Finding a derivative at a given number by simplifying a difference quotient

Let $f(x) = \boxed{-\frac{5}{x^2}.}$

(a) Find the difference quotient $\dfrac{f(x)-f(5)}{x-5}$.

Simplify your answer so that the resulting expression is defined at $x=5$.

$$\frac{f(x)-f(5)}{x-5} = \square$$

$$\frac{-\frac{5}{x^2} - \left(-\frac{1}{5}\right)}{x-5}$$

(b) Find $f'(5)$.

$$f'(5) = \square$$

$$\frac{-\frac{5}{x^2} - \left(-\frac{1}{5}\right)}{x-5}$$

$$\frac{\frac{-25+x^2}{5x^2}}{x-5} \rightarrow \frac{\frac{x^2-25}{5x^2}}{x-5}$$

$$\frac{\cancel{(x-5)}(x+5)}{5x^2} \times \frac{1}{\cancel{(x-5)}} = \boxed{\frac{x+5}{5x^2}}$$

Finding a derivative at a given number by simplifying a difference quotient

Let $f(x) = -\dfrac{5}{x^2}$.

(a) Find the difference quotient $\dfrac{f(x) - f(5)}{x - 5}$.

Simplify your answer so that the resulting expression is defined at $x = 5$.

$$\frac{f(x) - f(5)}{x - 5} = \square$$

$$\boxed{\frac{x+5}{5x^2}}$$

(b) Find $f'(5)$.

$$f'(5) = \square$$

$$\frac{10}{125} = \frac{2}{25}$$

Relating information about a function and its derivative to the equation of a tangent line

Suppose $y = 5x - 2$ is the equation of the tangent line to the graph of a function $y = f(x)$ at $x = \frac{1}{4}$.

$$y = 5\left(\tfrac{1}{4}\right) - 2$$

Find $f\left(\frac{1}{4}\right)$ and $f'\left(\frac{1}{4}\right)$.

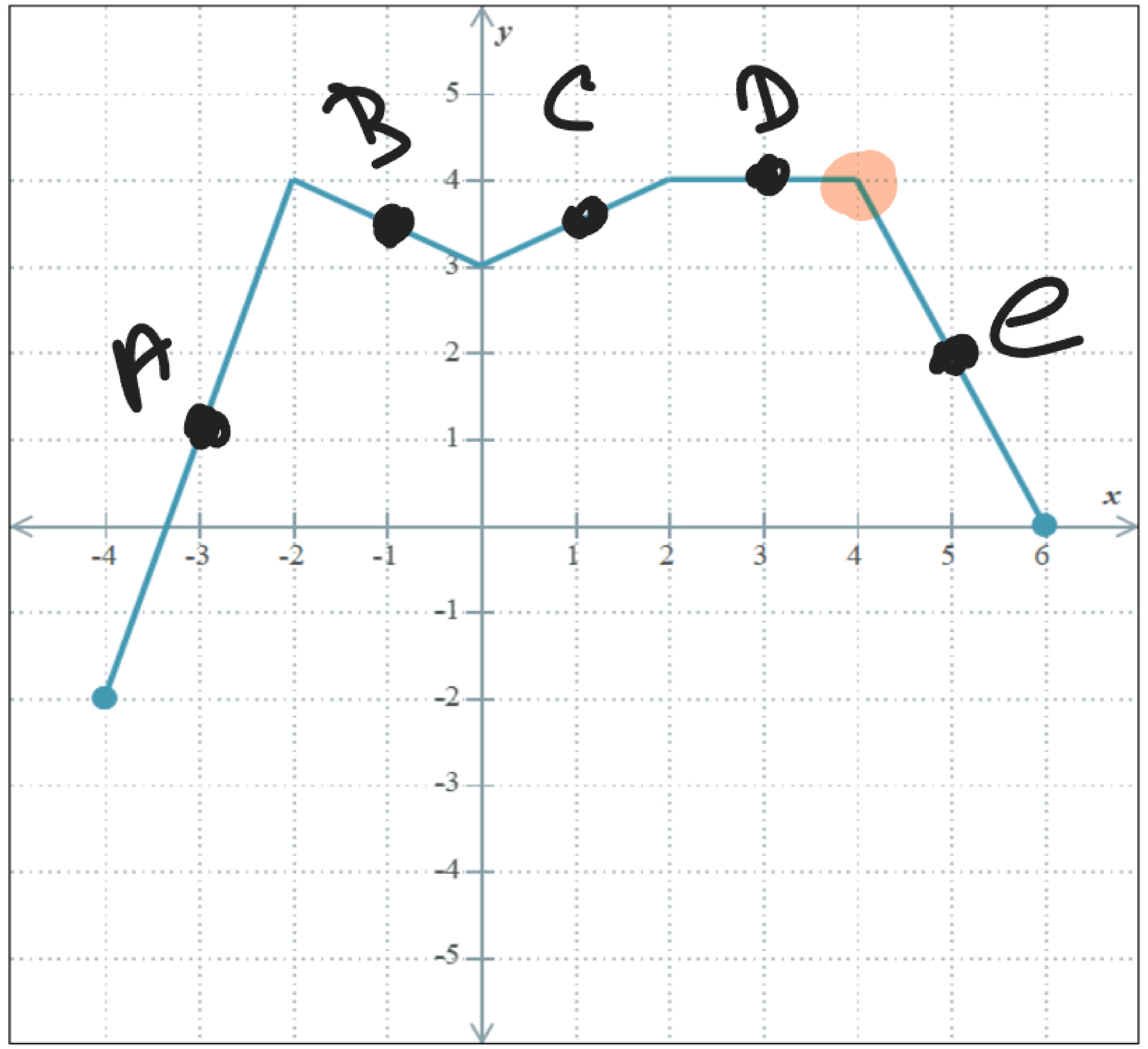
$f\left(\frac{1}{4}\right) = \square$ $-\frac{3}{4}$

$f'\left(\frac{1}{4}\right) = \square$ 5

Slope

Interpreting the graph of a function to order the values of its derivative at several values

The graph of a function f is shown below. Based on the graph, order the given values from least to greatest.



a) $f'(-3) \Rightarrow$ steep, positive

b) $f'(-1) \Rightarrow$ shawlower, negative

c) $f'(1) \rightarrow$ less than $f'(3)$, positive

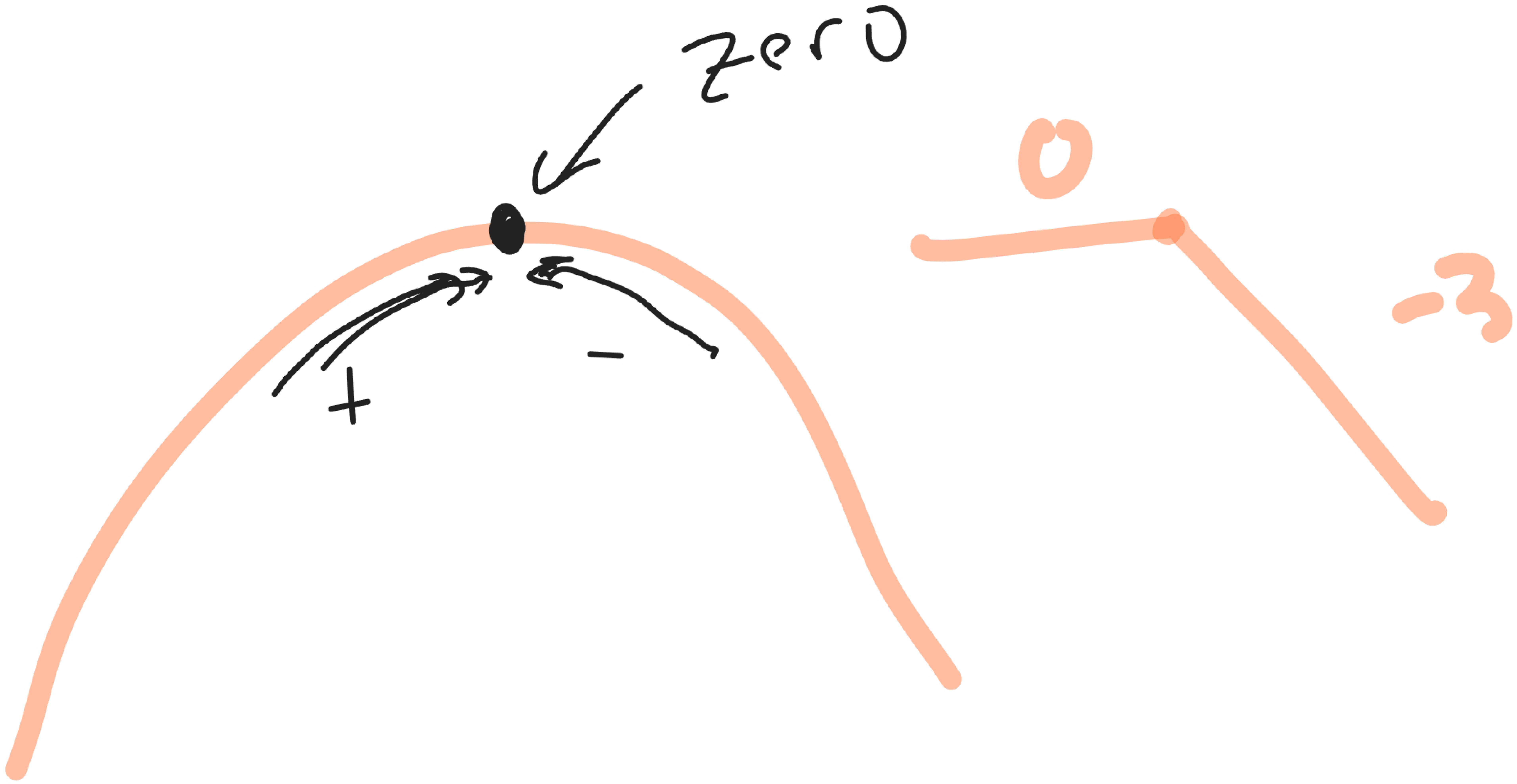
d $f'(3) \rightarrow$ zero

	a	b	c	d	e
Values:	$f'(-3)$	$f'(-1)$	$f'(1)$	$f'(3)$	$f'(5)$

e < b < d < c < a

f'

e) $f'(5) \rightarrow$ steeper than $f'(-1)$, negative


zero
+
-
0
-3

Finding a derivative by simplifying a difference quotient

Let the function g be defined as follows.

$$g(t) = 4 - 6t$$

Answer the following.

(a) Let a be constant. For $t \neq a$, find the difference quotient $\frac{g(t) - g(a)}{t - a}$.

