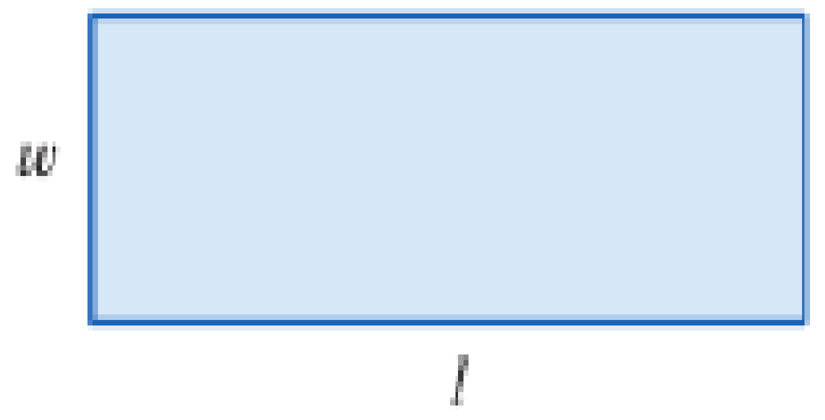
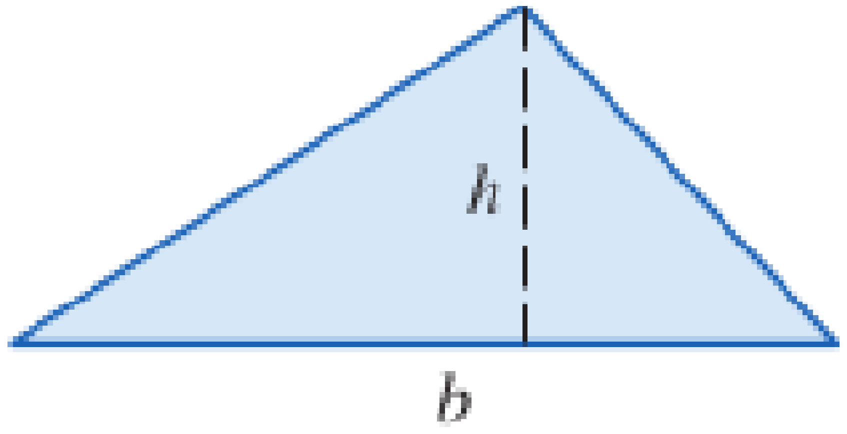
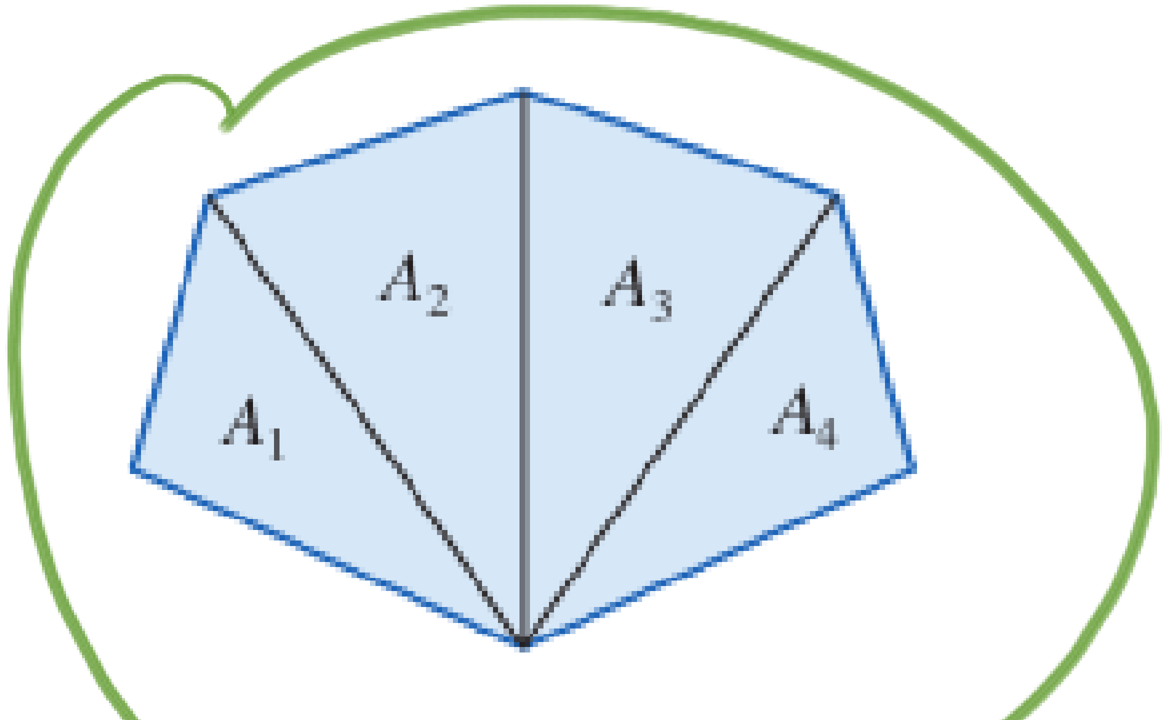


