$$x^2 + y^2 = 25$$

$$y = \sqrt{x^3 + 1}$$

$$y = x \sin x$$

$$x^3 + y^3 = 6xy$$

$$x^2 + y^2 = 25$$

$$y^2 = 25 - x^2$$

$$y = \pm\sqrt{25 - x^2}$$

$$y = \sqrt{25 - x^2}$$

$$u = 25 - x^2 \quad u' = -2x$$

$$y = u^{1/2} \quad y' = \frac{1}{2\sqrt{u}} u'$$

$$y' = \frac{-2x}{2\sqrt{25 - x^2}} = \frac{\pm x}{\sqrt{25 - x^2}}$$

$$x^3 + y^3 = 6xy$$

$$x^3 = 6xy - y^3$$

$$x^3 = y(6x - y^2)$$

If $x^2 + y^2 = 25$, find $\frac{dy}{dx}$. Then find an equation of the tangent to the circle $x^2 + y^2 = 25$ at the point $(3, 4)$.

$$x^2 + y^2 = 25$$

$$\frac{d}{dx}(x^2 + y^2) = \frac{d}{dx}(25)$$

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d}{dx}(25)$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$\boxed{\frac{dy}{dx} = \frac{-x}{y}}$$

$$y' = \frac{-x}{y}$$

$$x^2 + y^2 = 25$$

$$y = \pm\sqrt{25 - x^2}$$

$$y' = \frac{\pm x}{\sqrt{25 - x^2}}$$

(a) Find y' if $x^3 + y^3 = 6xy$.

(b) Find the tangent to the folium of Descartes $x^3 + y^3 = 6xy$ at the point $(3, 3)$

(c) At what point in the first quadrant is the tangent line horizontal?

$$x^3 + y^3 = 6xy$$

$$3x^2 + 3y^2 y' = 6y + 6xy'$$

$6xy$

$f(x) = 6x$

$g(x) = y$

$f'(x) = 6$

$g'(x) = y'$

$$3x^2 + 3y^2 y' = 6y + 6xy'$$

$$x^2 + y^2 y' = 2y + 2xy'$$

$$x^2 - 2y = 2xy' - y^2 y'$$

$$x^2 - 2y = (2x - y^2)\, y'$$

$$\boxed{\frac{x^2 - 2y}{2x - y^2} = y'}$$

$$y' = \frac{x^2 - 2y}{2x - y^2} \qquad (3,3)$$

$$y' = \frac{(3)^2 - 2(3)}{2(3) - (3)^2} = \frac{3}{-3} = -1$$

$$y - y_1 = m(x - x_1)$$

$$y - 3 = -1(x - 3)$$

$$\boxed{y = -x + 6}$$

$$\frac{x^2-2y}{2x-y^2} = y' = 0$$

$$x^2-2y=0 \longleftrightarrow y=\frac{x^2}{2}$$

$$x^3+\left(\frac{1}{2}x^2\right)^3 = 6x\left(\frac{1}{2}x^2\right)$$

$$x^3+\frac{1}{8}x^6 = 3x^3$$

$$\frac{1}{8}x^6 = 2x^3$$

$$x^6 = 16x^3$$

$$x^3 = 16$$

$$x = \sqrt[3]{16}$$

$$x = 2^{4/3}$$

$$y = \frac{2^{8/3}}{2} = 2^{5/3}$$

Find y' if $\boxed{\sin(x+y)} = \boxed{y^2 \cos x}$

$f(x) = y^2$

$f'(x) = 2y\,y'$

$g(x) = \cos x$

$g'(x) = -\sin x$

$u = x + y \qquad u' = 1 + y'$

$\sin u \to \cos u \; u'$

$\cos(x+y)(1+y') = 2yy'\cos x + (-\sin x)(y^2)$

$\cos(x+y) + \cos(x+y)\,y' = 2yy'\cos x - y^2 \sin x$

$\cos(x+y) + y^2 \sin x = 2yy'\cos x - \cos(x+y)\,y'$

$$\cos(x+y) + y^2 \sin x = 2yy'\cos x - \cos(x+y)y'$$

$$\cos(x+y) + y^2 \sin x = y'(2y\cos x - \cos(x+y))$$

$$\boxed{\frac{\cos(x+y) + y^2 \sin x}{2y\cos x - \cos(x+y)} = y'}$$

Find y'' if $x^4 + y^4 = 16$

$y' \rightarrow 4x^3 + 4y^3 y' = 0$

$$y' = \frac{-4x^3}{4y^3} = \boxed{\frac{-x^3}{y^3}}$$

$$y' = \frac{-x^3}{y^3}$$

$$f(x) = -x^3 \qquad f'(x) = -3x^2$$

$$g(x) = y^3 \qquad g'(x) = 3y^2 \boxed{y'}$$

$$g'(x) = 3y^2\left(\frac{-x^3}{y^3}\right)$$

$$g'(x) = \frac{-3x^3}{y}$$

$$y'' = \frac{-3x^2(y^3) - \left(\frac{-3x^3}{y}\right)(-x^3)}{y^6}$$

$$y'' = \frac{-3x^2y^3 - \frac{3x^6}{y}}{y^6}$$

$$y'' = \frac{-3x^2y^3 - \frac{3x^6}{y}}{y^6} = \frac{\frac{-3x^2y^4 - 3x^6}{y}}{y^6}$$

$$= \frac{-3x^2y^4 - 3x^6}{y^7} = \frac{-3x^2(y^4+x^4)}{y^7} \quad 16$$

$$= \boxed{\frac{-48x^2}{y^7}}$$

$$x^2 - 4xy + y^2 = 4$$

$$e^x \sin y = x + y$$

$$2xe^y + ye^x = 3$$

$$x^2 + 4y^2 = 4$$