Simplify your answer so that the resulting expression is defined at $t = a$.

$$\boxed{\frac{g(t) - g(a)}{t - a} = \square} \quad -6$$

(b) Find $g'(a)$.

$$g'(a) = \square \quad -6$$

$$\frac{(4-6t) - (4-6a)}{t-a}$$

$$4 - 6t - 4 + 6a$$

$$\frac{-6t + 6a}{t - a}$$

$$\frac{-6\cancel{(t-a)}}{\cancel{t-a}} = -6$$

Let the function g be defined as follows.

$$g(t) = 5\sqrt{t}$$

Answer the following.

(a) For $h \neq 0$, find the difference quotient $\dfrac{g(t+h)-g(t)}{h}$.

Simplify your answer so that the resulting expression is defined at $h = 0$.

$$\frac{g(t+h)-g(t)}{h} = \square$$

(b) Find $g'(t)$.

$$g'(t) = \square$$

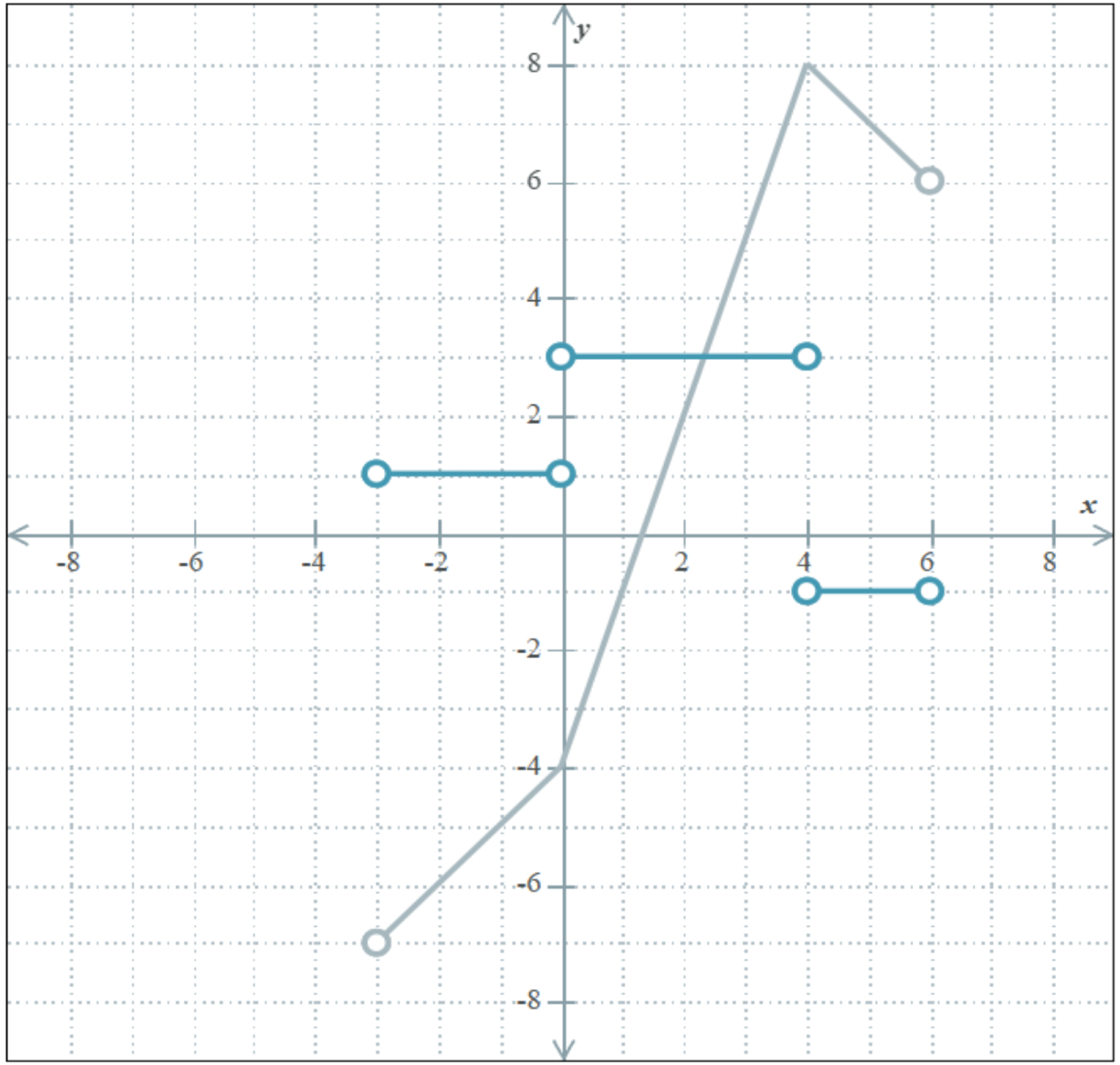
$$\frac{5\sqrt{t+h} - 5\sqrt{t}}{h}$$

$$\frac{5\sqrt{t+h} - 5\sqrt{t}}{h} \cdot \frac{\left(5\sqrt{t+h} + 5\sqrt{t}\right)}{\left(5\sqrt{t+h} + 5\sqrt{t}\right)}$$

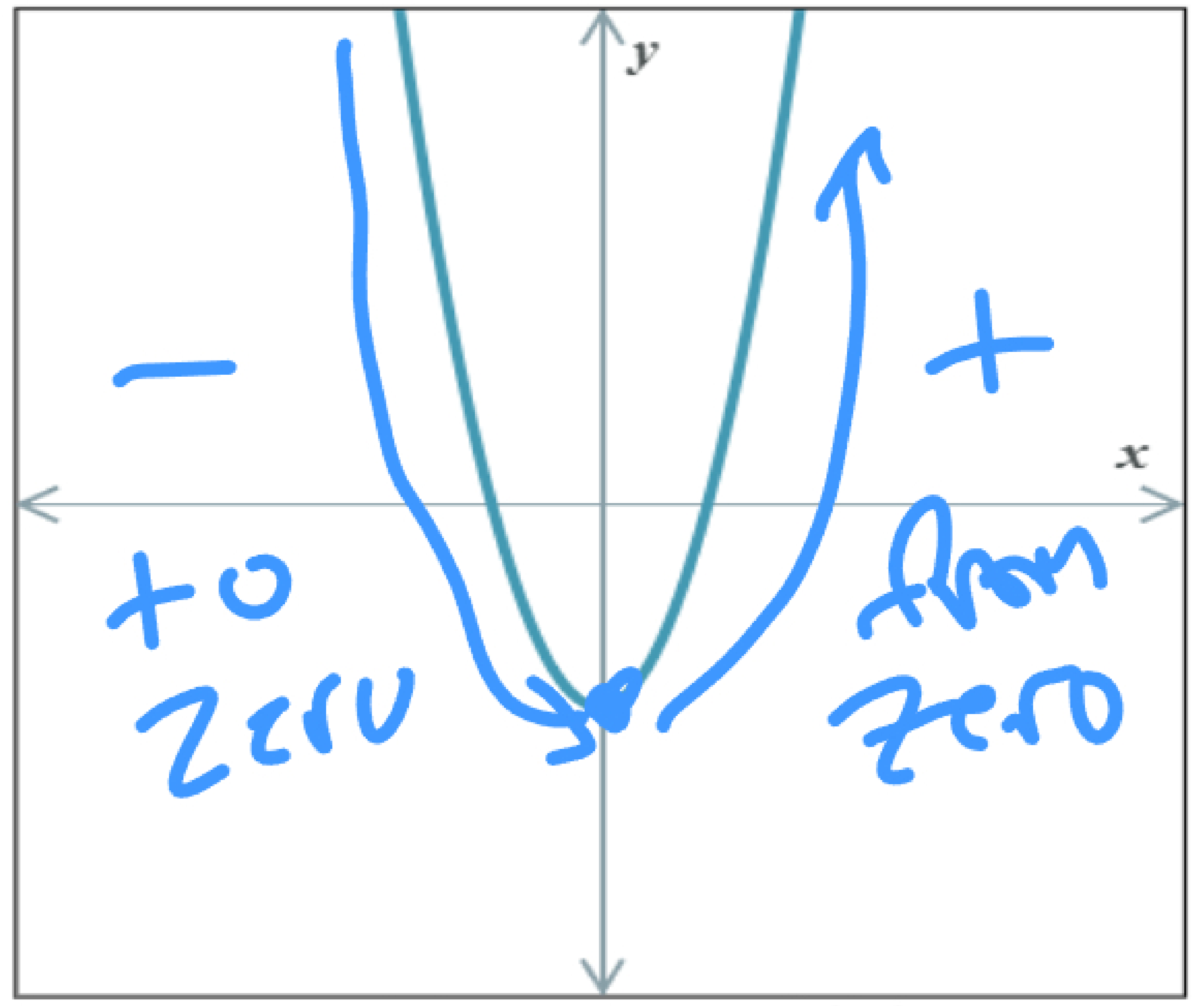
$$\cancel{h}$$

$$\frac{25(t+h) - 25t}{\cancel{h}\left(5\sqrt{t+h} + 5\sqrt{t}\right)} = \boxed{\frac{25}{5\sqrt{t+\underset{\substack{\uparrow \\ 0}}{h}} + 5\sqrt{t}}}$$

