6.1 Using Properties of Exponents

Goals
- Use properties of exponents to evaluate and simplify expressions involving powers.
- Use exponents and scientific notation to solve real-life problems.

Your Notes

VOCABULARY

Scientific notation A number expressed in scientific notation is in the form $c \times 10^n$ where $1 \le c < 10$ and n is an integer.

PROPERTIES OF EXPONENTS

Let a and b be real numbers and let m and n be integers.

Product of powers property	$a^m \cdot a^n = a^{m+n}$
Power of a power property	$(a^m)^n = a^{mn}$
Power of a product property	$(ab)^m = a^m b^m$
Negative exponent property	$a^{-m} = \frac{1}{a^m}$, $a \neq 0$
Zero exponent property	$a^0 = 1$, $a \neq 0$
Quotient of powers property	$\frac{a^m}{a^n} = a^{m-n}$, $a \neq 0$
Power of a quotient property	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, $b \neq 0$

Example 1 ***Evaluating Numerical Expressions***

a. $\frac{7^6}{7^3} = 7^{6-3} = 7^3 = 343$

b. $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

c. $(3^2)^2 = 3^{2 \cdot 2} = 3^4 = 81$

Your Notes

Example 2 *Simplifying Algebraic Expressions*

a. $\left(\dfrac{x^{-3}}{y^2}\right)^4 = \dfrac{(x^{-3})^4}{(y^2)^4}$ — **Power of a quotient property**

$= \dfrac{x^{-12}}{y^8}$ — **Power of a power property**

$= \dfrac{1}{x^{12}y^8}$ — **Negative exponent property**

b. $(-3a)^3a^9a^{-7} = (-3)^3a^3\,a^9a^{-7}$ — **Power of a product property**

$= -27a^{3+9-7}$ — **Product of powers property**

$= -27a^5$ — **Simplify exponent.**

c. $\dfrac{(c^4d)^2}{c^9d^2} = \dfrac{(c^4)^2d^2}{c^9d^2}$ — **Power of a product property**

$= \dfrac{c^8d^2}{c^9d^2}$ — **Power of a power property**

$= c^{8-9}d^{2-2}$ — **Quotient of powers property**

$= c^{-1}d^0$ — **Simplify exponents.**

$= c^{-1}$ — **Zero exponent property**

$= \dfrac{1}{c}$ — **Negative exponent property**

Checkpoint **Complete the following exercises.**

1. Evaluate $(2^{-2})^3(2^5)$.

$\dfrac{1}{2}$

2. Simplify $\dfrac{(jk^2)^2}{(j^{-1}k)^3}$.

j^5k

Your Notes

Example 3 *Comparing Real-Life Volumes*

The radius of a basketball is about 5.7 times greater than the radius of a golf ball. How many times as great as the golf ball's volume is the basketball's volume?

Solution

Let r represent the radius of the golf ball.

$$\frac{\text{Basketball's volume}}{\text{Golf ball's volume}} = \frac{\frac{4}{3}\pi\left(\boxed{5.7r}\right)^3}{\frac{4}{3}\pi r^3}$$ **The volume of a sphere is $\frac{4}{3}\pi r^3$.**

$$= \frac{\frac{4}{3}\pi\,\boxed{5.7^3r^3}}{\frac{4}{3}\pi r^3}$$ **Power of a product property**

$$= \underline{5.7^3r^0}$$ **Quotient of powers property**

$$= \underline{5.7^3}$$ **Zero exponent property**

$$\approx \underline{185}$$ **Approximate power.**

The basketball's volume is about 185 times as great as the golf ball's volume.

Example 4 *Using Scientific Notation in Real Life*

Greenland covers about 2.2×10^6 square kilometers and has approximately 5.6×10^4 people. About how many square kilometers are there per person?

Solution

$$\frac{\text{Land area}}{\text{Population}} = \frac{2.2 \times 10^6}{5.6 \times 10^4}$$ **Divide land area by population.**

$$= \frac{2.2}{5.6} \times 10^2$$ **Quotient of powers property**

$$\approx 0.393 \times 10^2$$ **Use a calculator.**

$$= 39.3$$ **Write in standard notation.**

There are about 39 square kilometers per person.

Homework