Lecture Notes for Chapter 7-1

Dont Forget to Record.

Integration by Parts (1 of 4)

Every differentiation rule has a corresponding integration rule. For instance, the Substitution Rule for integration corresponds to the Chain Rule for differentiation. The rule that corresponds to the Product Rule for differentiation is called the rule for *integration by parts*.

The Product Rule states that if f and g are differentiable functions, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

$$f'(x)g(x) + f(x)g'(x)$$

Integration by Parts (2 of 4)

In the notation for indefinite integrals this equation becomes

$$\int [f(x)g'(x) + g(x)f'(x)]\, dx = f(x)g(x)$$

or

$$\int f(x)g'(x)\, dx + \int g(x)f'(x)\, dx = f(x)g(x)$$

We can rearrange this equation as

$$\textbf{1} \quad \boxed{\int f(x)g'(x)\, dx = f(x)g(x) - \int g(x)f'(x)\, dx}$$

Formula 1 is called the **formula for integration by parts**.

Integration by Parts (3 of 4)

It is perhaps easier to remember in the following notation.

Let $u = f(x)$ and $v = g(x)$. Then the differentials are $du = f'(x)dx$ and $dv = g'(x)dx$,

so, by the Substitution Rule, the formula for integration by parts becomes

$$\mathbf{2} \qquad \int u\,dv = uv - \int v\,du$$

$$\int u\,dv = \boxed{uv - \int v\,du}$$

$u =$ $\boxed{v =}$

$\boxed{du =}$ $dv =$

$$\int x \sin x \, dx.$$

$$(x)(\sin x)$$

$u = x \qquad v = -\cos x$

$du = 1\,dx \qquad dv = \sin x\,dx$

$$\int u\,dv = uv - \int v\,du$$

$$= (x)(-\cos x) - \int -\cos x\,dx$$

$$= -x\cos x - (-\sin x) + C$$

$$= \boxed{-x\cos x + \sin x + C}$$

using integration by parts is the LIATE principle of precedence for u:

$\underline{\text{L}}$ogarithmic

$\underline{\text{I}}$nverse trigonometric

$\underline{\text{A}}$lgebraic

$\underline{\text{T}}$rigonometric

$\underline{\text{E}}$xponential

g them a u which appears as high as possible on the list. For example, in $\int xe$

tion (x) and an exponential function (e^{2x}). Since $\underline{\text{A}}$lgebraic appears before $\underline{\text{E}}$xp

imilar regardless of the selection of u and dv, but it is advisable to refer to LIATE

$$\int (\ln x)(1\,dx)$$

$$\boxed{\begin{array}{ll} u = \ln x & v = x \\ du = \frac{1}{x}dx & dv = 1\,dx \end{array}}$$

$$uv - \int v\,du$$

$$\int 1\,dx$$

$$(\ln x)(x) - \int (x)\left(\frac{1}{x}\right)dx$$

$$\boxed{x\ln x - x + C}$$

$$\int x^2 e^x \, dx \qquad u = x^2 \qquad v = e^x$$

$$du = 2x\,dx \qquad dv = e^x\,dx$$

$$x^2 e^x - \int (e^x)(2x)\,dx \qquad u = 2x \qquad v = e^x\,dx$$

$$du = 2\,dx \qquad du = e^x\,dx$$

$$x^2 e^x - \left[2x\,e^x - \int e^x\,2\,dx \right]$$

$$x^2 e^x - 2xe^x + 2e^x + C$$

$$\int 2x\sqrt{1+x^2}\,dx \qquad u = 1+x^2 \quad du = 2x\,dx$$

$$\int u^{1/2}\,du = \frac{2u^{3/2}}{3}$$

$$= \boxed{\frac{2}{3}\left(1+x^2\right)^{3/2} + C}$$

$$\int e^x \sin x \, dx$$

$u = \sin x \qquad v = e^x$

$du = \cos x \, dx \qquad dv = e^x \, dx$

$$\int e^x \sin x \, dx = e^x \sin x - \int e^x \cos x \, dx$$

$$\int e^x \sin x\,dx = e^x \sin x - \boxed{\int e^x \cos x\,dx}$$

$$u = \cos x \qquad v = e^x$$

$$du = -\sin x\,dx \qquad dv = e^x\,dx$$

$$\int e^x \sin x\,dx = e^x \sin x - \left[e^x \cos x + \int e^x(+\sin x)\,dx\right]$$

$$\int e^x \sin x\,dx = e^x \sin x - e^x \cos x - \int e^x \sin x\,dx$$

$$\int e^x \sin x\,dx = e^x \sin x - e^x \cos x - \int e^x \sin x\,dx$$

$$\begin{array}{r} + \int e^x \sin x\,dx \qquad\qquad + \int e^x \sin x\,dx \\ \hline \end{array}$$

$$2\int e^x \sin x\,dx = e^x \sin x - e^x \cos x$$

$$\int e^x \sin x\,dx = \frac{e^x \sin x - e^x \cos x}{2} + C$$

$$\int_0^1 \tan^{-1} x \, dx$$

$u = \tan^{-1} x \qquad v = x$

$du = \frac{1}{1+x^2} dx \qquad dv = dx$

$$x \tan^{-1} x - \int (x)\left(\frac{1}{1+x^2}\right) dx$$

$m = 1 + x^2$

$dm = 2x \, dx$

$\frac{1}{2} dm = x \, dx$

$\frac{1}{2} \int \frac{1}{m} \, dm = \frac{1}{2} \ln m$

$$\boxed{x \tan^{-1} x - \frac{1}{2} \ln(1+x^2) + C}$$

$$\left[x\tan^{-1}x - \frac{1}{2}\ln(1+x^2)\right]_0^1$$

$$\left[1(\tan^{-1}1) - \frac{1}{2}\ln(2)\right] - \left[0 - 0\right]$$

$$\boxed{\frac{\pi}{4} - \frac{1}{2}\ln(2)}$$

7. $\displaystyle\int x \sin 10x \, dx$

$u = x \qquad v = \frac{-1}{10}\cos 10x$

$du = dx \qquad dv = \sin 10x \, dx$

$$\int x \sin 10x \, dx = \frac{-1}{10} x \cos 10x \boxed{- \int -\frac{1}{10} \cos 10x \, dx}$$

$$= \frac{-1}{10} x \cos 10x + \frac{1}{100} \sin 10x + C$$

$$+ \frac{1}{10} \int \cos 10x \, dx$$

$$\int \sin 10x \, dx$$

$$u = 10x$$

$$du = 10\,dx$$

$$\frac{1}{10}\int \sin u \, du$$

$$\frac{1}{10}\,du = dx$$

$$\frac{1}{10}(-\cos u)$$

$$\boxed{-\frac{1}{10}\cos 10x}$$