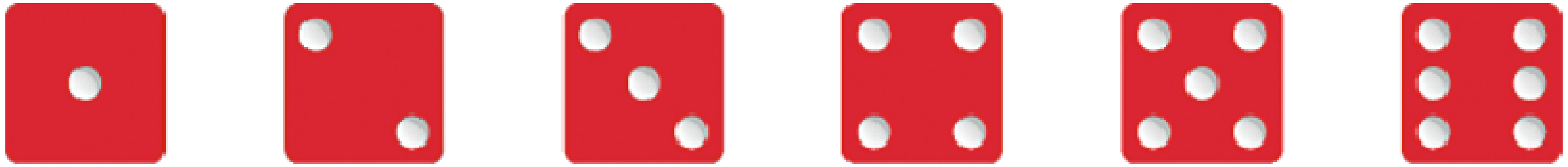
***Rolling a Die*** When a balanced die is rolled once, six equally likely outcomes are possible, a displayed in Fig. 4.22.

## Figure 4.22



Sample space for rolling a die once

Let $P(5 \mid odd) = \frac{1}{3}$ $P(5) = 1/6$

$F$ = event a 5 is rolled, and $P(odd) = \frac{3}{6}$

$O$ = event the die comes up odd.

## Definition 4.6: Conditional Probability

The probability that event $B$ occurs given that event $A$ occurs is called a **conditional probability**. It is denoted $\boldsymbol{P(B \mid A)}$ which is read "the probability of $B$ given $A$." We call $A$ the **given event.**

**Rank**

| | | Full professor $R_1$ | Associate professor $R_2$ | Assistant professor $R_3$ | Instructor $R_4$ | **Total** |
|---|---|---|---|---|---|---|
| | Under 30 $A_1$ | 2 | 3 | 57 | 6 | 68 |
| | 30–39 $A_2$ | 52 | 170 | 163 | 17 | 402 |
| **Age (yr)** | 40–49 $A_3$ | 156 | 125 | 61 | 6 | 348 |
| | 50–59 $A_4$ | 145 | 68 | 36 | 4 | 253 |
| | 60 & over $A_5$ | 75 | 15 | 3 | 0 | 93 |
| | **Total** | 430 | 381 | 320 | 33 | 1164 |

$P(A_2 \mid R_4)$

$$\frac{17}{33}$$

$P(A_2 \text{ and } R_4)$

$$\frac{17}{1164}$$

$P(A_2 \text{ or } R_4)$

$418/1164$

# Formula 4.4: The Conditional Probability Rule

If $A$ and $B$ are any two events with $P(A) > 0$, then

$$P(B \mid A) = \frac{P(A \,\&\, B)}{P(A)}.$$

***Marital Status and Gender*** From *America's Families and Living Arrangements*, a publication of the U.S. Census Bureau, we obtained a joint probability distribution for the marital status of U.S. adults by gender, as shown in Table 4.9. We used "Single" to mean "Never married."

**Table 4.9 Joint probability distribution of marital status and gender**

| | | **Marital status** | | | | |
|---|---|---|---|---|---|---|
| | | Single $M_1$ | Married $M_2$ | Widowed $M_3$ | Divorced $M_4$ | $P(S_i)$ |
| **Gender** | Male $S_1$ | 0.147 | 0.281 | 0.013 | $\boxed{0.044}$ | 0.485 |
| | Female $S_2$ | 0.121 | 0.284 | 0.050 | 0.060 | 0.515 |
| | $P(M_j)$ | 0.268 | 0.565 | 0.063 | 0.104 | 1.000 |

**a.** Determine the probability that the adult selected is divorced, given that the adult selected is a male.

**b.** Determine the probability that the adult selected is a male, given that the adult selected is divorced.

a) $P(A) \Rightarrow$ male

$P(B) \Rightarrow$ Divorced

$P(B|A)$

$$\frac{P(A \text{ and } B)}{P(A)} = \frac{0.044}{0.485} = \boxed{0.091}$$

**a.** Determine the probability that the adult selected is divorced, given that the adult selected is a male.

**b.** Determine the probability that the adult selected is a male, given that the adult selected is divorced.

$$P(\text{Male} \mid \text{Divorced}) = \frac{0.044}{0.104} = 0.423$$

Race

| Gender | | White $R_1$ | Black $R_2$ | Other $R_3$ | **Total** |
|---|---|---|---|---|---|
| | Male $G_1$ | 30 | 26 | 0 | 56 |
| | Female $G_2$ | 210 | 121 | 20 | 351 |
| | **Total** | 240 | 147 | 20 | 407 |

