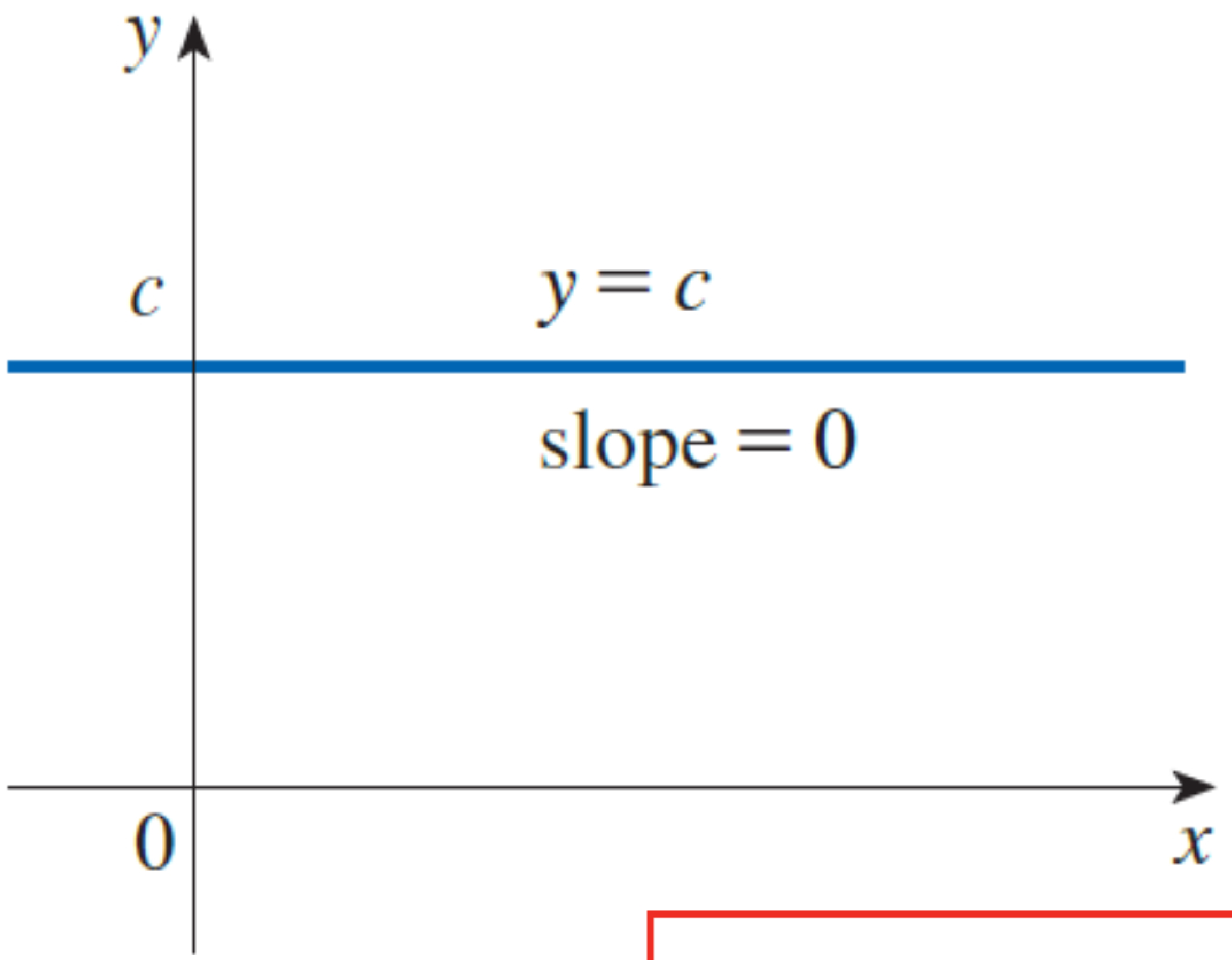
3 Differentiation Rules

3.1 Derivatives of Polynomials and Exponential Functions



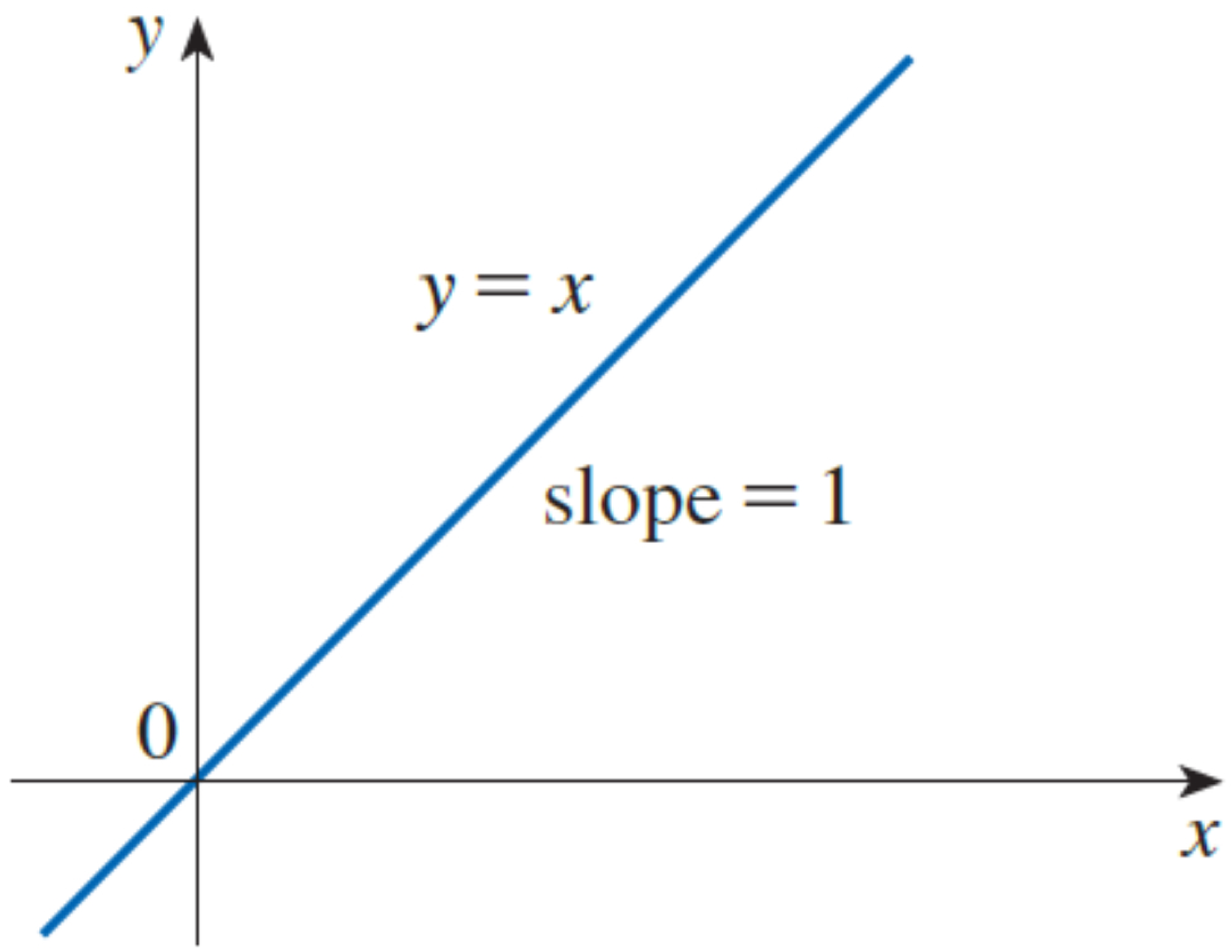
$y = 3$

$y = 5$

$y = c$

Derivative of a Constant Function

$$\frac{d}{dx}(c) = 0$$



$$f(x) = x$$

$$f'(x) = 1$$

$$f(x) = 2x$$

$$f'(x) = 2$$

expand $(x+h)^4$

$$= x^4 + h^4 + 5x^2h^2 + h^2x^2 + 4x^3h + 4xh^3$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^4 - x^4}{h}$$

$$= \cancel{x^4} + h^4 + 5x^2h^2 + h^2x^2 + 4x^3h + 4xh^3 \cancel{- x^4}$$

$$\frac{\cancel{h}(h^3 + 6x^2h + \boxed{4x^3} + 4xh^2)}{\cancel{h}}$$

$f(x) = x^n \qquad f'(x) = nx^{n-1}$

Power Rule

The Power Rule (General Version) If n is any real number, then

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

$$f(x) = x^4 \longrightarrow f'(x) = 4x^3$$

$$f(x) = x^7 \longrightarrow f'(x) = 7x^6$$

$$f(x) = \frac{1}{x^2} = x^{-2} \longrightarrow f'(x) = -2x^{-3} = \frac{-2}{x^3}$$

$$f(x) = \sqrt{x} = x^{1/2} \longrightarrow f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$$

