3.7 Rates of Change in the Natural and Social Sciences

EXAMPLE 1 The position of a particle is given by the equation

$$s = f(t) = t^3 - 6t^2 + 9t$$

where t is measured in seconds and s in meters.

$V(t) \Rightarrow m/s$

(a) Find the velocity at time t. $V(t) = f'(t) = 3t^2 - 12t + 9$

(b) What is the velocity after 2 s? After 4 s? $V(2) = -3$ $V(4) = 9$

(c) When is the particle at rest?

(d) When is the particle moving forward (that is, in the positive direction)?

(f) Find the total distance traveled by the particle during the first five seconds.

c) $0 = 3t^2 - 12t + 9$

$0 = 3(t^2 - 4t + 3)$

$0 = 3(t-3)(t-1)$

$t = 3\text{sec}$

$t = 1\text{sec}$

$t=0$ $t=2$ $t=4$

$(+)$ 1 $(-)$ 3 $(+)$

$(0,1) \cup (3,\infty)$

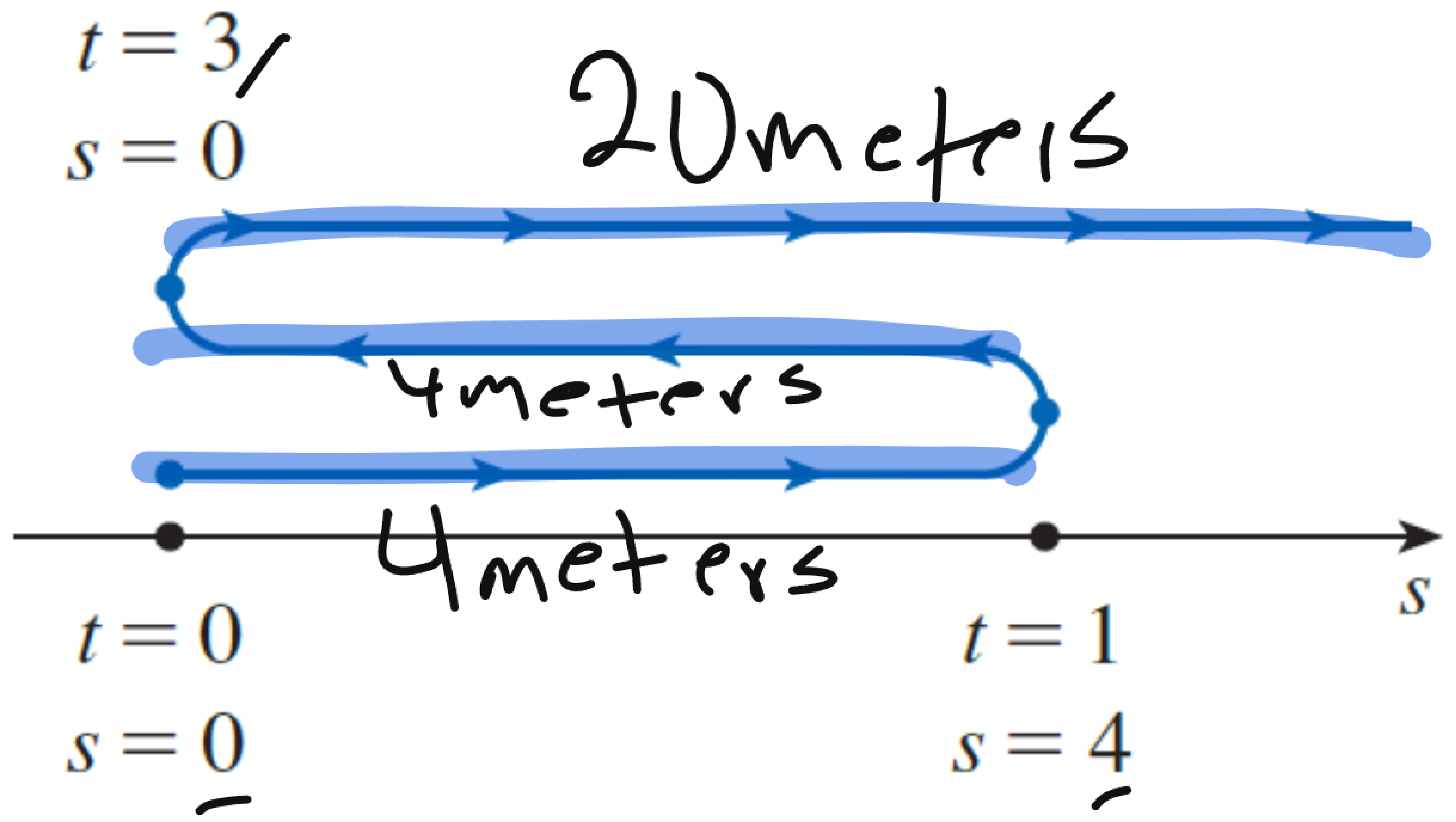
EXAMPLE 1 The position of a particle is given by the equation

