Find the derivative of

$$f(x) = 5x^3 + e^x + \ln|x| + 6$$

$$f'(x) = 15x^2 + e^x + \frac{1}{x}$$

$\int$

Antiderivative

or

Integral $\rightarrow \int$

	$5x^3$	$\rightarrow$	$15x^2$
f'	x^n	$\rightarrow$	$n x^{n-1}$
$\int$	$\frac{x^n}{n+1}$		x^n

Integral Power Rule

$$\int x^n = \frac{x^{n+1}}{n+1}$$

$$\int 10x^2 = 10\left(\frac{x^3}{3}\right) = \frac{10}{3}x^3 + C$$

$$\int 5 = 5\left(\frac{x^1}{1}\right) = 5x + C$$

$$\int \frac{1}{x^2} = \int x^{-2} = \frac{x^{-1}}{-1} = -\frac{1}{x} + C$$

$$\int \frac{1}{x} = \int x^{-1} = \cancel{\frac{x^0}{0}} = \ln|x| + C$$

2 Table of Antidifferentiation Formulas

Function	Particular antiderivative	Function	Particular antiderivative
$cf(x)$	$cF(x)$	$\sin x$	$-\cos x$
$f(x) + g(x)$	$F(x) + G(x)$	$\sec^2 x$	$\tan x$
$x^n \ (n \neq -1)$	$\dfrac{x^{n+1}}{n+1}$	$\sec x \ \tan x$	$\sec x$
$\dfrac{1}{x}$	$\ln \lvert x \rvert$	$\dfrac{1}{\sqrt{1 - x^2}}$	$\sin^{-1} x$
e^x	e^x	$\dfrac{1}{1 + x^2}$	$\tan^{-1} x$
b^x	$\dfrac{b^x}{\ln b}$		
$\cos x$	$\sin x$		

Integral

$\sin x$

$\cos x$

$-\sin x$

$-\cos x$

Derivati

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EXAMPLE 2 Find all functions g such that

$$g'(x) = 4\sin x + \boxed{\frac{2x^5 - \sqrt{x}}{x}} \rightarrow \frac{2x^5}{x} - \frac{x^{1/2}}{x}$$

$$\boxed{= 2x^4 - x^{-1/2}}$$

$$\int 4\sin x = 4(-\cos x) = -4\cos x$$

$$\int 2x^4 = \frac{2x^5}{5} = \frac{2}{5}x^5$$

$$\int -x^{-1/2} = -\left(\frac{x^{1/2}}{1/2}\right) = -2x^{1/2} = -2\sqrt{x}$$

$$g(x) = -4\cos x + \frac{2}{5}x^5 - 2\sqrt{x} + C$$

$$20\int \frac{1}{1+x^2}\,dx = 20\tan^{-1}(x)$$

$$f(x) = e^x + 20\tan^{-1}(x) + C \qquad f(0) = -2$$

$$f(0) = e^0 + 20\tan^{-1}(0) + C = -2$$

$$= 1 + 20(0) + C = -2$$

$$1 + C = -2$$

$$C = -3$$

$$\boxed{f(x) = e^x + 20\tan^{-1}(x) - 3}$$

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$$f'(x) = \int f''(x)\,dx = 4x^3 + 3x^2 - 4x + C$$

$$f(x) = \int f'(x)\,dx = x^4 + x^3 - 2x^2 + Cx + D$$

$$f(x) = x^4 + x^3 - 2x^2 + Cx + D$$

$$f(0) = (0)^4 + (0)^3 - 2(0)^2 + C(0) + D = 4 \rightarrow D = 4$$

$$f(1) = (1)^4 + (1)^3 - 2(1)^2 + C(1) + 4 = 1$$

$$C + 4 = 1$$

$$C = -3$$

$$\boxed{f(x) = x^4 + x^3 - 2x^2 - 3x + 4}$$

$\int \rightarrow$ Indefinte

$\int_a^b \rightarrow \int_2^5$ Definte

EXAMPLE 6 A particle moves in a straight line and has acceleration given by $a(t) = 6t + 4$. Its initial velocity is $v(0) = -6$ cm/s and its initial displacement is $s(0) = 9$ cm. Find its position function $s(t)$.

