Differentiate $y = e^{\sin x}$

$\sin x \xrightarrow{d/dx} \cos x$

$u \longleftrightarrow \sin x$

$e^u \xrightarrow{d/dx} e^u$

$y' = u' \cdot e^u$

$y' = \cos x \left(e^{\sin x}\right)$

$$f(x) = (3x^3 - 5x^2 + 2)^8$$

$u = 3x^3 - 5x^2 + 2$

$u' =$

$f'(x) = u'(8u^7)$

$= (9x^2 - 10x)(8(3x^3 - 5x^2 + 2)^7)$

$$y = \sqrt{3x^k - 4x}$$

$u = 3x^k - 4x$

$u' = 3kx^{k-1} - 4$

$y' = u' \cdot \frac{1}{2\sqrt{u}}$

$\sqrt{u} = u^{1/2} \xrightarrow{d/dx} \frac{1}{2}u^{-1/2} = \frac{1}{2\sqrt{u}}$

$$y' = \frac{3kx^{k-1} - 4}{2\sqrt{3x^k - 4x}}$$

1 $\ln x^r = r \ln x$

e e

2 $x^r = e^{r \ln x}$

3 $x^r = e^{\ln x^r}$ $\leftrightarrow$ $b^x = e^{\ln b^x}$

$$\frac{d}{dx}$$

$$= \frac{d}{dx} e^{x \ln b}$$

$U = x \ln b$	e^U
$U' = \ln b$	e^U

$$= U' \cdot e^U$$

$$= \ln b \cdot e^{x \ln b}$$

$$= \ln b \cdot e^{\ln b^x}$$

$$= (\ln b)(b^x)$$

$$\frac{d}{dx} b^x = (\ln b) b^x$$

$$\frac{d}{dx}(2^x) = (\ln 2)(2^x)$$

$$\frac{d}{dx}(e^x) = (\ln e)(e^x) = e^x$$

EXAMPLE 10 Find the derivative of each of the functions.

(a) $g(x) = 2^x$ (b) $h(x) = 5^{x^2}$

a) $g'(x) = (\ln 2)(2^x)$

b) $h'(x) = (2x)(\ln 5)(5^{x^2})$ $u = x^2$ $u' = 2x$

$$y = \sqrt{25 - x^2}$$

$$u = 25 - x^2$$

$$u' = -2x$$

$$y' = \frac{-2x}{2\sqrt{25-x^2}} = \boxed{\frac{-x}{\sqrt{25-x^2}}}$$

$$\frac{d}{dx}\sqrt{u} = \frac{1}{2\sqrt{u}}$$

$$y' = \frac{-(-4)}{\sqrt{25-(-4)^2}} = \frac{4}{\sqrt{25-16}} = \frac{4}{\sqrt{9}} = \frac{4}{3}$$

$$x = -4$$

$$x^2 + y^2 = 25$$

$$\sqrt{y^2} = \sqrt{25 - x^2}$$

$$y = \pm\sqrt{25 - x^2}$$

$$x^3 + y^3 = 6xy$$

$$\frac{d}{dx}\left(x^2 + y^2 = 25\right)$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}y^2 = \frac{d}{dx}(25)$$

$$2x + 2y\,y' = 0$$

$$\frac{2y\,y'}{2y} = -\frac{2x}{2y}$$

$$y' = -\frac{x}{y}$$

$$x^2 + y^2 = 25 \xrightarrow{d/dx} y' = -\frac{x}{y}$$

$$(x, y) \to (-4, 3) \quad y' = \frac{4}{3}$$

$$3x^2 + 3y^2 y' = 6y + 6x y'$$
$$-3x^2 \quad -6xy' \quad -3x^2 \quad -6xy'$$

$$3y^2 y' - 6xy' = 6y - 3x^2$$

$$y'(3y^2 - 6x) = 6y - 3x^2$$

$$\frac{y'(3y^2 - 6x)}{3y^2 - 6x} = \frac{6y - 3x^2}{3y^2 - 6x}$$

$$y' = \frac{6y - 3x^2}{3y^2 - 6x} = \boxed{\frac{2y - x^2}{y^2 - 2x}}$$

$-19x^2 \rightarrow -38x$

$y^2 \rightarrow 2yy'$

$$\frac{d}{dx}(-19x^2y^2) = \frac{d}{dx}(-4)$$

$$-38xy^2 + (-19x^2)(2yy') = 0$$

$$-38xy^2 - 38x^2yy' = 0$$

$$\frac{-38x^2yy'}{-38x^2y} = \frac{38xy^2}{-38x^2y}$$

$$\boxed{y' = -\frac{y}{x}}$$

$$-5y^7 - 8x^9 y + x^2 = 3$$

$$-35y^6 y' - 72x^8 y - 8x^9 y' + 2x = 0$$

$$-35y^6 y' - 8x^9 y' = 72x^8 y - 2x$$

$$y'(-35y^6 - 8x^9) = 72x^8 y - 2x$$

$$y' = \frac{72x^8 y - 2x}{-35y^6 - 8x^9}$$

$$\frac{d}{dx}(-5y^7) = -35y^6 y'$$

$$\frac{d}{dx}(-8x^9 y) = -72x^8 y - 8x^9 y'$$

$-8x^9 \rightarrow -72x^8$

$y \rightarrow y'$

$$\frac{d}{dx}(x^2) = 2x$$

$$\frac{d}{dx}(3) = 0$$

$$\frac{d}{dx}\left((7x-5y)^{1/2}\right)$$

$u = 7x - 5y \qquad u' = 7 - 5y'$

$u^{1/2} \rightarrow \frac{1}{2\sqrt{u}}$

$$\frac{7-5y'}{2\sqrt{7x-5y}} = 2y + 2xy'$$

$$\frac{d}{dx}(2xy)$$

$2x \Rightarrow 2$

$y \Rightarrow y'$

$2y + 2xy'$

$2\sqrt{7x-5y}$

$$7-5y' = 2\sqrt{7x-5y}\,(2y+2xy')$$

$$7-5y' = 4y\sqrt{7x-5y} + 4xy'\sqrt{7x-5y}$$

$$7-4y\sqrt{7x-5y} = 4xy'\sqrt{7x-5y} + 5y'$$

$$7-4y\sqrt{7x-5y} = y'(4x\sqrt{7x-5y} + 5)$$

$$\frac{7-4y\sqrt{7x-5y}}{4x\sqrt{7x-5y}+5} = y'$$