3. $\lim_{x \to a} [cf(x)] = c \lim_{x \to a} f(x)$

$$f(x) = 4x^3 \rightarrow 4 \frac{d}{dx}(x^3)$$

$$= 4(3x^2)$$

$$= 12x^2$$

The Sum and Difference Rules If f and g are both differentiable, then

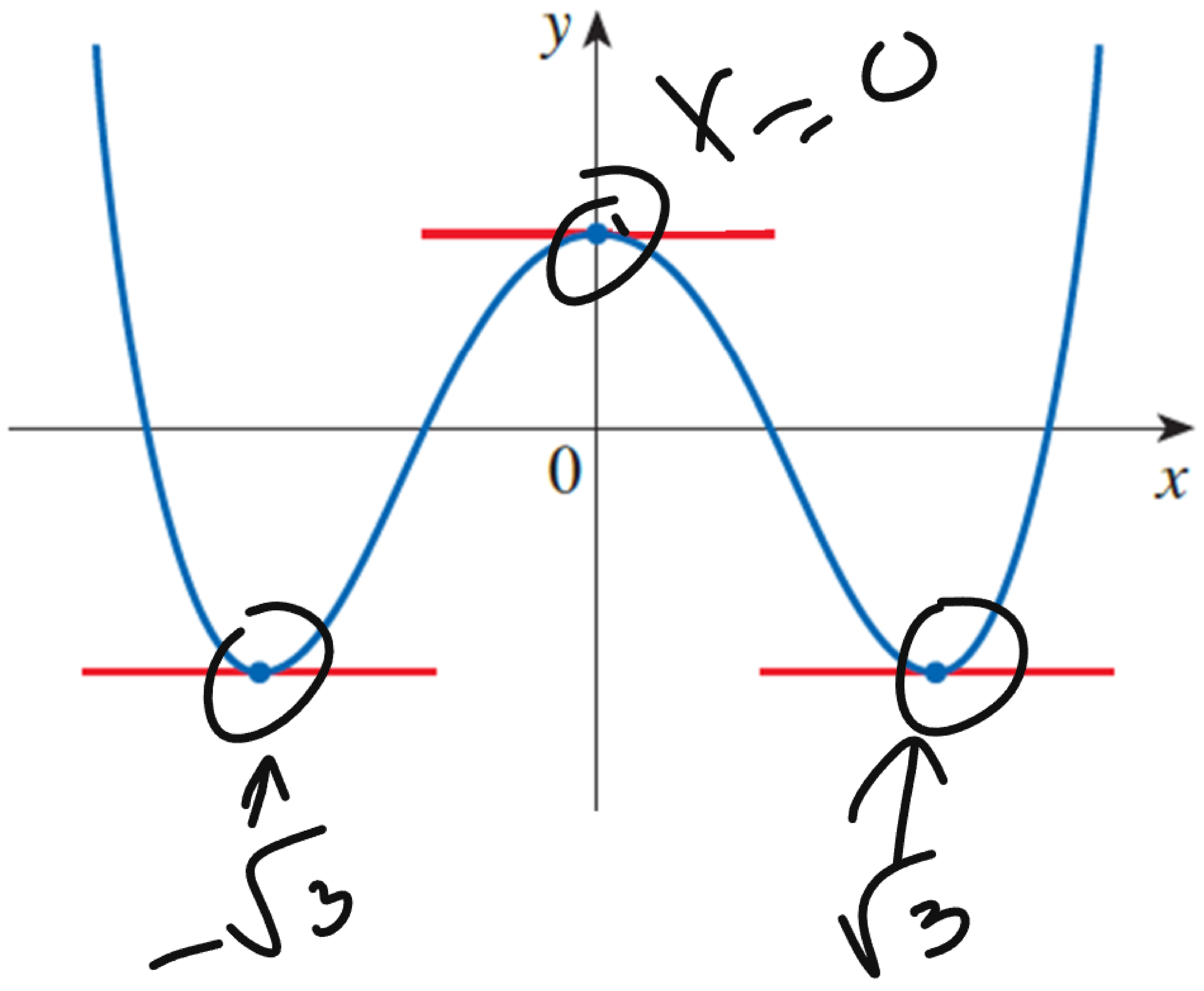
$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

