



Chapter 7

Techniques of Integration

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7.4 Integration of Rational Functions by Partial Fractions

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Integration of Rational Functions by Partial Fractions (1 of 16)

In this section we show how to integrate any rational function (a ratio of polynomials) by expressing it as a sum of simpler fractions, called *partial fractions*, that we already know how to integrate.

To illustrate the method, observe that by taking the fractions $\frac{2}{x-1}$ and $\frac{1}{x+2}$ to a common denominator we obtain

$$\frac{2}{x-1} - \frac{1}{x+2} = \frac{2(x+2) - (x-1)}{(x-1)(x+2)} = \frac{x+5}{x^2+x-2}$$

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Integration of Rational Functions by Partial Fractions (2 of 16)

If we now reverse the procedure, we see how to integrate the function on the right side of this equation:

$$\int \frac{x+5}{x^2+x-2} dx = \int \left(\frac{2}{x-1} - \frac{1}{x+2} \right) dx$$

$$= 2\ln|x-1| - \ln|x+2| + C$$



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Integration of Rational Functions by Partial Fractions (3 of 16)

To see how the method of partial fractions works in general, let's consider a rational function

$$f(x) = \frac{P(x)}{Q(x)}$$

where P and Q are polynomials. It's possible to express f as a sum of simpler fractions provided that the degree of P is less than the degree of Q . Such a rational function is called *proper*.



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Integration of Rational Functions by Partial Fractions (4 of 16)

We know that if

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

where $a_n \neq 0$, then the degree of P is n and we write $\deg(P) = n$.

If f is *improper*, that is, $\deg(P) \geq \deg(Q)$, then we must take the preliminary step of dividing Q into P (by long division) until a remainder $R(x)$ is obtained such that $\deg(R) < \deg(Q)$.



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Integration of Rational Functions by Partial Fractions (5 of 16)

The division statement is

$$1 \ f(x) = \frac{P(x)}{Q(x)} = S(x) + \frac{R(x)}{Q(x)}$$

where S and R are also polynomials.

As the next example illustrates, sometimes this preliminary step is all that is required.



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Example 1

Find $\int \frac{x^3 + x}{x-1} dx$.

Solution:

Since the degree of the numerator is greater than the degree of the denominator, we first perform the long division.

This enables us to write

$$\begin{aligned} \int \frac{x^3 + x}{x-1} dx &= \int \left(x^2 + x + 2 + \frac{2}{x-1} \right) dx \\ &= \frac{x^3}{3} + \frac{x^2}{2} + 2x + 2 \ln|x-1| + C \end{aligned}$$



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Integration of Rational Functions by Partial Fractions (6 of 16)

The next step is to factor the denominator $Q(x)$ as far as possible.

It can be shown that any polynomial Q can be factored as a product of linear factors (of the form $ax + b$) and irreducible quadratic factors (of the form $ax^2 + bx + c$, where $b^2 - 4ac < 0$).

For instance, if $Q(x) = x^4 - 16$, we could factor it as

$$Q(x) = (x^2 - 4)(x^2 + 4) = (x - 2)(x + 2)(x^2 + 4)$$



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Integration of Rational Functions by Partial Fractions (7 of 16)

The third step is to express the proper rational function $\frac{R(x)}{Q(x)}$ (from Equation 1) as a sum of **partial fractions** of the form

$$\frac{A}{(ax + b)^j} \quad \text{or} \quad \frac{Ax + B}{(ax^2 + bx + c)^j}$$

A theorem in algebra guarantees that it is always possible to do this. We explain the details for the four cases that occur.



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Integration of Rational Functions by Partial Fractions (8 of 16)

Case I The denominator $Q(x)$ is a product of distinct linear factors.

This means that we can write

$$Q(x) = (a_1x + b_1)(a_2x + b_2) \cdots (a_kx + b_k)$$

where no factor is repeated (and no factor is a constant multiple of another).



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Integration of Rational Functions by Partial Fractions (9 of 16)

In this case the partial fraction theorem states that there exist constants A_1, A_2, \dots, A_k such that

$$2 \quad \frac{R(x)}{Q(x)} = \frac{A_1}{a_1x + b_1} + \frac{A_2}{a_2x + b_2} + \cdots + \frac{A_k}{a_kx + b_k}$$

These constants can be determined as in the following example.



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Example 2

Evaluate $\int \frac{x^2 + 2x - 1}{2x^3 + 3x^2 - 2x} dx$.