$$\lim_{h \to 0} = \boxed{\frac{25}{10\sqrt{t}}} = \frac{5}{2\sqrt{t}}$$

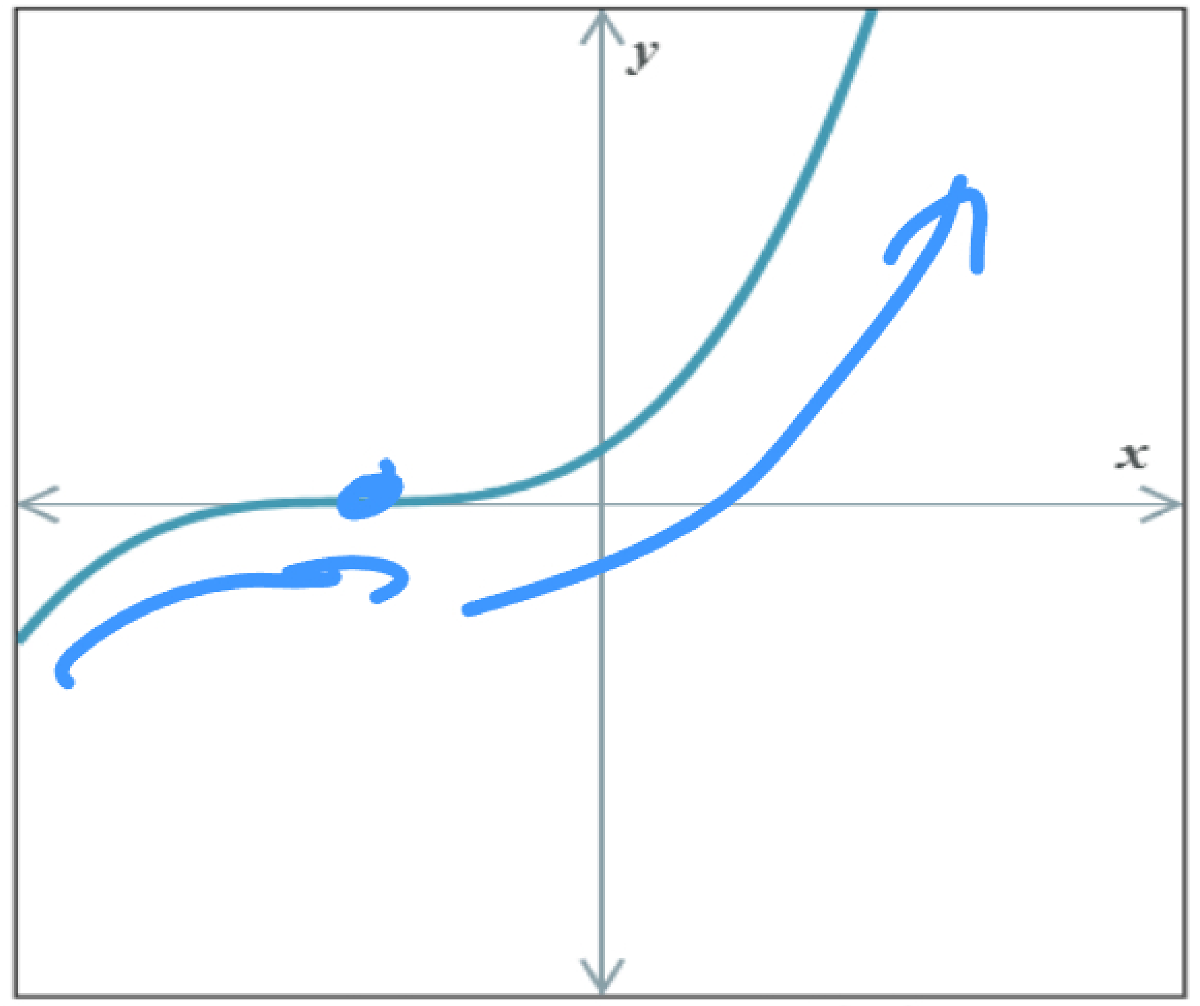

y
x
8
6
4
2
-2
-4
-6
-8
-8
-6
-4
-2
2
4
6
8

$f(x)$



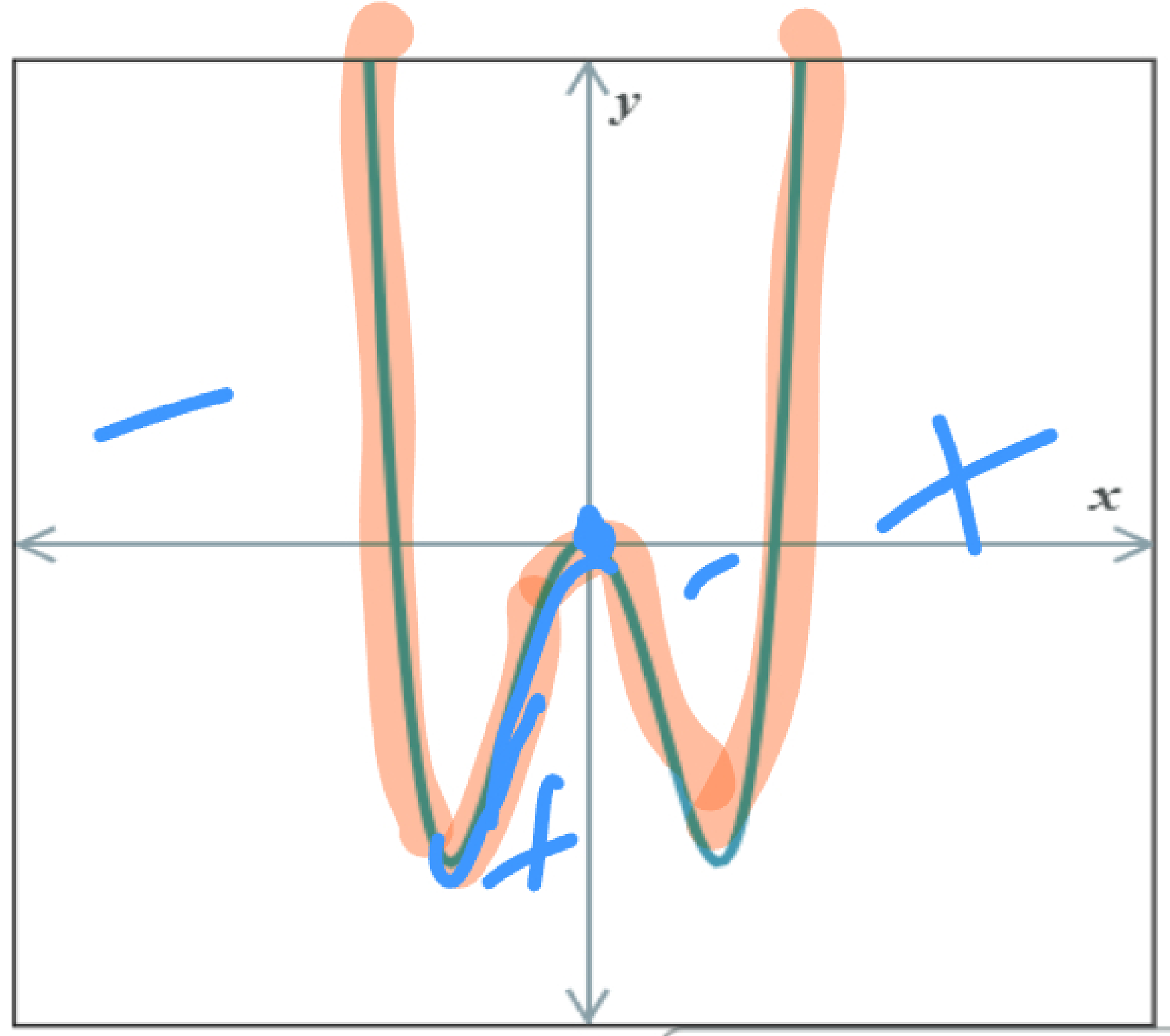
The graph of $f'(x)$ is (Choose one). D

$g(x)$



The graph of $g'(x)$ is (Choose one). C

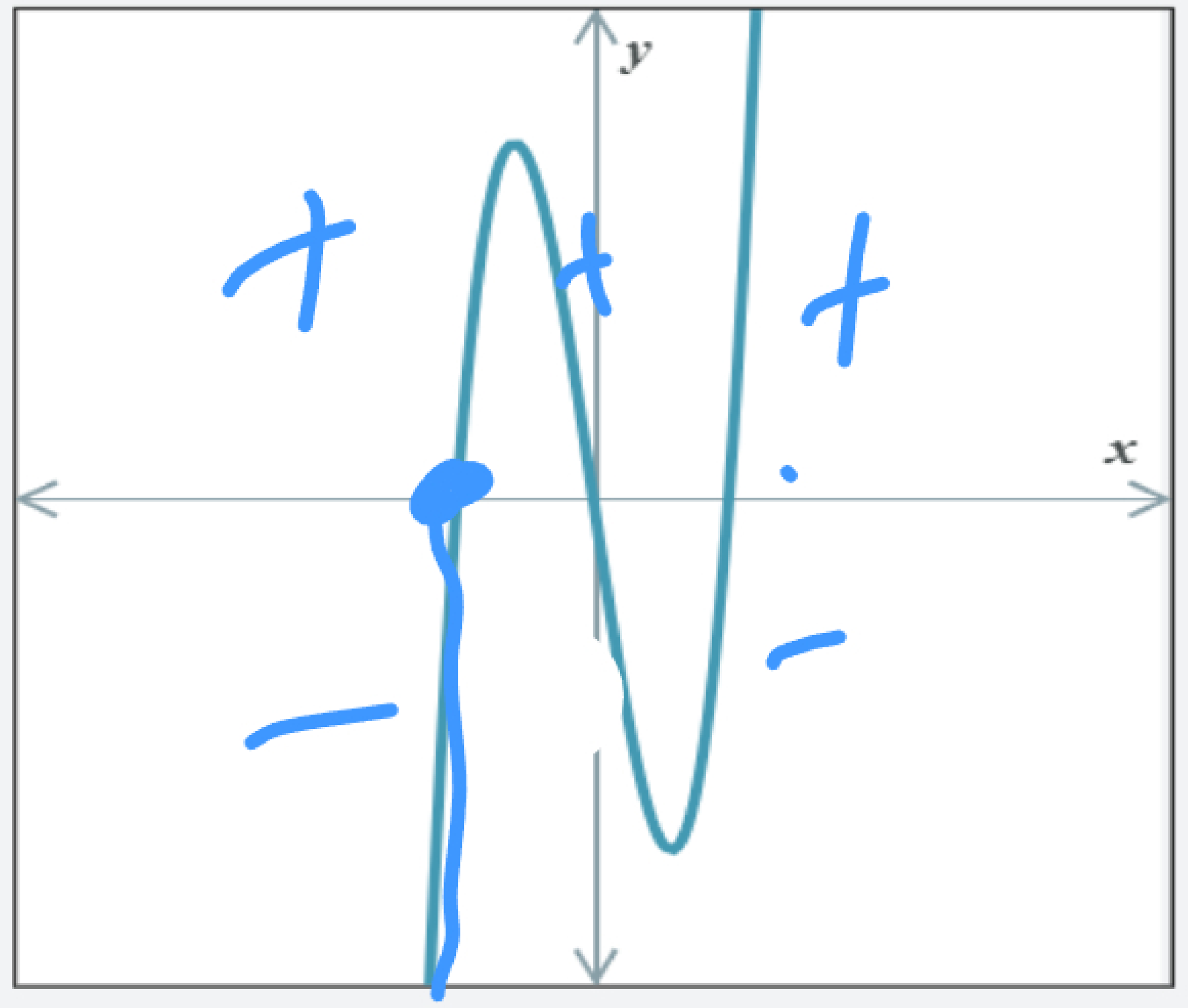
$h(x)$



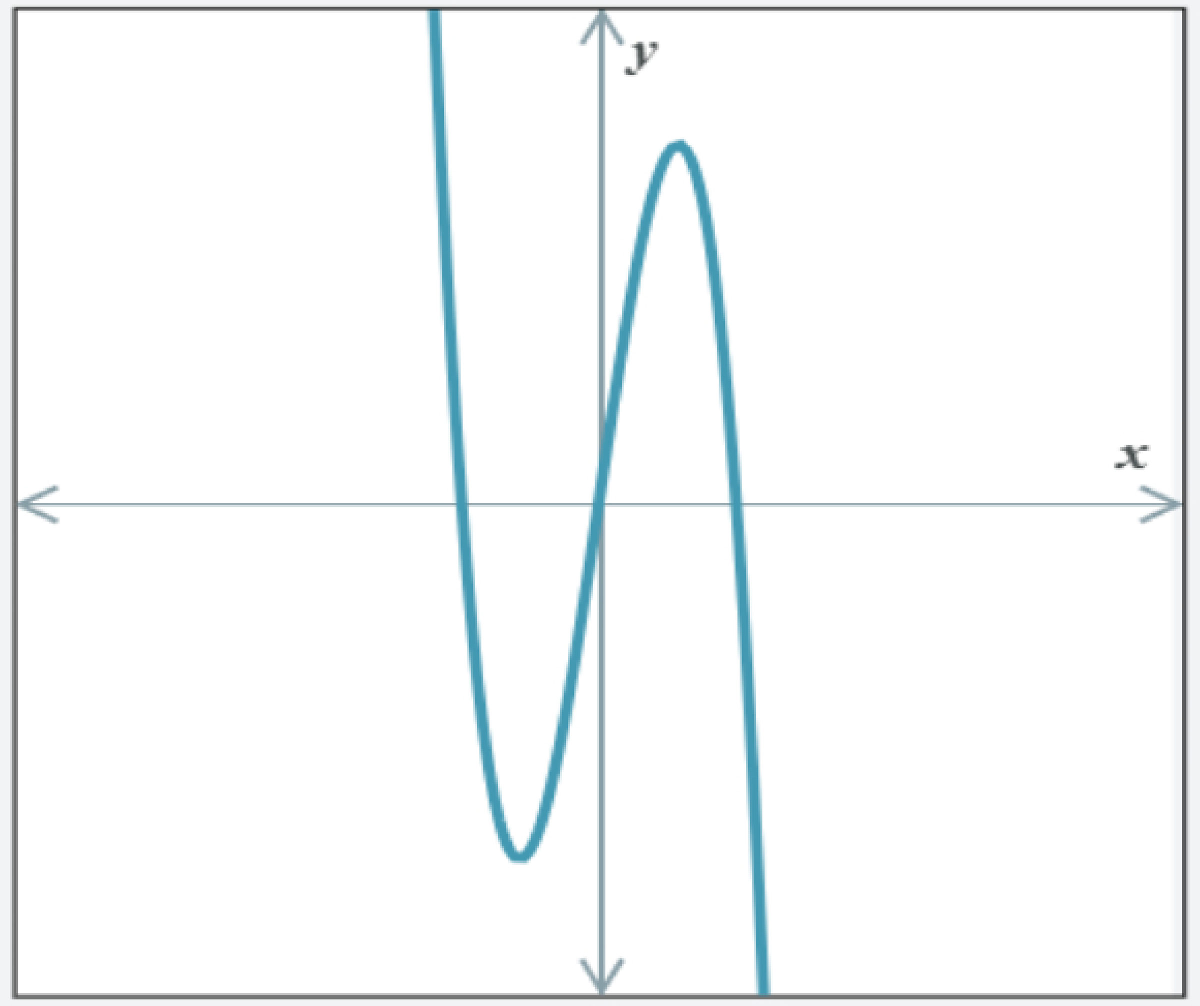
A

The graph of $h'(x)$ is (Choose one).