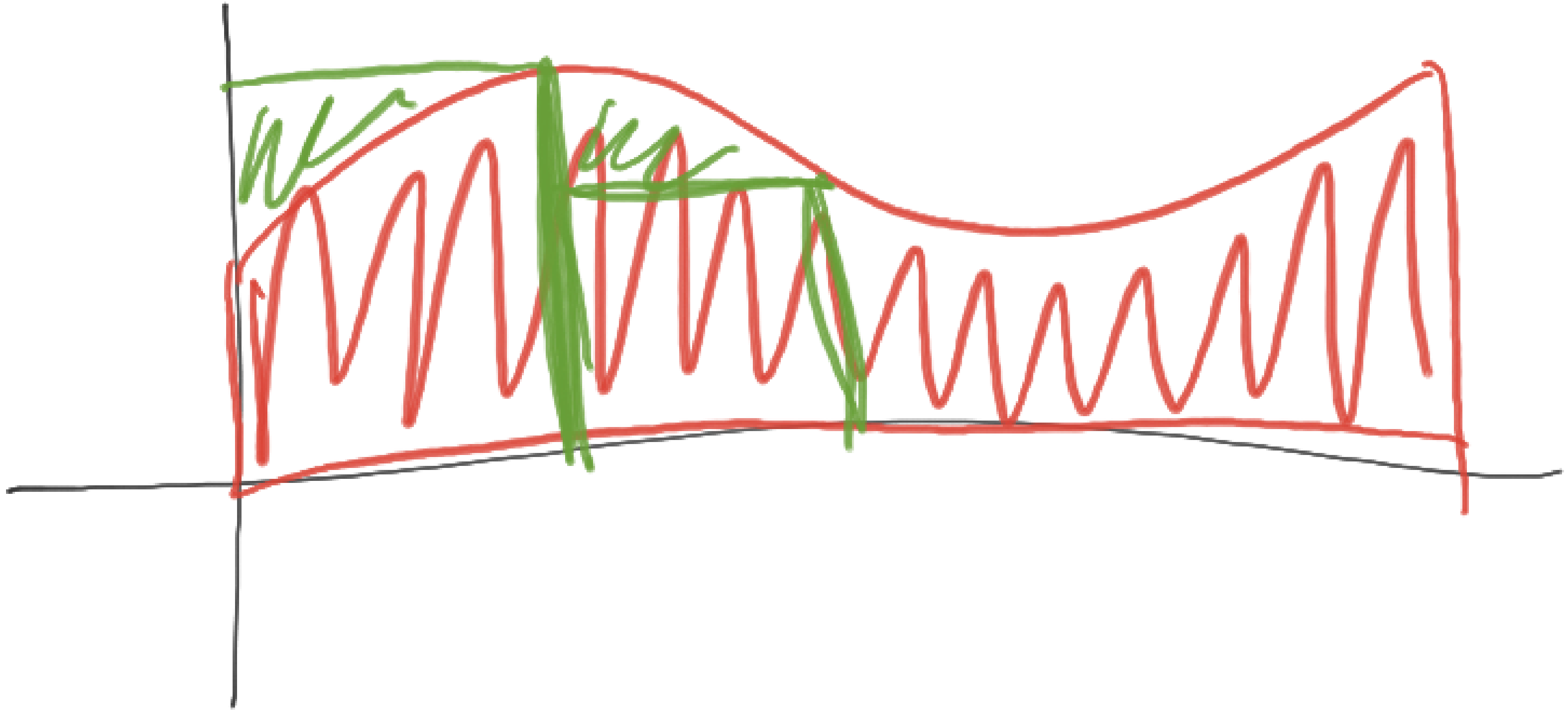
$A = lw$

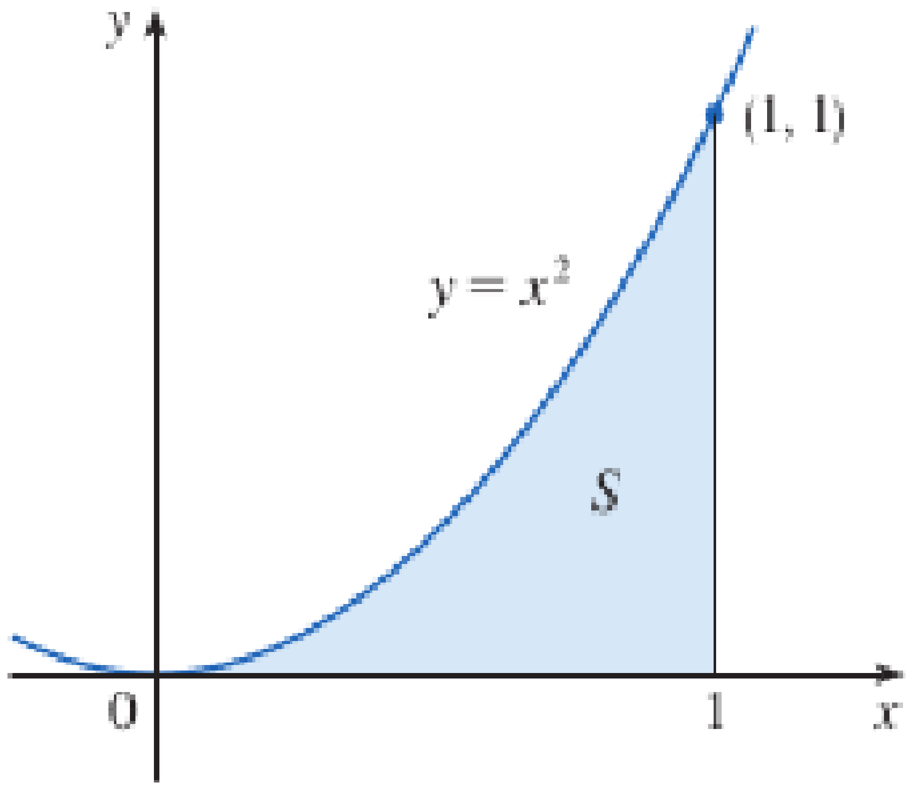


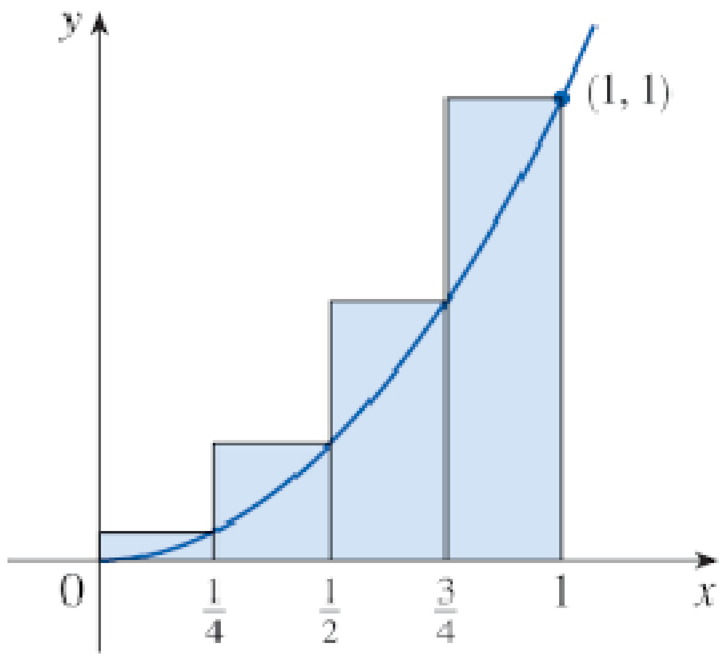
