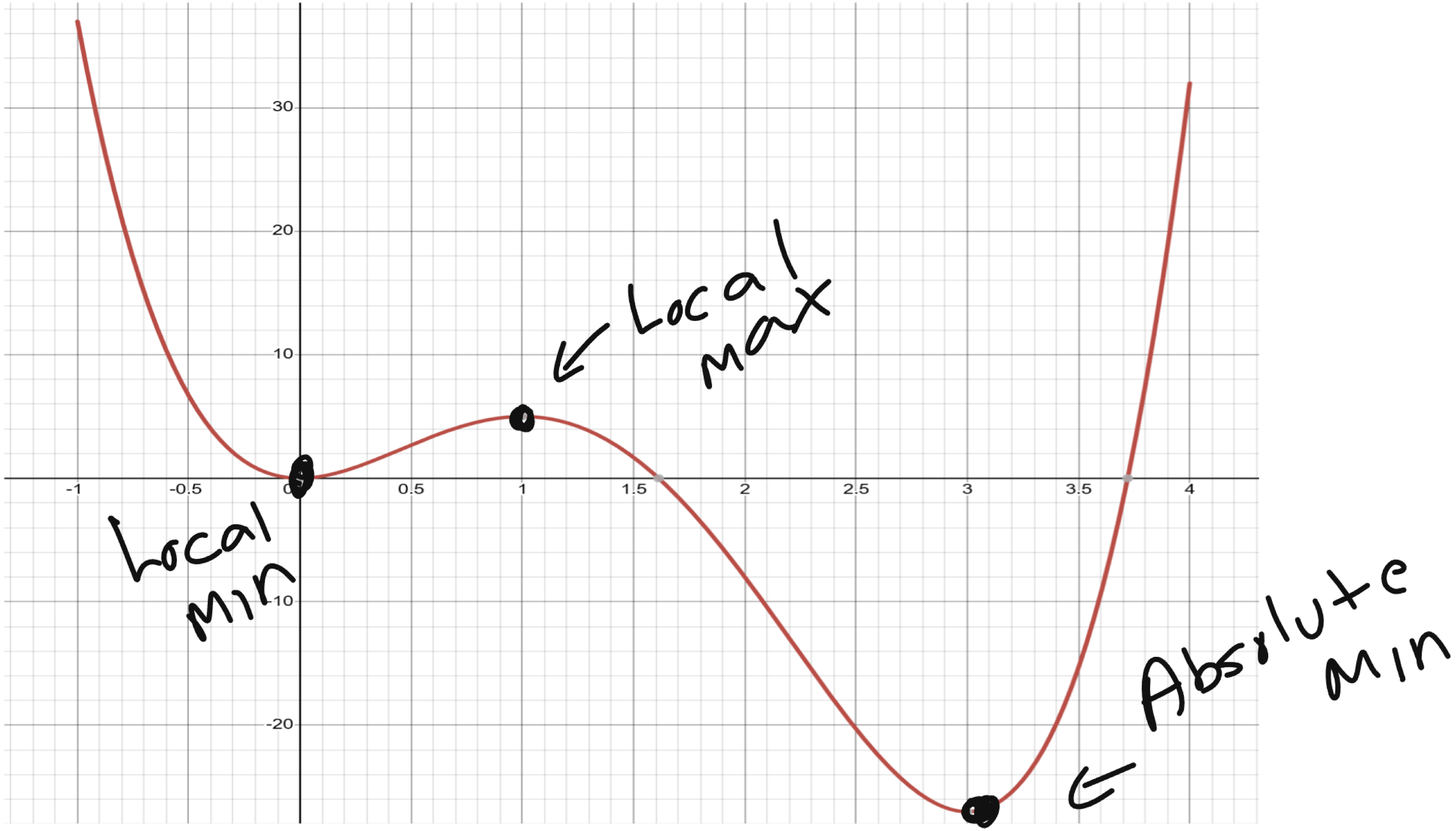
EXAMPLE 1 The graph of the function

$$f(x) = 3x^4 - 16x^3 + 18x^2 \qquad -1 \leqslant x \leqslant 4$$



1 **Definition** Let c be a number in the domain D of a function f. Then $f(c)$ is the

- **absolute maximum** value of f on D if $f(c) \geqslant f(x)$ for all x in D.
- **absolute minimum** value of f on D if $f(c) \leqslant f(x)$ for all x in D.