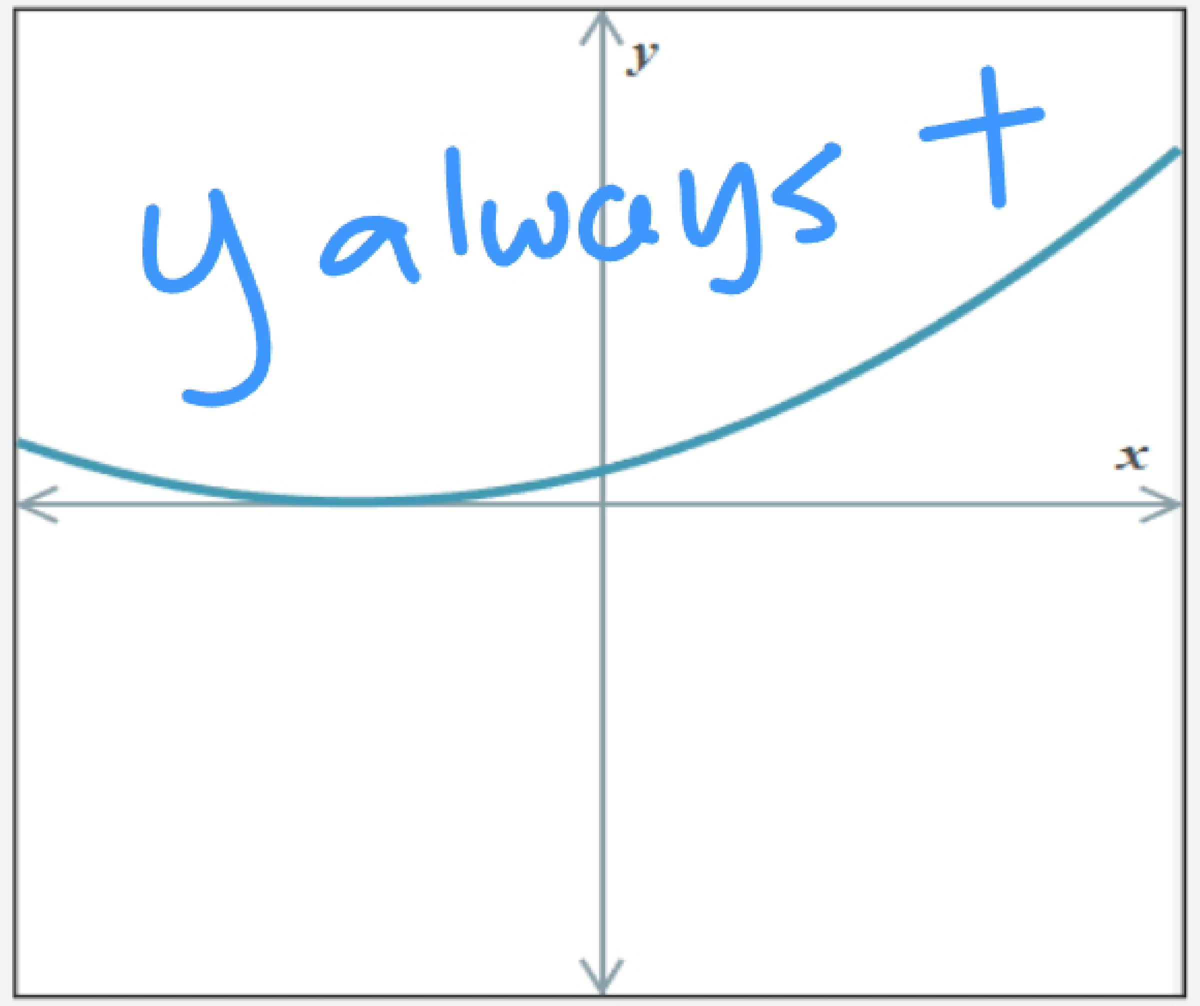
Graph A



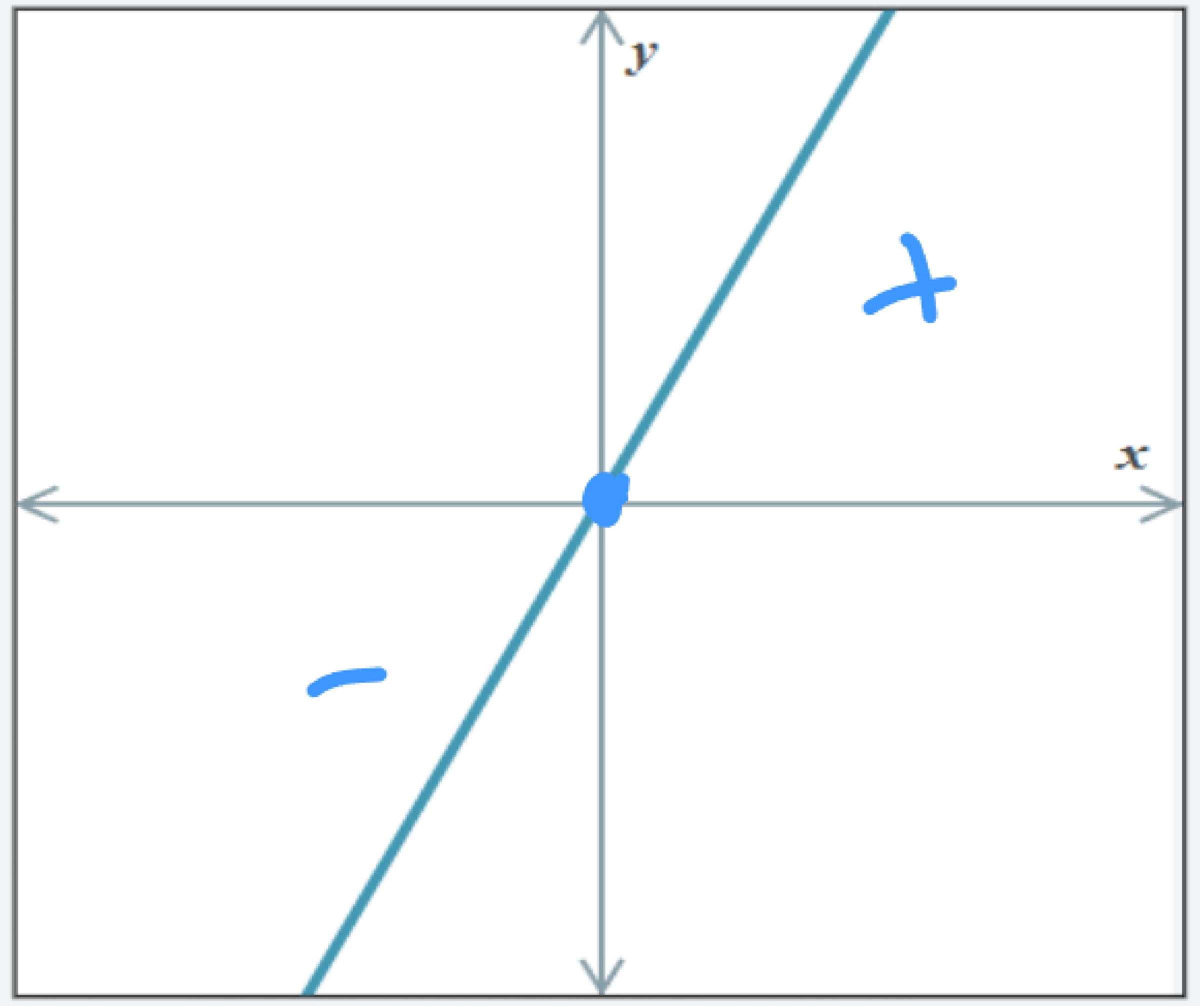
Graph B



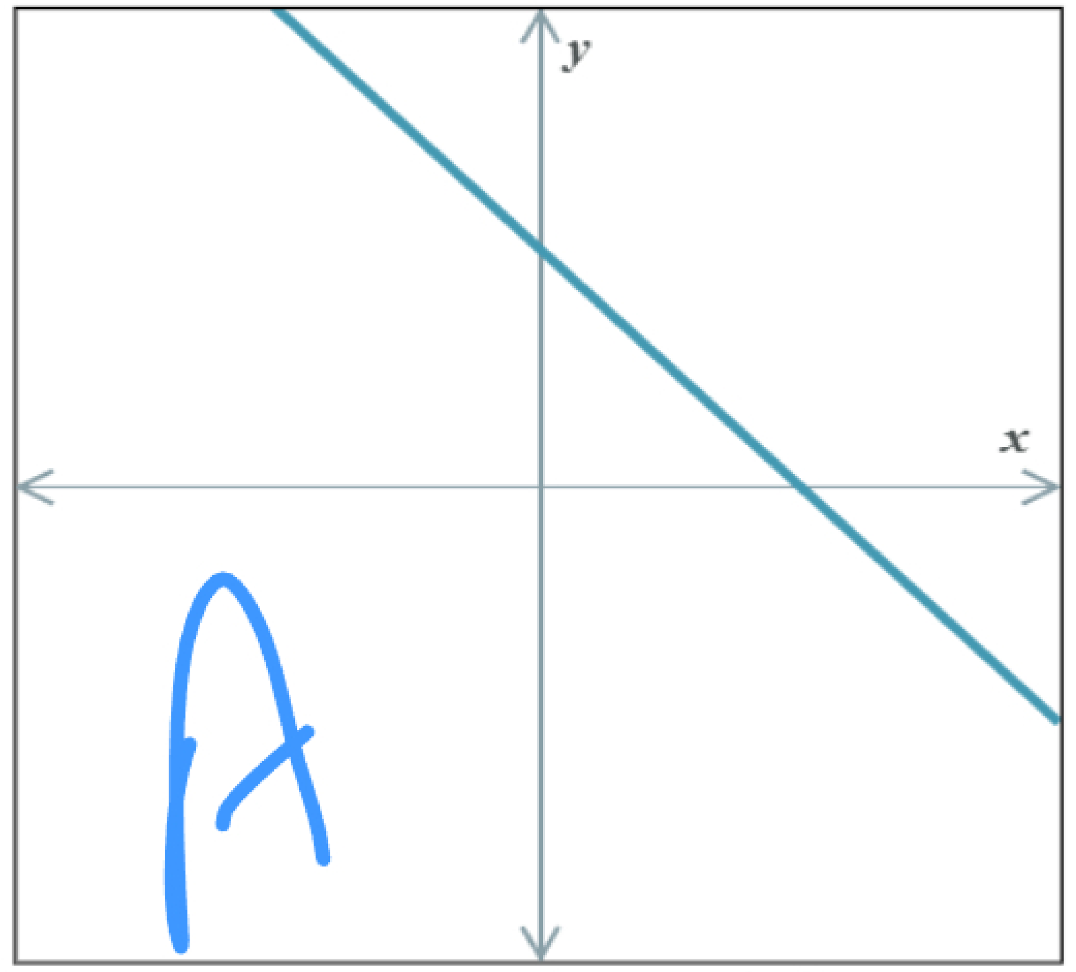
