

10-2 Angles of Rotation

Objectives

Draw angles in standard position.

Determine the values of the trigonometric functions for an angle in standard position.

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10-2 Angles of Rotation

Vocabulary

standard position
initial side
terminal side
angle of rotation
coterminal angle
reference angle

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10-2 Angles of Rotation

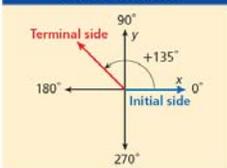
In Lesson 13-1, you investigated trigonometric functions by using acute angles in right triangles. The trigonometric functions can also be evaluated for other types of angles.

An angle is in **standard position** when its vertex is at the origin and one ray is on the positive x -axis. The **initial side** of the angle is the ray on the x -axis. The other ray is called the **terminal side** of the angle.

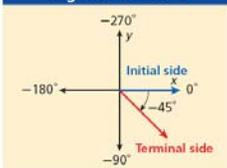
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Positive Rotation



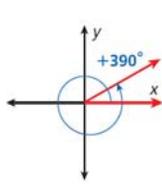
Negative Rotation



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An **angle of rotation** is formed by rotating the terminal side and keeping the initial side in place. If the terminal side is rotated counterclockwise, the angle of rotation is positive. If the terminal side is rotated clockwise, the angle of rotation is negative. The terminal side can be rotated more than 360° .



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Remember!

A 360° rotation is a complete rotation. A 180° rotation is one-half of a complete rotation.

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10-2 Angles of Rotation

Example 1: Drawing Angles in Standard Position

Draw an angle with the given measure in standard position.

A. 320° B. -110° C. 990°

Rotate the terminal side 320° counterclockwise. Rotate the terminal side -110° clockwise. Rotate the terminal side 990° counterclockwise.

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Check It Out! Example 1

Draw an angle with the given measure in standard position.

A. 210° B. 1020° C. -300°

Rotate the terminal side 210° counterclockwise. Rotate the terminal side 1020° counterclockwise. Rotate the terminal side 300° clockwise.

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Coterminal angles are angles in standard position with the same terminal side. For example, angles measuring 120° and -240° are coterminal.

There are infinitely many coterminal angles. One way to find the measure of an angle that is coterminal with an angle θ is to add or subtract integer multiples of 360° .

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Example 2A: Finding Coterminal Angles

Find the measures of a positive angle and a negative angle that are coterminal with each given angle.

$$\theta = 65^\circ$$

$$65^\circ + 360^\circ = 425^\circ \quad \text{Add } 360^\circ \text{ to find a positive coterminal angle.}$$

$$65^\circ - 360^\circ = -295^\circ \quad \text{Subtract } 360^\circ \text{ to find a negative coterminal angle.}$$

Angles that measure 425° and -295° are coterminal with a 65° angle.

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Example 2B: Finding Coterminal Angles

Find the measures of a positive angle and a negative angle that are coterminal with each given angle.

$$\theta = 410^\circ$$

$$410^\circ - 360^\circ = 50^\circ \quad \text{Subtract } 360^\circ \text{ to find a positive coterminal angle.}$$

$$410^\circ - 2(360^\circ) = -310^\circ \quad \text{Subtract a multiple of } 360^\circ \text{ to find a negative coterminal angle.}$$

Angles that measure 50° and -310° are coterminal with a 410° angle.

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Check It Out! Example 2a

Find the measures of a positive angle and a negative angle that are coterminal with each given angle.

$$\theta = 88^\circ$$

$$88^\circ + 360^\circ = 448^\circ \quad \text{Add } 360^\circ \text{ to find a positive coterminal angle.}$$

$$88^\circ - 360^\circ = -272^\circ \quad \text{Subtract } 360^\circ \text{ to find a negative coterminal angle.}$$

Angles that measure 448° and -272° are coterminal with an 88° angle.

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Check It Out! Example 2b

Find the measures of a positive angle and a negative angle that are coterminal with each given angle.

$$\theta = 500^\circ$$

$$500^\circ + 360^\circ = 860^\circ \quad \text{Add } 360^\circ \text{ to find a positive coterminal angle.}$$

$$500^\circ - 2(360^\circ) = -220^\circ \quad \text{Subtract a multiple of } 360^\circ \text{ to find a negative coterminal angle.}$$

Angles that measure 860° and -220° are coterminal with a 500° angle.

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Check It Out! Example 2c

Find the measures of a positive angle and a negative angle that are coterminal with each given angle.

$$\theta = -120^\circ$$

$$-120^\circ + 360^\circ = 240^\circ \quad \text{Add } 360^\circ \text{ to find a positive coterminal angle.}$$

$$-120^\circ - 360^\circ = -480^\circ \quad \text{Subtract } 360^\circ \text{ to find a negative coterminal angle.}$$

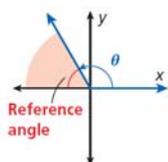
Angles that measure 240° and -480° are coterminal with a -120° angle.

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For an angle θ in standard position, the **reference angle** is the positive acute angle formed by the terminal side of θ and the x -axis. In Lesson 13-3, you will learn how to use reference angles to find trigonometric values of angles measuring greater than 90° or less than 0° .



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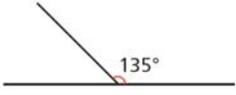
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Example 3: Finding Reference Angles

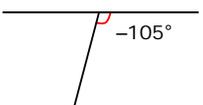
Find the measure of the reference angle for each given angle.

A. $\theta = 135^\circ$



The measure of the reference angle is 45° .

B. $\theta = -105^\circ$



The measure of the reference angle is 75° .

