Lecture Notes for Chapter 3-1

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Calculus Early Trans Multi Term Enh Web Assign Acc

Key Topics:

Constant Functions
Power Functions
New Derivatives from Old
Exponential Functions

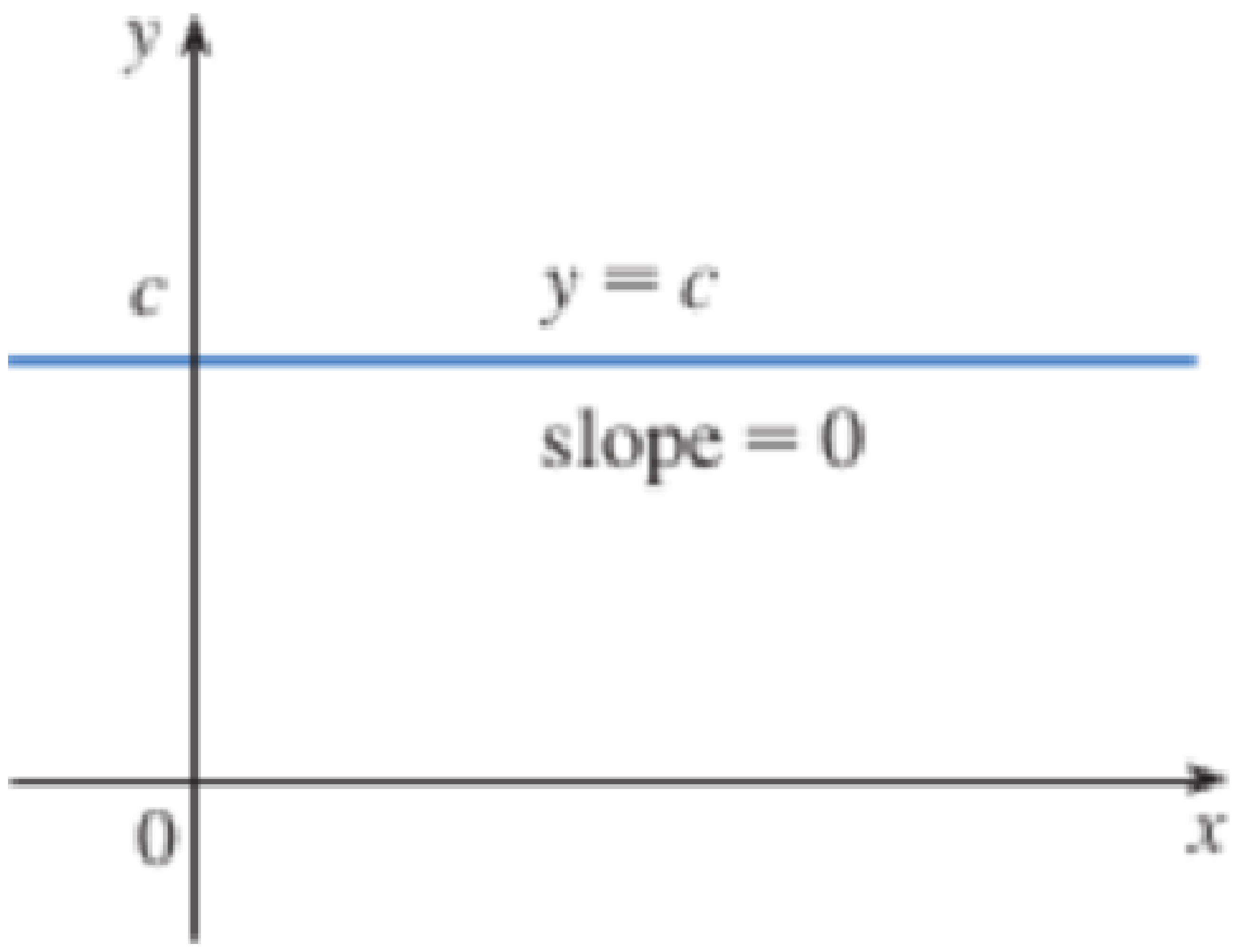
$$\lim_{h \to 0} \frac{(-2+h)^{-1} + 2^{-1}}{h}$$

$$(-2+h)^{-1} + 2^{-1} \to \frac{1}{-2+h} + \frac{1}{2}$$

$$= \frac{2 + (-2+h)}{2(-2+h)}$$

$$\lim_{h \to 0} \frac{\cancel{h}}{-4+2h} * \frac{1}{\cancel{h}} \to \frac{1}{-4+2h} = \frac{1}{-4}$$

Derivative of a Constant Function



$$\frac{d}{dx}(c) = 0$$

If n is any real number, then

$y = \sqrt[3]{x^2} = x^{2/3}$

$y' = \frac{2}{3} x^{-1/3}$

The Power Rule

$$\boxed{\frac{d}{dx}(x^n) = nx^{n-1}}$$

Example 1.

(a) If $f(x) = x^6$ $f'(x) = 6x^5$

(b) If $y = x^{1000}$ $y' = 1000x^{999}$

(c) If $y = t^4$ $y' = 4t^3$

(d) $\frac{d}{dr}(r^3) =$ $3r^2$

Example 2.

Differentiate:

$\frac{1}{x^2} = x^{-2}$

$-2x^{-3}$

$\frac{-2}{x^3} = f'(x)$

(a) $f(x) = \frac{1}{x^2}$

(b) $y = \sqrt[3]{x^2}$

$$y = \sqrt[3]{x^2} = x^{2/3}$$

$$y' = \frac{2}{3} x^{-1/3}$$

$$y' = \frac{2}{3} \cdot \frac{1}{x^{1/3}} = \boxed{\frac{2}{3\sqrt[3]{x}}}$$

Example 3.

