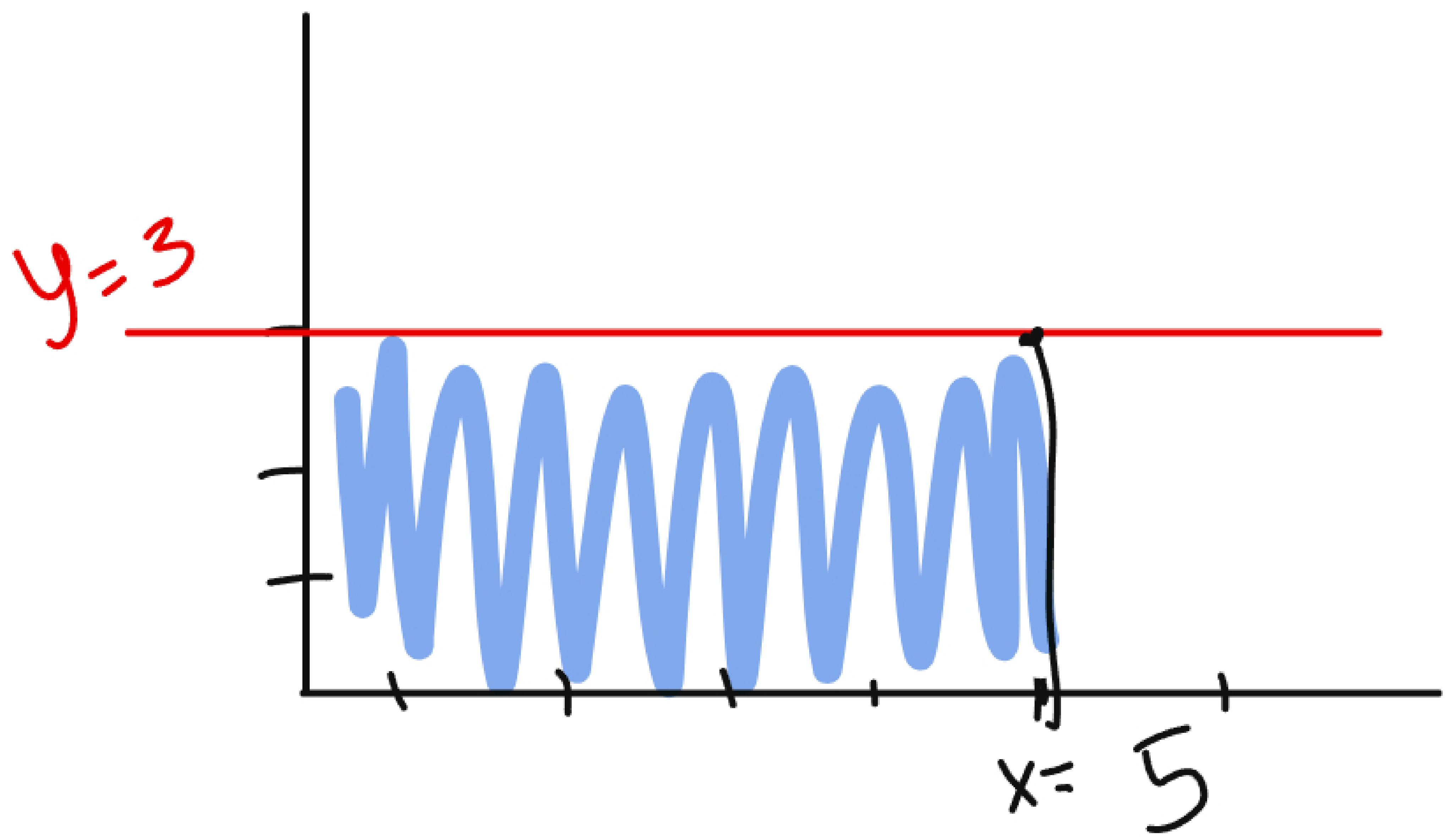
Antiderivative to Integral

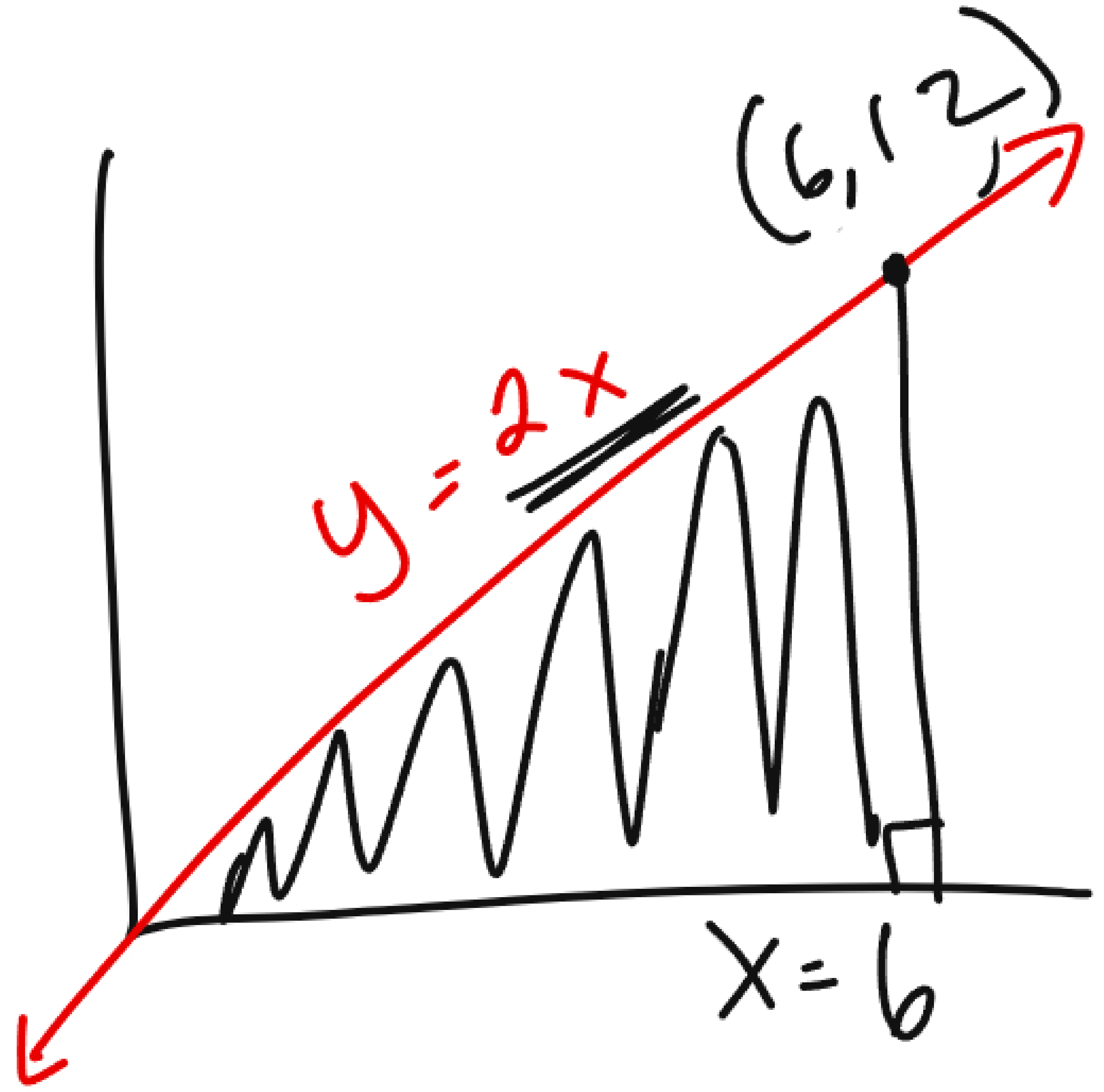


$A = lw$

$A = 3(5)$

$f'(x) = 3$ $\quad$ $f(x) = 3x + c$

$x = 5$ $\quad$ $f(5) = 15$



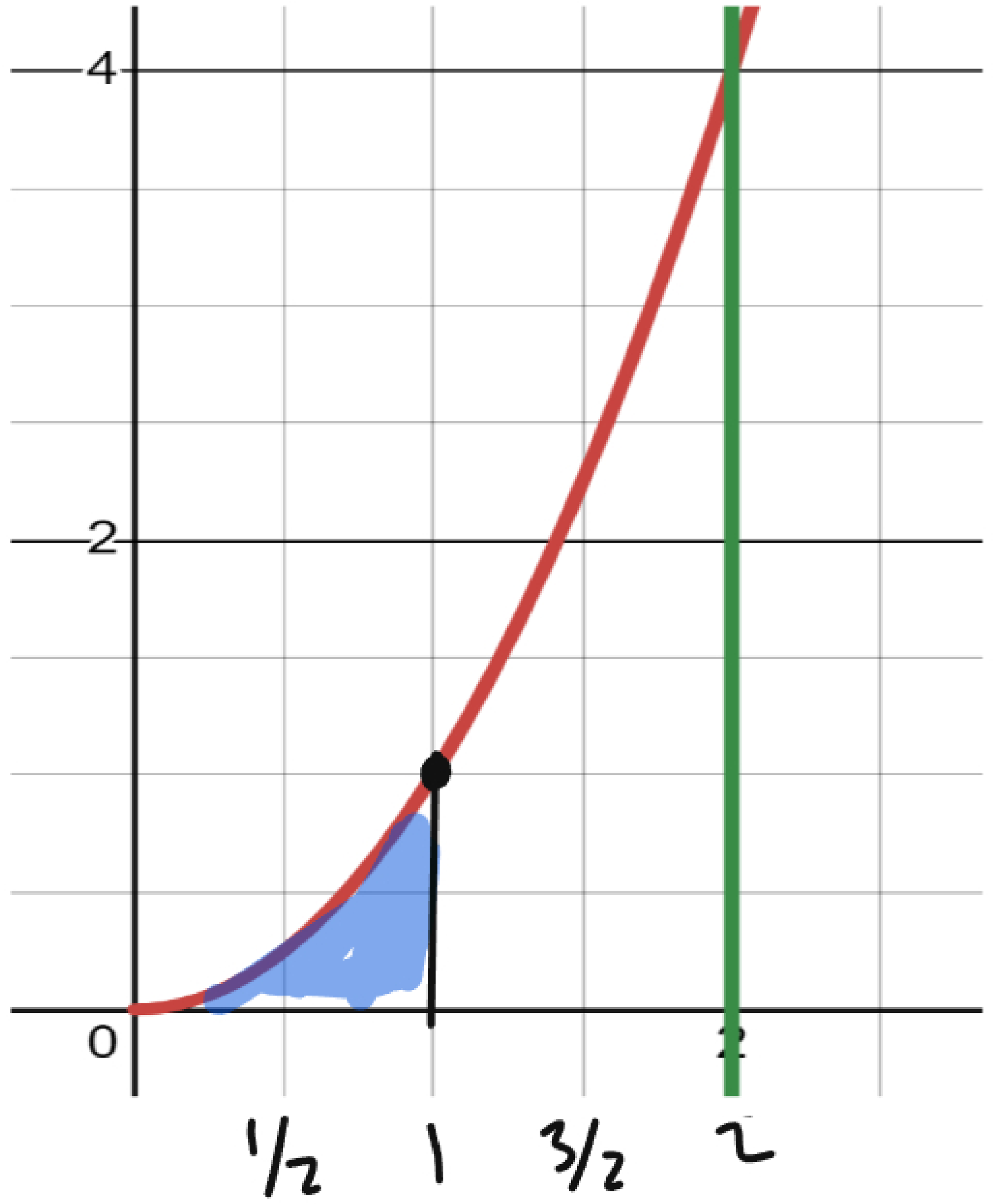
$$A = \frac{1}{2}bh$$

$$A = \frac{1}{2}(6)(12)$$

$$A = 36$$

