$$\frac{d}{dx}\left[\frac{2}{3}\boxed{(1+x^2)}^{3/2}\right]$$

$$U = 1 + x^2$$

$$U' = 2x$$

$$\frac{2}{3}(U)^{3/2} \rightarrow \frac{2}{3}\cdot\frac{3}{2}(U)^{1/2}\cdot U'$$

$$= (1+x^2)^{1/2}\cdot 2x$$

$$= 2x\sqrt{1+x^2}$$

$$\int 2x\sqrt{1+x^2}\,dx$$

$$u = 1 + x^2$$

$$du = \boxed{2x\,dx}$$

$$\int (1+x^2)^{1/2}\,\boxed{2x\,dx}$$

$$\int u^{1/2}\,du$$

$$\frac{2}{3}u^{3/2} + C \rightarrow \frac{2}{3}(1+x^2)^{3/2} + C$$

4 The Substitution Rule If $u = g(x)$ is a differentiable function whose range is an interval I and f is continuous on I, then

$$\int f(g(x))\,g'(x)\,dx = \int f(u)\,du$$

Evaluate $\int \sqrt{2x+1}\,dx$.

$$u = 2x + 1$$

$$du = 2\,dx$$

$$\frac{1}{2}du = dx$$

$$\int (2x+1)^{1/2} \boxed{dx}$$

$$\int u^{1/2} \boxed{\left(\frac{1}{2}\right) du}$$

$$\frac{1}{2}\int u^{1/2}\,du$$

$$\frac{1}{2}\left[\frac{2}{3}u^{3/2}\right] + C$$

$$\boxed{\frac{1}{3}(2x+1)^{3/2} + C}$$

Find $\int \frac{x}{\sqrt{1-4x^2}}\,dx.$

$u = 1 - 4x^2$

$du = -8\boxed{x\,dx}$

$-\frac{1}{8}\,du = \boxed{x\,dx}$

$-\frac{1}{8}\int u^{-1/2}\,du$

$-\frac{1}{8}\left[2u^{1/2}\right] + C \rightarrow -\frac{1}{4}(1-4x^2)^{1/2} + C$

Evaluate $\int e^{5x}\,dx$.

$u = 5x$

$du = 5\,dx$

$\frac{1}{5}du = dx$

$\frac{1}{5}\int e^{u}\,du$

$\frac{1}{5}e^{u} + C$

$\frac{1}{5}e^{5x} + C$

$\int e^{10x}\,dx \to \frac{1}{10}e^{10x} + C$

$$\int e^{x^2}\,dx$$

$$u = x^2$$

$$du = 2x\,dx$$

$$\frac{1}{2}\int u^{5/2} - 2u^{3/2} + u^{1/2}\, du \qquad u^{1/2}(u^2 - 2u + 1)$$

$$u^{5/2} - 2u^{3/2} + u^{1/2}$$

$$\frac{1}{2}\left[\frac{2}{7}u^{7/2} - \frac{4}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right] + C$$

$$\boxed{\frac{1}{7}(1+x^2)^{7/2} - \frac{2}{5}(1+x^2)^{5/2} + \frac{1}{3}(1+x^2)^{3/2} + C}$$

$$\int \frac{\boxed{\sin x}}{\cos x}\, \boxed{dx}$$

$$-du = \boxed{\sin x \, dx}$$

$$-\int \frac{1}{u}\, du$$

$$-\ln|u| + C$$

$$-\ln|\cos x| + C$$

or

$$\ln|\sec x| + C$$

$$\log x^a = a \log x$$

2 Table of Antidifferentiation Formulas

Function	Particular antiderivative	Function	Particular antiderivative
$cf(x)$	$cF(x)$	$\sin x$	$-\cos x$
$f(x) + g(x)$	$F(x) + G(x)$	$\sec^2 x$	$\tan x$
$x^n \ (n \neq -1)$	$\dfrac{x^{n+1}}{n+1}$	$\sec x \tan x$	$\sec x$
$\dfrac{1}{x}$	$\ln\lvert x\rvert$	$\dfrac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x$
e^x	e^x	$\dfrac{1}{1+x^2}$	$\tan^{-1} x$
b^x	$\dfrac{b^x}{\ln b}$	$\cosh x$	$\sinh x$
$\cos x$	$\sin x$	$\sinh x$	$\cosh x$

$$\boxed{\int \tan x \, dx = \ln\lvert\sec x\rvert + C}$$

Evaluate $\int_0^4 \sqrt{2x+1}\,dx$ $= \left.\frac{1}{3}(2x+1)^{3/2}\right|_0^4$

$\frac{1}{3}(2(4)+1)^{3/2} - \frac{1}{3}(2(0)+1)^{3/2}$

$\frac{1}{3}(9)^{3/2} - \frac{1}{3}(1)^{3/2}$

$\frac{1}{3}(27) - \frac{1}{3}(1) = \frac{26}{3}$

Evaluate $\int_0^4 \sqrt{2x+1}\,dx$

$u = 2x + 1$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$u(0) = 2(0) + 1 = 1$

$u(4) = 2(4) + 1 = 9$

$\frac{1}{2}\int_1^9 u^{1/2}\,du$

$\frac{1}{2}\left(\frac{2}{3}u^{3/2}\right)$

$\frac{1}{3}\left(u^{3/2}\right)\Big|_1^9 = \frac{1}{3}(9)^{3/2} - \frac{1}{3}(1)^{3/2}$

$= \frac{1}{3}(27) - \frac{1}{3} = \frac{26}{3}$

Evaluate $\int_1^2 \frac{dx}{(3-5x)^2}$

$u = 3-5x \rightarrow du = -5\,dx$

$-\frac{1}{5}du = dx$

$u(1) = -2$

$u(2) = -7$

$-\frac{1}{5}\int_{-2}^{-7} \frac{1}{u^2}\,du \rightarrow \frac{1}{5}\int_{-7}^{-2} \frac{1}{u^2}\,du$

$-\frac{1}{5}\left(-\frac{1}{u}\right)\Big|_{-2}^{-7} = -\frac{1}{5}\left[\frac{1}{7} - \frac{1}{2}\right] = -\frac{1}{5}\left[-\frac{5}{14}\right]$

$= \frac{1}{14}$