Graph C



Graph D

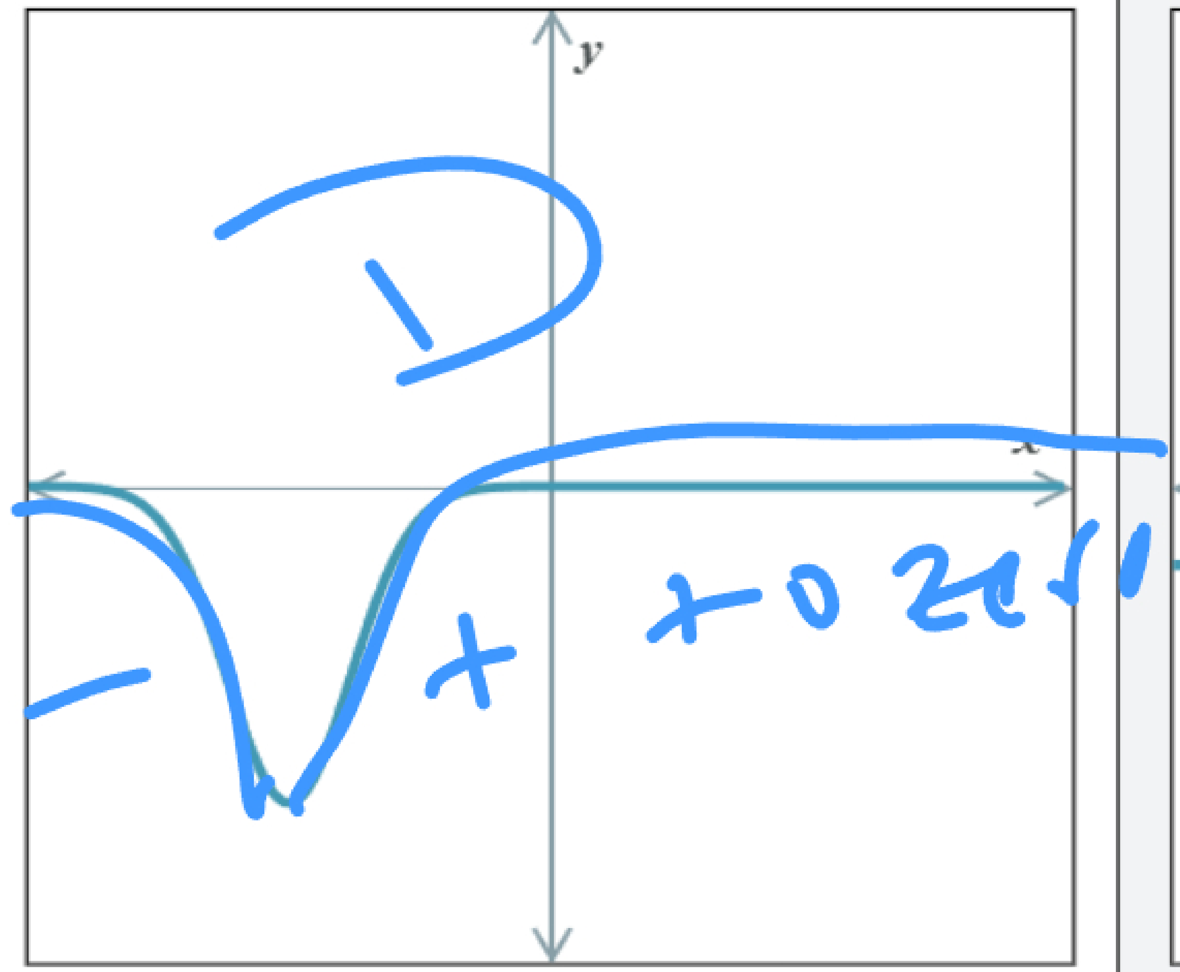


$f(x)$



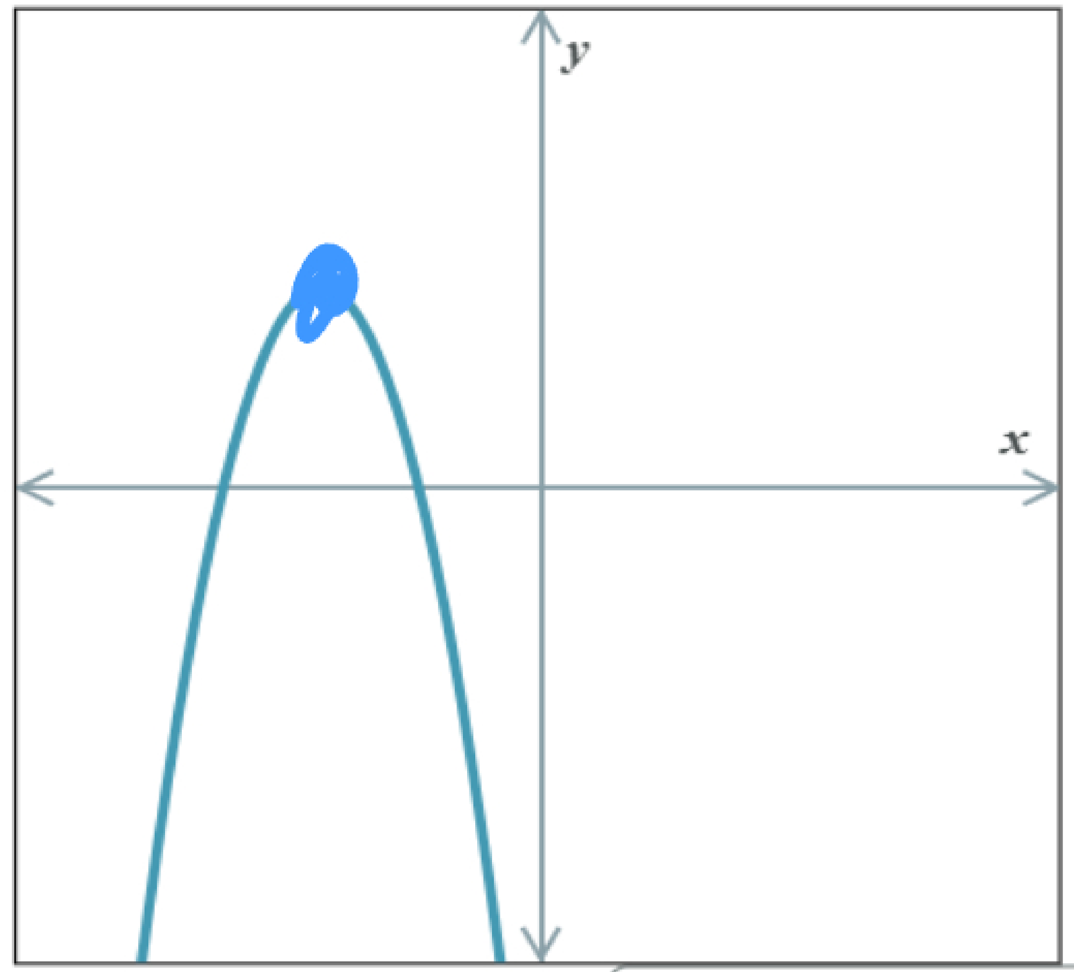
The graph of $f'(x)$ is (Choose one).