$$a(t) = 6t + 4 = s''(t)$$

$$v(t) = 3t^2 + 4t - 6 = s'(t)$$

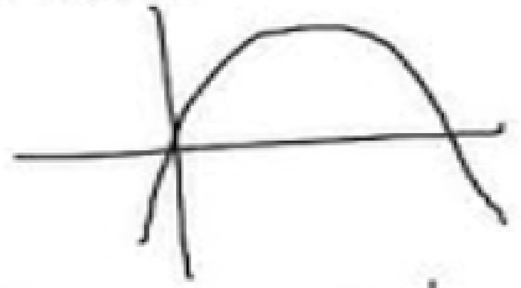
$$\boxed{s(t) = t^3 + 2t^2 - 6t + 9}$$

(initial) velocity   starting height

Example: An object is fired straight up from the top of a 150 ft. tower at a velocity of 75 ft./sec.

a. Write the vertical motion model to reflect the information above.

$h(t) = -16t^2 + 75t + 150$



b. Use the model to predict the height of the object after $\boxed{\text{3 seconds.}}$ $\rightarrow t$

$h(3) = -16(3^2) + 75(3) + 150$

$h(3) = 231$ ft

After 3 seconds, the object is 231 ft above the ground.

c. At what time(s) will the object be 235 ft. off of the ground? Why are there 2 positive answers?

EXAMPLE 7 A ball is thrown upward with a speed of 48 ft/s from the edge of a cliff, 432 ft above the ground. Find its height above the ground t seconds later. When does it reach its maximum height? When does it hit the ground?

$-9.8\,m/s^2 \rightarrow -32\,ft/s^2$

$a(t) = -32$

$v(t) = -32t + 48$

$s(t) = -16t^2 + 48t + 432$

$-32t + 48 = 0$

$t = \frac{48}{32}$

$= 3/2 = 1.5$

$$t = \frac{3 \pm 3\sqrt{13}}{2}$$

$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

47. $f''(\theta) = \sin\theta + \cos\theta, \quad f(0) = 3, \quad f'(0) = 4$

$-\cos\theta + \sin\theta + C = 4$

$-1 \quad + 0 \quad + C \qquad C = 5$

$f'(\theta) = \sin\theta - \cos\theta + 5$

$f(\theta) = -\cos\theta \; - \sin\theta + 5\theta + D$

$= 3$

$-1 \; - 0 \; + 0 \; + D$

$D = 4$

$$\boxed{f(\theta) = -\sin\theta - \cos\theta + 5\theta + 4}$$

80. A car is traveling at 50 mi/h when the brakes are fully applied, producing a constant deceleration of $\boxed{22\ \text{ft/s}^2.}$ What is the distance traveled before the car comes to a stop?

$$a(t) = -22$$

$$v(t) = -22t + \frac{220}{3}$$

$$s(t) = -11t^2 + \frac{220}{3}t$$

$$s\left(\frac{10}{3}\right) = -11\left(\frac{10}{3}\right)^2 + \frac{220}{3}\left(\frac{10}{3}\right)$$

$$s\left(\frac{10}{3}\right) = \frac{1100}{9} = 122.\overline{2}\ \text{ft}$$

$$\frac{50(5280)}{(60)(60)} = \frac{220}{3}$$

$$0 = -22t + \frac{220}{3}$$

$$22t = \frac{220}{3}$$

$$t = \frac{10}{3}$$

24. $g(v) = 2\cos v - \dfrac{3}{\sqrt{1-v^2}}$

$G(v) = 2\sin v - 3\sin^{-1}(v) + C$

26. $f(x) = \dfrac{2x^2+5}{x^2+1}$ Algebra

$f(x) = \dfrac{2(x^2+1)+3}{x^2+1}$

$f(x) = 2 + \dfrac{3}{x^2+1}$

$F(x) = 2x + 3\tan^{-1}(x) + C$

Find $y'(x)$.

