1 Theorem

If F is an antiderivative of f on an interval I , then the most general antiderivative of f on I is

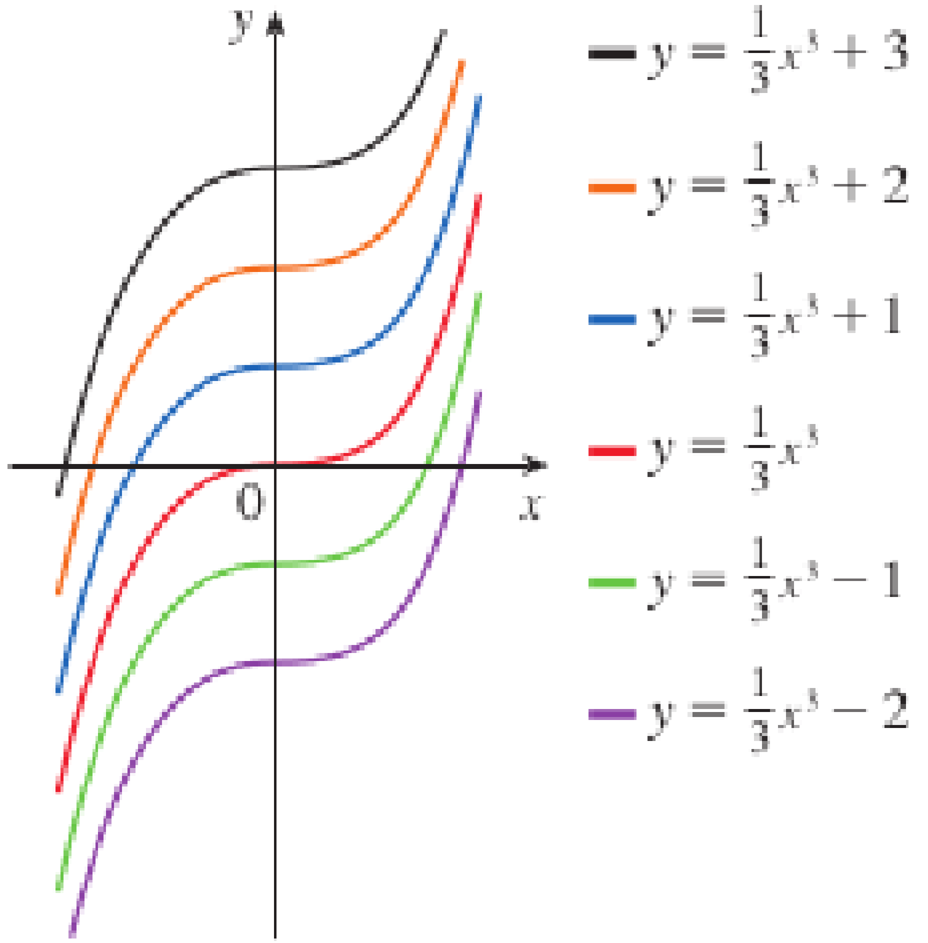
$$F(x) + C$$

where C is an arbitrary constant.

$F(x) + C$

$y' = x^2$

Figure 1



$y = \frac{1}{3}x^3 + 3$

$y = \frac{1}{3}x^3 - 2$

Example 1

Find the most general antiderivative of each of the following functions.

(a) $f(x) = \sin x$ $\qquad f'(x) = \sin x \qquad f(x) = -\cos x + C$

(b) $f(x) = 1/x$ $\qquad f'(x) = \frac{1}{x} \qquad f(x) = \ln x + C$

(c) $f(x) = x^n, n \neq -1$ $\qquad f'(x) = x^n \qquad f(x) =$

$f(x) = x^4$ $f'(x) = 4x^3$

x^5 $5x^4$

$$\boxed{\frac{x^{n+1}}{n+1}}$$

x^n

$x^n (n \neq -1)$	$\frac{x^{n+1}}{n+1}$
$\frac{1}{x}$	$\ln \lvert x \rvert$
e^x	e^x
b^x	$\frac{b^x}{\ln b}$
$\cos x$	$\sin x$