$g(x)$



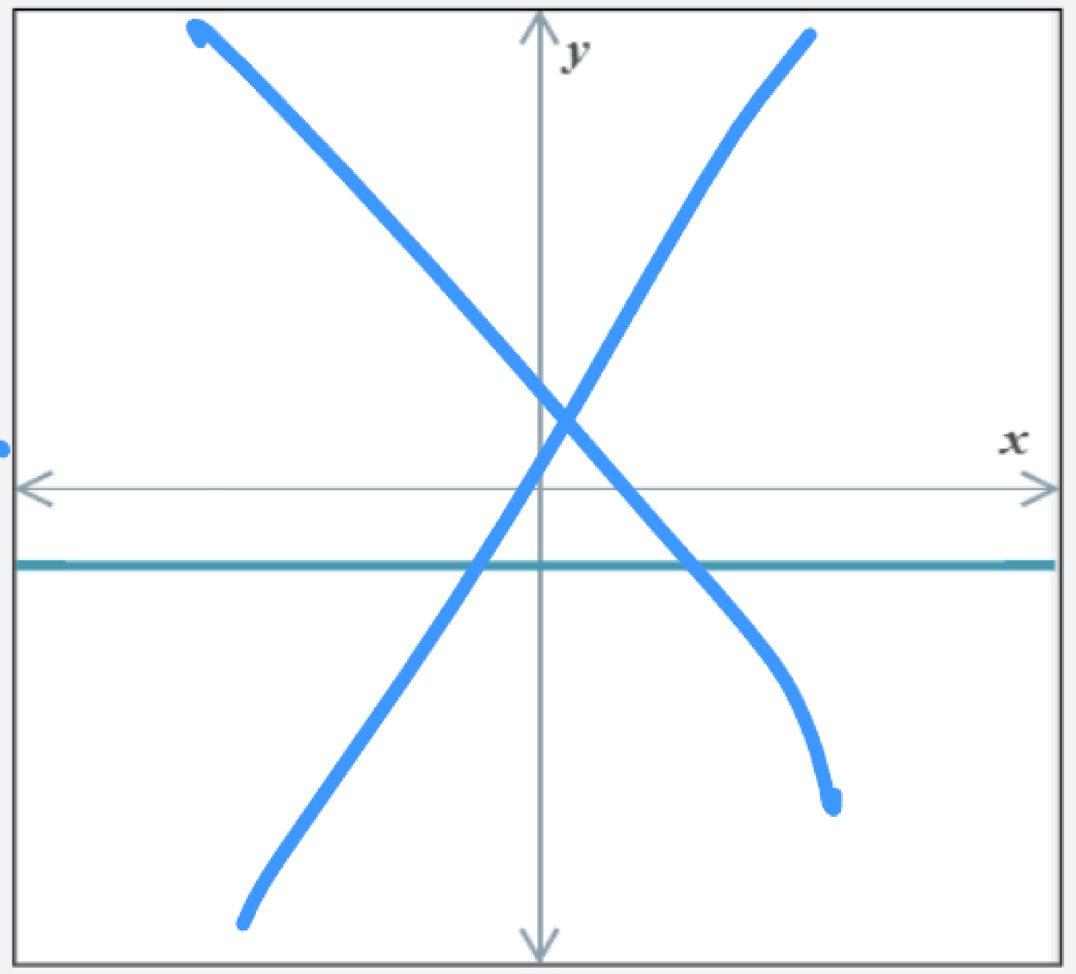
The graph of $g'(x)$ is (Choose one).

$h(x)$

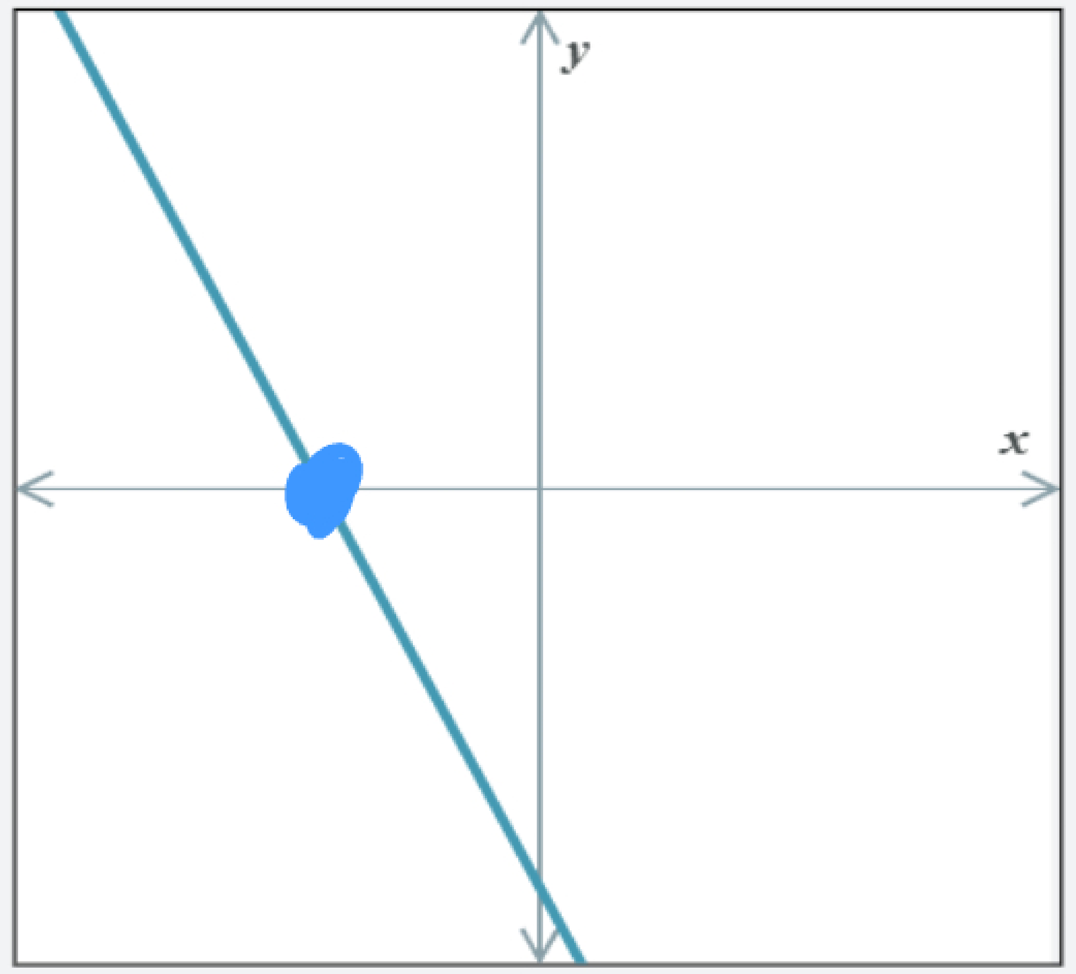


The graph of $h'(x)$ is (Choose one).