$A = \frac{1}{2}bh$



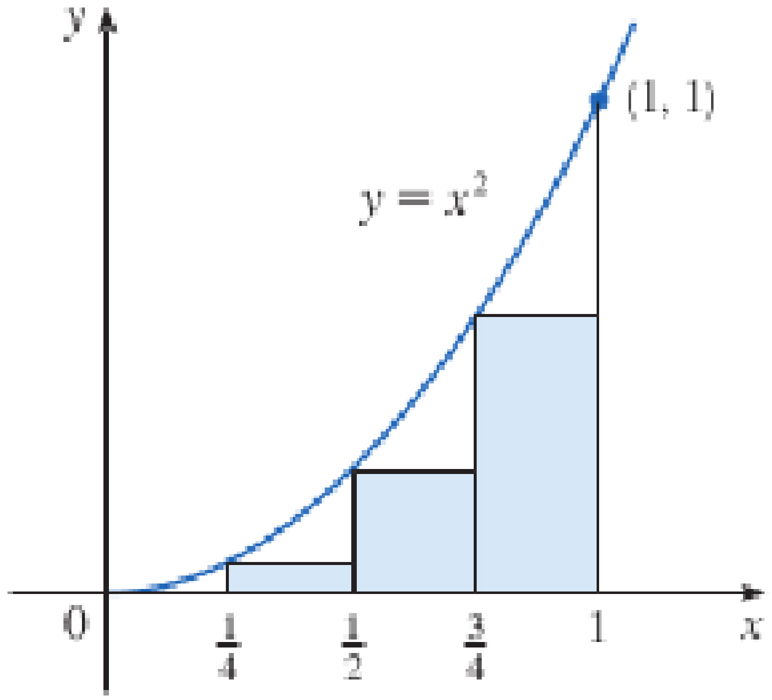
$A = A_1 + A_2 + A_3 + A_4$



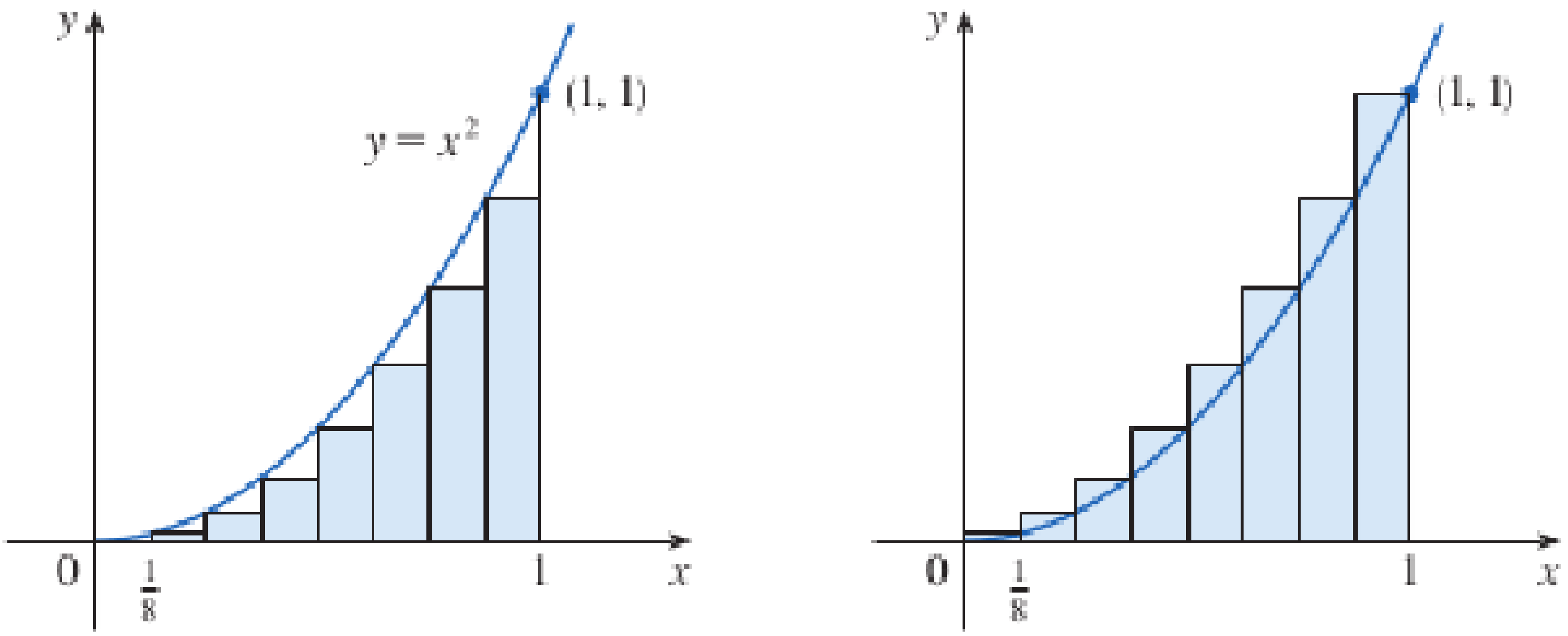




(b)



n	L_n	R_n
10	0.2850000	0.3850000
20	0.3087500	0.3587500
30	0.3168519	0.3501852
50	0.3234000	0.3434000
100	0.3283500	0.3383500
1000	0.3328335	0.3338335



