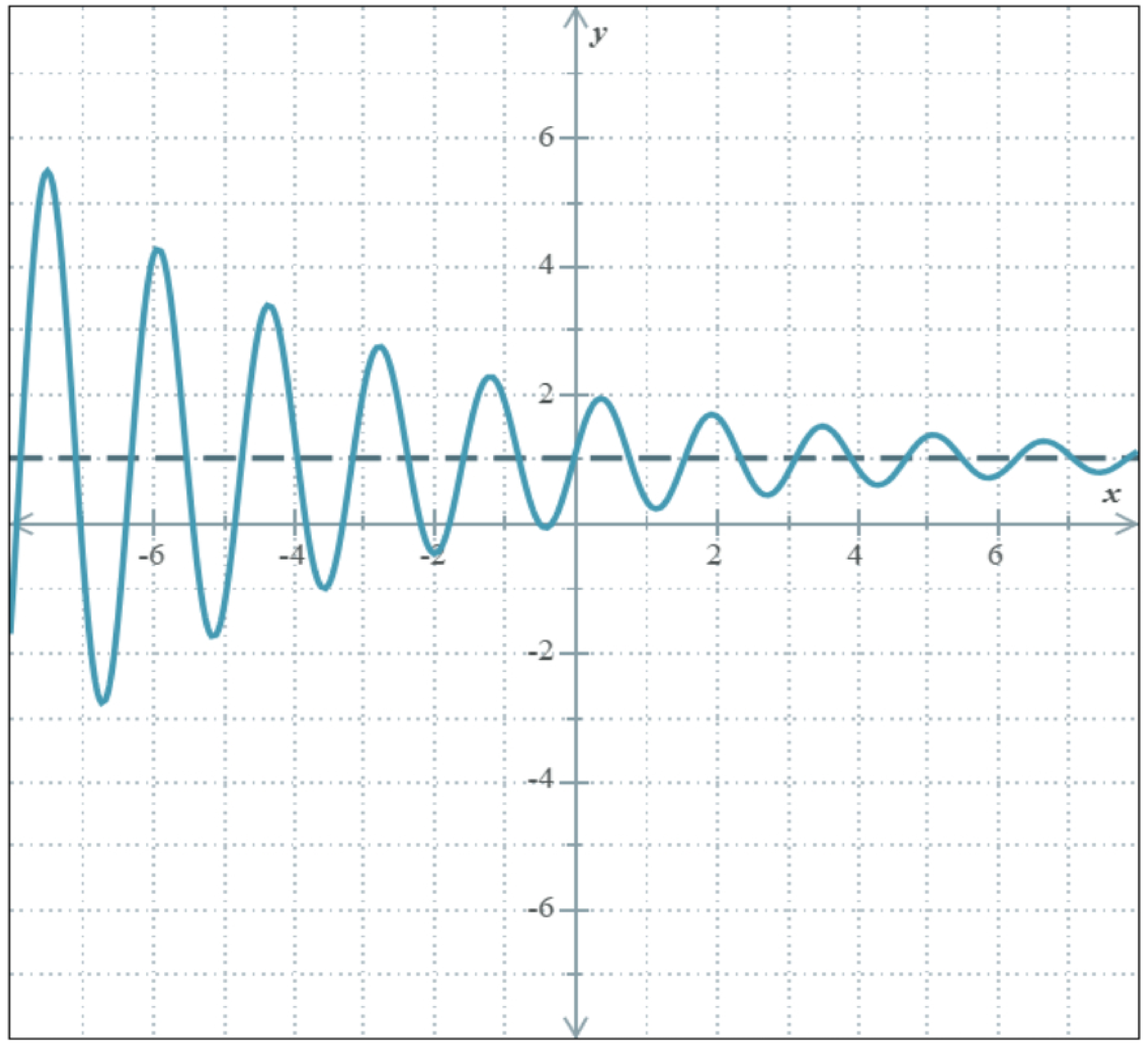
Use the graph to find the following limits.

If necessary, choose the most informative answer from ∞, $-\infty$, and "Does Not Exist".

$$\lim_{x \longrightarrow \infty} g(x) = \square \quad -2$$

$$\lim_{x \longrightarrow -\infty} g(x) = \square \quad 2$$

The graph of $h(x) = \sin(4x)e^{-0.2x} + 1$ is shown below, along with its asymptote.