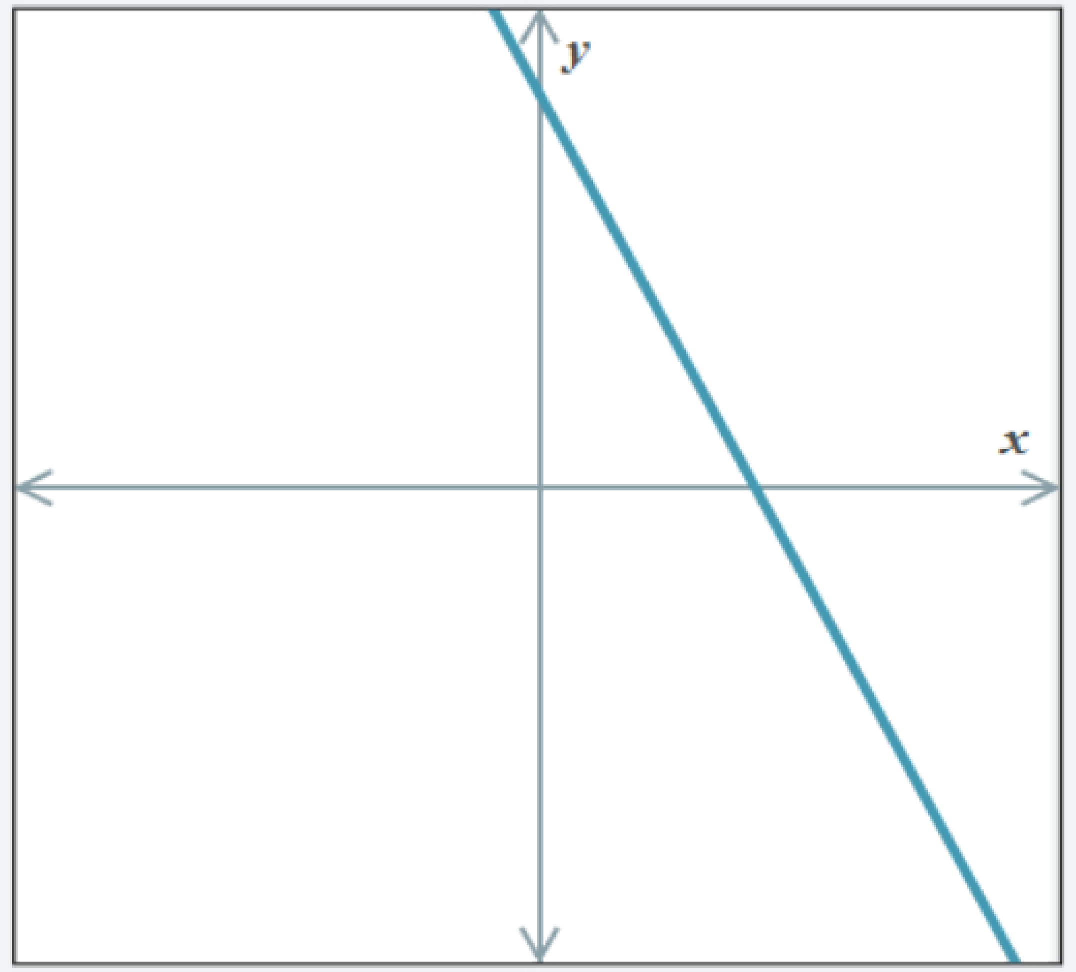
Graph A



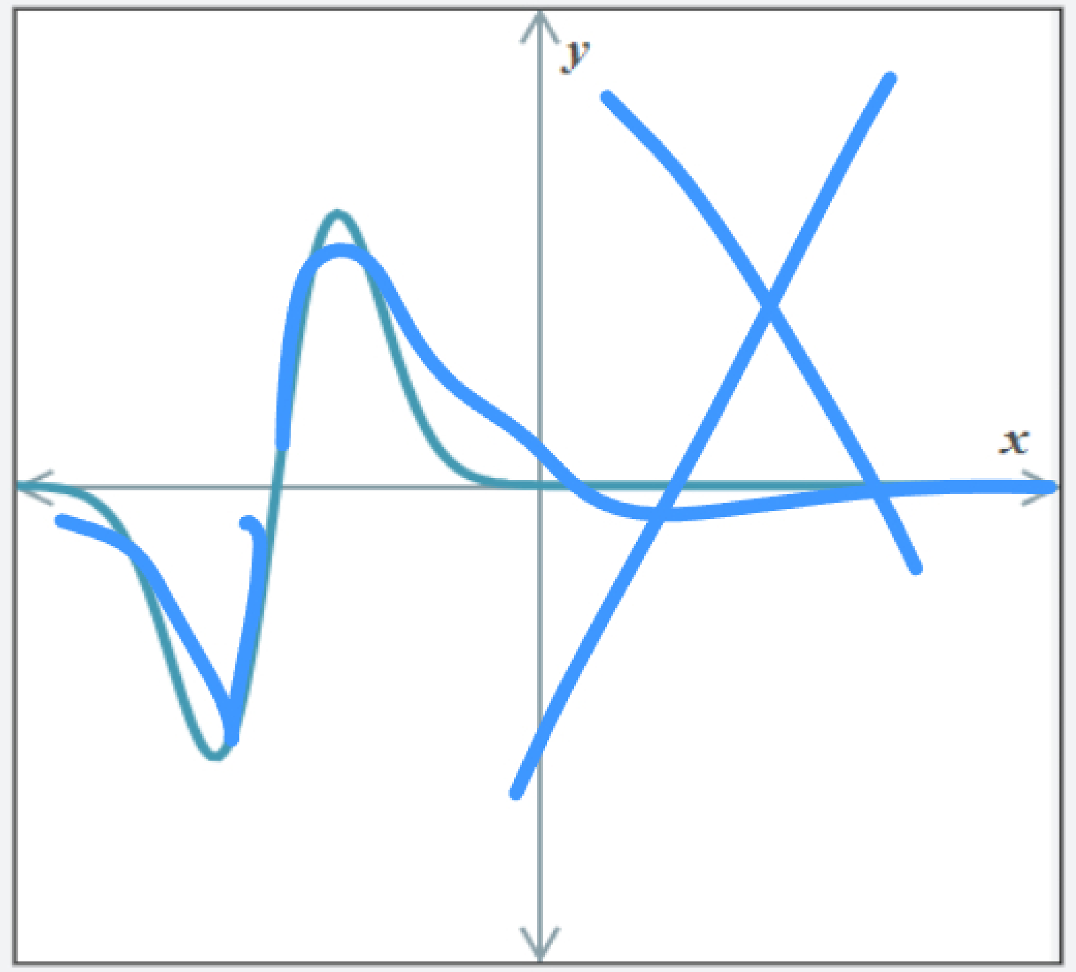
Graph B

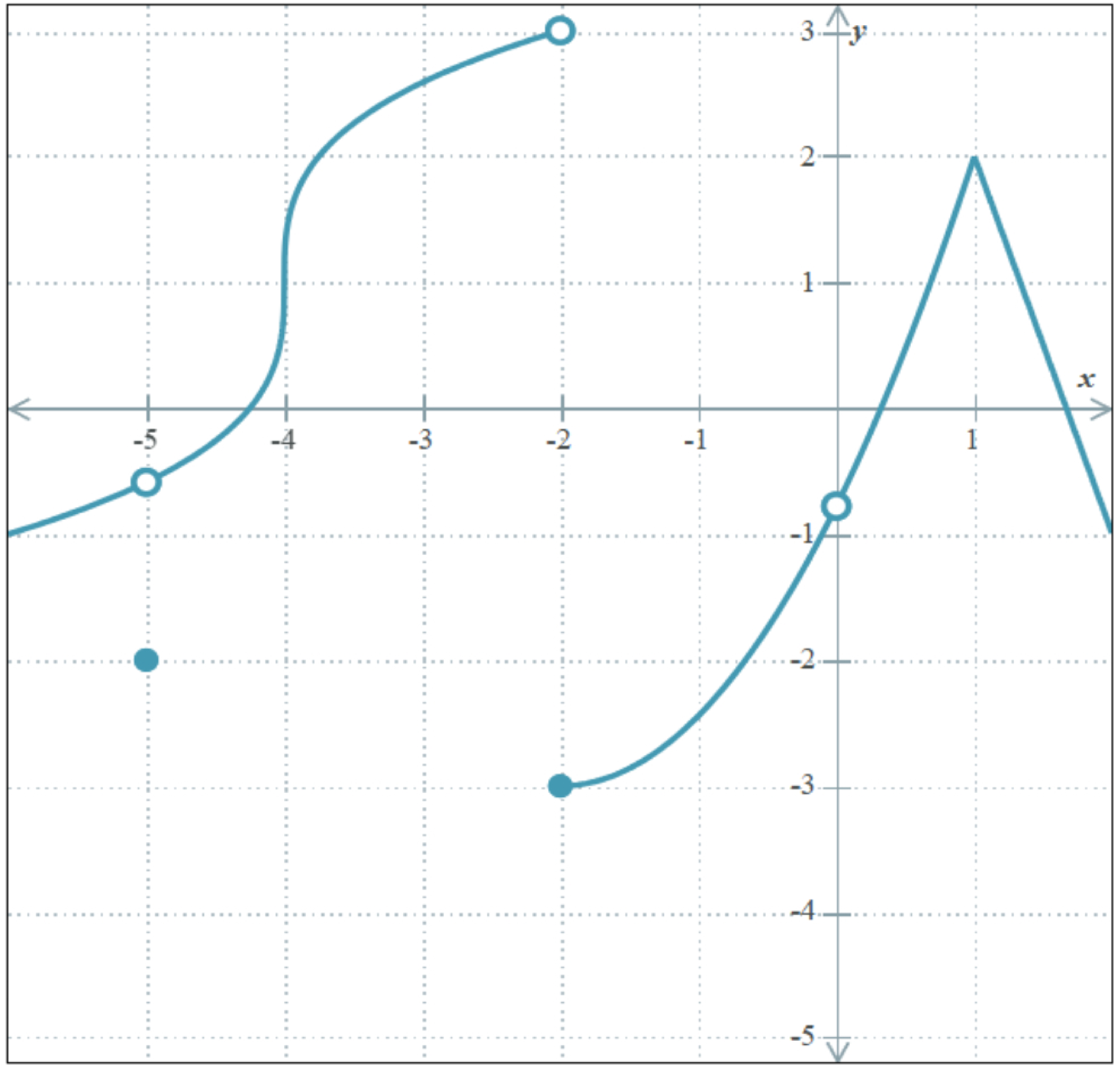


Graph C



Graph D





(a) From the choices of values of x below, choose all at which f appears to have a vertical tangent line.

☐ $x = -5$ ☐ $x = -4$ ☐ $x = -3$ ☐ $x = -2$
☐ $x = -1$ ☐ $x = 0$ ☐ $x = 1$ ☐ None

(b) From the choices of values of x below, choose all at which f appears to have a "corner" (or "kink").

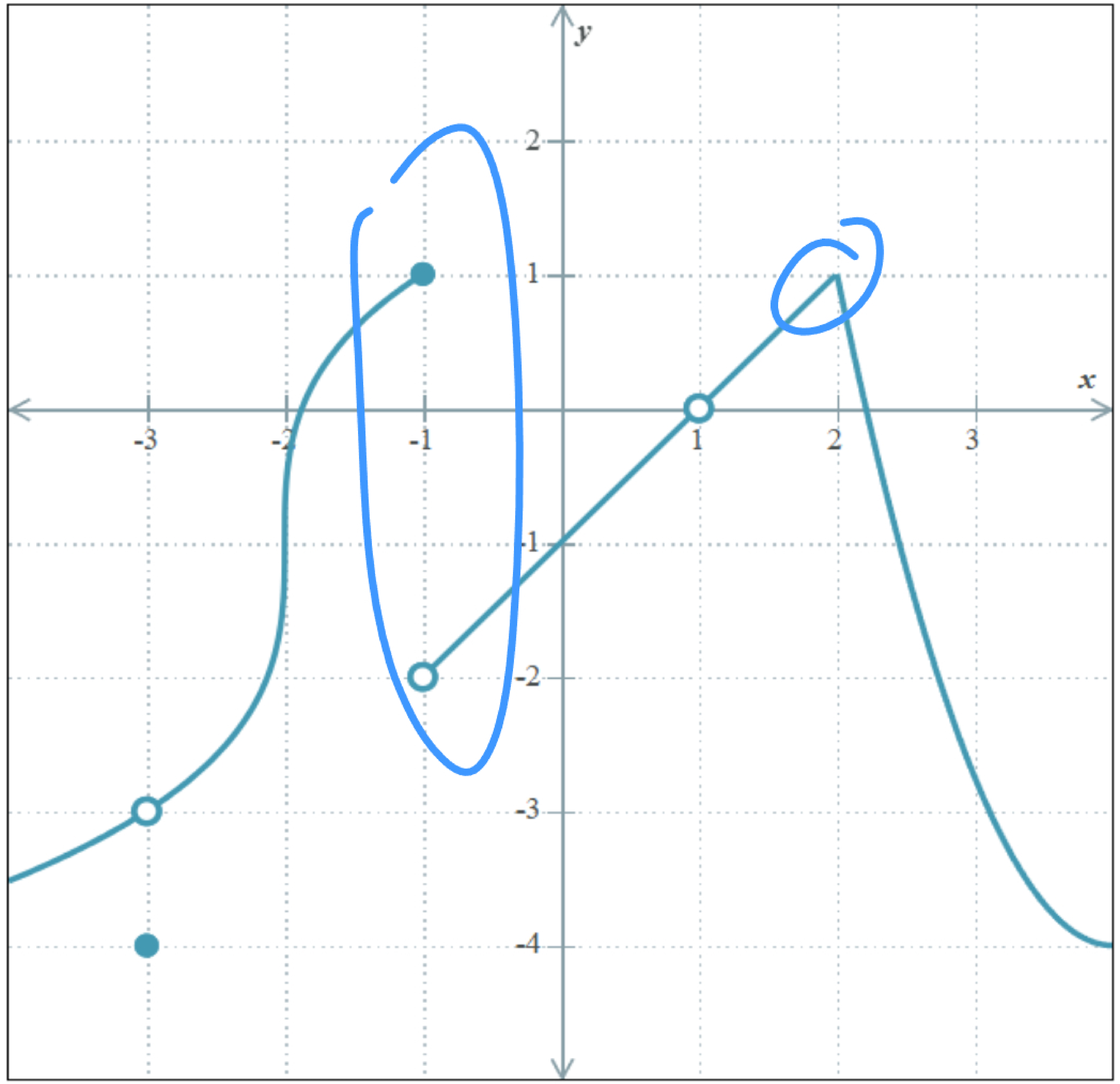
☐ $x = -5$ ☐ $x = -4$ ☐ $x = -3$ ☐ $x = -2$
☐ $x = -1$ ☐ $x = 0$ ☐ $x = 1$ ☐ None

(c) From the choices of values of x below, choose all at which f appears to have a discontinuity.

☐ $x = -5$ ☐ $x = -4$ ☐ $x = -3$ ☐ $x = -2$
☐ $x = -1$ ☐ $x = 0$ ☐ $x = 1$ ☐ None

(d) From the choices of values of x below, choose all at which f appears to be *not* differentiable.

