



Chapter 7 Techniques of Integration

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7.1 Integration by Parts

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Integration by Parts (1 of 4)

Every differentiation rule has a corresponding integration rule. For instance, the Substitution Rule for integration corresponds to the Chain Rule for differentiation. The rule that corresponds to the Product Rule for differentiation is called the rule for *integration by parts*.

The Product Rule states that if f and g are differentiable functions, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

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Integration by Parts (2 of 4)

In the notation for indefinite integrals this equation becomes

$$\int [f(x)g'(x) + g(x)f'(x)] dx = f(x)g(x)$$

or

$$\int f(x)g'(x) dx + \int g(x)f'(x) dx = f(x)g(x)$$

We can rearrange this equation as

$$1 \quad \int f(x)g'(x) dx = f(x)g(x) - \int g(x)f'(x) dx$$

Formula 1 is called the **formula for integration by parts**.



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Integration by Parts (3 of 4)

It is perhaps easier to remember in the following notation.

Let $u = f(x)$ and $v = g(x)$. Then the differentials are $du = f'(x)dx$ and $dv = g'(x)dx$, so, by the Substitution Rule, the formula for integration by parts becomes

$$2 \quad \int u dv = uv - \int v du$$



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Example 1

Find $\int x \sin x dx$.

Solution Using Formula 1:

Suppose we choose $f(x) = x$ and $g'(x) = \sin x$. Then $f'(x) = 1$ and $g(x) = -\cos x$.

(For g we can choose *any* antiderivative of g' .) Thus, using Formula 1, we have

$$\begin{aligned} \int x \sin x dx &= f(x)g(x) - \int g(x)f'(x) dx \\ &= x(-\cos x) - \int (-\cos x) dx \\ &= x(-\cos x) - \int (-\cos x) dx \\ &= -x \cos x + \sin x + C \end{aligned}$$



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Example 1 – Solution (1 of 2)

It's wise to check the answer by differentiating it. If we do so, we get $x \sin x$, as expected.

Solution Using Formula 2:

Let $u = x \quad dv = \sin x \, dx$

Then $du = dx \quad v = -\cos x$



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Example 1 – Solution (2 of 2)

So

$$\begin{aligned} \int x \sin x \, dx &= \int \overbrace{x}^u \overbrace{\sin x \, dx}^{dv} \\ &= \overbrace{x}^u \overbrace{(-\cos x)}^v - \int \overbrace{(-\cos x)}^u \overbrace{dx}^{du} \\ &= -x \cos x + \int \cos x \, dx \\ &= -x \cos x + \sin x + C \end{aligned}$$



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Integration by Parts (4 of 4)

If we combine the formula for integration by parts with Part 2 of the Fundamental Theorem of Calculus, we can evaluate definite integrals by parts. Evaluating both sides of Formula 1 between a and b , assuming f' and g' are continuous, and using the Fundamental Theorem, we obtain

$$6 \quad \int_a^b f(x)g'(x) \, dx = f(x)g(x) \Big|_a^b - \int_a^b g(x)f'(x) \, dx$$



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