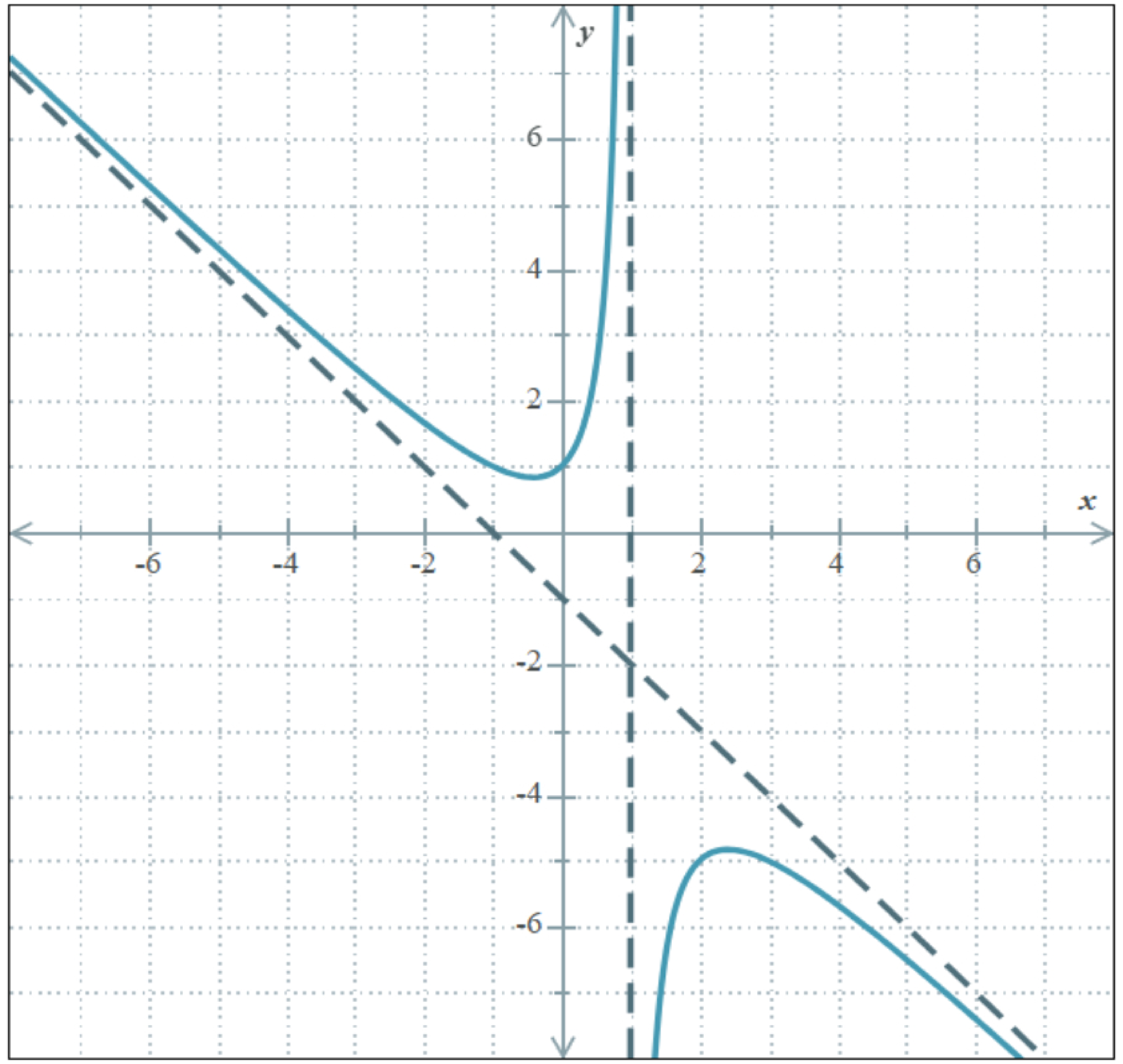
Use the graph to find the following limits.

If necessary, choose the most informative answer from ∞, $-\infty$, and "Does Not Exist".

$$\lim_{x \longrightarrow -\infty} h(x) = \square$$ DNE

$$\lim_{x \longrightarrow \infty} h(x) = \square$$ 1

The graph of $g(x) = \dfrac{x^2+1}{1-x}$ is shown below, along with its asymptotes.



$$\lim_{x \to \infty} g(x) = -\infty$$

$$\lim_{x \to -\infty} g(x) = \infty$$

Use the graph to find the following limits.