$$s = f(t) = t^3 - 6t^2 + 9t$$

where t is measured in seconds and s in meters.

$v(t) = 3t^2 - 12t + 9$

$s(5) = 20$



$t = 5$
$s = 20$

$\boxed{\text{28 meters in 5 secs}}$

EXAMPLE 1 The position of a particle is given by the equation

$$s = f(t) = t^3 - 6t^2 + 9t$$

where t is measured in seconds and s in meters.

$v(t) = 3t^2 - 12t + 9$

$a(t) = 6t - 12$

$a(4) = 12$ meter/sec^2

(g) Find the acceleration at time t and after 4 s.

$(2, \infty)$ $(0, 2)$

(i) When is the particle speeding up? When is it slowing down?

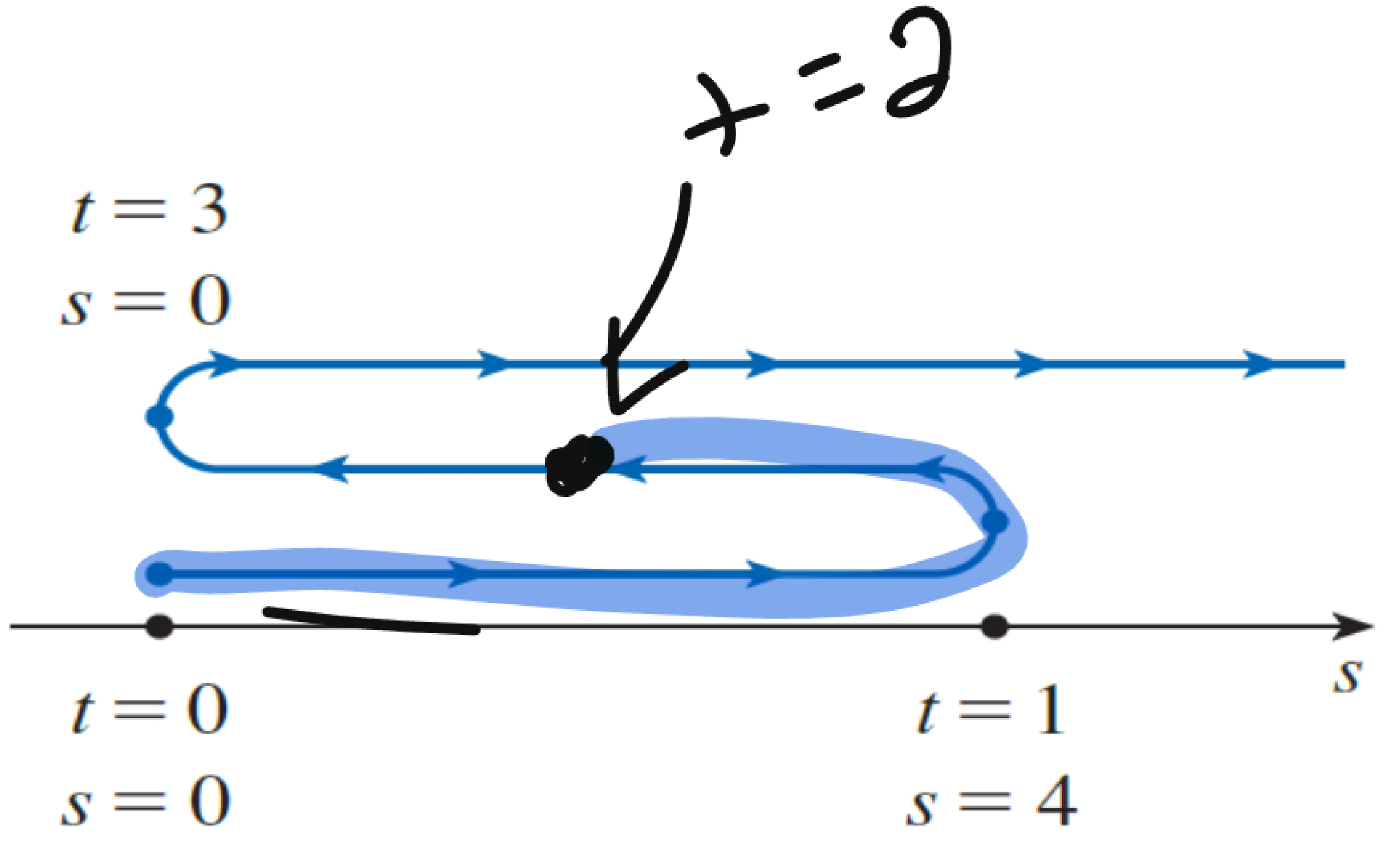
$0 = 6t - 12$

$12 = 6t$

$2 = t$

$t = 0$ $t = 4$

$(-)$ 2 $(+)$



-324

Answer the following questions to describe the motion of the particle at time $t = 6$ seconds.

Where is the particle located at $t = 6$ seconds?

$y(6) = -324$

- ○ at the origin
- X to the left of the origin
- ○ to the right of the origin

In which direction is the particle traveling at $t = 6$ seconds?

$v(6) = 108$

- ○ to the left
- X to the right
- ○ in neither direction (it is at rest)

How is the speed of the particle changing at $t = 6$ seconds?

$a(6) = 54$

- X speeding up
- ○ slowing down
- ○ not changing

A particle is moving along the y-axis. Its position y as a function of time t (in seconds) is given by the following:

$$y(t) = -5t^3 + 18t^2 - 30t + 3 \qquad \text{for } t \geq 0$$

$y(t) = -5t^3 + 18t^2 - 30t + 3$

Complete the parts below.

(a) Find $v(t)$, the velocity (in units per second) of the particle at time t.

$v(t) =$ $v(t) = -15t^2 + 36t - 30$

(b) Find $a(t)$, the acceleration (in units per second per second) of the particle at time t.

$a(t) =$ $a(t) = -30t + 36$

(c) List all times t when the velocity of the particle is decreasing at a rate of 9 units per second per second.

If there is more than one value, separate them with commas. Click on "None" if applicable.

All t when velocity is decreasing 9 units per second per second: $t =$ $\boxed{1.5}$

$a(t) = -9 = -30t + 36 \rightarrow -45 = -30t \rightarrow 1.5 = t$

3.8 Exponential Growth and Decay

In general, if $y(t)$ is the value of a quantity y at time t and if the rate of change of y with respect to t is proportional to its size $y(t)$ at any time, then

1
$$\boxed{\frac{dy}{dt} = ky}$$

where k is a constant. Equation 1 is sometimes called the **law of natural growth** (if $k > 0$) or the **law of natural decay** (if $k < 0$). It is called a *differential equation* because it involves an unknown function y and its derivative dy/dt.