Solution:

Since the degree of the numerator is less than the degree of the denominator, we don't need to divide.

We factor the denominator as

$$\begin{aligned} 2x^3 + 3x^2 - 2x &= x(2x^2 + 3x - 2) \\ &= x(2x - 1)(x + 2) \end{aligned}$$



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Example 2 – Solution (1 of 4)

Since the denominator has three distinct linear factors, the partial fraction decomposition of the integrand (2) has the form

$$3 \quad \frac{x^2 + 2x - 1}{x(2x - 1)(x + 2)} = \frac{A}{x} + \frac{B}{2x - 1} + \frac{C}{x + 2}$$

To determine the values of A , B , and C , we multiply both sides of this equation by the product of the denominators, $x(2x - 1)(x + 2)$, obtaining

$$4 \quad x^2 + 2x - 1 = A(2x - 1)(x + 2) + Bx(x + 2) + Cx(2x - 1)$$



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Example 2 – Solution (2 of 4)

Expanding the right side of Equation 4 and writing it in the standard form for polynomials, we get

$$5 \quad x^2 + 2x - 1 = (2A + B + 2C)x^2 + (3A + 2B - C)x - 2A$$

The polynomials in Equation 5 are identical, so their coefficients must be equal. The coefficient of x^2 on the right side, $2A + B + 2C$, must equal the coefficient of x^2 on the left side—namely, 1.

Likewise, the coefficients of x are equal and the constant terms are equal.



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Example 2 – Solution (3 of 4)

This gives the following system of equations for A , B , and C :

$$2A + B + 2C = 1$$

$$3A + 2B - C = 2$$

$$-2A = -1$$

Solving, we get, $A = \frac{1}{2}$, $B = \frac{1}{5}$, and $C = -\frac{1}{10}$, and so

$$\int \frac{x^2 + 2x - 1}{2x^3 + 3x^2 - 2x} dx = \int \left(\frac{1}{2} \frac{1}{x} + \frac{1}{5} \frac{1}{2x-1} - \frac{1}{10} \frac{1}{x+2} \right) dx$$



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Example 2 – Solution (4 of 4)

$$= \frac{1}{2} \ln|x| + \frac{1}{10} \ln|2x-1| - \frac{1}{10} \ln|x+2| + K$$

In integrating the middle term we have made the mental substitution $u = 2x - 1$, which gives $du = 2dx$ and $dx = \frac{1}{2} du$.



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Integration of Rational Functions by Partial Fractions (10 of 16)**Note:**

We can use an alternative method to find the coefficients A , B , and C in Example 2. Equation 4 is an identity; it is true for every value of x . Let's choose values of x that simplify the equation.

If we put $x = 0$ in Equation 4, then the second and third terms on the right side vanish and the equation then becomes $-2A = -1$, or $A = \frac{1}{2}$.

Likewise, $x = \frac{1}{2}$ gives $\frac{5B}{4} = \frac{1}{4}$ and $x = -2$ gives $10C = -1$, so $B = \frac{1}{5}$ and $C = -\frac{1}{10}$.



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Integration of Rational Functions by Partial Fractions (11 of 16)

Case II $Q(x)$ is a product of linear factors, some of which are repeated.

Suppose the first linear factor $(a_1x + b_1)$ is repeated r times; that is, $(a_1x + b_1)^r$ occurs in the factorization of $Q(x)$. Then instead of the single term $\frac{A_1}{(a_1x + b_1)}$ in Equation 2, we would use

$$7 \quad \frac{A_1}{a_1x + b_1} + \frac{A_2}{(a_1x + b_1)^2} + \cdots + \frac{A_r}{(a_1x + b_1)^r}$$



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Integration of Rational Functions by Partial Fractions (12 of 16)

By way of illustration, we could write

$$\frac{x^3 - x + 1}{x^2(x-1)^3} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} + \frac{D}{(x-1)^2} + \frac{E}{(x-1)^3}$$

but we prefer to work out in detail a simpler example.



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Example 4

Find $\int \frac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1} dx$.

Solution:

The first step is to divide. The result of long division is

$$\frac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1} = x + 1 + \frac{4x}{x^3 - x^2 - x + 1}$$



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Example 4 – Solution (1 of 4)

The second step is to factor the denominator $Q(x) = x^3 - x^2 - x + 1$.