$$\lim_{x \to -\infty} \frac{6 - 5x^{2}}{2x^{3} - 7}$$

$y = 0$

$N > D$ No HA

$N < D$ $y = 0$

$N = D$ $y = \frac{L.C.}{L.C}$

Degree

$$\lim_{x \to \infty} \frac{1}{x} = 0$$

$$\lim_{x \to \infty} \frac{1}{x^n} = 0$$

5 Theorem If $r > 0$ is a rational number, then

$$\lim_{x \to \infty} \frac{1}{x^r} = 0$$

If $r > 0$ is a rational number such that x^r is defined for all x, then

$$\lim_{x \to -\infty} \frac{1}{x^r} = 0$$

$$\lim_{x \to -\infty} \frac{6 - 5x^2}{2x^3 - 7}$$

$$\frac{\frac{6}{x^3} - \frac{5x^2}{x^3}}{\frac{2x^3}{x^3} - \frac{7}{x^3}} = \frac{\frac{6}{x^3} - \frac{5}{x}}{2 - \frac{7}{x^3}}$$

$$\frac{\lim\limits_{x \to -\infty} \frac{6}{x^3} - \lim\limits_{x \to -\infty} \frac{5}{x}}{\lim\limits_{x \to -\infty} 2 - \lim\limits_{x \to -\infty} \frac{7}{x^3}} = \frac{0 - 0}{2 - 0} = 0$$

$$\lim_{x \to \infty} \frac{1/x - 3x^{2}/x}{3x/x - 5/x} = \frac{1/x - 3x}{3 - 5/x} = \frac{0 - \infty}{3 - 0} = -\infty$$

$$\lim_{x \to \infty} \frac{5x^2/x^2 - 6/x^2}{5/x^2 - 2x^2/x^2} = \lim_{x \to \infty} \frac{5 - 6/x^2}{5/x^2 - 2} = \frac{5 - 0}{0 - 2} = \boxed{\frac{-5}{2}}$$

$$\lim_{x \to \infty} \frac{\sqrt{7x + 6x^2}}{3x - 8} = \boxed{\frac{\sqrt{6}}{3}}$$

$$x = \sqrt{x^2}$$

$$\frac{\dfrac{\sqrt{7x+6x^2}}{x}}{\dfrac{3x-8}{x}} = \frac{\sqrt{\dfrac{7x}{x^2} + \dfrac{6x^2}{x^2}}}{\dfrac{3x}{x} - \dfrac{8}{x}} = \frac{\sqrt{\dfrac{7}{x} + 6}}{3 - 8/x} = \frac{\sqrt{0+6}}{3-0}$$

$$\lim_{x\to\infty} \frac{\dfrac{7-5x}{x}}{\dfrac{\sqrt{6x^2-5}}{\sqrt{x^2}}} = \frac{\dfrac{7}{x}-5}{\sqrt{6-\dfrac{5}{x^2}}} = \frac{0-5}{\sqrt{6-0}}$$

$$= \frac{-5}{\sqrt{6}} = \frac{-5\sqrt{6}}{6}$$

$$\frac{-5\cdot\sqrt{6}}{\sqrt{6}\cdot\sqrt{6}} = \frac{-5\sqrt{6}}{\sqrt{36}} = \frac{-5\sqrt{6}}{6}$$

$$\lim_{x \to \infty} \frac{\left(\sqrt{4x^6 + 9} - 2x^3\right)}{1} \frac{\left(\sqrt{4x^6+9} + 2x^3\right)}{\left(\sqrt{4x^6+9} + 2x^3\right)}$$

$$\lim_{x \to \infty} \frac{(4x^6+9) - 4x^6}{\sqrt{4x^6+9} + 2x^3} = \frac{9}{\sqrt{4x^6+9} + 2x^3} = \frac{9}{\infty} = \boxed{0}$$

$$\lim_{x \to -\infty} \frac{\left(\sqrt{16x^2 + 5x} + 4x\right)}{1} \cdot \frac{\left(\sqrt{16x^2+5x} - 4x\right)}{\sqrt{16x^2+5x} - 4x}$$

$$\lim_{x \to -\infty} \frac{16x^2 + 5x - 16x^2}{\sqrt{16x^2+5x} - 4x} = \frac{5x}{\sqrt{\frac{16x^2+5x}{x^2}} - 4x} \Big/ x$$

$$\lim_{x \to -\infty} \frac{5}{\sqrt{16 + 5/x} - 4} = \frac{5}{-\sqrt{16} - 4} = \frac{5}{-8}$$

$$f(x) = \frac{7+3x}{\sqrt{x^2-3}}$$

$\lim_{x \to \infty} f(x) =$ 3

$\lim_{x \to -\infty} f(x) =$ -3

$\lim_{x \to \sqrt{3}^+} f(x) =$ ∞

$\lim_{x \to -\sqrt{3}^-} f(x) =$ ∞

Vertical asymptote(s): $-\sqrt{3}$, $\sqrt{3}$

Horizontal asymptote(s): 3, -3

$$\frac{\frac{7+3x}{x}}{\sqrt{\frac{x^2}{x^2}-\frac{3}{x^2}}} = \frac{\frac{7}{x}+3}{\sqrt{1-3/x^2}} = \frac{3}{-\sqrt{1}} = 3$$

$$\frac{7 \overset{-}{+} 3(\sqrt{3})}{\sqrt{3-3}} = \frac{\quad}{0}$$