(a) Using left endpoints (b) Using right endpoints

$$y' = x^2 \rightarrow y = \frac{1}{3}x^3$$

$$\rightarrow y(1) = \boxed{\frac{1}{3}}$$

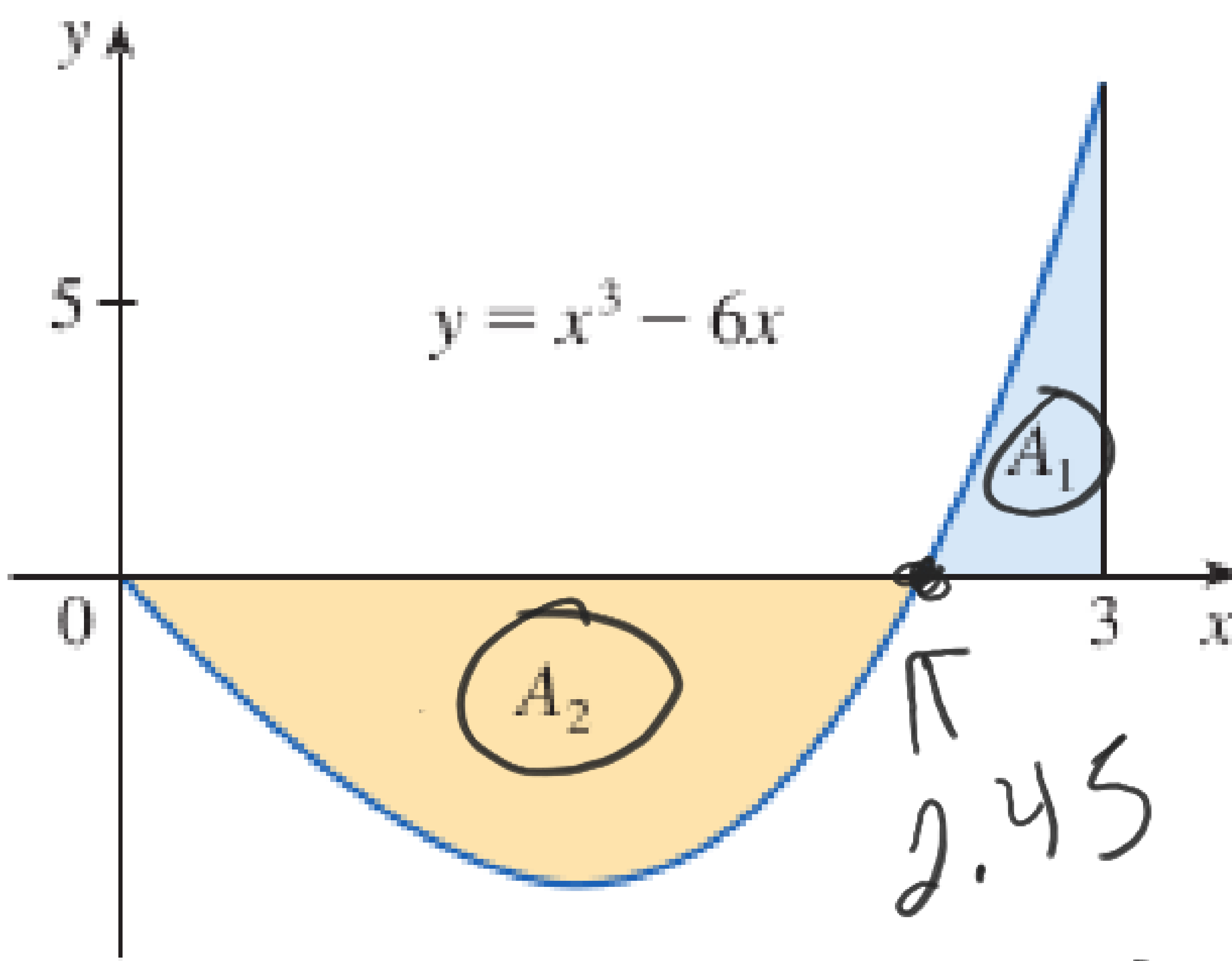
2 Definition of a Definite Integral

If f is a function defined for $a \leqslant x \leqslant b$, we divide the interval $[a, b]$ into n subintervals of equal width $\Delta x = (b - a)/n$. We let $x_0(= a), x_1, x_2, \cdots, x_n(= b)$ be the endpoints of these subintervals and we let $x_1^*, x_2^*, \ldots, x_n^*$ be any **sample points** in these subintervals, so x_i^* lies in the i th subinterval $[x_{i-1}, x_i]$. Then the **definite integral of f from a to b** is

$$\int_a^b f(x)\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\, \Delta x$$

provided that this limit exists and gives the same value for all possible choices of sample points. If it does exist, we say that f is **integrable** on $[a, b]$.

