



Video Tutor

Using Transformations to Graph Quadratic Functions

Extension: Graphing Quadratic Functions in Vertex Form

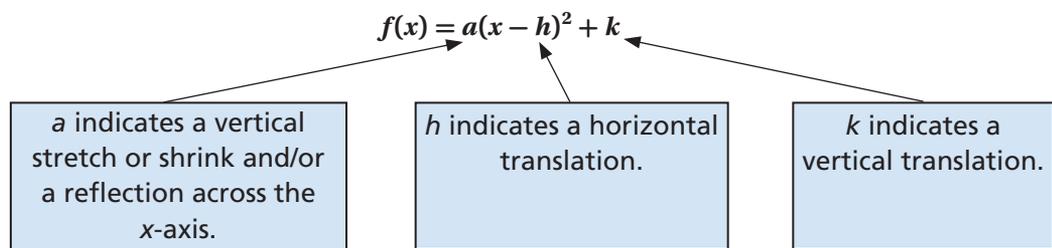
Essential question: How can you graph the function $f(x) = a(x - h)^2 + k$?

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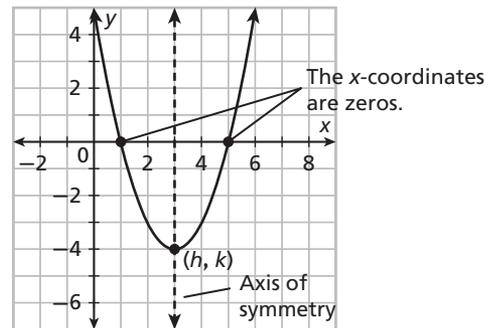
1 ENGAGE

Understanding Vertex Form

The **vertex form** of a quadratic function is $f(x) = a(x - h)^2 + k$. The vertex of the graph of a quadratic function in vertex form is (h, k) .



A **zero of a function** is an input value x that makes the output value $f(x)$ equal 0. You can estimate the zeros of a quadratic function by observing where the graph crosses the x -axis.



REFLECT

1a. For the function $f(x) = 2(x - 3)^2 + 1$, what are the values of a , h , and k ? What do each of these values indicate about the graph of the function?

1b. Explain why the vertex of the graph of a quadratic function in vertex form is (h, k) .

1c. If you estimate a zero of a quadratic function from a graph, how could you use algebra to check your answer?

2 EXAMPLE Graphing $f(x) = a(x - h)^2 + k$

Graph the function $f(x) = 2(x + 1)^2 - 2$. Identify the vertex, minimum or maximum, axis of symmetry, and zeros of the function.

- A** Identify and graph the vertex.

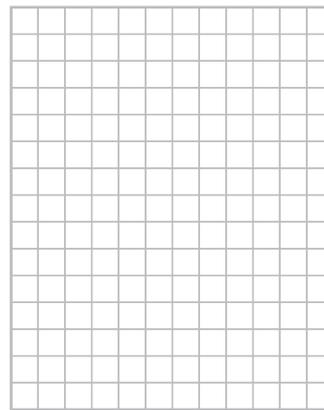
$$h = \square$$

$$k = \square$$

The vertex of the graph is _____.

- B** Identify the coordinates of points to the left and right of the vertex.

x	-3	-2	0	1
f(x)				



- C** Graph the points and connect them with a smooth curve.

- D** Identify the minimum or maximum.

The graph opens upward, so the function has a _____.

The minimum is _____.

- E** Identify the axis of symmetry.

The axis of symmetry is the vertical line $x =$ _____.

- F** Identify the zeros of the function.

The graph appears to cross the x -axis at the points _____ and _____, so the zeros of the function appear to be _____ and _____.

REFLECT

- 2a.** How could you use the value of a to determine whether the function $f(x) = 2(x + 1)^2 - 2$ has a minimum or a maximum?

- 2b.** How could you use the table in part B to confirm that you correctly identified the zeros of the function from its graph?

Write the vertex form of the quadratic function whose graph is shown.

- A** Use the vertex of the graph to identify the values of h and k .