2 **Definition** The number $f(c)$ is a

- **local maximum** value of f if $f(c) \geqslant f(x)$ when x is near c.
- **local minimum** value of f if $f(c) \leqslant f(x)$ when x is near c.

3 **The Extreme Value Theorem** If f is continuous on a closed interval $[a, b]$, then f attains an absolute maximum value $f(c)$ and an absolute minimum value $f(d)$ at some numbers c and d in $[a, b]$.

$\boxed{4}$ Fermat's Theorem If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.

Let the function f be defined as follows.

$$f(x) = x^3 - 3x^2 - 9x + 4$$

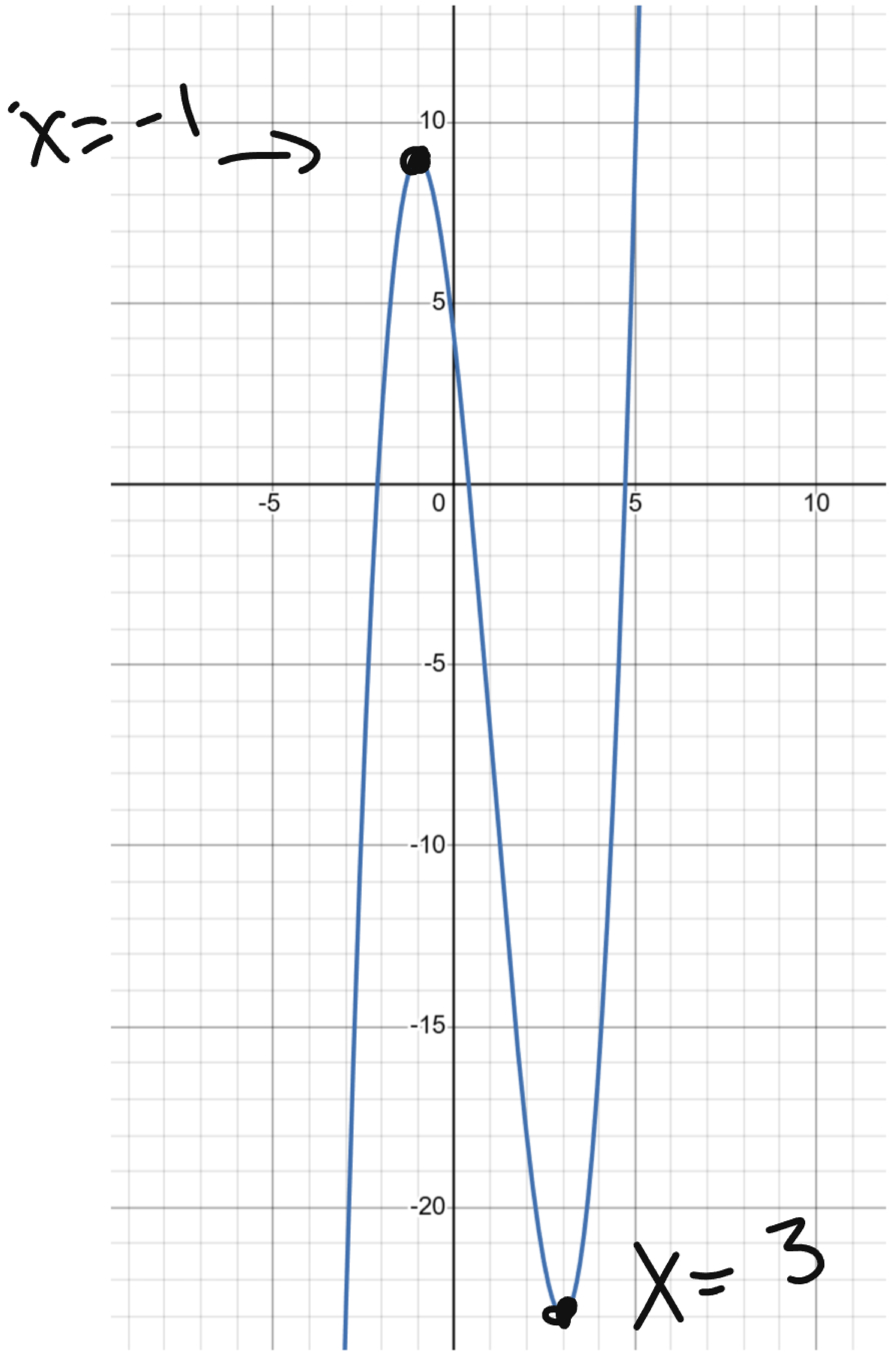
$$f'(x) = 3x^2 - 6x - 9$$

$$0 = 3x^2 - 6x - 9$$

$$0 = x^2 - 2x - 3$$

$$0 = (x-3)(x+1)$$

$$x = 3, -1$$



$$f(x) = x^3$$

$$f'(x) = 3x^2$$

$$0 = 3x^2$$

$$x = 0$$

$$f(x) = |x|$$

$$f(x) = \begin{cases} -x & x < 0 \\ x & x \geq 0 \end{cases}$$

$$f'(x) = \begin{cases} -1 & x < 0 \\ 1 & x > 0 \\ \text{DNE} & x = 0 \end{cases}$$

6 **Definition** A **critical number** of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

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Let the function f be defined as follows.

$$f(x) = \frac{1}{2}x^2 + \frac{81}{x^3} + 2$$

C.N. $x = 3$

$$f'(x) = x - 243x^{-4} = x - \frac{243}{x^4}$$

$$0 = x - \frac{243}{x^4} \Rightarrow x = \frac{243}{x^4}$$

$$\sqrt[5]{x^5} = \sqrt[5]{243}$$

$$x = 3$$

DNE

$$x = 0$$

$-4x^2 + \frac{1}{12}$

$$f'(x) = \frac{(-3x^2)(-4x^2 + 1/12) - (-x^3)(-8x)}{(-4x^2 + 1/12)^2}$$

$$0 = 12x^4 - 1/4\,x^2 - 8x^4$$

$$0 = 4x^4 - 1/4\,x^2$$

$$0 = \frac{1}{4}x^2(16x^2 - 1) = \frac{1}{4}x^2(4x-1)(4x+1)$$

$$\boxed{x = 0, \frac{1}{4}, -\frac{1}{4}}$$