☐ $x = -5$ ☐ $x = -4$ ☐ $x = -3$ ☐ $x = -2$
☐ $x = -1$ ☐ $x = 0$ ☐ $x = 1$ ☐ None



(a) Which statement below best corresponds to the meaning of the following?

$$f(x_0),\ \lim_{x \to x_0^-} f(x), \text{ and } \lim_{x \to x_0^+} f(x) \text{ all exist, but } \lim_{x \to x_0} f(x) \text{ does not exist}$$

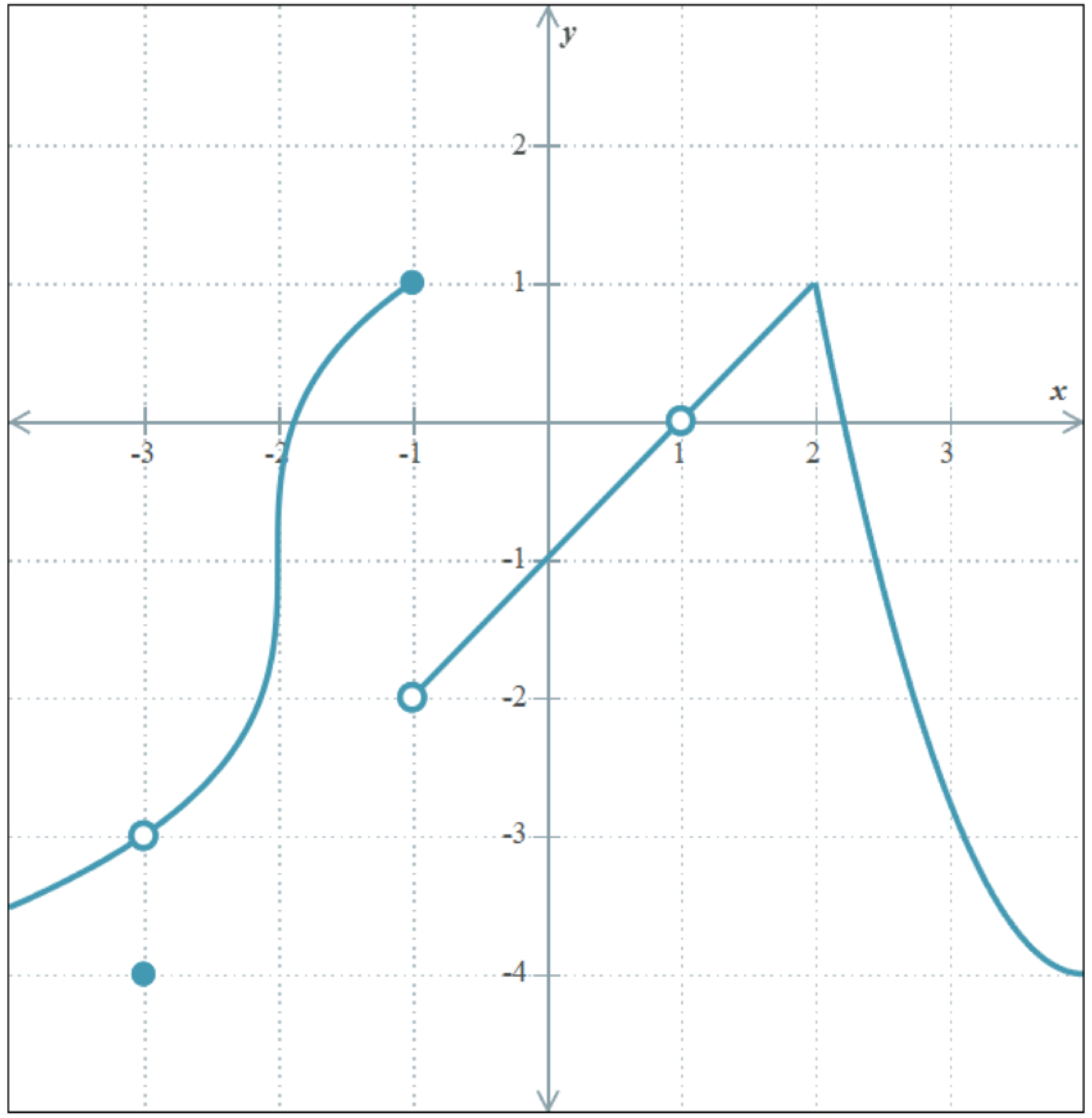
- ○ f is continuous at x_0.
- ● x_0 is in the domain of f, and f has a jump discontinuity at x_0.
- ○ x_0 is in the domain of f, and f has a removable discontinuity at x_0.
- ○ x_0 is not in the domain of f.

$\lim_{x \to x_0}$

$f(x_0) \neq$

For which values of x_0 does this statement appear to be true? Select all that apply.

☐ $x_0 = -3$	☐ $x_0 = -2$	☒ $x_0 = -1$	☐ $x_0 = 0$
☐ $x_0 = 1$	☐ $x_0 = 2$	☐ $x_0 = 3$	☐ None



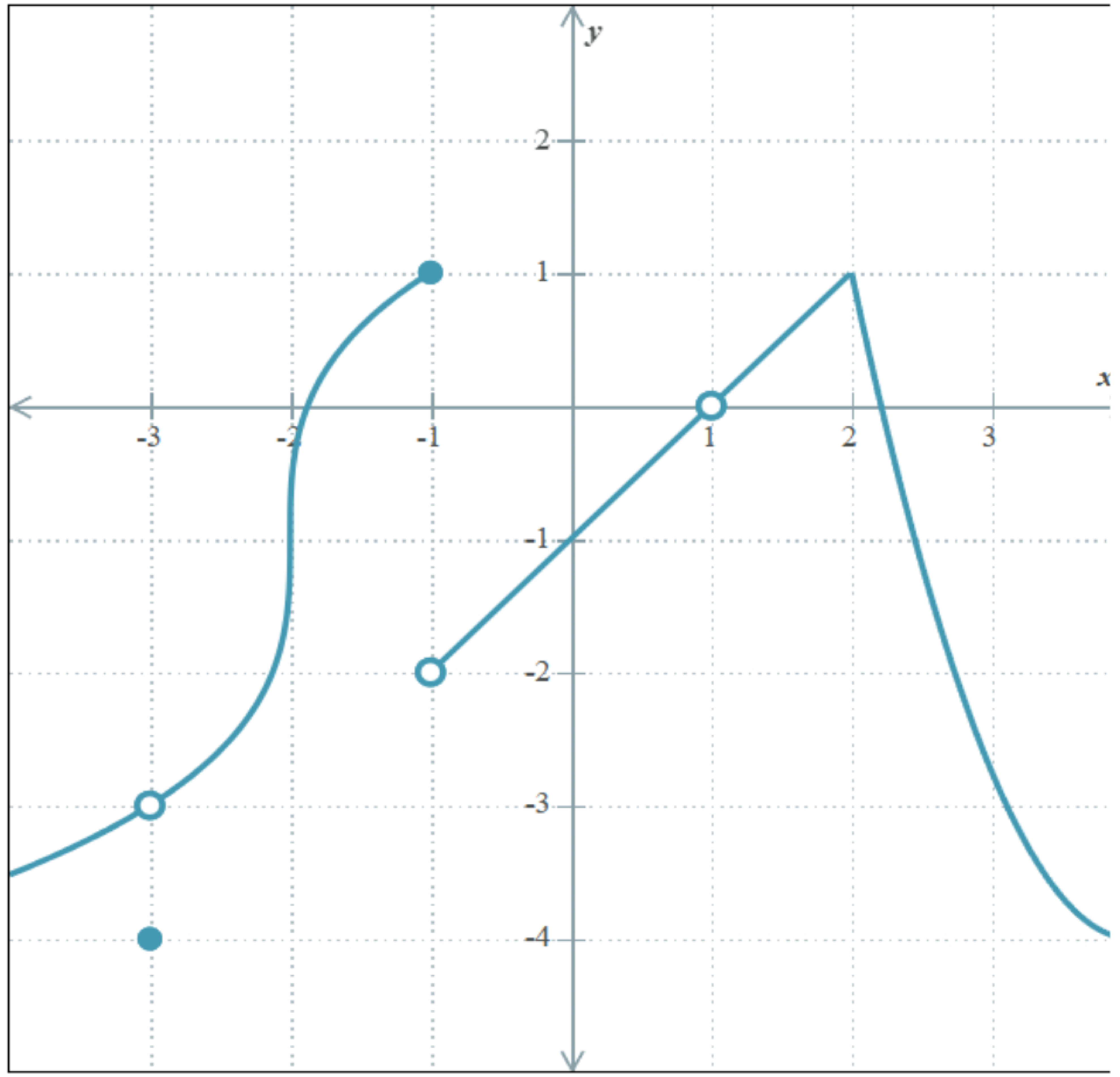
(b) Which statement below best corresponds to the meaning of the following?

$$\lim_{x \longrightarrow x_0} f(x) = f(x_0) \text{ and } \lim_{x \longrightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0} = d, \text{ where } d \text{ is a real number}$$

- ○ f is continuous at x_0, but f ~~does not have a tangent line at x_0~~
- ○ f is continuous at x_0, but f ~~has a vertical tangent line at x_0~~.
- ○ f is continuous at x_0, and f has a non-vertical tangent line at x_0.
- ○ f ~~is not continuous at x_0~~, and so f does not have a tangent line at x_0.

For which values of x_0 does this statement appear to be true? Select all that apply.

☐ $x_0 = -3$	☐ $x_0 = -2$	☐ $x_0 = -1$	☐ $x_0 = 0$
☐ $x_0 = 1$	☐ $x_0 = 2$	☐ $x_0 = 3$	☐ None



(c) Which statement below best corresponds to the meaning of the following?

$$\lim_{x \to x_0} f(x) \neq f(x_0) \text{ so there is no real number } d \text{ such that } \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} = d$$

- ○ f ~~is continuous at x_0,~~ but f does not have a tangent line at x_0.
- ○ f ~~is continuous at x_0,~~ but f has a vertical tangent line at x_0.
- ○ f ~~is continuous at x_0~~, and f has a non-vertical tangent line at x_0.
- ○ f is not continuous at x_0, and so f does not have a tangent line at x_0

For which values of x_0 does this statement appear to be true? Select all that apply.

☐ $x_0 = -3$	☐ $x_0 = -2$	☐ $x_0 = -1$	☐ $x_0 = 0$
☐ $x_0 = 1$	☐ $x_0 = 2$	☐ $x_0 = 3$	☐ None

(d) From the choices of x_0 below, choose all at which f does not appear to be differentiable.

☐ $x_0 = -3$	☐ $x_0 = -2$	☐ $x_0 = -1$	☐ $x_0 = 0$
☐ $x_0 = 1$	☐ $x_0 = 2$	☐ $x_0 = 3$	☐ None