

2-6 The Quadratic Formula

Objectives

Solve quadratic equations using the Quadratic Formula.

Holt McDougal Algebra 2 Copyright © by Holt Mc Dougal. All Rights Reserved.

2-6 The Quadratic Formula

Vocabulary

discriminant

Holt McDougal Algebra 2 Copyright © by Holt Mc Dougal. All Rights Reserved.

2-6 The Quadratic Formula

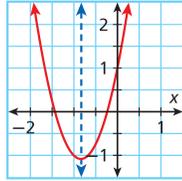
You have learned several methods for solving quadratic equations: graphing, making tables, factoring, using square roots, and completing the square. Another method is to use the *Quadratic Formula*, which allows you to solve a quadratic equation in standard form.

By completing the square on the standard form of a quadratic equation, you can determine the Quadratic Formula.

Holt McDougal Algebra 2 Copyright © by Holt Mc Dougal. All Rights Reserved.

2-6 The Quadratic Formula

The symmetry of a quadratic function is evident in the last step, $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$. These two zeros are the same distance, $\frac{\sqrt{b^2 - 4ac}}{2a}$, away from the axis of symmetry, $x = -\frac{b}{2a}$, with one zero on either side of the vertex.



Holt McDougal Algebra 2 Copyright © by Holt Mc Dougal. All Rights Reserved.

2-6 The Quadratic Formula

The Quadratic Formula

If $ax^2 + bx + c = 0$ ($a \neq 0$), then the solutions, or roots, are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

You can use the Quadratic Formula to solve any quadratic equation that is written in standard form, including equations with real solutions or complex solutions.

Holt McDougal Algebra 2 Copyright © by Holt Mc Dougal. All Rights Reserved.

2-6 The Quadratic Formula

Example 1: Quadratic Functions with Real Zeros

Find the zeros of $f(x) = 2x^2 - 16x + 27$ using the Quadratic Formula.

$$2x^2 - 16x + 27 = 0$$

Set $f(x) = 0$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Write the Quadratic Formula.

$$x = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(2)(27)}}{2(2)}$$

Substitute 2 for a, -16 for b, and 27 for c.

$$x = \frac{16 \pm \sqrt{256 - 216}}{4} = \frac{16 \pm \sqrt{40}}{4}$$

Simplify.

$$x = \frac{16 \pm 2\sqrt{10}}{4} = 4 \pm \frac{\sqrt{10}}{2}$$

Write in simplest form.

Holt McDougal Algebra 2 Copyright © by Holt Mc Dougal. All Rights Reserved.

2-6 The Quadratic Formula

Check It Out! Example 1a

Find the zeros of $f(x) = x^2 + 3x - 7$ using the Quadratic Formula.

$$x^2 + 3x - 7 = 0 \quad \text{Set } f(x) = 0.$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Write the Quadratic Formula.}$$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(1)(-7)}}{2(1)} \quad \text{Substitute 1 for } a, 3 \text{ for } b, \text{ and } -7 \text{ for } c.$$

$$x = \frac{-3 \pm \sqrt{9 + 28}}{2} \quad \text{Simplify.}$$

$$x = \frac{-3 \pm \sqrt{37}}{2} \quad \text{Write in simplest form.}$$

Holt McDougal Algebra 2 Copyright © by Holt Mc Dougal. All Rights Reserved.

2-6 The Quadratic Formula

Check It Out! Example 1b

Find the zeros of $f(x) = x^2 - 8x + 10$ using the Quadratic Formula.

$$x^2 - 8x + 10 = 0 \quad \text{Set } f(x) = 0.$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Write the Quadratic Formula.}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(10)}}{2(1)} \quad \text{Substitute 1 for } a, -8 \text{ for } b, \text{ and } 10 \text{ for } c.$$

$$x = \frac{8 \pm \sqrt{64 - 40}}{2} = \frac{8 \pm \sqrt{24}}{2} \quad \text{Simplify.}$$

$$x = \frac{8 \pm 2\sqrt{6}}{2} = 4 \pm \sqrt{6} \quad \text{Write in simplest form.}$$

Holt McDougal Algebra 2 Copyright © by Holt Mc Dougal. All Rights Reserved.

2-6 The Quadratic Formula

Example 2: Quadratic Functions with Complex Zeros

Find the zeros of $f(x) = 4x^2 + 3x + 2$ using the Quadratic Formula.

$$f(x) = 4x^2 + 3x + 2 \quad \text{Set } f(x) = 0.$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \text{Write the Quadratic Formula.}$$

$$x = \frac{-3 \pm \sqrt{(3)^2 - 4(4)(2)}}{2(4)} \quad \text{Substitute 4 for } a, 3 \text{ for } b, \text{ and } 2 \text{ for } c.$$

$$x = \frac{-3 \pm \sqrt{9 - 32}}{2(4)} = \frac{-3 \pm \sqrt{-23}}{8} \quad \text{Simplify.}$$

$$x = \frac{-3 \pm \sqrt{-23}}{8} = -\frac{3}{8} \pm \frac{\sqrt{23}}{8}i \quad \text{Write in terms of } i.$$

Holt McDougal Algebra 2 Copyright © by Holt Mc Dougal. All Rights Reserved.