Evaluate $\int_0^3 (x^3 - 6x)\,dx$.



$$\int_0^3 (x^3 - 6x) = \frac{1}{4}x^4 - 3x^2$$

$$\frac{1}{4}x^4 - 3x^2\Big]_0^3 = \left(\frac{1}{4}(3)^4 - 3(3)^2\right) - \left(\frac{1}{4}(0)^4 - 3(0)^2\right)$$

$$\frac{81}{4} - \frac{108}{4} = \frac{-27}{4} = 6.75$$

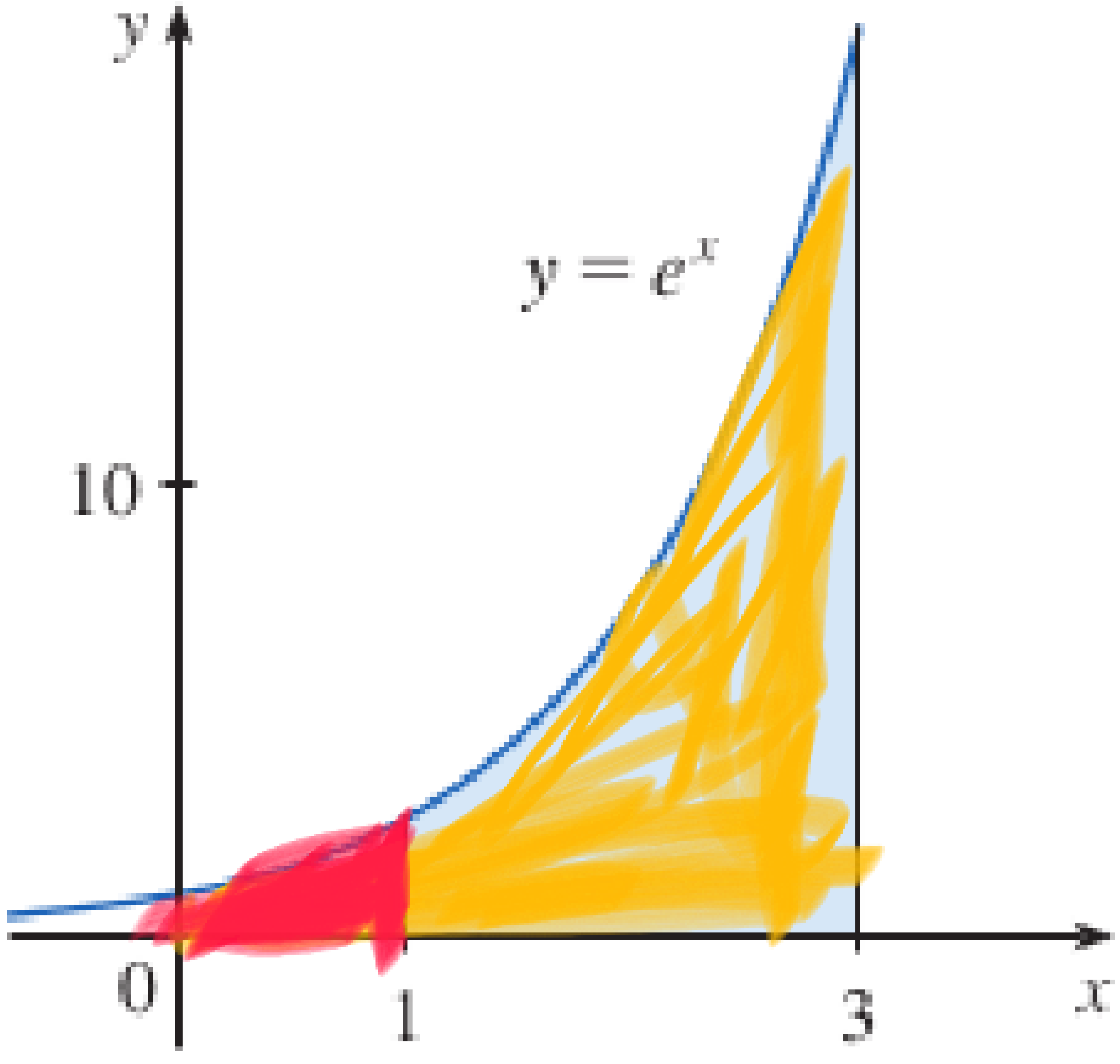
$A_2) \int_0^{2.45} x^3 - 6x \rightarrow \frac{1}{4}x^4 - 3x^2 \Big]_0^{2.45} = \boxed{-9}$