Since $Q(1) = 0$, we know that $x - 1$ is a factor and we obtain

$$\begin{aligned} x^3 - x^2 - x + 1 &= (x-1)(x^2 - 1) \\ &= (x-1)(x-1)(x+1) \\ &= (x-1)^2(x+1) \end{aligned}$$



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Example 4 – Solution (2 of 4)

Since the linear factor $x - 1$ occurs twice, the partial fraction decomposition is

$$\frac{4x}{(x-1)^2(x+1)} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{x+1}$$

Multiplying by the least common denominator, $(x-1)^2(x+1)$, we get

$$8 \quad 4x = A(x-1)(x+1) + B(x+1) + C(x-1)^2$$



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Example 4 – Solution (3 of 4)

$$= (A+C)x^2 + (B-2C)x + (-A+B+C)$$

Now we equate coefficients:

$$\begin{aligned} A + C &= 0 \\ B - 2C &= 4 \\ -A + B + C &= 0 \end{aligned}$$



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Example 4 – Solution (4 of 4)

Solving, we obtain $A = 1$, $B = 2$, and $C = -1$, so

$$\begin{aligned}\int \frac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1} dx &= \int \left[x + 1 + \frac{1}{x-1} + \frac{2}{(x-1)^2} - \frac{1}{x+1} \right] dx \\ &= \frac{x^2}{2} + x + \ln|x-1| - \frac{2}{x-1} - \ln|x+1| + K \\ &= \frac{x^2}{2} + x - \frac{2}{x-1} + \ln \left| \frac{x-1}{x+1} \right| + K\end{aligned}$$



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Integration of Rational Functions by Partial Fractions (13 of 16)

Case III $Q(x)$ contains irreducible quadratic factors, none of which is repeated.

If $Q(x)$ has the factor $ax^2 + bx + c$, where $b^2 - 4ac < 0$, then, in addition to the partial fractions in Equations 2 and 7, the expression for $\frac{R(x)}{Q(x)}$ will have a term of the form

$$9 \quad \frac{Ax + B}{ax^2 + bx + c}$$

where A and B are constants to be determined.



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Integration of Rational Functions by Partial Fractions (14 of 16)

For instance, the function given by $f(x) = \frac{x}{(x-2)(x^2+1)(x^2+4)}$ has a partial fraction decomposition of the form

$$\frac{x}{(x-2)(x^2+1)(x^2+4)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{x^2+4}$$

The term given in (9) can be integrated by completing the square (if necessary) and using the formula

$$10 \quad \int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + C$$



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Example 6

Evaluate $\int \frac{4x^2 - 3x + 2}{4x^2 - 4x + 3} dx$.

Solution:

Since the degree of the numerator is *not less than* the degree of the denominator, we first divide and obtain

$$\frac{4x^2 - 3x + 2}{4x^2 - 4x + 3} = 1 + \frac{x - 1}{4x^2 - 4x + 3}$$



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Example 6 – Solution (1 of 3)

Notice that the quadratic $4x^2 - 4x + 3$ is irreducible because its discriminant is $b^2 - 4ac = -32 < 0$. This means it can't be factored, so we don't need to use the partial fraction technique.

To integrate the given function we complete the square in the denominator:

$$4x^2 - 4x + 3 = (2x - 1)^2 + 2$$

This suggests that we make the substitution $u = 2x - 1$.



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Example 6 – Solution (2 of 3)

Then $du = 2 dx$ and $x = \frac{1}{2}(u + 1)$, so

$$\begin{aligned} \int \frac{4x^2 - 3x + 2}{4x^2 - 4x + 3} dx &= \int \left(1 + \frac{x - 1}{4x^2 - 4x + 3} \right) dx \\ &= x + \frac{1}{2} \int \frac{\frac{1}{2}(u + 1) - 1}{u^2 + 2} du \\ &= x + \frac{1}{4} \int \frac{u - 1}{u^2 + 2} du \end{aligned}$$



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Example 6 – Solution (3 of 3)

$$\begin{aligned}
 &= x + \frac{1}{4} \int \frac{u}{u^2 + 2} du - \frac{1}{4} \int \frac{1}{u^2 + 2} du \\
 &= x + \frac{1}{8} \ln(u^2 + 2) - \frac{1}{4} \cdot \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + C \\
 &= x + \frac{1}{8} \ln(4x^2 - 4x + 3) - \frac{1}{4\sqrt{2}} \tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right) + C
 \end{aligned}$$



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Integration of Rational Functions by Partial Fractions (15 of 16)**Note:**

Example 6 illustrates the general procedure for integrating a partial fraction of the form

$$\frac{Ax + B}{ax^2 + bx + c} \text{ where } b^2 - 4ac < 0$$

We complete the square in the denominator and then make a substitution that brings the integral into the form

$$\int \frac{Cu + D}{u^2 + a^2} du = C \int \frac{u}{u^2 + a^2} du + D \int \frac{1}{u^2 + a^2} du$$

Then the first integral is a logarithm and the second is expressed in terms of \tan^{-1} .