2-6 The Quadratic Formula

Check It Out! Example 2

Find the zeros of $g(x) = 3x^2 - x + 8$ using the Quadratic Formula.

$3x^2 - x + 8 = 0$ *Set $f(x) = 0$*

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ *Write the Quadratic Formula.*

$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(3)(8)}}{2(3)}$ *Substitute 3 for a, -1 for b, and 8 for c.*

$x = \frac{1 \pm \sqrt{1 - 96}}{2(3)} = \frac{1 \pm \sqrt{-95}}{6}$ *Simplify.*

$x = \frac{1 \pm \sqrt{-95}}{6} = -\frac{1}{6} \pm \frac{\sqrt{95}}{6}i$ *Write in terms of i.*

Holt McDougal Algebra 2 Copyright © by Holt Mc Dougal. All Rights Reserved.

2-6 The Quadratic Formula

Example 4: Sports Application

An athlete on a track team throws a shot put. The height y of the shot put in feet t seconds after it is thrown is modeled by $y = -16t^2 + 24.6t + 6.5$. The horizontal distance x in between the athlete and the shot put is modeled by $x = 29.3t$. To the nearest foot, how far does the shot put land from the athlete?

Holt McDougal Algebra 2 Copyright © by Holt Mc Dougal. All Rights Reserved.

2-6 The Quadratic Formula

Example 4 Continued

Step 1 Use the first equation to determine how long it will take the shot put to hit the ground. Set the height of the shot put equal to 0 feet, and the use the quadratic formula to solve for t .

$y = -16t^2 + 24.6t + 6.5$

$0 = -16t^2 + 24.6t + 6.5$ *Set y equal to 0.*

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ *Use the Quadratic Formula.*

$t = \frac{-(24.6) \pm \sqrt{(24.6)^2 - 4(-16)(6.5)}}{2(-16)}$ *Substitute -16 for a, 24.6 for b, and 6.5 for c.*

Holt McDougal Algebra 2 Copyright © by Holt Mc Dougal. All Rights Reserved.

2-6 The Quadratic Formula

Example 4 Continued

$$t = \frac{-24 \pm \sqrt{1021.16}}{-32} \approx \frac{-24 \pm 31.96}{-32}$$

Simplify.

$$t \approx \frac{-56.56}{-32} \approx 1.77 \text{ or } t \approx \frac{7.36}{-32} \approx -0.23$$

The time cannot be negative, so the shot put hits the ground about 1.8 seconds after it is released.

Holt McDougal Algebra 2 Copyright © by Holt Mc Dougal. All Rights Reserved.

2-6 The Quadratic Formula

Example 4 Continued

Step 2 Find the horizontal distance that the shot put will have traveled in this time.

$$x = 29.3t$$

$$x \approx 29.3(1.77) \quad \textit{Substitute 1.77 for t.}$$

$$x \approx 51.86 \quad \textit{Simplify.}$$

$$x \approx 52$$

The shot put will have traveled a horizontal distance of about 52 feet.

Holt McDougal Algebra 2 Copyright © by Holt Mc Dougal. All Rights Reserved.

2-6 The Quadratic Formula

Example 4 Continued

Check Use substitution to check that the shot put hits the ground after about 1.77 seconds.

$$y = -16t^2 + 24.6t + 6.5$$

$$y \approx -16(1.77)^2 + 24.6(1.77) + 6.5$$

$$y \approx -50.13 + 43.54 + 6.5$$

$$y \approx -0.09 \quad \checkmark \quad \textit{The height is approximately equal to 0 when } t = 1.77.$$

Holt McDougal Algebra 2 Copyright © by Holt Mc Dougal. All Rights Reserved.

2-6 The Quadratic Formula

Check It Out! Example 4

A pilot of a helicopter plans to release a bucket of water on a forest fire. The height y in feet of the water t seconds after its release is modeled by $y = -16t^2 - 2t + 500$. the horizontal distance x in feet between the water and its point of release is modeled by $x = 91t$.

The pilot's altitude decreases, which changes the function describing the water's height to $y = -16t^2 - 2t + 400$. To the nearest foot, at what horizontal distance from the target should the pilot begin releasing the water?

Holt McDougal Algebra 2 Copyright © by Holt Mc Dougal. All Rights Reserved.

2-6 The Quadratic Formula

Check It Out! Example 4 Continued

Step 1 Use the equation to determine how long it will take the water to hit the ground. Set the height of the water equal to 0 feet, and then use the quadratic formula for t .