Consider the function defined by the equation below.

$$y = e^{-4x}(x+1)^9$$

$y = e^{-4x}(x+1)^9$

$x = -1$

$x = 5/4$

Find the absolute minimum and absolute maximum values of y on the interval $\left[-\frac{7}{5}, 2\right]$. Give exact answers, not decimal approximations. If there is no absolute minimum or absolute maximum value, select "None".

$f = e^{-4x}$ $\quad$ $f' = -4e^{-4x}$

$g = (x+1)^9$ $\quad$ $g' = 9(x+1)^8$

$y' = \boxed{-4e^{-4x}\boxed{(x+1)}^9} + \boxed{e^{-4x}(9\boxed{(x+1)}^8)}$

$0 = \boxed{e^{-4x}(x+1)^8}\left[-4(x+1) + 9\right]$

Consider the function defined by the equation below:

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Find the absolute minimum and absolute maximum values of y on the interval $\left[-\frac{7}{5}, 2\right]$. Give exact answers, not decimal approximations. If there is no absolute minimum or absolute maximum value, select "None".

$$y = e^{-4\left(-\frac{7}{5}\right)}\left(-\frac{7}{5}+1\right)^9 = e^{28/5}\left(-\frac{2}{5}\right)^9 \leftarrow \text{min}$$

$$y = e^{-4(-1)}(-1+1)^9 = e^5(0) = 0$$

$$y = e^{-4(5/4)}\left(\frac{5}{4}+1\right)^9 = e^{-5}\left(9/4\right)^9 \leftarrow \text{max}$$

$$y = e^{-4(2)}(2+1)^9 = e^{-8}(3)^9$$

8

9

10

Consider the function defined by the equation below.

$$y = 5\sin^2 x + 5\cos x$$

$[\pi/4, \pi/2]$

$y = 5(\sin x)^2 + 5\cos x$

$x = \frac{\pi}{3}$

$y' = 5[2(\sin x)(\cos x)] - 5\sin x$

$0 = 10\sin x \cos x - 5\sin x$

$0 = 5\sin x\,(2\cos x - 1)$

$5\sin x = 0$

$\cancel{\sin x = 0}$

$2\cos x - 1 = 0$

$\cos x = 1/2$

Consider the function defined by the equation below.