$$\frac{d}{dx}(x^8 + 12x^5 - 4x^4 + 10x^3 - 6x + 5)$$

$$8x^7 + 12(5x^4) - 4(4x^3) + 10(3x^2) - 6(1) + 0$$

$$\boxed{8x^7 + 60x^4 - 16x^3 + 30x^2 - 6}$$



$$y = x^4 - 6x^2 + 4$$

$$y' = 4x^3 - 12x$$

$$0 = 4x^3 - 12x$$

$$0 = 4x(x^2 - 3)$$

$$4x = 0 \qquad x^2 - 3 = 0$$

$$x = 0 \qquad \sqrt{x^2} = \sqrt{3}$$

$$x = \pm\sqrt{3}$$

$$h(x) = x^{\sqrt{2}} \rightarrow h'(x) = \sqrt{2}\, x^{\sqrt{2}-1}$$

$$g(x) = x^{\pi} \rightarrow g'(x) = \pi\, x^{\pi-1}$$

$$m(x) = x^{e} \rightarrow m'(x) = e\, x^{e-1}$$

$$f(x) = e^x \rightarrow f'(x) = e^x$$

Definition of the Number *e*

$$e \text{ is the number such that } \lim_{h \to 0} \frac{e^h - 1}{h} = 1$$

Derivative of the Natural Exponential Function

$$\frac{d}{dx}(e^x) = e^x$$

$$f(x) = e^x - x,$$

$$f'(x) = e^x - 1$$

$$f''(x) = e^x$$

EXAMPLE 7 The equation of motion of a particle is $s = 2t^3 - 5t^2 + 3t + 4$, where s is measured in centimeters and t in seconds. Find the acceleration as a function of time. What is the acceleration after 2 seconds?

$$s = 2t^3 - 5t^2 + 3t + 4$$

$$s' = v = 6t^2 - 10t + 3$$

$$s'' = v' = a = 12t - 10 \qquad a(2) = 14 \text{ cm/s}^2$$

19. $f(x) = x^3(x + 3)$

$^3 + 9x^2$

24. $y = \dfrac{\sqrt{x} + x}{x^2}$

$y = x^{-2}(x^{1/2} + x)$

$$\boxed{y = \frac{x^{1/2}}{x^2} + \frac{x}{x^2}}$$

$y = x^{-3/2} + x^{-1}$

$y' = -\frac{3}{2}x^{-5/2} - 1x^{-2}$

26. $G(t) = \sqrt{5t} + \dfrac{\sqrt{7}}{t}$

$$\sqrt{5}\sqrt{t} + \sqrt{7}\,t^{-1}$$

$$\sqrt{5}\,t^{1/2} + \sqrt{7}\,t^{-1}$$

$$\sqrt{5}\left(\frac{1}{2}t^{-1/2}\right) + \sqrt{7}\left(-1\,t^{-2}\right)$$

$$G'(t) = \frac{\sqrt{5}}{2\sqrt{t}} - \frac{\sqrt{7}}{t^2}$$

31. $D(t) = \dfrac{1 + 16t^2}{(4t)^3} = \dfrac{1 + 16t^2}{64t^3}$

$$= \frac{1}{64t^3} + \frac{16t^2}{64t^3}$$

$$= \tfrac{1}{64}t^{-3} + \tfrac{1}{4}t^{-1}$$

$$= \frac{1}{64}\left(-3t^{-4}\right) + \frac{1}{4}\left(-1t^{-2}\right)$$

$$= \boxed{\frac{-3}{64t^4} - \frac{1}{4t^2}}$$

30. $G(q) = (1 + q^{-1})^2$ $\rightarrow$ $(1 + x^{-1})(1 + x^{-1})$

$$1 + x^{-1} + x^{-1} + x^{-2}$$

$$1 + 2x^{-1} + x^{-2}$$

$$G'(q) = -2x^{-2} - 2x^{-3}$$

34. $y = e^{x+1} + 1$

e^{x+1}

$e^1 e^x$

$y = e e^x + 1$

$y = e e^x = e^{x+1}$