$\sec x \tan x$	$\sec x$
$\frac{1}{\sqrt{1 - x^2}}$	$\sin^{-1} x$
$\frac{1}{1 + x^2}$	$\tan^{-1} x$
$\cosh x$	$\sinh x$
$\sinh x$	$\cosh x$

$$g'(x) = 4\sin x + \frac{2x^5 - \sqrt{x}}{x}$$

$$\frac{2x^5}{x} - \frac{x^{1/2}}{x}$$

$$g'(x) = 4\sin x + 2x^4 - x^{-1/2}$$

$$g(x) = 4(-\cos x) + 2\left(\frac{x^5}{5}\right) - \left(\frac{x^{1/2}}{1/2}\right)$$

$$g(x) = -4\cos x + \frac{2}{5}x^5 - 2\sqrt{x} + C$$

Find f if $f'(x) = e^x + 20(1 + x^2)^{-1}$ and $\boxed{f(0) = -2}$

$$f'(x) = e^x + 20(1+x^2)^{-1}$$

$$= e^x + 20\left(\frac{1}{1+x^2}\right)$$

$$f(x) = e^x + 20\tan^{-1}x + C \quad f(0) = -2$$

$$f(0) = -2 = e^0 + 20\tan^{-1}(0) + C$$

$$-2 = 1 + C$$

$$-1 \quad -1$$

$$-3 = C$$

$$f(x) = \boxed{e^x + 20\tan^{-1}(x) - 3}$$

Find f if $f''(x) = 12x^2 + 6x - 4$, $f(0) = 4$, and $f(1) = 1$

$$f''(x) = 12x^2 + 6x - 4x^0$$

$$f'(x) = 12\left(\frac{x^3}{3}\right) + 6\left(\frac{x^2}{2}\right) - 4x + C$$

$$= 4x^3 + 3x^2 - 4x + C$$

$$f(x) =$$

$$= 4x^3 + 3x^2 - 4x + C$$

$$f(x) = 4\left(\frac{x^4}{4}\right) + 3\left(\frac{x^3}{3}\right) - 4\left(\frac{x^2}{2}\right) + Cx + D$$

$f(0) = 4$

$f(1) = 1$

$$f(x) = x^4 + x^3 - 2x^2 + Cx + D$$

$$f(0) = 4 = D$$

$$f(x) = x^4 + x^3 - 2x^2 + Cx + 4$$

$1 = C + 4$

$-3 = C$

$$\boxed{f(x) = x^4 + x^3 - 2x^2 - 3x + 4}$$

A particle moves in a straight line and has acceleration given by $a(t) = 6t + 4$. Its initial velocity is $v(0) = -6$ cm/s and its initial displacement is $s(0) = 9$ cm. Find its position function $s(t)$.

$$\boxed{\begin{array}{l} \text{position } s(t) \\ \text{velocity } v(t) = s'(t) \\ \text{acceleration } a(t) = s''(t) \end{array}}$$

$a(t) = 6t + 4$

$v(0) = -6$

$s(0) = 9$

$s(t) =$

$a(t) = 6t + 4$

$v(0) = -6$

$s(0) = 9$

$s(t) =$

$v(t) = 6\left(\frac{t^2}{2}\right) + 4t + C$

$v(t) = 3t^2 + 4t + C$

$v(0) = -6$

$C = -6$

$v(t) = 3t^2 + 4t - 6$

$$v(t) = 3t^2 + 4t - 6$$

$$s(t) = 3\left(\frac{t^3}{3}\right) + 4\left(\frac{t^2}{2}\right) - 6t + C$$

$$s(t) = t^3 + 2t^2 - 6t + C$$

$$s(0) = 9$$

$$C = 9$$

$$\boxed{s(t) = t^3 + 2t^2 - 6t + 9}$$