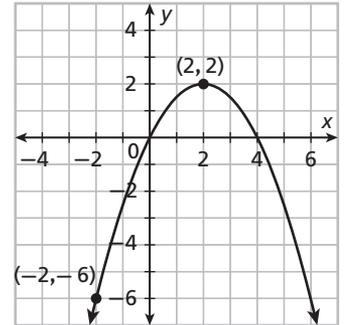
The vertex of the graph is _____.

$$h = \square$$

$$k = \square$$

Substitute the values of h and k into the vertex form:

$$f(x) = a(x - \square)^2 + \square$$



- B** Use the point $(-2, -6)$ to identify the value of a .

$$f(x) = a(x - 2)^2 + 2 \quad \text{Vertex form}$$

$$\square = a(\square - 2)^2 + 2 \quad \text{Substitute } -6 \text{ for } f(x) \text{ and } -2 \text{ for } x.$$

$$-6 = a(\square) + 2 \quad \text{Simplify.}$$

$$\square = a(16) \quad \text{Subtract 2 from both sides.}$$

$$\square = a \quad \text{Divide both sides by 16.}$$

Substitute the value of a into the vertex form:

$$f(x) = \square(x - 2)^2 + 2$$

So, the vertex form of the function shown in the graph is

_____.

REFLECT

- 3a.** How can you tell by looking at the graph that the value of a is negative?

- 3b.** Describe the graph of the given function as a transformation of the parent quadratic function.

4 EXAMPLE Modeling Quadratic Functions in Vertex Form

The shape of a bridge support can be modeled by $f(x) = -\frac{1}{600}(x - 300)^2 + 150$, where x is the horizontal distance in feet from the left end of the bridge and $f(x)$ is the height in feet above the bridge deck. Sketch a graph of the support. Then determine the maximum height of the support above the bridge deck and the width of the support at the level of the bridge deck.

A Graph the function.

- The vertex of the graph is _____.
- Find the point at the left end of the support ($x = 0$).

Since $f(0) = \underline{\hspace{2cm}}$, the point _____ represents the left end.

- Use symmetry to find the point at the right end of the support.

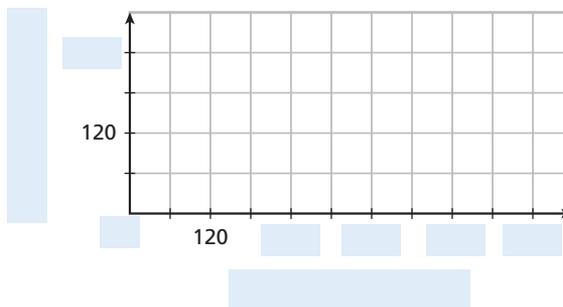
Since the left end is 300 feet to the left of the vertex, the right end will be 300 feet to the right of the vertex.

The point _____ represents the right end.

- Find two other points on the support.

$\left(120, \square\right)$ and $\left(480, \square\right)$

- Sketch the graph.



B Determine the maximum height of the support.

The maximum of the function is _____.

So, the maximum height of the bridge support is _____ feet.

C Determine the width of the bridge support at the level of the bridge deck.

The distance from the left end to the right end is _____ feet.

So, the width is _____ feet at the level of the bridge deck.

REFLECT

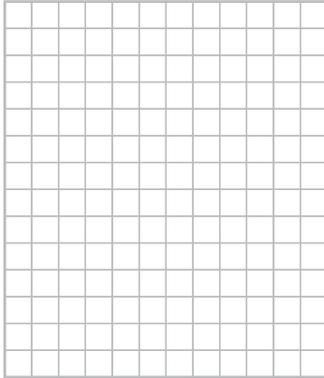
4a. Explain how you know that the y -coordinate of the right end of the support is 0.

4b. What does the vertex represent in this situation?

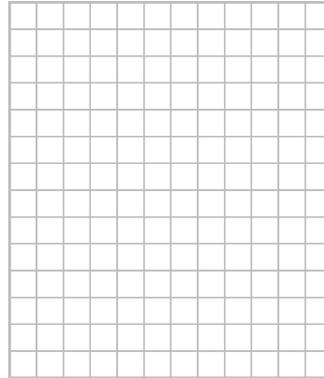
PRACTICE

Graph each quadratic function. Identify the vertex, minimum or maximum, axis of symmetry, and zeros of the function.

