Lecture Notes for Chapter 2-7 and 2-8

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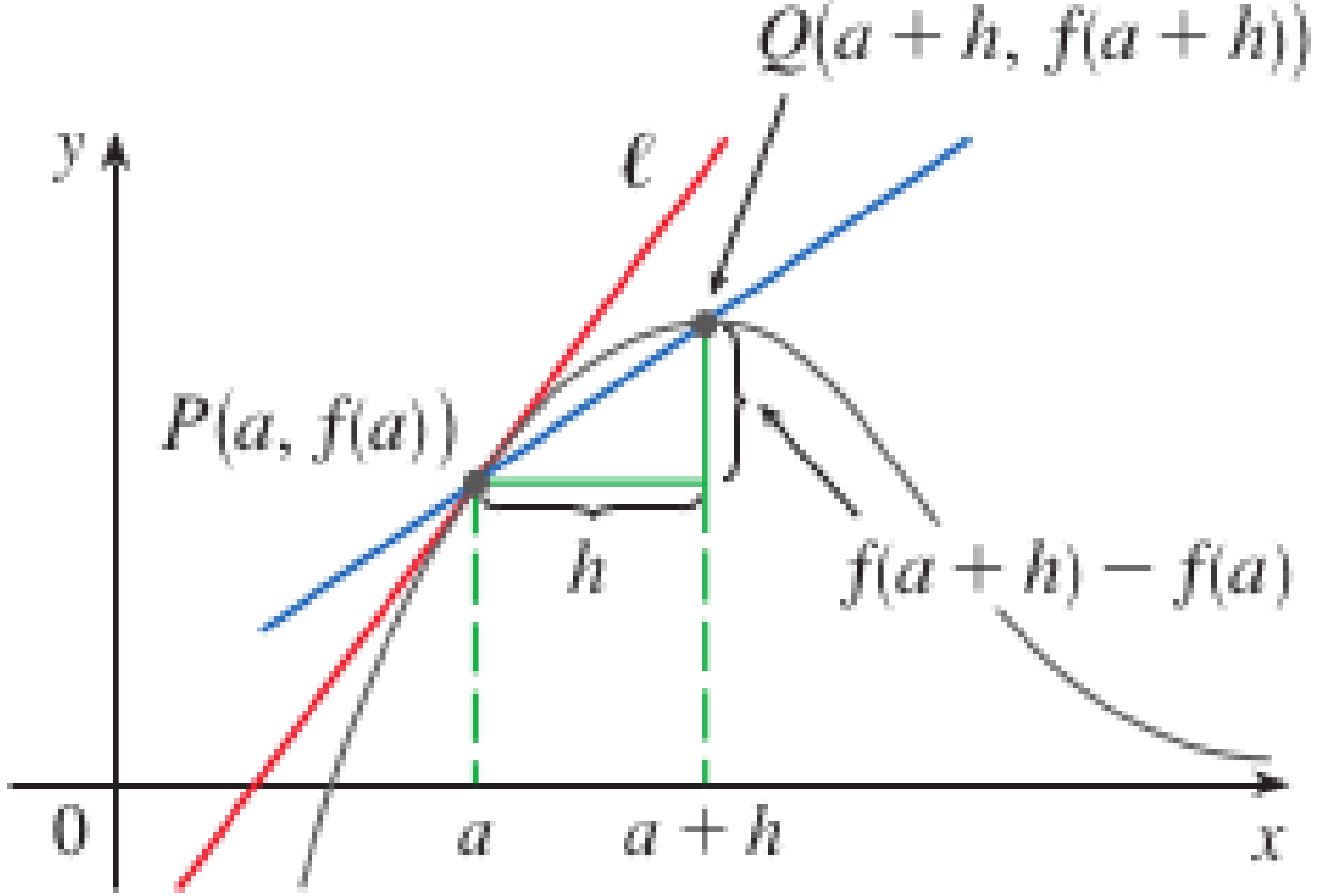
Calculus Early Trans Multi Term Enh Web Assign Acc

Key Topics:
Derivatives and Rates of Change
The Derivative as a Function



$Q(a+h, f(a+h))$
y
ℓ
$P(a, f(a))$
h
$f(a+h) - f(a)$
0
a
$a+h$
x

Tangents

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

Example 1.

