3 Differentiation Rules

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Find the derivative of the function.

$$u = 6^5$$

Derivative of a Constant Function

$$\frac{d}{dx}(c) = 0$$

$\frac{d}{dx}$ or $\frac{du}{dx}$

find $\frac{d}{dx}$ of $5x$

$$\frac{d}{dx} = 5$$

$\frac{d}{dx}$ of $3x^2$

The Power Rule (General Version) If n is any real number, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$3x^2 \rightarrow \frac{d}{dx}(3x^2) = 6x$$

$$3(2)x^{(2-1)}$$

Find the derivative of the function.

$$f(x) = 2x^9$$

$$f'(x) = 2(9)\,x^{9-1}$$

$$= \boxed{18x^8}$$

$$f'(x)$$

Find the derivative of the function.

$$y = z^3 + e^3$$

$$\frac{d}{dz}(z^3) + \frac{d}{dz}(e^3)$$

$$\frac{dy}{dz} = \square \quad 3z^2$$

$$f(z) = z^{-5} + z^{k}$$

$$\frac{d}{dx}\left(z^{-5}\right) + \frac{d}{dx}\left(z^{k}\right)$$

$$\boxed{f'(z) = -5z^{-6} + (k)z^{k-1}}$$

Derivative of the Natural Exponential Function

$$\frac{d}{dx}(e^x) = e^x$$

$$f(x) = 3e^x + x^5$$

$$f'(x) = 3e^x + 5x^4$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{b^{x+h} - b^x}{h}$$

$$= \lim_{h \to 0} \frac{b^x b^h - b^x}{h} = \lim_{h \to 0} \frac{b^x(b^h - 1)}{h}$$

The factor b^x doesn't depend on h, so we can take it in front of the limit:

$$f'(x) = b^x \lim_{h \to 0} \frac{b^h - 1}{h}$$

Notice that the limit is the value of the derivative of f at 0, that is,

$$\lim_{h \to 0} \frac{b^h - 1}{h} = f'(0)$$

Therefore we have shown that if the exponential function $f(x) = b^x$ is differentiable at 0, then it is differentiable everywhere and

$$\boxed{4} \qquad f'(x) = f'(0)\, b^x$$

$$\text{for } b = 2, \quad f'(0) = \lim_{h \to 0} \frac{2^h - 1}{h} \approx 0.693$$

$$\text{for } b = 3, \quad f'(0) = \lim_{h \to 0} \frac{3^h - 1}{h} \approx 1.099$$

om Equation 4 we have

$$\frac{d}{dx}(2^x) \approx (0.693)2^x \qquad \frac{d}{dx}(3^x) \approx (1.099)3^x$$

Definition of the Number e

$$e \text{ is the number such that } \lim_{h \to 0} \frac{e^h - 1}{h} = 1$$

$$y = t^2 + \frac{1}{e^5} + \frac{1}{k}\left(e^t\right)$$

$$\boxed{\frac{dy}{dt} = 2t + 0 + \frac{1}{k}e^t}$$

1. $\lim\limits_{x \to a} [f(x) + g(x)] = \lim\limits_{x \to a} f(x) + \lim\limits_{x \to a} g(x)$

2. $\lim\limits_{x \to a} [f(x) - g(x)] = \lim\limits_{x \to a} f(x) - \lim\limits_{x \to a} g(x)$

3. $\lim\limits_{x \to a} [cf(x)] = c \lim\limits_{x \to a} f(x)$

4. $\lim\limits_{x \to a} [f(x)\,g(x)] = \xcancel{\lim\limits_{x \to a} f(x) \cdot \lim\limits_{x \to a} g(x)}$

5. $\lim\limits_{x \to a} \dfrac{f(x)}{g(x)} = \xcancel{\dfrac{\lim\limits_{x \to a} f(x)}{\lim\limits_{x \to a} g(x)} \quad \text{if } \lim\limits_{x \to a} g(x) \neq 0}$

nd the third derivative of the function.