$$y = 5\sin^2 x + 5\cos x$$

$x = \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$

$y = 5(\sin x)^2 + 5\cos x$

$y\left(\frac{\pi}{4}\right) = 5\left(\sin\left(\frac{\pi}{4}\right)\right)^2 + 5\cos\frac{\pi}{4}$

$= 5\left(\frac{\sqrt{2}}{2}\right)^2 + 5\left(\frac{\sqrt{2}}{2}\right)$

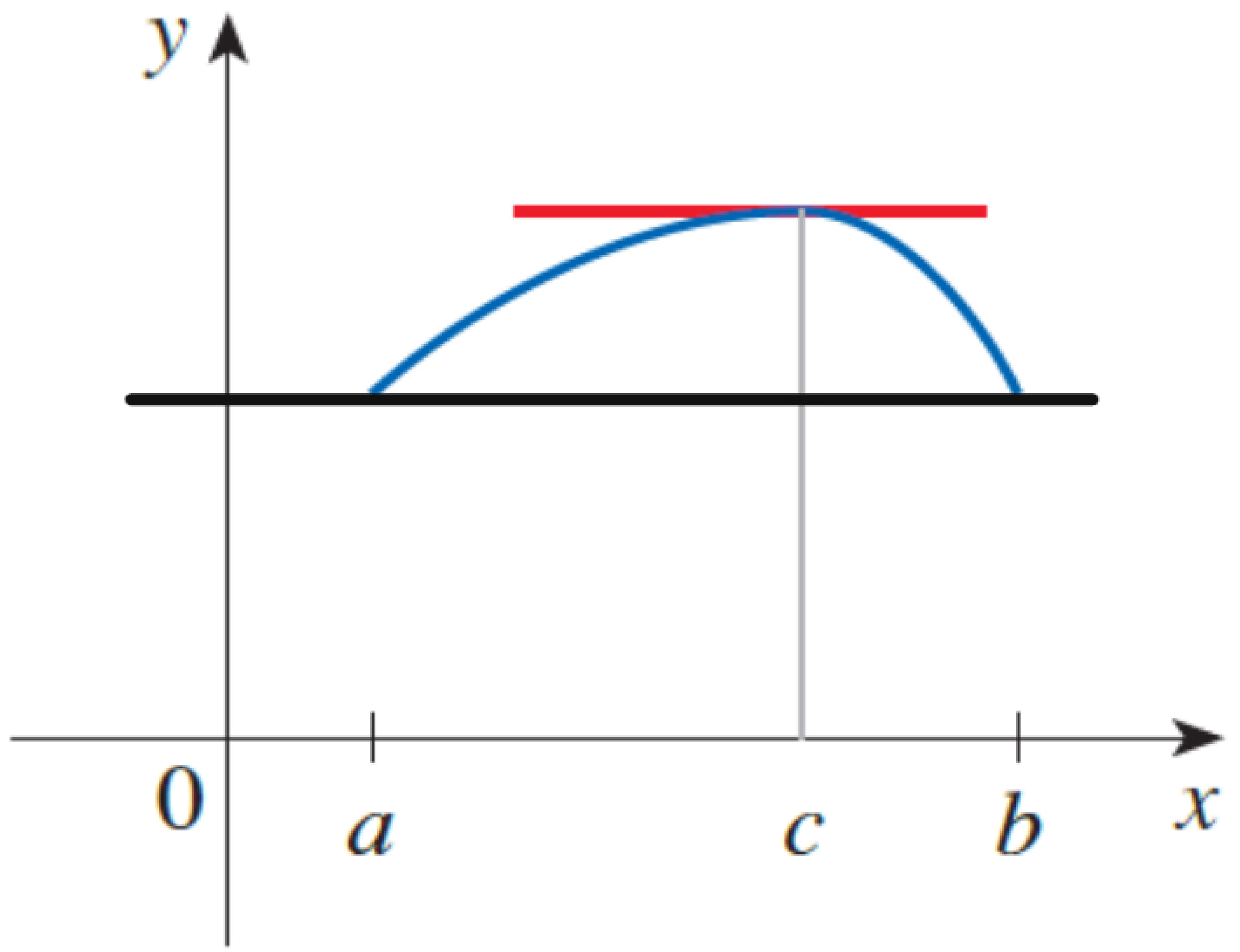
$= 5\left(\frac{1}{2}\right) + \frac{5\sqrt{2}}{2} = \frac{5}{2} + \frac{5\sqrt{2}}{2}$