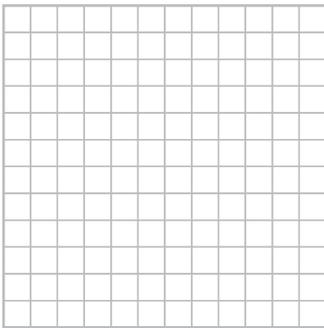
1. $f(x) = -2x^2 + 8$



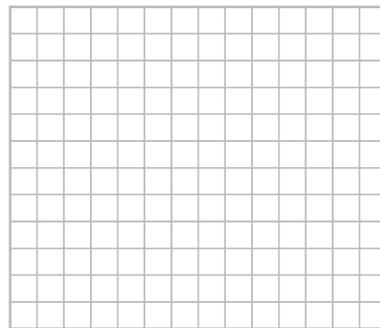
2. $f(x) = (x - 2)^2 - 4$



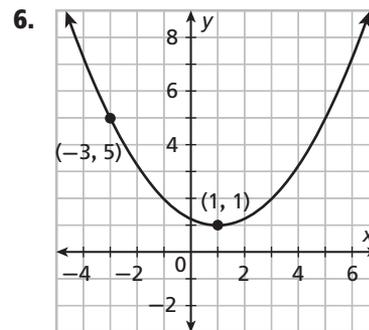
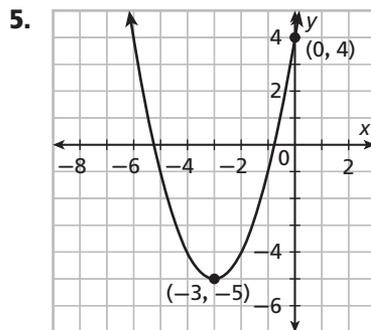
3. $f(x) = -(x + 4)^2 + 1$



4. $f(x) = \frac{1}{3}(x - 2)^2 - 3$



Write the vertex form of each quadratic function.

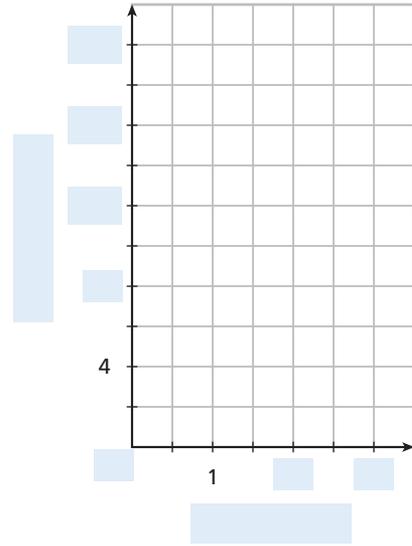


7. The function $f(x) = -16(x - 1)^2 + 16$ gives the height in feet of a football x seconds after it is kicked from ground level.

a. Sketch a graph of the function.

b. What is the maximum height that the ball reaches?

c. How long does the ball stay in the air? Explain how you determined your answer.

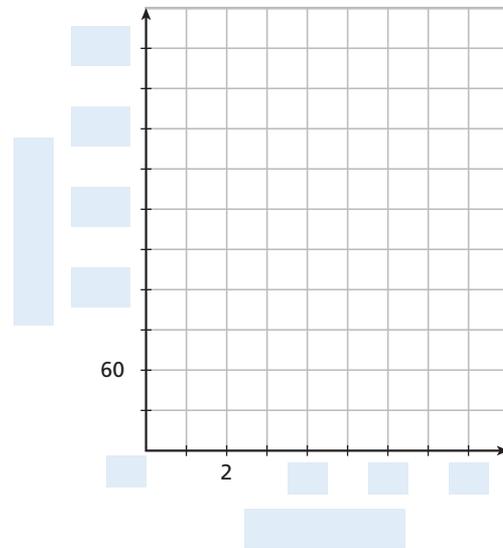


8. A technician is launching an aerial firework from a tower. The height of the firework in feet is modeled by the function $f(x) = -16(x - 3)^2 + 256$ where x is the time in seconds after the firework is launched.

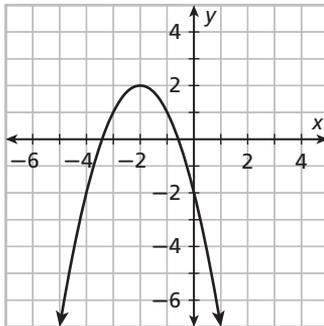
a. Sketch a graph of the function.

b. Professional fireworks are usually timed to explode as they reach their highest point. How high will the firework be when it reaches its highest point?

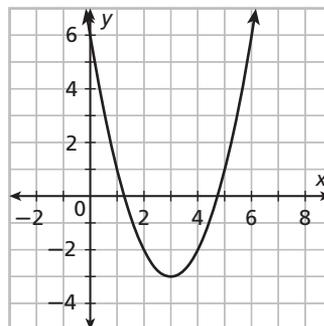
c. What is the height of the tower from which the firework is launched? Explain how you determined your answer.