$$g(t) = \sqrt{t} + \frac{1}{t^5}$$

$g'''(t) =$

$$g(t) = t^{1/2} + t^{-5}$$

$$g'(t) = \frac{1}{2}t^{-1/2} - 5t^{-6}$$

$$g''(t) = \frac{-1}{4}t^{-3/2} + 30t^{-7}$$

$$g'''(t) = \frac{3}{8}t^{-5/2} - 210t^{-8}$$

$y(1) = (1)^3 + 2(1)$

$y(1) = 3$

$\boxed{(1, 3)}$

point

slope

$y = x^3 + 2x$

$y' = 3x^2 + 2$

Tangent Slope

$y'(1) = 3(1)^2 + 2 = 5$

normal slope

$m = -\frac{1}{5}$

Point-slope

$(y - y_1) = m(x - x_1)$

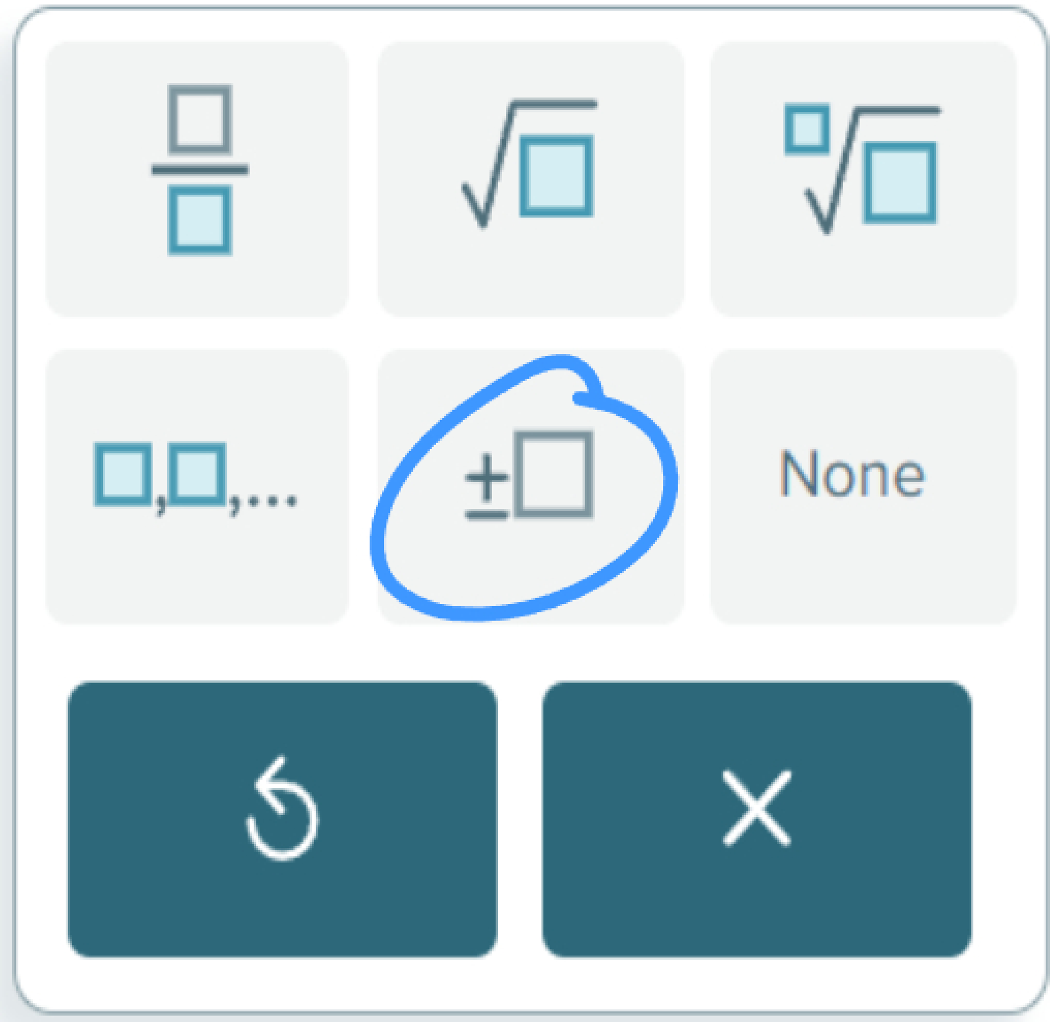
$y - 3 = -\frac{1}{5}(x - 1)$

d all values of x at which the curve has a horizontal tangent line.

$$y = -\frac{3}{2}x^4 - x^3 + 27x^2 + 27x - \frac{7}{4}$$

te your answers as exact values (not decimal approximations). If there is more than one such value, separate them with commas. If there o such value, click on None.