$y'(x) =$ $\boxed{\dfrac{2x+2}{2\sqrt{x^2+2x}} - 1}$

Find $y''(x)$.

$y''(x) =$ $\boxed{-\dfrac{1}{(x^2+2x)^{3/2}}}$

$$f(x) = \sqrt{x^2+5x} - x$$

$$\lim_{x\to\infty} \sqrt{x^2+5x} - x \quad \frac{\left(\sqrt{x^2+5x}+x\right)}{}$$

$(5x)$

$$\lim_{x\to\infty} \frac{(x^2+5x)-x^2}{\sqrt{x^2+5x}+x} \overset{H}{=} \frac{5}{\frac{x}{\sqrt{x^2+5x}}+1}$$

$$2x\left(\tfrac{1}{2}(x^2+5x)^{-1/2}\right)$$

Find $y'(x)$.

$x = 0$

$y'(x) =$ $\dfrac{2x+2}{2\sqrt{x^2+2x}} - 1$

Find $y''(x)$.

$y''(x) =$ $-\dfrac{1}{(x^2+2x)^{3/2}}$

$$\frac{2x+2}{2\sqrt{x^2+2x}} - 1 = 0$$

$$2x+2 = 2\sqrt{x^2+2x}$$

$$4x^2 + 8x + 4 = 4(x^2+2x)$$

$$4x^2 + 8x$$

$$4 = 0 \quad X$$

$-$ $\quad$ $\frac{12}{2\sqrt{35}}$ $\quad$ $+$

$-5 \quad 0 \quad 5$

$(-\infty, 0)$ D $\qquad$ $(0, \infty)$ I

$$\boxed{-\frac{1}{(x^2+2x)^{3/2}}}$$

$$(x^2+2x)^{3/2} = 0$$

$$x^2+2x=0$$

$$x(x+2)=0 \qquad x=0 \qquad x=-2$$

$-10 \qquad -1 \qquad 5$

-2 DNE 0

$(-) \qquad (-)$

concave down $(-\infty,-2) \cup (0,\infty)$

Consider the equation below. (If an answer does not exist, enter DNE.)