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Integration of Rational Functions by Partial Fractions (16 of 16)**Case IV $Q(x)$ contains a repeated irreducible quadratic factor.**

If $Q(x)$ has the factor $(ax^2 + bx + c)^r$, where $b^2 - 4ac < 0$, then instead of the single partial fraction (9), the sum

$$11 \quad \frac{A_1x + B_1}{ax^2 + bx + c} + \frac{A_2x + B_2}{(ax^2 + bx + c)^2} + \cdots + \frac{A_r x + B_r}{(ax^2 + bx + c)^r}$$

occurs in the partial fraction decomposition of $\frac{R(x)}{Q(x)}$. Each of the terms in (11) can be integrated by using a substitution or by first completing the square if necessary.



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Example 8

Evaluate $\int \frac{1-x+2x^2-x^3}{x(x^2+1)^2} dx$.

Solution:

The form of the partial fraction decomposition is

$$\frac{1-x+2x^2-x^3}{x(x^2+1)^2} = \frac{A}{x} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}$$

Multiplying by $x(x^2+1)^2$, we have

$$-x^3+2x^2-x+1 = A(x^2+1)^2 + (Bx+C)x(x^2+1) + (Dx+E)x$$



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Example 8 – Solution (1 of 2)

$$\begin{aligned} &= A(x^4+2x^2+1) + B(x^4+x^2) + C(x^3+x) + Dx^2 + Ex \\ &= (A+B)x^4 + Cx^3 + (2A+B+D)x^2 + (C+E)x + A \end{aligned}$$

If we equate coefficients, we get the system

$$A+B=0 \quad C=-1 \quad 2A+B+D=2 \quad C+E=-1 \quad A=1$$

which has the solution $A=1$, $B=-1$, $C=-1$, $D=1$ and $E=0$.



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Example 8 – Solution (2 of 2)

Thus

$$\begin{aligned} \int \frac{1-x+2x^2-x^3}{x(x^2+1)^2} dx &= \int \left(\frac{1}{x} - \frac{x+1}{x^2+1} + \frac{x}{(x^2+1)^2} \right) dx \\ &= \int \frac{dx}{x} - \int \frac{x}{x^2+1} dx - \int \frac{dx}{x^2+1} + \int \frac{xdx}{(x^2+1)^2} \\ &= \ln|x| - \frac{1}{2} \ln|x^2+1| - \tan^{-1} x - \frac{1}{2(x^2+1)} + K \end{aligned}$$



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Rationalizing Substitutions



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Rationalizing Substitutions

Some nonrational functions can be changed into rational functions by means of appropriate substitutions.

In particular, when an integrand contains an expression of the form $\sqrt[n]{g(x)}$, then the substitution $u = \sqrt[n]{g(x)}$ may be effective. Other instances appear in the exercises.



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Example 9

Evaluate $\int \frac{\sqrt{x+4}}{x} dx$.

Solution:

Let $u = \sqrt{x+4}$. Then $u^2 = x+4$, so $x = u^2 - 4$ and $dx = 2u du$. Therefore

$$\begin{aligned} \int \frac{\sqrt{x+4}}{x} dx &= \int \frac{u}{u^2-4} 2u du \\ &= 2 \int \frac{u^2}{u^2-4} du \\ &= 2 \int \left(1 + \frac{4}{u^2-4} \right) du \end{aligned}$$



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Example 9 – Solution

We can evaluate this integral either by factoring $u^2 - 4$ as $(u - 2)(u + 2)$ and using partial fractions or by using Formula 6 with $a = 2$:

$$\begin{aligned} \int \frac{\sqrt{x+4}}{x} dx &= 2 \int du + 8 \int \frac{du}{u^2 - 4} \\ &= 2u + 8 \cdot \frac{1}{2 \cdot 2} \ln \left| \frac{u-2}{u+2} \right| + C \\ &= 2\sqrt{x+4} + 2 \ln \left| \frac{\sqrt{x+4} - 2}{\sqrt{x+4} + 2} \right| + C \end{aligned}$$



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