$x = -\frac{1}{2}, 3, -3$



$y' = -6x^3 - 3x^2 + 54x + 27$

$-3x^2(2x+1) + 27(2x+1)$

$(2x+1)(-3x^2+27) = 0$

$2x+1=0$

$2x=-1$

$x=-1/2$

$-3x^2+27=0$

$-3x^2=-27$

$x^2=9$

$x=\pm 3$

$$y = (x+3)(5x^2-4)$$

$$y = 5x^3 - 4x + 15x^2 - 12$$

$$y' = 15x^2 - 4 + 30x$$

$$\frac{d}{dx}\left(f(x)\,g(x)\right) = f'(x)\,g(x) + f(x)\,g'(x)$$

$$(t+2) \xrightarrow{\frac{d}{dt}} 1$$

$$f(t) = (t+2)\left(2t^4 + 3t^2 + 2\right)$$

$$\left(2t^4+3t^2+2\right) \xrightarrow{\frac{d}{dt}} 8t^3 + 6t$$

$$f'(t) =$$

$$f'(t) = (1)\left(2t^4+3t^2+2\right) + (t+2)\left(8t^3+6t\right)$$
$$= 2t^4+3t^2+2 + (t+2)\left(8t^3+6t\right)$$

Find the derivative of the following function.

$$g(x) = e^x\left(9\sqrt{x} - 2x^2\right)$$

$$\frac{d}{dx}\left(f(x)g(x)\right) =$$

$$f'(x)g(x) + f(x)g'(x)$$

$$e^x \xrightarrow{\frac{d}{dx}} e^x$$

$$9x^{1/2} - 2x^2 \xrightarrow{\frac{d}{dx}} \frac{9}{2}x^{-1/2} - 4x$$

$$g'(x) = (e^x)(9x^{1/2} - 2x^2) + (e^x)(9/2\,x^{-1/2} - 4x)$$

nd the derivative of the function.

$$f(z) = (z^2 + 2)(2z^4 + 1)^{-1}$$

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$$

$$f(z) = \frac{z^2+2}{2z^4+1}$$

$$\boxed{f'(z) = \frac{(2z)(2z^4+1) - (z^2+2)(8z^3)}{(2z^4+1)^2}}$$

$$z^2 + 2 \rightarrow 2z$$

$$\frac{d}{dx}$$

$$2z^4 + 1 \rightarrow 8z^3$$

Applying rules of differentiation given information about two functions

Let $H(x) = \dfrac{g(x)}{f(x)}$ and $W(x) = 5f(x) + 2g(x)$. The following table gives some values of the functions f, g, f', and g'.

$f(7)$	$g(7)$	$f'(7)$	$g'(7)$
-7	-6	3	2

$$H'(x) = \frac{g'(x)f(x) - g(x)f'(x)}{f(x)^2}$$

Use the table to find $H'(7)$ and $W'(7)$.