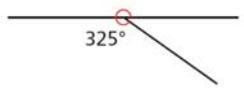
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Example 3: Finding Reference Angles

Find the measure of the reference angle for each given angle.

C. $\theta = 325^\circ$



The measure of the reference angle is 35° .

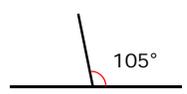
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Check It Out! Example 3

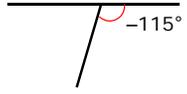
Find the measure of the reference angle for each given angle.

a. $\theta = 105^\circ$



The measure of the reference angle is 75° .

b. $\theta = -115^\circ$



The measure of the reference angle is 65° .

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Check It Out! Example 3

Find the measure of the reference angle for each given angle.

c. $\theta = 310^\circ$

The measure of the reference angle is 50°

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To determine the value of the trigonometric functions for an angle θ in standard position, begin by selecting a point P with coordinates (x, y) on the terminal side of the angle. The distance r from point P to the origin is given by $\sqrt{x^2 + y^2}$.

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Trigonometric Functions

For a point $P(x, y)$ on the terminal side of θ in standard position and $r = \sqrt{x^2 + y^2}$,

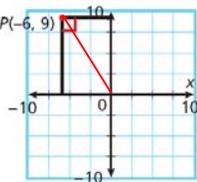
| SINE | COSINE | TANGENT |
|-----------------------------|-----------------------------|---------------------------------------|
| $\sin \theta = \frac{y}{r}$ | $\cos \theta = \frac{x}{r}$ | $\tan \theta = \frac{y}{x}, x \neq 0$ |

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Example 4: Finding Values of Trigonometric Functions

$P(-6, 9)$ is a point on the terminal side of θ in standard position. Find the exact value of the six trigonometric functions for θ .



Step 1 Plot point P , and use it to sketch a right triangle and angle θ in standard position. Find r .

$$r = \sqrt{(-6)^2 + 9^2} = \sqrt{36 + 81} = \sqrt{117} = 3\sqrt{13}$$

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Example 4 Continued

Step 2 Find $\sin \theta$, $\cos \theta$, and $\tan \theta$.

$$\begin{aligned} \sin \theta &= \frac{y}{r} & \cos \theta &= \frac{x}{r} & \tan \theta &= \frac{y}{x} \\ &= \frac{9}{3\sqrt{13}} & &= \frac{-6}{3\sqrt{13}} & &= \frac{9}{-6} \\ &= \frac{3\sqrt{13}}{13} & &= \frac{-2\sqrt{13}}{13} & &= -\frac{3}{2} \end{aligned}$$

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Example 4 Continued

Step 3 Use reciprocals to find $\csc \theta$, $\sec \theta$, and $\cot \theta$.

$$\begin{aligned} \csc \theta &= \frac{1}{\sin \theta} = \frac{\sqrt{13}}{3} \\ \sec \theta &= \frac{1}{\cos \theta} = \frac{\sqrt{13}}{-2} \\ \cot \theta &= \frac{1}{\tan \theta} = -\frac{2}{3} \end{aligned}$$

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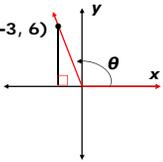
Helpful Hint
 Because r is a distance, its value is always positive, regardless of the sign of x and y .

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Check It Out! Example 4

$P(-3, 6)$ is a point on the terminal side of θ in standard position. Find the exact value of the six trigonometric functions for θ .



Step 1 Plot point P , and use it to sketch a right triangle and angle θ in standard position. Find r .

$$r = \sqrt{(-3)^2 + 6^2} = \sqrt{9 + 36} = \sqrt{45} = 3\sqrt{5}$$

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Check It Out! Example 4 Continued

Step 2 Find $\sin \theta$, $\cos \theta$, and $\tan \theta$.

$$\sin \theta = \frac{y}{r} \qquad \cos \theta = \frac{x}{r} \qquad \tan \theta = \frac{y}{x}$$

$$= \frac{6}{3\sqrt{5}} \qquad = \frac{-3}{3\sqrt{5}} \qquad = \frac{-6}{3}$$

$$= \frac{2\sqrt{5}}{5} \qquad = -\frac{\sqrt{5}}{5} \qquad = -2$$

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Check It Out! Example 4 Continued

Step 3 Use reciprocals to find $\csc \theta$, $\sec \theta$, and $\cot \theta$.

$$\csc \theta = \frac{1}{\sin \theta} = \frac{\sqrt{5}}{2}$$

$$\sec \theta = \frac{1}{\cos \theta} = -\sqrt{5}$$

$$\cot \theta = \frac{1}{\tan \theta} = -\frac{1}{2}$$

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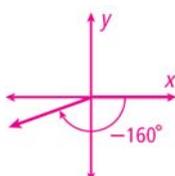
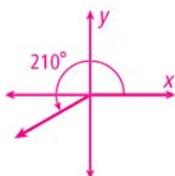
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Lesson Quiz: Part I

Draw an angle in standard position with the given measure.

1. 210°

2. -160°



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Lesson Quiz: Part II

Find the measure of the reference angle for each given angle.

3. $\theta = 290^\circ$ 70° 4. $\theta = -195^\circ$ 15°

5. $P(1, -1)$ is a point on the terminal side of θ in standard position. Find the exact value of the six trigonometric functions for θ .

$$\sin \theta = -\frac{2\sqrt{5}}{5}, \cos \theta = \frac{\sqrt{5}}{5}, \tan \theta = -2,$$

$$\csc \theta = -\frac{\sqrt{5}}{2}, \sec \theta = \sqrt{5}, \cot \theta = -\frac{1}{2}$$

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