$\boxed{2}$ **Theorem** The only solutions of the differential equation $dy/dt = ky$ are the exponential functions

$$y(t) = y(0)e^{kt}$$

$y(0) \rightarrow$ starting value

$k \rightarrow$ constant of change

EXAMPLE 1 Use the fact that the world population was 2560 million in 1950 and 3040 million in 1960 to model the population of the world in the second half of the 20th century. (Assume that the growth rate is proportional to the population size.) What is the relative growth rate? Use the model to estimate the world population in 1993 and to predict the population in the year 2025.

$P(0) = 2560 \quad t=0 \ (1950)$

$$P(t) = P(0)e^{kt}$$

k = relative growth rate

$P(t) = 2560e^{kt}$

$3040 = 2560e^{10k}$

$1.1875 = e^{10k}$

$$\frac{\ln 1.1875}{10} = \frac{10k}{10}$$

$0.0171850 = k$

EXAMPLE 1 Use the fact that the world population was 2560 million in 1950 and 3040 million in 1960 to model the population of the world in the second half of the 20th century. (Assume that the growth rate is proportional to the population size.) What is the relative growth rate? Use the model to estimate the world population in 1993 and to predict the population in the year 2025.

$$P(t) = 2560\, e^{0.017185t}$$

$$P(43) = 5360$$

$$P(75) = 9289$$

$$\frac{\ln 1.1875}{10} = \frac{10k}{10}$$

$$0.0171850 = k$$

Suppose that the velocity $v(t)$ (in $\mathrm{m/s}$) of a sky diver falling near the Earth's surface is given by the follo
measured in seconds.

$$v(t) = 49\left(1 - e^{-0.21t}\right)$$

Find the velocity of the sky diver after 3 seconds and after 5 seconds.
Round your answers to the nearest whole number as necessary.

An amount of $35,000 is borrowed for 14 years at 6.25% interest, compounded annually. Assuming that no payments are made, find the amount owed after 14 years.

Use the calculator provided and round your answer to the nearest dollar.

Compound $A = P\left(1 + \frac{r}{n}\right)^{nt}$ $n =$ # of compounds Annually

Continous $A = Pe^{rt}$

$35000\left(1 + \frac{.0625}{1}\right)^{1(14)} = \$81,785$

$35000\, e^{0.0625(14)} = \$83,961$

exponential growth

The rate of change of y is proportional to y. The curve $y = f(x)$ passes through $(0, 40)$ and $(1, 100)$. Find $y(2)$.

Write your answer without exponents or logarithms. Use exact values in your answer (not decimal approximations).

$$y(x) = f(0)e^{kx}$$

$$f(0) = 40$$

$$y(x) = 40e^{kx}$$

$$k = \ln\left(\frac{100}{40}\right)$$

$$100 = 40e^{k(1)}$$

$$y(x) = 40e^{\ln\left(\frac{100}{40}\right)x}$$

$$\frac{100}{40} = e^{k}$$

$$y(2) = 250$$

$$\ln(100/40) = k$$

(a) Fill in the blanks to write an initial-value problem that models the spread of the rumor. In the differential equation, write y rather than $y(t)$.

Differential equation: $\frac{dy}{dt} =$ $0.055(510-y)$ ky

Initial condition: $y(0) =$ 10

(b) Find the particular solution to the differential equation that satisfies the initial condition. Write your answer in the form $y=f(t)$, where all the terms involving t are on the right-hand side. Use exact values in your answer (not decimal approximations).

$$y(t) = 510 - 500e^{-0.055t}$$

(c) How many students have heard the rumor 18 minutes after it begins to spread? Round your answer to the nearest whole number.

324 students

What is the rate of change of the number of students who have heard the rumor at the moment when 254 students have heard the rumor? Do not round your answer.

$0.055(510-254)$