$$H'(7) = \frac{(2)(-7) - (-6)(3)}{(-7)^2} = \frac{4}{49}$$

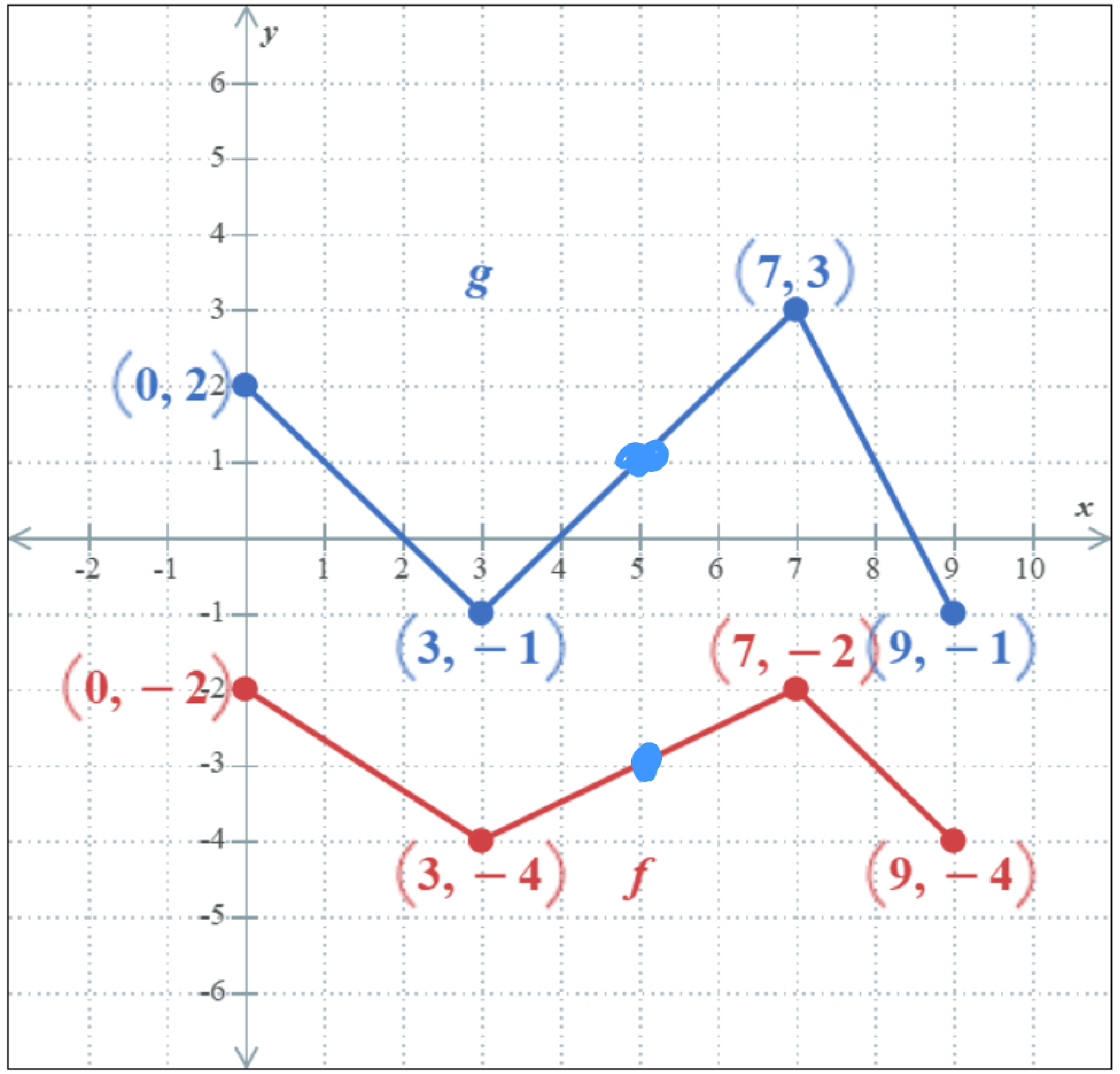
(a) $H'(7) =$ ☐

(b) $W'(7) =$ ☐

$$W'(7) = 5(3) + 2(2) = 19$$

The graphs of the piecewise linear functions f and g are show

Let $u(x) = f(x)g(x)$. Find $u'(5)$.



$u'(5) =$ ☐

$u(x) = f(x)\, g(x)$

$u'(x) = f'(x) g(x) + f(x) g'(x)$

$f(5) = -3$ $\qquad f'(5) = \frac{2}{4} = \frac{1}{2}$

$g(5) = 1$ $\qquad g'(5) = \frac{4}{4} = 1$

$u'(5) = \left(\frac{1}{2}\right)(1) + (-3)(1)$

$= -\frac{5}{2}$

The Power Rule (General Version) If n is any real number, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Derivative of the Natural Exponential Function

$$\frac{d}{dx}(e^x) = e^x$$

Product Rule

$$\frac{d}{dx}(f(x)g(x)) = f'(x)g(x) + f(x)g'(x)$$

Quotient Rule $\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$