$$f(x) = (x^2 + 3x)(7x^3 + 5)$$

$$f'(x) = 35x^4 + 84x^3 + 10x + 15$$

$$f(x) = x^3 \qquad g(x) = x^2$$

$$fg(x) = (x^3)(x^2)$$

$$(fg)'(x) = (3x^2)(2x) = 6x^3$$

$$(fg)(x) = x^5 \rightarrow (fg)'(x) = 5x^4$$

The Product Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = g(x)\frac{d}{dx}[f(x)] + f(x)\frac{d}{dx}[g(x)]$$

$$f'g + fg'$$

$$(fg)(x) = (x^3)(x^2)$$

$$(fg)'(x) = (3x^2)(x^2) + (x^3)(2x)$$

$$= 3x^4 + 2x^4 = 5x^4$$

$$f(x) = (x^2 + 3x)(7x^3 + 5)$$

$$m(x) = (x^2 + 3x) \rightarrow m'(x) = 2x + 3$$

$$n(x) = (7x^3 + 5) \rightarrow n'(x) = 21x^2$$

$$f'(x) = (2x+3)(7x^3+5) + (x^2+3x)(21x^2)$$

$$14x^4 + 10x + 21x^3 + 15 + 21x^4 + 63x^3$$

$$35x^4 + 84x^3 + 10x + 15$$

$$m(x) = xe^x$$

$$f(x) = x \rightarrow f'(x) = 1$$

$$g(x) = e^x \rightarrow g'(x) = e^x$$

$$m'(x) = f'g + fg'$$

$$= (1)(e^x) + (x)(e^x)$$

$$= \boxed{e^x + xe^x = e^x(1 + x)}$$

$m(x) = \left(x + 2\sqrt{x}\right) e^x$

$f(x) = (x + 2\sqrt{x}) \rightarrow f'(x) = 1 + 2\left(\frac{1}{2}x^{-1/2}\right)$

$(x + 2x^{1/2})$

$= 1 + \frac{1}{\sqrt{x}}$

$g(x) = e^x \rightarrow g'(x) = e^x$

$m'(x) = \left(1 + \frac{1}{\sqrt{x}}\right)(e^x) + (x + 2\sqrt{x})(e^x)$

The Quotient Rule If f and g are differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\dfrac{d}{dx}[f(x)] - f(x)\dfrac{d}{dx}[g(x)]}{[g(x)]^2}$$

$$\frac{f'g - fg'}{g^2}$$

$$f(x) = x^2 + x - 2 \rightarrow f'(x) = 2x + 1$$

$$g(x) = x^3 + 6 \rightarrow g'(x) = 3x^2$$

$$y' = \frac{\boxed{(2x+1)(x^3+6) - (x^2+x-2)(3x^2)}}{(x^3+6)^2}$$

$$2x^4 + 12x + x^3 + 6 - (3x^4 + 3x^3 - 6x^2)$$

$$-x^4 - 2x^3 + 6x^2 + 12x + 6$$

$$y' = \frac{-x^4 - 2x^3 + 6x^2 + 12x + 6}{(x^3+6)^2}$$

$$y = e^x/(1 + x^2)$$

$f = e^x \to f' = e^x$

$g = (1+x^2) \to g' = 2x$

$$y' = \frac{e^x(1+x^2) - e^x(2x)}{(1+x^2)^2}$$

$$= e^x(1 - 2x + x^2) \to \frac{e^x(1-x)^2}{(1+x^2)^2}$$

$$J(u) = \left(\frac{1}{u} + \frac{1}{u^2}\right)\left(u + \frac{1}{u}\right)$$

$$J'(u) = -\frac{1}{u^2} - \frac{2}{u^3} - \frac{3}{u^4}$$

$$J(u) = (u^{-1} + u^{-2})(u + u^{-1})$$

$$= 1 + u^{-2} + u^{-1} + u^{-3}$$

$$= -2u^{-3} + (-1)u^{-2} + (-3)u^{-4}$$

$$= \frac{-1}{u^2} - \frac{2}{u^3} - \frac{3}{u^4}$$

BitPaper Share Knowledge

$$f(x) = \frac{\sqrt[3]{x}}{x-3}$$

$f = x^{1/3}$ $\quad f' = \frac{1}{3}x^{-2/3}$

$g = x-3$ $\quad g' = 1$

$$f'(t) = \frac{(t-3)\left(\frac{1}{3}t^{-2/3}\right) - t^{1/3}(1)}{(t-3)^2} = \frac{\frac{1}{3}t^{1/3} - t^{-2/3} - t^{1/3}}{(t-3)^2} = \frac{-\frac{2}{3}t^{1/3} - t^{-2/3}}{(t-3)^2}$$

$$= \frac{\frac{-2t}{3t^{2/3}} - \frac{3}{3t^{2/3}}}{(t-3)^2} = \frac{-2t-3}{3t^{2/3}(t-3)^2}$$

the specified point.