Find an equation of the tangent line to the parabola $y = x^2$ at the point $P(1, 1)$

$y = x^2 \quad f(x) = x^2$

$f(1) = 1 \quad a = 1$

$$m = \lim_{x \to 1} \frac{x^2 - 1}{x - 1}$$

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} \to \frac{(x-1)(x+1)}{(x-1)} \to x+1$$

$$\lim_{x \to 1} x + 1 = \boxed{2 = m}$$ at $(1, 1)$ on $y = x^2$

Point-slope

$$(y - y_1) = m(x - x_1)$$

$$\boxed{(y - 1) = 2(x - 1)}$$

Find an equation of the tangent line to the hyperbola

$y = 3/x$ at the point $(3, 1)$

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$f(x) = 3/x$

$a = 3 \quad f(3) = 1$

$f(3+h) = 3/(3+h)$

$$m = \lim_{h \to 0} \frac{\frac{3}{3+h} - 1}{h}$$

$$m = \lim_{h \to 0} \frac{\frac{3}{3+h} - 1}{h}$$

$$\frac{3}{3+h} - 1 = \frac{3-(3+h)}{3+h}$$

$$= \frac{-h}{3+h}$$

$$\lim_{h \to 0} \frac{\frac{-h}{3+h}}{h} = \frac{-\cancel{h}}{3+h} \cdot \frac{1}{\cancel{h}}$$

$$\lim_{h \to 0} \frac{-1}{3+h} = \boxed{\frac{-1}{3}}$$

$(3, 1)$ $m = -1/3$

$$y - 1 = \frac{-1}{3}(x - 3)$$

The **derivative of a function** $\boldsymbol{f}$ **at a number** $\boldsymbol{a}$, denoted by $f'(a)$, is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

if this limit exists.

Use Definition 4 to find the derivative of the function

$f(x) = x^2 - 8x + 9$ at the

number (a) 2 and (b) a .

$f(x) = x^2 - 8x + 9$

$f'(2) = -4$

$f'(a) =$

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \frac{((2+h)^2 - 8(2+h) + 9) - (-3)}{h}$$

$$\lim_{h \to 0} \frac{((a+h)^2 - 8(a+h) + 9) - (a^2 - 8a + 9)}{h}$$

$f(x) = x^2 - 8x + 9$

$f(2+h) = (2+h)^2 - 8(2+h) + 9 \quad f(2) = -3$

Top $\left((2+h)^2 - 8(2+h) + 9\right) - (-3)$

$4 + 4h + h^2 - 16 - 8h + 9 + 3$

$h^2 - 4h = \boxed{h(h-4)}$

$\lim_{h \to 0} \frac{\cancel{h}(h-4)}{\cancel{h}} = h - 4 \to \boxed{-4}$

$$\lim_{h\to 0} \frac{\left((a+h)^2-8(a+h)+9\right) - (a^2-8a+9)}{h}$$

$$\cancel{a^2}+2ah+h^2\cancel{-8a}-8h\cancel{+9}\ \boxed{\cancel{-a^2}\cancel{+8a}\cancel{-9}}$$

$$2ah+h^2-8h \to \frac{h(2a+h-8)}{h}$$

$$\lim_{h\to 0} 2a+h-8 = \boxed{2a-8}$$

A manufacturer produces bolts of a fabric with a fixed width. The cost of producing x yards of this fabric is $C = f(x)$ dollars.

(a) What is the meaning of the derivative $f'(x)$? What are its units?

(b) In practical terms, what does it mean to say that $f'(1000) = 9$?

(c) Which do you think is greater, $f'(50)$ or $f'(500)$? What about $f'(5000)$?