$$f'(x) = 2x \quad \to \quad f(x) = x^2$$

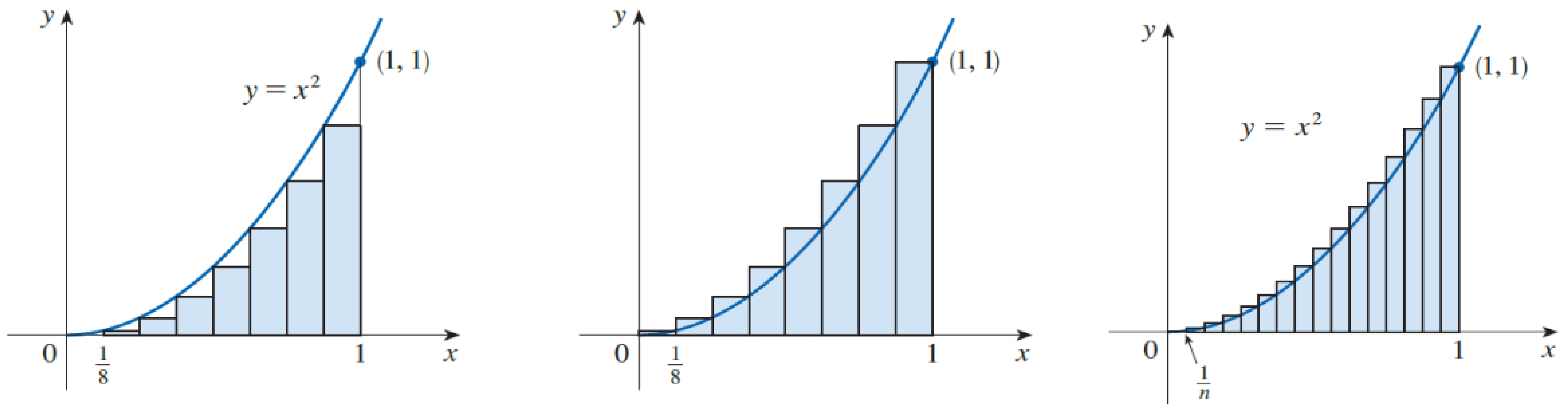
$$x = 6 \quad f(6) = 36$$



$$y = x^2$$

$$y = \frac{x^3}{3} + C$$

$$y = \frac{1}{3}$$



(a) Using left endpoints (b) Using right endpoints

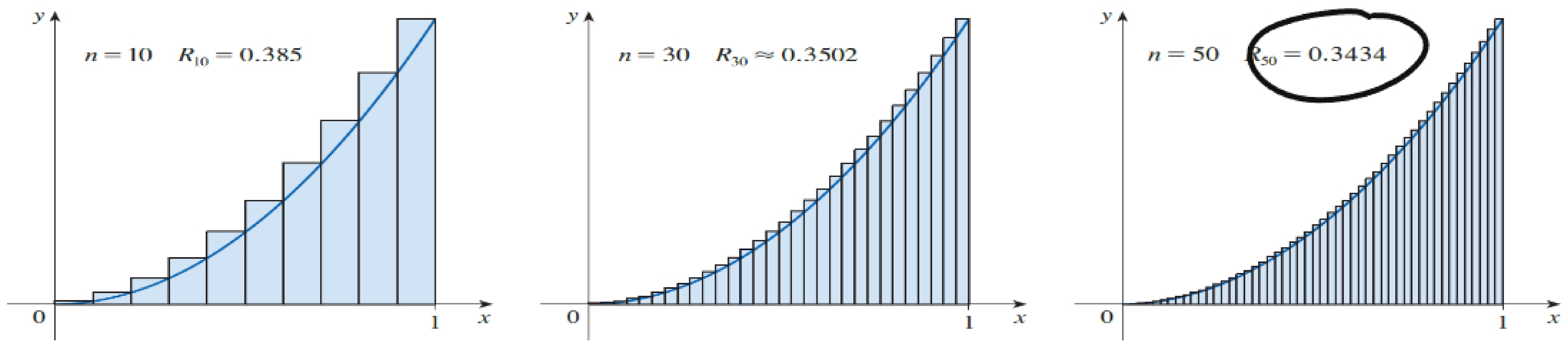


FIGURE 8 Right endpoints produce upper estimates because $f(x) = x^2$ is increasing.