$A_1) \int_{2.45}^{3} x^3 - 6x \rightarrow \frac{1}{4}x^4 - 3x^2 \Big]_{2.45}^{3} = -6.75 - (-9)$

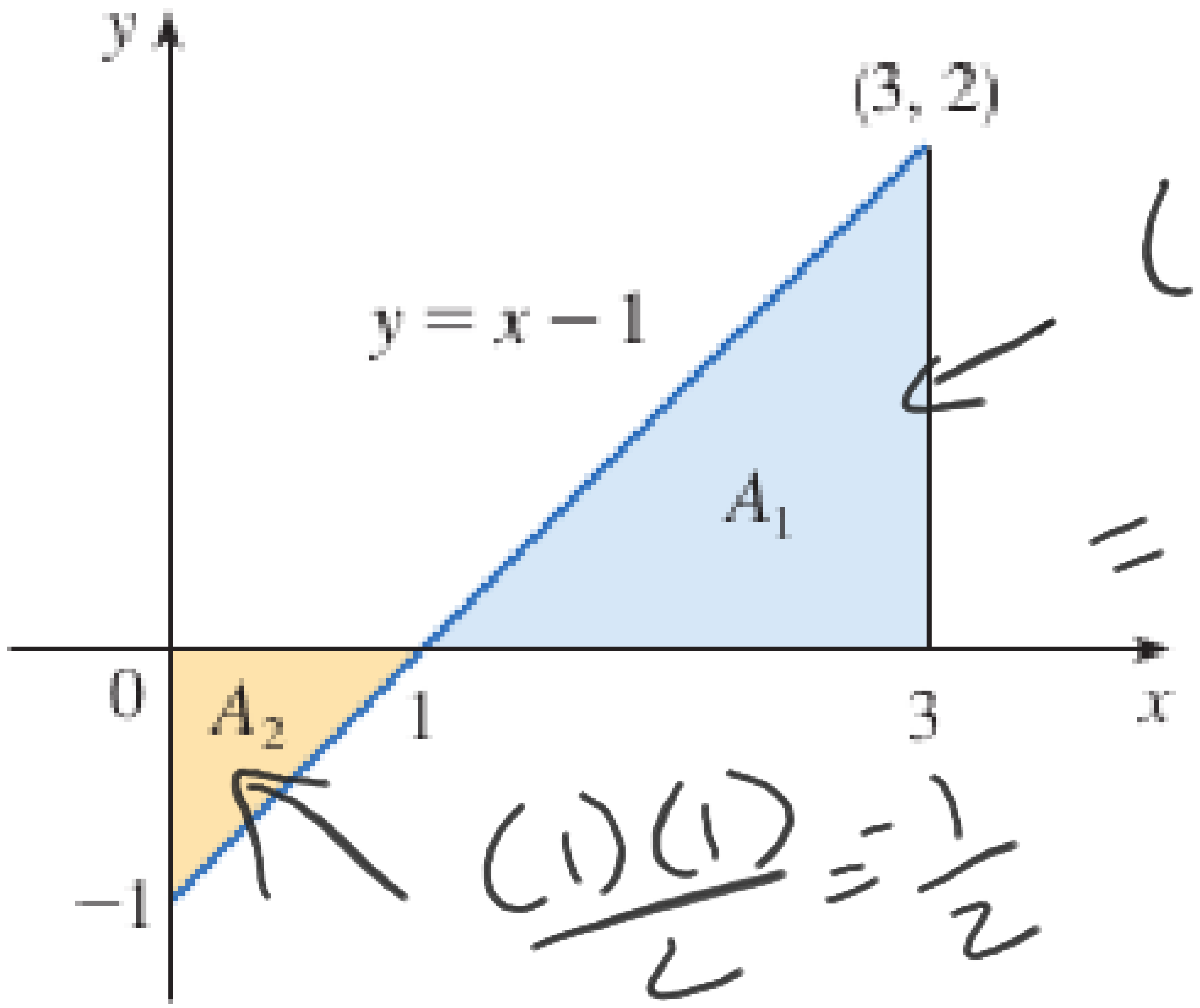
$= \boxed{2.25}$

$$\int_1^3 e^x \, dx = e^x \Big]_1^3$$

$$\boxed{e^3 - e} \approx 17.4$$



$$\int_0^3 (x-1)\,dx =$$



$$\frac{(2)(2)}{2} = 2$$

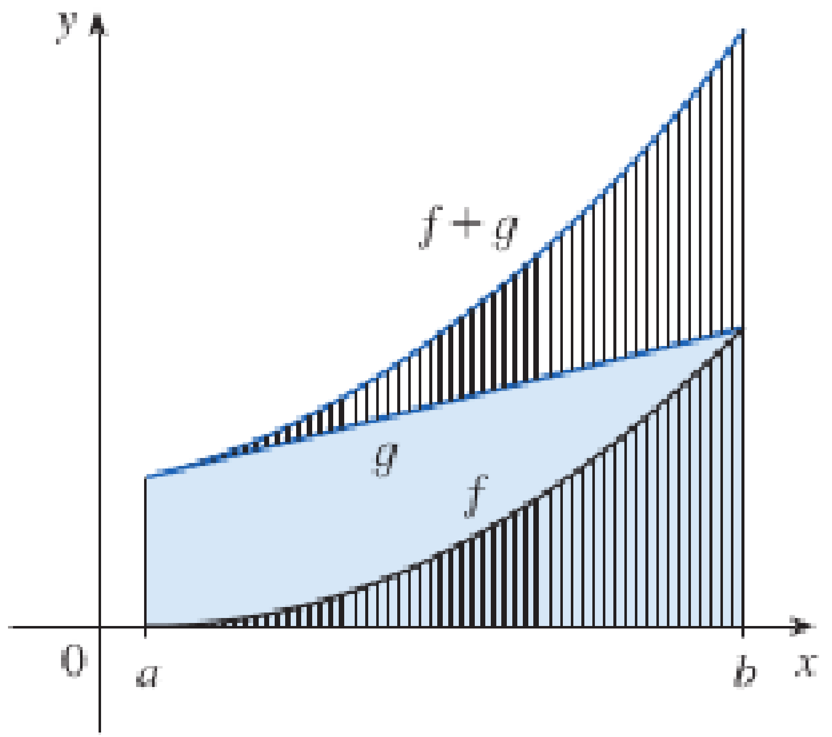
$$\frac{(1)(1)}{2} = -\frac{1}{2}$$

$$\left.\frac{1}{2}x^2 - x\right]_0^3$$

$$\frac{1}{2}(3)^2 - 3$$

$$\frac{9}{2} - \frac{6}{2} = \boxed{\frac{3}{2}}$$

1. $\int_a^b c \, dx = c(b-a)$, where c is any constant



2. $\int_a^b [f(x) + g(x)] \, dx = \int_a^b f(x) \, dx + \int_a^b g(x) \, dx$

▸ Details

$$\int_a^b [f(x) + g(x)] \, dx = \int_a^b f(x) \, dx + \int_a^b g(x) \, dx$$