$y = -16t^2 - 2t + 400$
 $0 = -16t^2 - 2t + 400$ *Set y equal to 0.*

$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ *Write the Quadratic Formula.*

$t = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-16)(400)}}{2(-16)}$ *Substitute -16 for a, -2 for b, and 400 for c.*

Holt McDougal Algebra 2 Copyright © by Holt Mc Dougal. All Rights Reserved.

2-6 The Quadratic Formula

Check It Out! Example 4

$t = \frac{2 \pm \sqrt{25604}}{-32}$ *Simplify.*

$t \approx -5.063$ or $t \approx 4.937$

The time cannot be negative, so the water lands on a target about 4.937 seconds after it is released.

Holt McDougal Algebra 2 Copyright © by Holt Mc Dougal. All Rights Reserved.

2-6 The Quadratic Formula

Check It Out! Example 4 Continued

Step 2 The horizontal distance x in feet between the water and its point of release is modeled by $x = 91t$. Find the horizontal distance that the water will have traveled in this time.

$x = 91t$

$x \approx 91(4.937)$ *Substitute 4.937 for t .*

$x \approx 449.267$ *Simplify.*

$x \approx 449$

The water will have traveled a horizontal distance of about 449 feet. Therefore, the pilot should start releasing the water when the horizontal distance between the helicopter and the fire is 449 feet.

Holt McDougal Algebra 2 Copyright © by Holt Mc Dougal. All Rights Reserved.

2-6 The Quadratic Formula

Check It Out! Example 4 Continued

Check Use substitution to check that the water hits the ground after about 4.937 seconds.

$y = -16t^2 - 2t + 400$

$y \approx -16(4.937)^2 - 2(4.937) + 400$

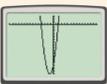
$y \approx -389.983 - 9.874 + 400$

$y \approx 0.143$ ✓ *The height is approximately equal to 0 when $t = 4.937$.*

Holt McDougal Algebra 2 Copyright © by Holt Mc Dougal. All Rights Reserved.

2-6 The Quadratic Formula

Properties of Solving Quadratic Equations

Method	When to Use	Examples
Graphing	Only approximate solutions or the number of real solutions is needed.	$2x^2 + 5x - 14 = 0$  $x \approx -4.2$ or $x \approx 1.7$
Factoring	$c = 0$ or the expression is easily factorable.	$x^2 + 4x + 3 = 0$ $(x + 3)(x + 1) = 0$ $x = -3$ or $x = -1$
Square roots	The variable side of the equation is a perfect square.	$(x - 5)^2 = 24$ $\sqrt{(x - 5)^2} = \pm\sqrt{24}$ $x - 5 = \pm 2\sqrt{6}$ $x = 5 \pm 2\sqrt{6}$

Holt McDougal Algebra 2 Copyright © by Holt Mc Dougal. All Rights Reserved.

2-6 The Quadratic Formula

Properties of Solving Quadratic Equations

Method	When to Use	Examples
Completing the square	$a = 1$ and b is an even number.	$x^2 + 6x = 10$ $x^2 + 6x + \blacksquare = 10 + \blacksquare$ $x^2 + 6x + \left(\frac{6}{2}\right)^2 = 10 + \left(\frac{6}{2}\right)^2$ $(x + 3)^2 = 19$ $x = -3 \pm \sqrt{19}$
Quadratic Formula	Numbers are large or complicated, and the expression does not factor easily.	$5x^2 - 7x - 8 = 0$ $x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(5)(-8)}}{2(5)}$ $x = \frac{7 \pm \sqrt{209}}{10}$

Holt McDougal Algebra 2 Copyright © by Holt Mc Dougal. All Rights Reserved.

2-6 The Quadratic Formula

Helpful Hint

No matter which method you use to solve a quadratic equation, you should get the same answer.

Holt McDougal Algebra 2 Copyright © by Holt Mc Dougal. All Rights Reserved.

2-6 The Quadratic Formula

Lesson Quiz: Part I

Find the zeros of each function by using the Quadratic Formula.

- $f(x) = 3x^2 - 6x - 5$ $1 \pm \frac{2\sqrt{6}}{3}$
- $g(x) = 2x^2 - 6x + 5$ $\frac{3}{2} \pm \frac{1}{2}i$

Find the type and member of solutions for each equation.

- $x^2 - 14x + 50$ 2 distinct nonreal complex
- $x^2 - 14x + 48$ 2 distinct real

Holt McDougal Algebra 2 Copyright © by Holt Mc Dougal. All Rights Reserved.

2-6 The Quadratic Formula**Lesson Quiz: Part II**

5. A pebble is tossed from the top of a cliff. The pebble's height is given by $y(t) = -16t^2 + 200$, where t is the time in seconds. Its horizontal distance in feet from the base of the cliff is given by $d(t) = 5t$. How far will the pebble be from the base of the cliff when it hits the ground?

about 18 ft