$y\left(\frac{\pi}{3}\right) = 5\left(\frac{\sqrt{3}}{2}\right)^2 + 5\left(\frac{1}{2}\right) = \frac{15}{4} + \frac{5}{2} = \frac{25}{4}$ max

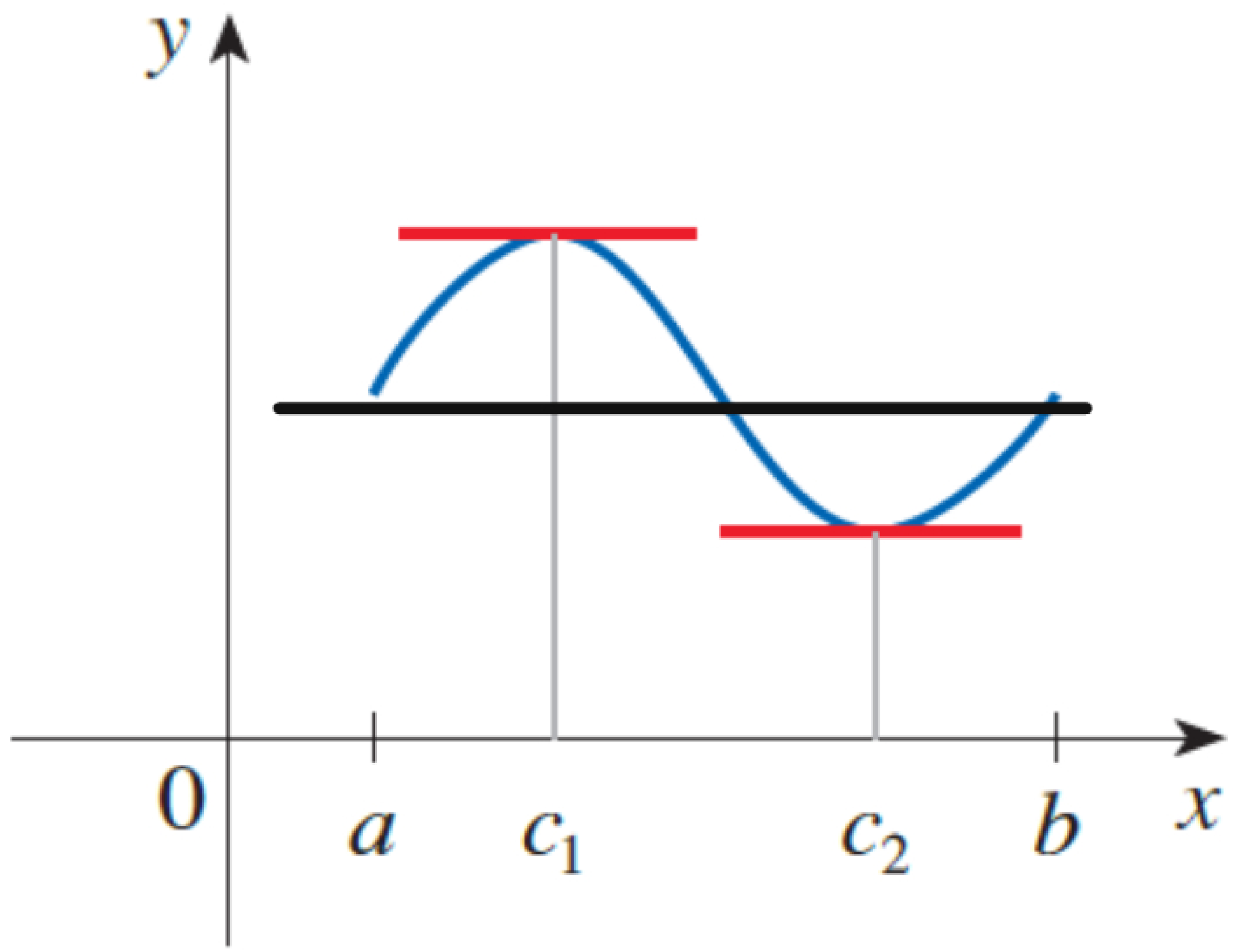
Rolle's Theorem Let f be a function that satisfies the following three hypotheses:

1. f is continuous on the closed interval $[a, b]$.
2. f is differentiable on the open interval (a, b).
3. $f(a) = f(b)$

Then there is a number c in (a, b) such that $f'(c) = 0$.



