

Quantitative Literacy: Thinking Between the Lines

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Chapter 1: Critical Thinking

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Lesson Plan

- ▶ Public policy and Simpson's paradox: Is "average" always average?
- ▶ Logic and informal fallacies: Does that argument hold water?
- ▶ Formal logic and truth tables: Do computers think?
- ▶ Sets and Venn diagrams: Pictorial logic
- ▶ Critical thinking and number sense: What do these figures mean?



Chapter 1 Critical Thinking

1.1 Public policy and Simpson's paradox: Is "average" always average?

Learning Objectives:

- ▶ View with a critical eye conclusions based on averages
 - ▶ Understand Simpson's paradox and test scores
 - ▶ Understand the Berkeley gender discrimination case

Chapter 1 Critical Thinking

1.1 Public policy and Simpson's paradox: Is "average" always average?

- ▶ **Example:** Suppose a certain high school gave a math proficiency exam to its students and that the percentage who passed was below the statewide average. After examining the figures further, the school decided to report its test data by separating them into students from low-income families and students from higher-income families.

	Local school		Statewide	
	Students tested	Passed	Students tested	Passed
Low income	400	260	200,000	128,000
High Income	700	532	1,100,000	825,000
Total	1100	792	1,300,000	953,000

Show that the local school outperformed statewide students in both the low-income and high-income categories but had a lower passing percentage overall than the statewide rate.

Chapter 1 Critical Thinking

1.1 Public policy and Simpson's paradox: Is "average" always average?

- **Solution:** The local school's pass percentage for low-income families:

$$\begin{aligned}\text{Local low - income average} &= \frac{\text{Number passes}}{\text{Numer tests taken}} \times 100\% \\ &= \frac{2260}{200} \times 100\% = 65.0\%.\end{aligned}$$

	Local school % pass	Statewide % pass
Low income	260/400 or 65.0%	128,000/200,000 or 64.0%
High income	532/700 or 76.0%	825,000/1,100,000 or 75.0%
Total	792/1100 or 72.0%	953,000/1,300,000 or 73.3%

Overall the pass rate of the local school is *lower* than the statewide rate by more than one percentage point.

Chapter 1 Critical Thinking

1.1 Public policy and Simpson's paradox: Is "average" always average?

▶ **Simpson's paradox**

Simpson's paradox occurs:

1. when combining, or aggregating, data masks underlying patterns.
2. when a factor distorts the overall picture, but distortion goes away when underlying data are examined.

- ▶ **Example:** Separating test scores by the economic level of the students may show that at a local school students at each economic level perform better than the statewide average for students at the same level. But, if school has more students at lower economic levels, its test scores overall may be lower than the state average. Such results can be **counterintuitive**. Without careful consideration, one can be led to an incorrect conclusion.

Chapter 1 Critical Thinking

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▶ **The Berkeley gender discrimination case:** Data from a 1973 study showed persuasive evidence that the University of California at Berkeley was practicing gender discrimination in graduate school admissions.

	Applicants	Accepted	% accepted
Male	8442	3714	44.0%
Female	4321	1512	35.0%



Students on the University of California at Berkeley campus.

Graduate school admissions are based on departmental selections. Let's study how the male and female applicants might have been divided among two departments, Math and English.

Chapter 1 Critical Thinking

1.1 Public policy and Simpson's paradox: Is "average" always average?

	Males			Females		
	Applicants	Accepted	% accepted	Applicants	Accepted	% accepted
Math	2000	500	25.0%	3000	780	26.0%
English	6442	3214	49.9%	1321	732	55.4%
Total	8442	3714	44.0%	4321	1512	35.0%

The two departments actually accepted a larger percentage of female applicants than male applicants.

Most women applied to the Math Department, where it is more difficult to be accepted, but most men applied to English, where it is less difficult.

Chapter 1 Critical Thinking

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- ▶ **Example:** The following table shows the hitting records of two major league baseball players, Derek Jeter and David Justice, in 1995 and 1996:

	1995		1996	
	At-bats	Hits	At-bats	Hits
Jeter	48	12	582	183
Justice	411	104	140	45

1. Which batter had the higher average in 1995?
2. Which had the higher average in 1996?
3. Which had the higher average over the two-year period?

Chapter 1 Critical Thinking

1.1 Public policy and Simpson's paradox: Is "average" always average?

- **Solution:** 1. Jeter's average in 1995 = $\frac{\text{Hits}}{\text{At-bats}} = \frac{12}{48} = 0.250$

We calculate average for Jeter and Justice over each of the two years.

	1995	1996
Average for Jeter	12/48 or 0.250	183/582 or 0.314
Average for Justice	104/411 or 0.253	45/140 or 0.321

2. Jeter's average over two – year peirod = $\frac{\text{Hits}}{\text{At-bats}}$
 $= \frac{12+183}{48+582} = \frac{195}{630} = 0.310$

3. Justice gives his average over the two-year period as $\frac{149}{551} = 0.270$. Justice had the higher batting average in each of the two years, but Jeter had the higher batting average over the two-year period.

Chapter 1 Critical Thinking

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Quick Review Calculating Percentages

To calculate P% of a quantity, we multiply the quantity by $\frac{P}{100}$:

$$P\% \text{ of quantity} = \frac{P}{100} \times \text{Quantity}$$

To find what percentage of a whole a part:

$$\text{Percentage} = \frac{\text{Part}}{\text{Whole}} \times 100\%$$

- ▶ **Example:** Find 45% of 500.

$$45\% \text{ of } 500 = \frac{45}{100} \times 500 = 225.$$

- ▶ **Example:** Find what percentage of 140 is 35.

$$\text{Percentage} = \frac{\text{Part}}{\text{Whole}} \times 100\% = \frac{35}{140} \times 100\% = 25\%.$$

Chapter 1 Critical Thinking: **Chapter Summary**

- ▶ Public policy and Simpson's paradox: Is "average" always average?
 - ▶ Understand that Simpson's paradox is a striking example of the need for critical thinking skills.
 - ▶ Overall average may lead to *invalid* conclusion.
- ▶ Logic and informal fallacies: Does that argument hold water?
 - ▶ Logical argument involves: Premises, Conclusion
 - ▶ Informal fallacies: *fallacies of relevance*,
fallacies of presumption
 - ▶ Deductive arguments and Inductive arguments



Chapter 1 Critical Thinking: **Chapter Summary**

- ▶ Formal logic and truth tables: Do computers think?
 - ▶ Formal logic: The truth table
 - ▶ Operations on statements: Negation, conjunction, disjunction, conditional or implication.
- ▶ Sets and Venn diagrams: Pictorial logic
 - ▶ The Venn diagrams: Analyze logical statements.
- ▶ Critical thinking and number sense: What do these figures mean?
 - ▶ Relative sizes of numbers are indicated using *magnitudes* or powers of 10.
 - ▶ Estimation: To avoid complicated computations

