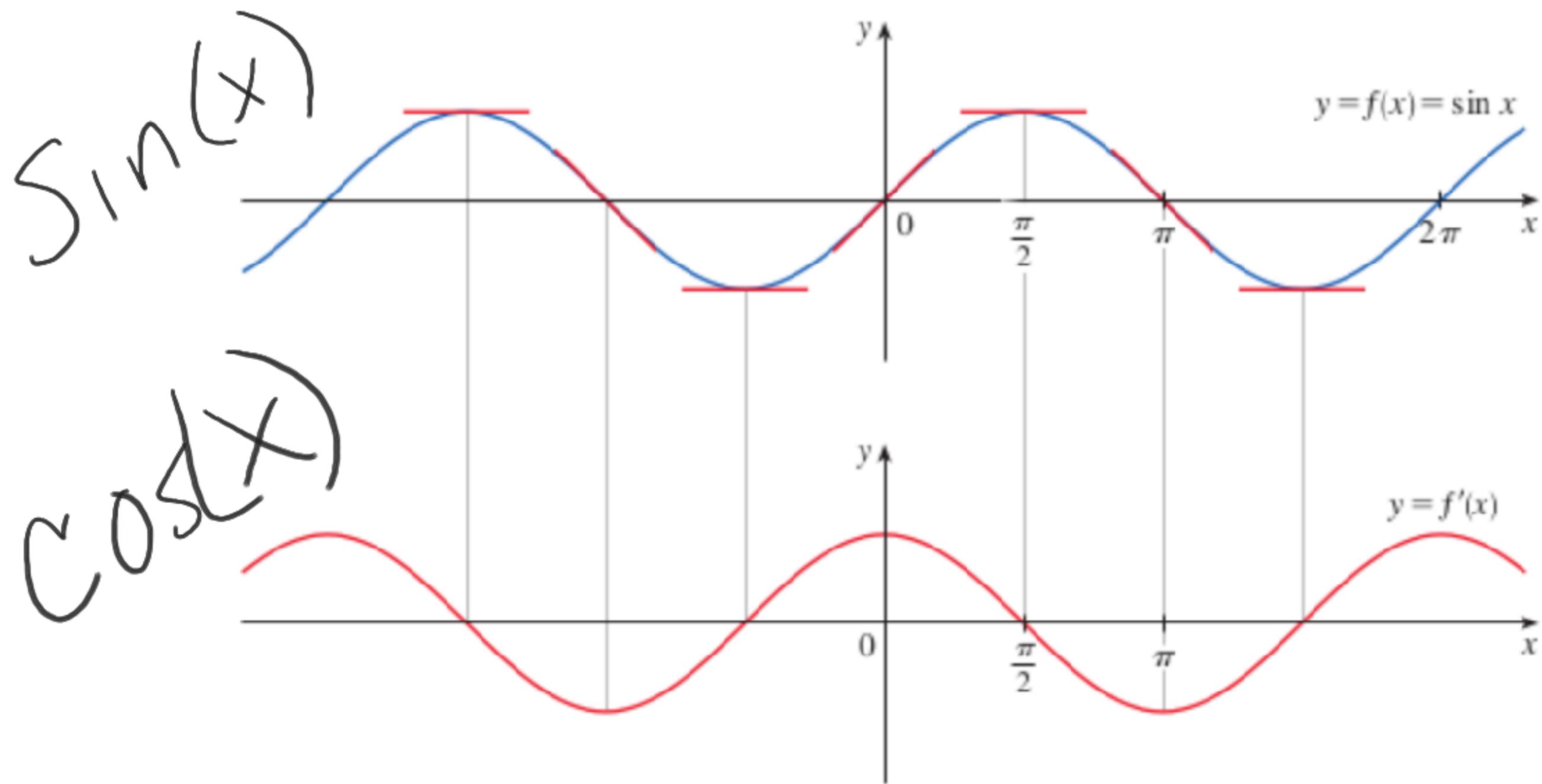


Figure 1.



$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sin(x+h) - \sin x}{h}$$

$$\sin(x+h) = \sin x \cos h + \cos x \sin h$$

$$\lim_{h \rightarrow 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$\lim_{h \rightarrow 0} \frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h}$$

$$\lim_{h \rightarrow 0} \frac{\sin x \cosh - \sin x}{h} + \frac{\cos x \sinh}{h}$$

$$\lim_{h \rightarrow 0} \sin x \left(\frac{\cosh - 1}{h} \right) + \cos x \left(\frac{\sinh}{h} \right)$$

$$\sin x \lim_{h \rightarrow 0} \left(\frac{\cosh - 1}{h} \right) + \cos x \lim_{h \rightarrow 0} \frac{\sinh}{h}$$

$$\sin x (0) + \cos x (1)$$

$$f(x) = \sin x \quad f'(x) = \cos x$$

$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$$

$$\lim_{\theta \rightarrow 0} \frac{\cos \theta - 1}{\theta} = 0$$

Differentiate $y = x^2 \sin x$ $y' =$

$$f(x) = x^2 \quad f'(x) = 2x$$

$$g(x) = \sin x \quad g'(x) = \cos x$$

$$y' = (2x) \sin x + (\cos x)(x^2)$$

$$= 2x \sin x + x^2 \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$y = \sin x$$