35. $y = \dfrac{x^2}{1+x}, \quad \left(1, \frac{1}{2}\right)$ $\qquad y' = \dfrac{(1+x)(2x) - x^2(1)}{(1+x)^2}$

$$y'(1) = \frac{(2)(2) - (1)(1)}{(2)^2} = \frac{4-1}{4} = \frac{3}{4}$$

$$y - y_1 = m(x - x_1)$$

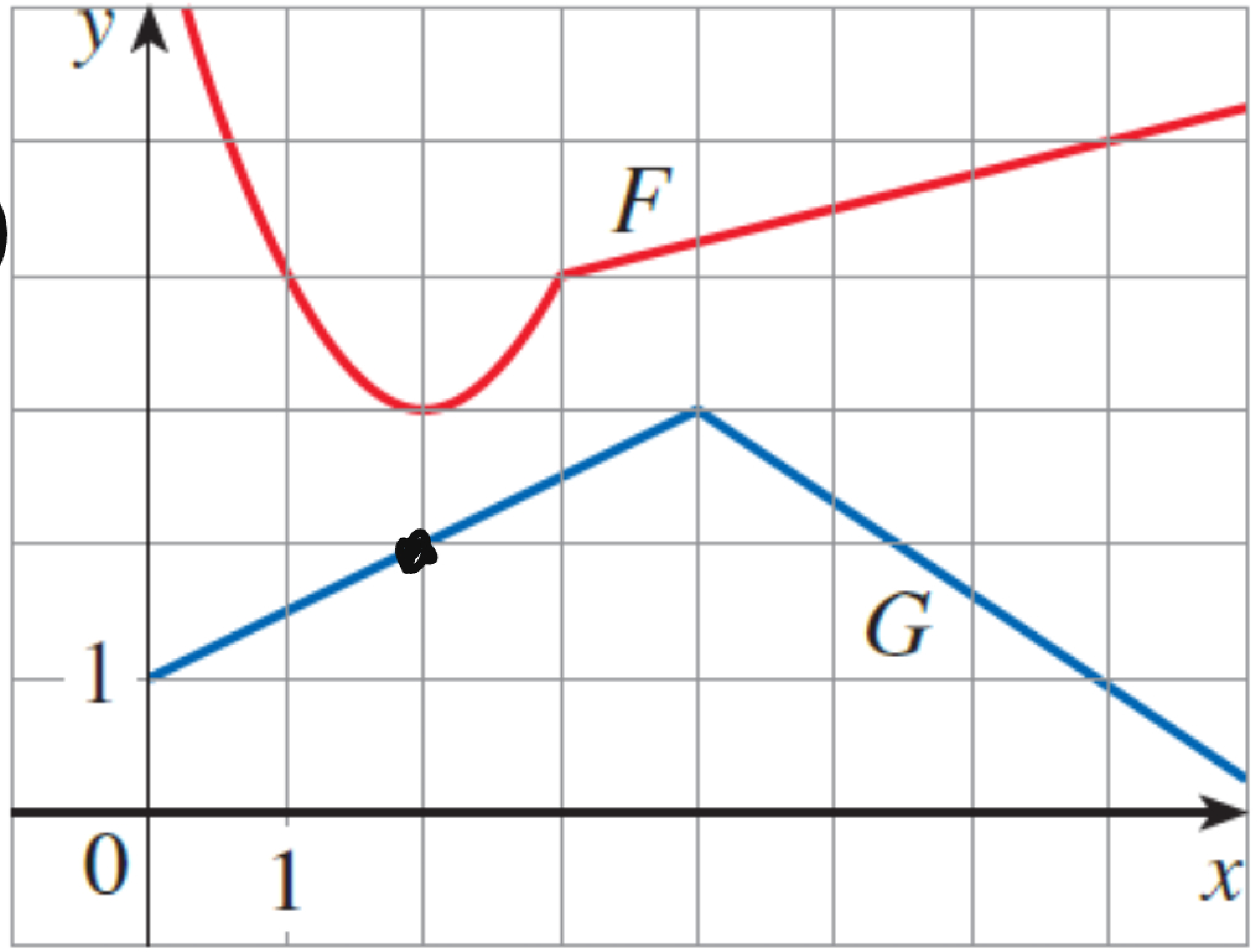
$$y - \frac{1}{2} = \frac{3}{4}(x - 1)$$

$$y - \frac{1}{2} = \frac{3}{4}x - \frac{3}{4} \rightarrow \boxed{y = \frac{3}{4}x - \frac{1}{4}}$$

Let $P(x) = F(x)G(x)$ and $Q(x) = F(x)/G(x)$, where F and G are the functions whose graphs are shown.

(a) Find $P'(2)$. (b) Find $Q'(7)$.

$f'g + fg'$



$P'(2) =$

$(0)(2) + (3)\left(\frac{1}{2}\right)$

$= \frac{3}{2}$

$f(2) = 3$

$g(2) = 2$

$f'(2) = 0$

$g'(2) = \frac{1}{2}$

The curve $y = 1/(1 + x^2)$ is called a **witch of Maria Agnesi**. Find an equation of the tangent line to this curve at the point $\left(-1, \frac{1}{2}\right)$.

$$y = \frac{1}{1+x^2} \rightarrow y' \frac{0(1+x^2) - (1)(2x)}{(1+x^2)^2}$$

$f = 1 \quad f' = 0$

$g = 1 + x^2 \quad g' = 2x$

$$y' = \frac{-2x}{(1+x^2)^2} \qquad y'(-1) = \frac{2}{4} = \frac{1}{2}$$

$$y - \frac{1}{2} = \frac{1}{2}(x+1) \rightarrow \boxed{y = \frac{1}{2}x + 1}$$