3. $\int_a^b cf(x) \, dx = c \int_a^b f(x) \, dx$, where c is any constant

4. $\int_a^b [f(x) - g(x)] \, dx = \int_a^b f(x) \, dx - \int_a^b g(x) \, dx$

$$\int_0^1 (4 + 3x^2)\, dx = 4x + x^3 \Big]_0^1$$

$$4(1) + (1)^3 = \boxed{5}$$

$$\int_0^1 4\, dx + \int_0^1 3x^2\, dx$$

$$4x + 3\int_0^1 x^2\, dx$$

$$\int_0^3 \sqrt{x-1}\, dx$$

$$\int_0^3 \sqrt{2x}\, dx = \int_0^3 \sqrt{2}\,\sqrt{x}$$

$$\sqrt{2}\int_1^3 x^{1/2} = \sqrt{2}\left(\frac{x^{3/2}}{3/2}\right) = \sqrt{2}\left(\frac{2x^{3/2}}{3}\right)\Bigg]_0^3$$

$$\int_{-3}^{2} x^2 - 2x + 3 \, dx$$

$$\left. \frac{1}{3}x^3 - x^2 + 3x \right]_{-3}^{2}$$

$$\overset{14/3}{\left(\frac{8}{3} - 4 + 6\right)} - \overset{-27}{\left(-9 - 9 - 9\right)}$$

$$14/3 + 81/3 = \boxed{95/3}$$