$$y' = \cos x \quad (1)$$

$$y'' = -\sin x \quad (2)$$

$$y^{(3)} = -\cos x \quad (3)$$

$$y^{(4)} = \sin x \quad (4)$$

Find the 27 th derivative of $\cos x$

$$\frac{27}{4} = 6 \text{ r } 3$$

$$3^{\text{rd}} = -\cos x$$

$$27^{\text{th}} = -\cos x$$

$$\frac{d}{dx} (\tan x) = \frac{\sin x}{\cos x}$$

$$f(x) = \sin x \quad f'(x) = \cos x$$

$$g(x) = \cos x \quad g'(x) = -\sin x$$

$$y' = \frac{(\cos x)(\cos x) - (-\sin x)(\sin x)}{\cos^2 x}$$

$$y' = \frac{1}{\cos^2 x} = \boxed{\sec^2 x}$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

Differentiate $f(x) = \frac{\sec x}{1 + \tan x}$. For what values of x does the graph of f have a horizontal tangent?

$$m(x) = \sec x \quad m'(x) = \sec x \tan x$$

$$n(x) = 1 + \tan x \quad n'(x) = \sec^2 x$$

$$f'(x) = \frac{(\sec x \tan x)(1 + \tan x) - (\sec^2 x)(\sec x)}{(1 + \tan x)^2}$$

$$= \frac{\sec x (\tan x + \tan^2 x - \sec^2 x)}{(1 + \tan x)^2}$$

$$= \frac{\sec x (\tan x + \tan^2 x - \sec^2 x)}{(1 + \tan x)^2}$$

$$\begin{aligned}\sec^2 x &= \tan^2 x + 1 \\ \sec^2 x - \tan^2 x &= 1 \\ \tan^2 x - \sec^2 x &= -1\end{aligned}$$

$$= \frac{\sec x (\tan x - 1)}{(1 + \tan x)^2} = f'(x)$$

$$\sec x (\tan x - 1)$$

$$\begin{aligned}\tan x &= 1 \\ x &= \frac{\pi}{4} + n\pi\end{aligned}$$

Find $\lim_{x \rightarrow 0} \frac{\sin 7x}{4x}$ $\frac{7}{4}$

$\lim_{\theta \rightarrow 0}$

$\frac{\sin \theta}{\theta} = 1$

$\lim_{x \rightarrow 0} \frac{7 \sin 7x}{4 \cdot 7x} = \frac{7}{4} \left[\lim_{x \rightarrow 0} \frac{\sin 7x}{7x} = 1 \right]$

$\lim_{x \rightarrow 0} \frac{\sin 7x}{4x} = \frac{7}{4}$

Calculate $\lim_{x \rightarrow 0} x \cot x$

\rightarrow

$$\frac{x \cos x}{\sin x}$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\frac{\frac{x \cos x}{x}}{\frac{\sin x}{x}} =$$

$$\frac{\cos x}{\sin x / x} = \frac{1}{1} = 1$$

$$g(x) = 3x + x^2 \cos x$$

$$m(x) = x^2 \quad m'(x) = 2x$$

$$n(x) = \cos x \quad n'(x) = -\sin x$$

$$g'(x) = 3 + \left[(2x)(\cos x) + (-\sin x)(x^2) \right]$$

$$g'(x) = 3 + 2x \cos x - x^2 \sin x$$

$$f(t) = \frac{\cot t}{e^t}$$

$$m(x) = \cot x \quad m'(x) = -\operatorname{csc}^2 x$$

$$n(x) = e^x \quad n'(x) = e^x$$

$$f'(x) = \frac{(-\operatorname{csc}^2 x)(e^x) - (e^x)(\cot x)}{(e^x)^2}$$

$$f'(x) = \frac{-\operatorname{csc}^2 x - \cot x}{e^x}$$

$$y = x + \sin x, \quad (\pi, \pi)$$

$$y' = 1 + \cos x \quad x = \pi \quad y' = 1 + \cos \pi \quad (-1)$$
$$y' = 0$$

Tangent Line

$$y = 0x + b$$

$$\pi = 0 \cdot \pi + b$$

$$\pi = b$$

$$\boxed{y = \pi}$$

$f(x) = e^x \cos x$, find $f'(x)$ and $f''(x)$

$$m(x) = e^x \quad m'(x) = e^x$$
$$n(x) = \cos x \quad n'(x) = -\sin x$$

$$f'(x) = e^x \cos x + (-\sin x)(e^x)$$

$$f'(x) = e^x \cos x - e^x \sin x$$

$$p(x) = \sin x$$
$$p'(x) = \cos x$$

$$f''(x) = [e^x \cos x - e^x \sin x] - [e^x \sin x + e^x \cos x]$$

$$f''(x) = -2e^x \sin x$$

$$e^x (\cos x - \sin x)$$

$$m(x) = e^x \quad m'(x) = e^x$$

$$n(x) = \boxed{\cos x} - \boxed{\sin x} \quad n'(x) = -\sin x + (-\cos x)$$

$$e^x (\cancel{\cos x} - \sin x) + e^x (-\sin x - \cancel{\cos x})$$

$$\boxed{-2e^x \sin x}$$