$$f(x) = 6\cos^2(x) - 12\sin(x), \qquad 0 \le x \le 2\pi$$

$$f'(x) = (12\cos x)(-\sin(x)) - 12\cos x$$

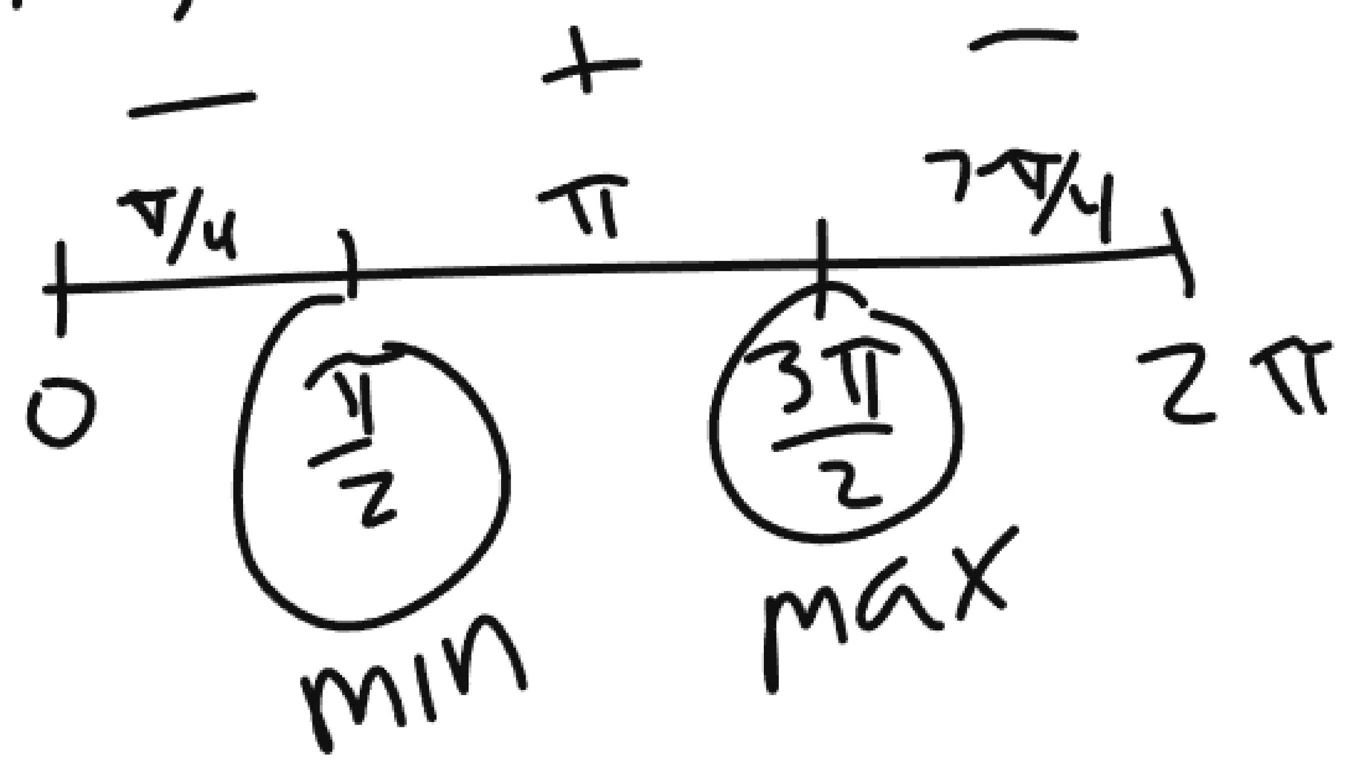
$$= -12\cos x \sin x - 12\cos x$$

$$0 = -12\cos x(\sin x + 1)$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$D \rightarrow \left(0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right)$$

$$I \rightarrow \left(\pi/2, 3\pi/2\right)$$



$-12\sin^2 x$

$(1-\sin^2 x)$

$-12+12\sin^2 x)$

$m = \sin x$

$= 24\sin^2 x + 12\sin x - 12$

$2m^2 + m - 1$

$= 12(2\sin^2 x + \sin x - 1)$

$(2m-1)(m+1)$

$= 12(2\sin x - 1)(\sin x + 1)$

$2\sin x - 1 = 0$

$\sin x = 1/2$

$x = \frac{\pi}{6}, \frac{5\pi}{6}$

$\sin x + 1 = 0$

$\sin x = -1$

$x = \frac{3\pi}{2}$

Find the interval on which *f* is concave up. (Enter your an

$\left(\frac{\pi}{6}, \frac{5\pi}{6}\right)$

Find the interval on which *f* is concave down. (Enter your

$\left(0, \frac{\pi}{6}\right), \left(\frac{5\pi}{6}, \frac{3\pi}{2}\right), \left(\frac{3\pi}{2}, 2\pi\right)$