Find equations of the tangent line and normal line to the curve $y = x\sqrt{x}$ at the point $(1, 1)$. Illustrate by graphing the curve and these lines.

$y = x\sqrt{x} = x\,x^{1/2} = x^{3/2}$ $\quad y' = \frac{3}{2}x^{1/2}$

Tangent at $(1,1)$ $\quad m = \frac{3}{2}(1)^{1/2} = \frac{3}{2}$

$\boxed{(y-1) = \frac{3}{2}(x-1)}$ $\quad \boxed{y-1 = \frac{-2}{3}(x-1)}$

Tangent $\quad$ Normal

The Constant Multiple Rule

If c is a constant and f is a differentiable function, then

$$\frac{d}{dx}[cf(x)] = c\frac{d}{dx}f(x)$$

Example 4.

(a) $\frac{d}{dx}(3x^4) = 3\frac{d}{dx}x^4 = 3 \cdot 4x^3 = 12x^3$

(b) $\frac{d}{dx}(-x) = -1\frac{d}{dx}x = -1 \cdot 1x^0 = -1$

The Sum and Difference Rules

If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)+g(x)]=\frac{d}{dx}f(x)+\frac{d}{dx}g(x)$$

$$\frac{d}{dx}[f(x)-g(x)]=\frac{d}{dx}f(x)-\frac{d}{dx}g(x)$$

$$\frac{d}{dx}(x^8 + 12x^5 - 4x^4 + 10x^3 - 6x + 5)$$

$$\frac{d}{dx}x^8 + \frac{d}{dx}12x^5 - \frac{d}{dx}4x^4 + \frac{d}{dx}10x^3 - \frac{d}{dx}6x + \frac{d}{dx}5$$

$$8x^7 + 12\cdot 5x^4 - 4\cdot 4x^3 + 10\,3x^2 - 6x^0 + 0$$

$$\boxed{8x^7 + 60x^4 - 16x^3 + 30x^2 - 6}$$

The equation of motion of a particle is $s = 2t^3 - 5t^2 + 3t + 4$, where s is measured in centimeters and t in seconds. Find the acceleration as a function of time. What is the acceleration after 2 seconds?

$S = 2t^3 - 5t^2 + 3t + 4$ position

$S' = V(s) = 6t^2 - 10t + 3$ velocity

$S'' = a(s) = 12t - 10$ acceleraton

$= \boxed{a(2) = 14 \text{ cm}/s^2}$

Derivative of the Natural Exponential Function

$$\frac{d}{dx}(e^x) = e^x$$

Example 8.

If $f(x) = e^x - x$

$f'(x) = e^x - 1$ $f''(x) = e^x$

5.

$$f(x) = x^{75} - x + 3$$

$75x^{74} - 1$

7.

$$f(t) = -2e^t$$

$-2e^t$

13.

$$s(t) = \frac{1}{t} + \frac{1}{t^2}$$

$$s(t) = t^{-1} + t^{-2}$$

$$s'(t) = -1t^{-2} + -2t^{-3}$$

$$= \frac{-1}{t^2} - \frac{2}{t^3}$$

17.

$$g(x) = \frac{1}{\sqrt{x}} + \sqrt[4]{x}$$

$$g(x) = x^{-1/2} + x^{1/4}$$

$$g'(x) = \frac{-1}{2}x^{-3/2} + \frac{1}{4}x^{-3/4}$$

$$= \frac{-1}{2\sqrt{x^3}} + \frac{1}{4\sqrt[4]{x^3}}$$

26.

$$G(t) = \sqrt{5t} + \frac{\sqrt{7}}{t}$$

$$\sqrt{5}\, t^{1/2} + \sqrt{7}\, t^{-1}$$

$$G^{-1}(t) = \sqrt{5} \cdot \frac{1}{2} t^{-1/2} + \sqrt{7} \cdot -1\, t^{-2}$$

$$= \frac{\sqrt{5}}{2\sqrt{t}} - \frac{\sqrt{7}}{t^{2}}$$

29.

$$F(z) = \frac{A + Bz + Cz^2}{z^2}$$

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$$F(z) = \frac{A}{z^2} + \frac{Bz}{z^2} + \frac{Cz^2}{z^2}$$

$$= Az^{-2} + Bz^{-1} + C$$

$$F'(z) = -2Az^{-3} - 1Bz^{-2}$$

$$= \boxed{\frac{-2A}{z^3} - \frac{B}{z^2}}$$

35–36 Find dy/dx and dy/dt

35. $y = tx^2 + t^3x$

$$\frac{dy}{dx} = 2tx + t^3$$

$$\frac{dy}{dt} = x^2 + 3t^2x$$

$$y = x + \frac{2}{x} \quad , \quad (2, 3)$$

$m = 1 - \frac{2}{4} = \frac{1}{2}$

$y = x + 2x^{-1}$

$\boxed{(y - 3) = \frac{1}{2}(x - 2)}$

$y' = 1 - 2x^{-2}$

$= 1 - \frac{2}{x^2}$

$$\sqrt{2\cdot 6} = \sqrt{2}\cdot\sqrt{6}$$

$$\sqrt{5x} = \sqrt{5}\cdot\sqrt{x}$$

$$\sqrt{9x} = 3\sqrt{x} = 3\cdot x^{1/2}$$