9. Which quadratic function has a greater maximum: the function $f(x) = -(x - 5)^2 + 4$ or the function graphed below?



10. Which quadratic function has a lesser minimum: a function whose graph has a vertex at $(-5, -1)$ or the function graphed below?

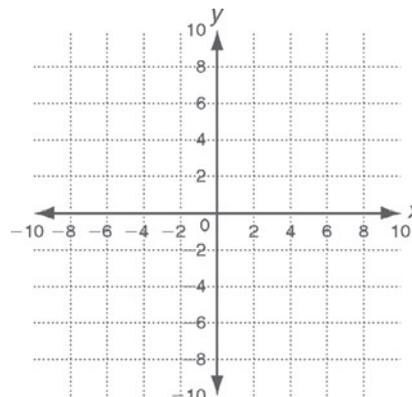


Additional Practice

Graph the function by using a table.

1. $f(x) = x^2 + 2x - 1$

x	$f(x) = x^2 + 2x - 1$	$(x, f(x))$
-2		
-1		
0		
1		
2		

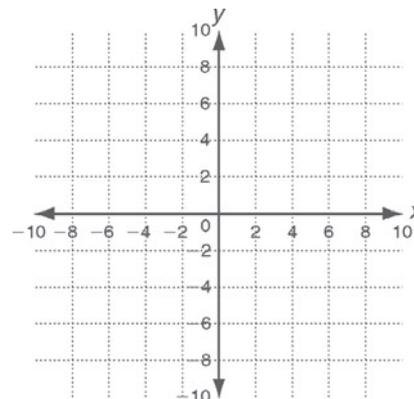


Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph each function. Label each function on the graph.

2. $h(x) = (x - 2)^2 + 2$

3. $h(x) = -(3x)^2$

4. $h(x) = \left(\frac{1}{2}x\right)^2$



Use the description to write a quadratic function in vertex form.

5. The parent function $f(x) = x^2$ is reflected across the x -axis, horizontally stretched by a factor of 3 and translated 2 units down to create function g .

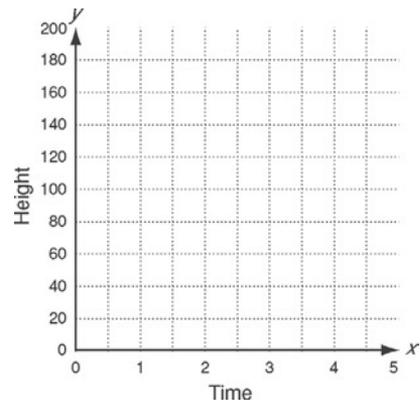
6. A ball dropped from the top of tower A can be modeled by the function $h(t) = -9.8t^2 + 400$, where t is the time after it is dropped and $h(t)$ is its height at that time. A ball dropped from the top of tower B can be modeled by the function $h(t) = -9.8t^2 + 200$. What transformation describes this change? What does this transformation mean?

Problem Solving

Christa and Jelani are standing at the top of the Leaning Tower of Pisa in Italy, 185 feet above the ground. Jelani wonders what the path of a dropped object would be as it falls to the ground from the top of the tower. The height of an object after t seconds is given by the function, $f(t) = -16t^2 + 185$.

- Complete the table to show the height, $f(t)$, of the object for different values of t .
- Plot the ordered pairs from the table and draw the graph to show the path of the object.

Time (t)	$f(t) = -16t^2 + 185$	$(t, f(t))$
0	$f(0) = -16(0)^2 + 185$	
1	$f(1) = -16(1)^2 + 185$	
2		
3		
4		



- What is the parent function for the graph?

- What is the name for this U-shaped curve?

- Describe the transformations of the parent function into $f(t) = -16t^2 + 185$, which describes the path of an object falling from 185 feet.

Choose the letter for the best answer.

- Mario dropped a wrench from the top of a sailboat mast 58 feet high. Which function describes the path of the falling wrench?
 - $f(t) = 16(t - 58)^2 - 185$
 - $f(t) = -16(t - 58)^2 + 185$
 - $f(t) = 16t^2 - 58$
 - $f(t) = -16t^2 + 58$
- Delle wants to transform the parent function $f(t) = t^2$ into $f(t) = -4(t - 0.6)^2 + 6$. Which is NOT a step in that transformation?
 - Translation 6 units up
 - Translation 0.6 unit left
 - Reflection across the x -axis
 - Vertical stretch by a factor of 4