.87 .823

**a.** a Black.

**b.** a white female.

**c.** a male, given that the filer was white.

**d.** a male, given that the filer was Black.

a) 147/407 c) 30/240

b) 210/407 d) 26/147

c) 0.125 d) 0.177

# Formula 4.5: The General Multiplication Rule

If $A$ and $B$ are any two events, then

$$P(A \,\&\, B) = P(A) \cdot P(B \mid A).$$

***U.S. Congress*** The U.S. Congress, Joint Committee on Printing, provides information on the composition of the Congress in the *Congressional Directory*. For the 113th Congress, 18.7% of the members are senators and 53% of the senators are Democrats. What is the probability that a randomly selected member of the 113th Congress is a Democratic senator?

535 members

100 senators

$$P(D \text{ and } S) = P(S) \cdot P(D \mid S)$$

$$= .187 \times .530$$

$$= 0.099 \rightarrow \boxed{9.9\%}$$

***Gender of Students*** In Professor Weiss's introductory statistics class, the number of males and females are as shown in the frequency distribution presented in Table 4.10. Two students are selected at random from the class. The first student selected is not returned to the class for possible reselection; that is, the sampling is without replacement. Find the probability that the first student selected is female and the second is male.

**Table 4.10 Frequency distribution of males and females in Professor Weiss's introductory statistics class**

| Gender | Frequency |
|---|---|
| Male | 17 |
| Female | 23 |
| | 40 |

# Definition 4.7: Independent Events

Event $B$ is said to be **independent** of event $A$ if $P(B \mid A) = P\left(B\right)$.

***Playing Cards*** Consider again the experiment of randomly selecting one card from a deck of 52 playing cards. Let

$F$ = event a face card is selected,

$K$ = event a king is selected, and

$H$ = event a heart is selected.

$P(K) = \frac{4}{52} = .077$

NOT

**a.** Determine whether event $K$ is independent of event $F$.

$P(K|F) = \frac{4}{12} = .333$

**b.** Determine whether event $K$ is independent of event $H$. Independent

$P(K) = .077$

$P(K|H) = \frac{1}{13} = .077$

## Formula 4.6: The Special Multiplication Rule (for Two Independent Events)

If $A$ and $B$ are independent events, then

$$P(A \,\&\, B) = P(A) \cdot P(B),$$

and conversely, if $P(A \,\&\, B) = P(A) \cdot P(B)$, then $A$ and $B$ are independent events.

or → addition

and → multiplication

***Roulette*** An American roulette wheel contains 38 numbers, of which 18 are red, 18 are black, and 2 are green. When the roulette wheel is spun, the ball is equally likely to land on any of the 38 numbers. In three plays at a roulette wheel, what is the probability that the ball will land on green the first time and on black the second and third times?

$$\left(\frac{2}{38}\right)\left(\frac{18}{38}\right)\left(\frac{18}{38}\right) = 0.012$$

$$\boxed{1.2\%}$$

***License Plates*** The license plates of a state consist of three letters followed by three digits.

**a.** How many different license plates are possible?

**b.** How many possibilities are there for license plates on which no letter or digit is repeated?

## Definition 4.8: Factorials

The product of the first $k$ positive integers (counting numbers) is called $\boldsymbol{k}$ **factorial** and is denoted $\boldsymbol{k!}$. In symbols,

$$k! = k\,(k-1)\cdots 2\cdot 1.$$

We also define $0! = 1$.

# Formula 4.10: The Permutations Rule

The number of possible permutations of $r$ objects from a collection of $m$ objects is given by the formula

$$_mP_r = \frac{m!}{(m-r)!}$$

***Exacta Wagering*** In an exacta wager at the race track, a bettor picks the two horses that he or she thinks will finish first and second in a specified order. For a race with 12 entrants, determine the number of possible exacta wagers.

## Formula 4.12: The Combinations Rule

The number of possible combinations of $r$ objects from a collection of $m$ objects is given by the formula

$$_mC_r = \frac{m!}{r!(m-r)!}.$$

***CD-Club Introductory Offer*** To recruit new members, a compact-disc (CD) club advertises a special introductory offer: A new member agrees to buy 1 CD at regular club prices and receives free any 4 CDs of his or her choice from a collection of 69 CDs. How many possibilities does a new member have for the selection of the 4 free CDs?