$C = f(x)$

$f'(x)$ $\qquad \dfrac{\text{dollars}}{\text{yards}}$

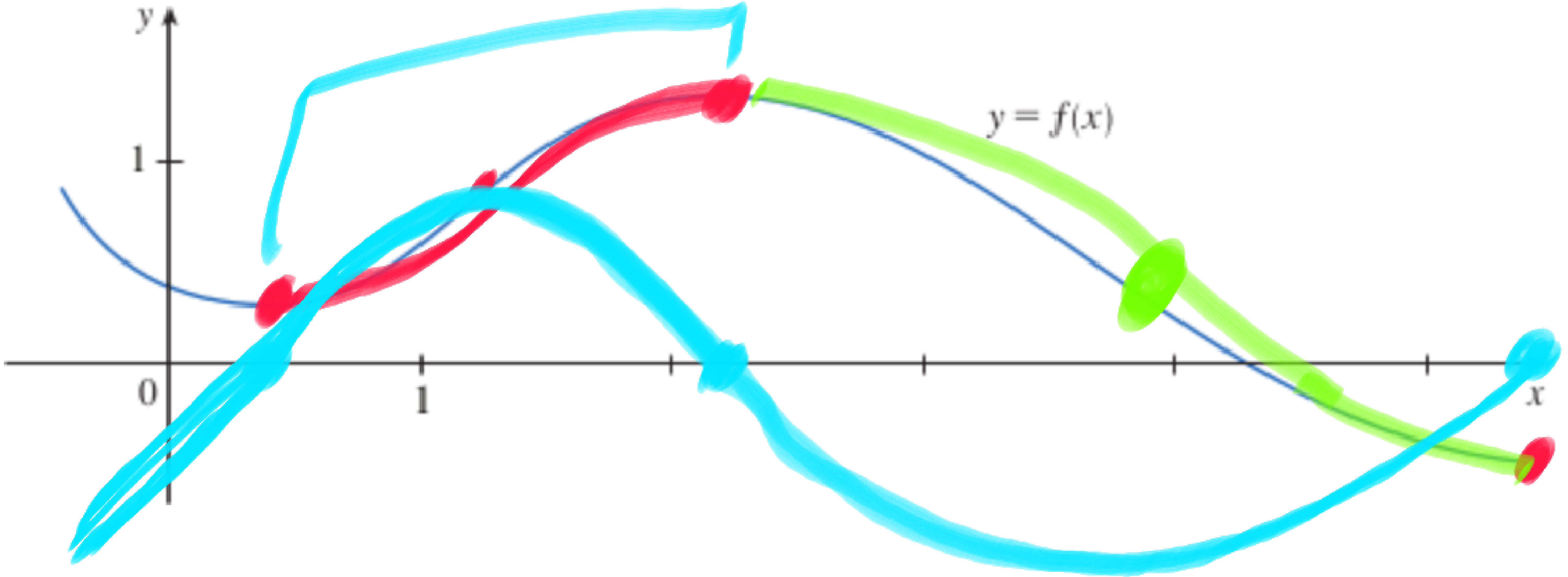
$f'(1000) = 9$

$f'(50) > f'(500)$

$f'(50) > f'(500) > f'(5000)$

The graph of a function f is given in Figure 1. Use it to sketch the graph of the derivative f' .

Figure 1.



(a) If $f(x) = x^3 - x$, find a formula for $f'(x)$.

(b) Illustrate this formula by comparing the graphs of f and f'

$f(x) = x^3 - x$

$f(a) = a^3 - a$

$f(a+h) = (a+h)^3 - (a+h)$

$$\lim_{h \to 0} \frac{((a+h)^3 - (a+h)) - (a^3 - a)}{h}$$

$$[(a+h)^3 - (a+h)] - [a^3 - a]$$

$$(a^2 + 2ah + h^2)(a+h)$$

$$a^3 + 2a^2h + ah^2 + a^2h + 2ah^2 + h^3 - a - h$$

$$a^3 + 3a^2h + 3ah^2 + h^3 - a - h - a^3 + a$$

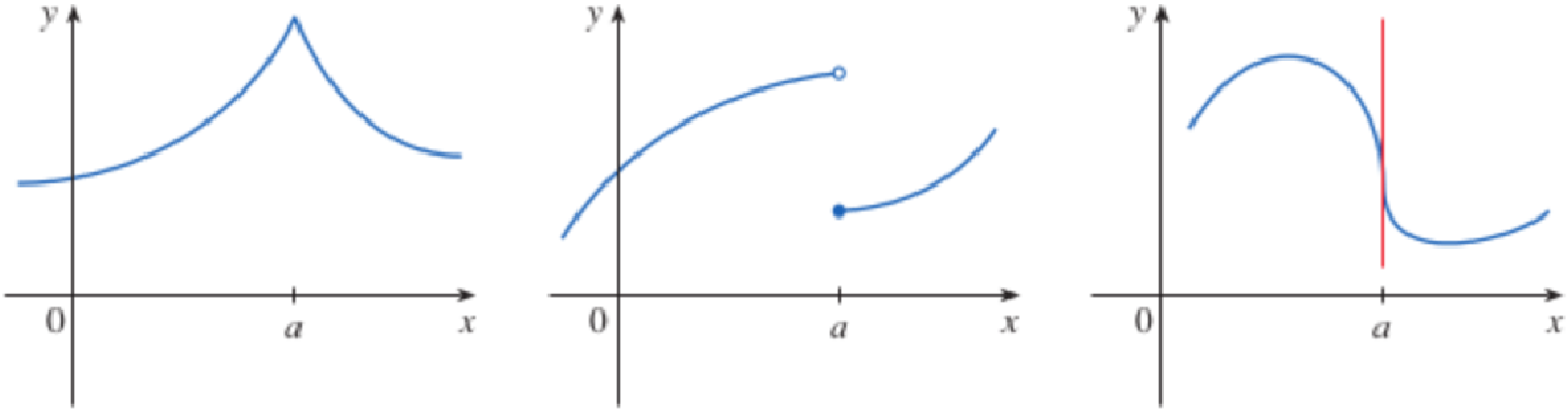
$$\frac{3a^2h + 3ah^2 + h^3 - h}{h} = \frac{\cancel{h}(3a^2 + 3ah + h^2 - 1)}{\cancel{h}}$$

$$\lim_{h \to 0} 3a^2 + 3ah + h^2 - 1 \to \boxed{3a^2 - 1}$$

Other Notations

If we use the traditional notation $y = f(x)$ to indicate that the independent variable is x and the dependent variable is y, then some common alternative notations for the derivative are as follows:

$$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx}f(x) = Df(x) = D_x f(x)$$



(a) A corner

(b) A discontinuity

(c) A vertical tangent

Three ways for f not to be differentiable at a