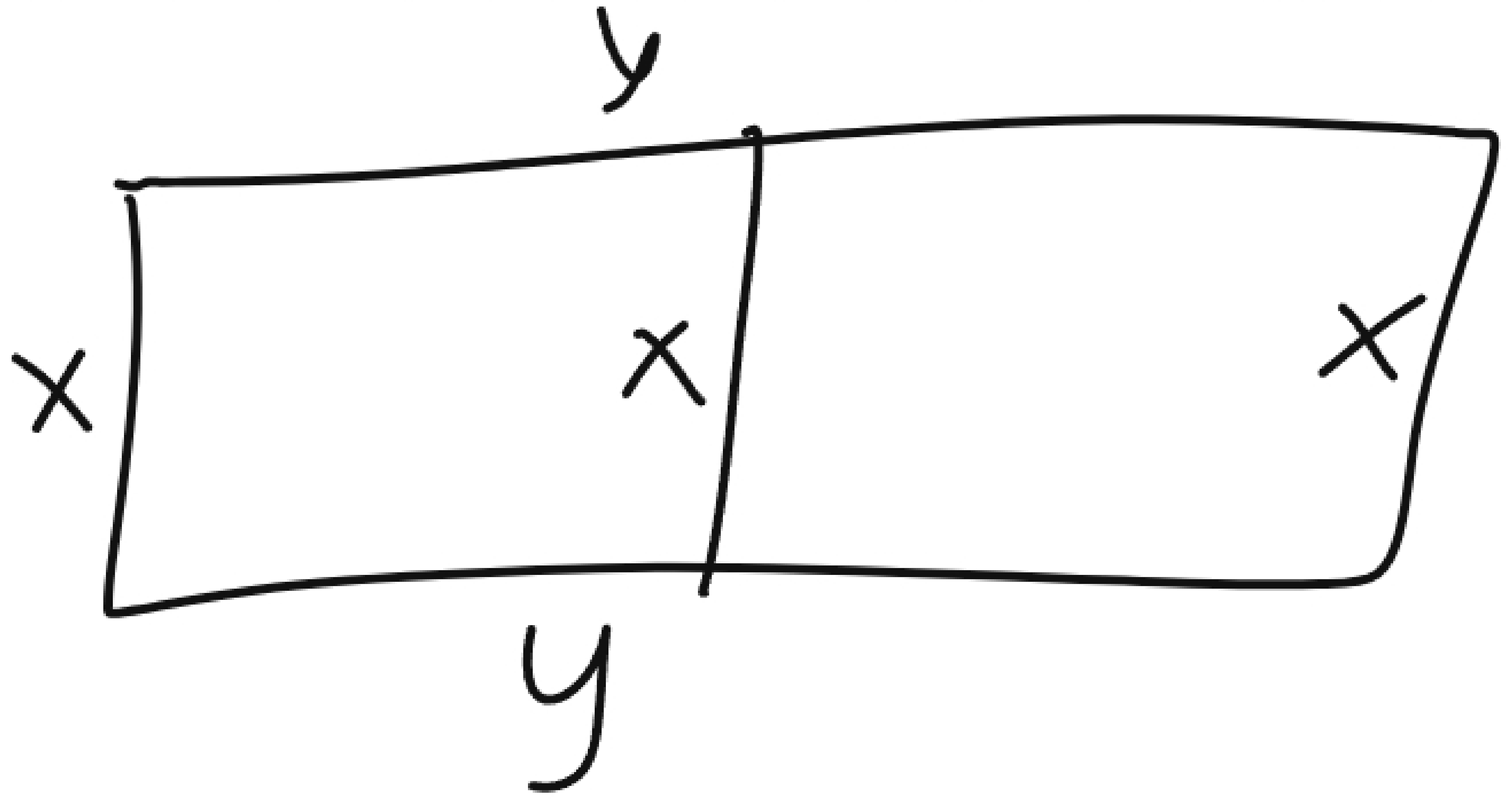
$$\underline{1200b}\boxed{-\frac{b^3}{4}} \qquad 0 < b <$$

$$\frac{b^3}{4} = 1200b$$

$$b^3 = 4800b$$

$$\sqrt{b^2} = \sqrt{4800} = \sqrt{1600 \cdot 3} = 40\sqrt{3}$$

A farmer wants to fence an area of 6 million square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. Let y represent the length (in feet) of a side perpendicular to the dividing fence, and let x represent the length (in feet) of a side parallel to the dividing fence.



$$A = xy$$

$$\frac{37.5\times10^6}{x} = y$$

$$P = 3x + 2y = 3x + 2\left(\frac{37.5\times10^6}{x}\right)$$

$$P = 3x + \frac{75\times10^6}{x} \qquad x^{-1} \to -1x^{-2}$$

$$P' = 3 + (75\times10^6)\left(\frac{-1}{x^2}\right)$$

$$P' = 3 - \frac{75\times10^6}{x^2} = 0$$

$$75\times10^6 = 3x^2$$

$$\sqrt{25\times10^6 = x^2}$$