(b)



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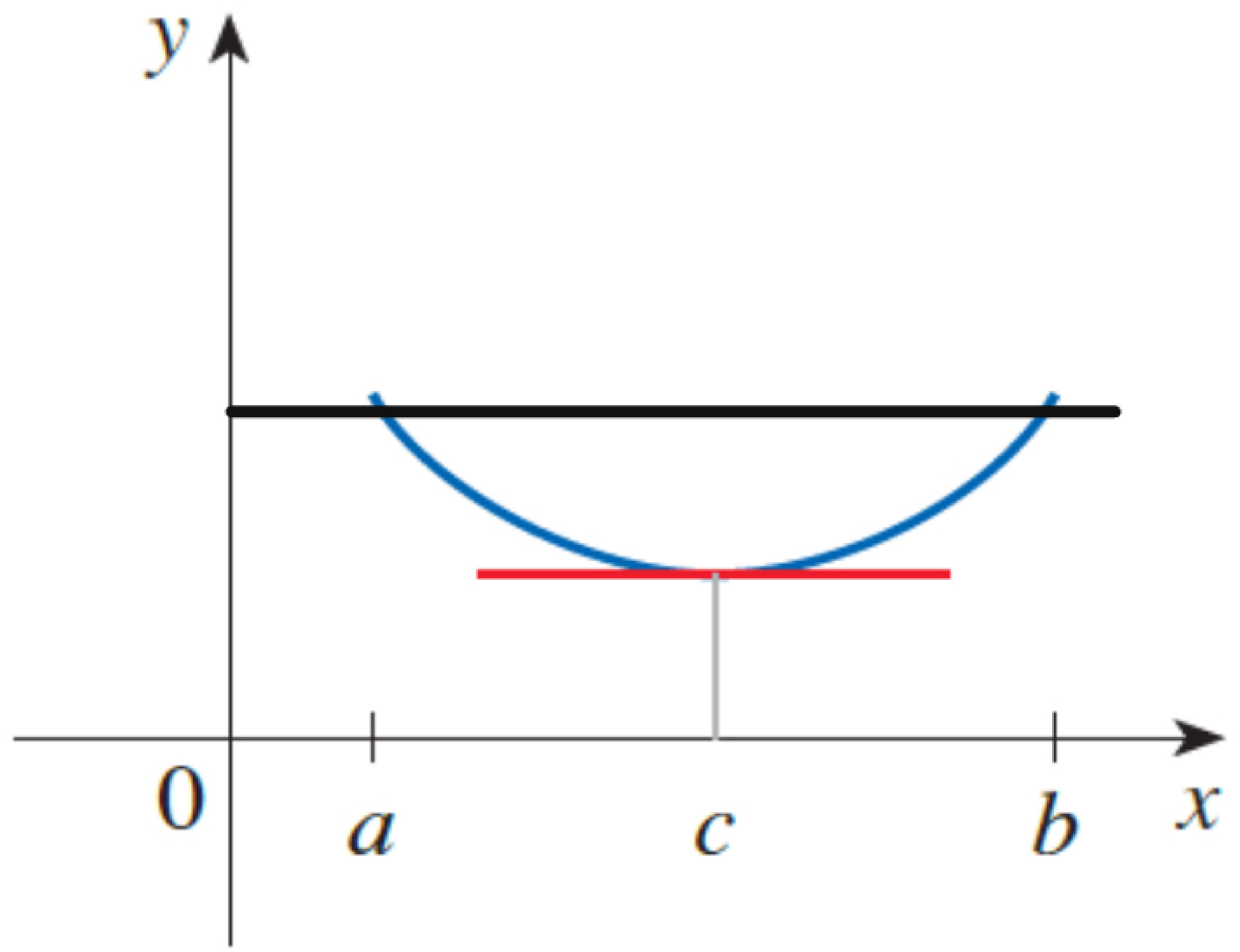

(d)

The Mean Value Theorem Let f be a function that satisfies the following hypotheses:

1. f is continuous on the closed interval $[a, b]$.
2. f is differentiable on the open interval (a, b).

Then there is a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a} \tag{1}$$

or, equivalently,

$$f(b) - f(a) = f'(c)(b - a) \tag{2}$$