Find $F'(x)$ if $F(x) = \sqrt{x^2 + 1}$

$$\cancel{(x^2+1)^{1/2} \rightarrow (x^2)^{1/2} + (1)^{1/2}}$$

$$\sqrt{(x^2+4x+4)} = \sqrt{(x+2)^2} = x+2$$

The Chain Rule

If g is differentiable at x and f is differentiable at $g(x)$, then the composite function $F = f \circ g$ defined by $F(x) = f(g(x))$ is differentiable at x and F' is given by the product

$$F'(x) = f'(g(x)) \cdot g'(x)$$

$\boxed{1}$

$$F(x) = f(g(x))$$

$$F'(x) = f'(g(x)) \cdot g'(x)$$

In Leibniz notation, if $y = f(u)$ and $u = g(x)$ are both differentiable functions, then

$\boxed{2}$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Find $F'(x)$ if $F(x) = \sqrt{x^2 + 1}$

$g(x) = x^2 + 1$

$f(x) = \sqrt{u} = u^{1/2}$

$u = x^2 + 1$

$f'(x) = \dfrac{1}{2\sqrt{u}}$

$g'(x) = 2x$

$f'(x) = \dfrac{1}{2\sqrt{x^2+1}} \cdot 2x = \boxed{\dfrac{x}{\sqrt{x^2+1}}}$

(a) $y = \sin(x^2)$

(b) $y = \sin^2 x = (\sin x)^2$

b) $u = \sin x \rightarrow u' = \cos x$

$y = u^2 \rightarrow y' = 2u \cdot u'$

$y' = 2(\sin x)(\cos x)$

a) $y = \sin(x^2)$

$u = x^2 \rightarrow u' = 2x$

$y = \sin(u)$

$y' = \cos(u) \cdot u'$

$= \cos(x^2) \cdot 2x$

Differentiate $y = (x^3 - 1)^{100}$

$u = x^3 - 1 \qquad u' = 3x^2$

$y = u^{100} \qquad y' = 100u^{99} \cdot u'$

$= 100(x^3-1)^{99} \cdot 3x^2$

$= \boxed{300x^2(x^3-1)^{99}}$

$$f(x) = \frac{1}{\sqrt[3]{x^2+x+1}} = (x^2+x+1)^{-1/3}$$

$$u = x^2+x+1 \qquad u' = 2x+1$$

$$y = u^{-1/3} \qquad y' = -\frac{1}{3}u^{-4/3} \cdot u'$$

$$y' = -\frac{1}{3}(x^2+x+1)^{-4/3}(2x+1)$$

$$y' = \frac{-2x-1}{3\sqrt[3]{(x^2+x+1)^4}}$$

$$g(t) = \left(\frac{t-2}{2t+1}\right)^9$$

$f(x) = x-2 \quad f'(x) = 1$

$g(x) = 2x+1 \quad g'(x) = 2$

$$u = \frac{x-2}{2x+1} \qquad u' = \frac{1(2x+1) - 2(x-2)}{(2x+1)^2} = \frac{5}{(2x+1)^2}$$

$$y = u^9 \qquad y' = 9u^8 \cdot u' = 9\left(\frac{x-2}{2x+1}\right)^8 \cdot \frac{5}{(2x+1)^2}$$

$$y' = \frac{45(x-2)^8}{(2x+1)^8(2x+1)^2} = \boxed{\frac{45(x-2)^8}{(2x+1)^{10}}}$$

$$y = (2x+1)^5(x^3 - x + 1)^4$$

$f(x) = (2x+1)^5$

$f'(x) = 5(2x+1)^4 \cdot 2$

$f'(x) = 10(2x+1)^4$

$g(x) = (x^3 - x + 1)^4$

$g'(x) = 4(x^3 - x + 1)^3(3x^2 - 1)$

$$y' = 10(2x+1)^4(x^3 - x + 1)^4 + 4(x^3 - x + 1)^3(3x^2-1)(2x+1)^5$$

$$y' = 10(2x+1)^4(x^3-x+1)^4 + 4(x^3-x+1)^3(3x^2-1)(2x+1)^5$$

$6x^3+3x^2-2x-1$

$$(x^3-x+1)^3(2x+1)^4\left(10(x^3-x+1)+4(3x^2-1)(2x+1)\right)$$

$10x^3-10x+10+24x^3+12x^2-8x-4$

$$\boxed{(x^3-x+1)^3(2x+1)^4(34x^3+12x^2-18x+6)}$$

te $y = e^{\sin x}$

$u = \sin x \qquad u' = \cos x$

$y = e^{u} \qquad y' = e^{u} \cdot u'$

$y' = (e^{\sin x})(\cos x)$

If $f(x) = \sin(\cos(\tan x))$

$\boxed{u = \cos(\tan x)}$ $\quad u' = \boxed{-\sin(\tan x)\cdot \sec^2 x}$

$y = \sin u \qquad y' = \cos u \cdot u'$

$y' = \cos(\cos(\tan x)) \cdot [-\sin(\tan x)](\sec^2 x)$

$u = \cos(\tan x)$

$m = \tan x \qquad m' = \sec^2 x$

$u = \cos m \qquad u' = -\sin(m) \cdot m'$

e $y = e^{\sec 3\theta}$

$U = \sec 3\theta$ $\qquad$ $u' = \sec 3\theta \cdot \tan 3\theta \cdot 3$

$m = 3\theta$ $\qquad$ $m' = 3$

$u = \sec m$ $\qquad$ $u' = \sec m \tan m \cdot m'$

$y = e^{u}$ $\qquad$ $y' = e^{u} \cdot u'$

$$y' = e^{\sec 3\theta} \cdot 3 \sec 3\theta \tan 3\theta$$

$$\boxed{b^x} = e^{(\ln b)x}$$

$$y = e^{(\ln b)x}$$

$u = (\ln b)x \qquad u' = \ln b$

$$y' = \boxed{e^{(\ln b)(x)}} \cdot \ln b$$

$$\boxed{y' = b^x \cdot \ln b}$$

$$f(x) = e^x$$

$$f'(x) = e^x$$

$$g(x) = 2^x$$

$$g'(x) \overset{?}{=} 2^x$$

$$= 2^x \ln 2$$

a. $g(x) = 2^x$

$g(x) = 2^x \quad g'(x) = 2^x \cdot \ln 2$

b. $h(x) = 5^{x^2}$

$y = 5^{x^2}$

$u = x^2 \quad u' = 2x$

$y = 5^u \quad y' = 5^u \cdot \ln 5 \cdot u'$

$$\boxed{y' = (5^{x^2})(\ln 5) \cdot (2x)}$$