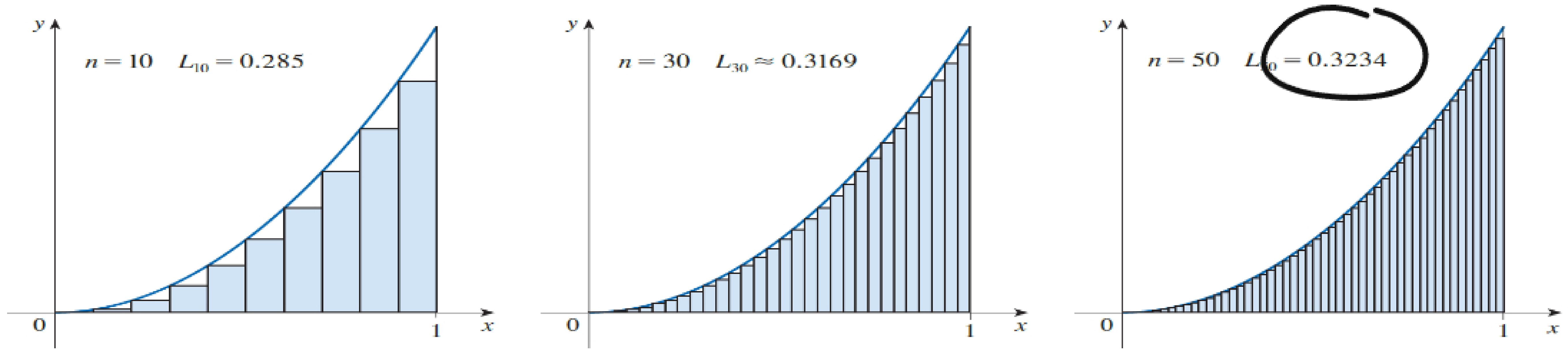


FIGURE 9 Left endpoints produce lower estimates because $f(x) = x^2$ is increasing.

$$f'(x) = x^2 \longrightarrow f(x) = \frac{x^3}{3}$$

$$\boxed{f(1) = \frac{1}{3}}$$

$$\sum_{n=0}^{\infty} n$$

function

Change in x

$$\int_a^b f(x)\ dx$$

Interval

$$\int_0^1 x^2\,dx = \frac{x^3}{3}\Big|_0^1$$

$$\frac{(1)^3}{3} - \frac{(0)^3}{3} = \frac{1}{3}$$

$$\int_1^2 x^2\,dx = \frac{x^3}{3}\Big|_1^2 = \frac{(2)^3}{3} - \frac{(1)^3}{3} = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$$

$$\int_0^2 x^2\,dx = \frac{x^3}{3}\Big|_0^2 = \frac{(2)^3}{3} - \frac{(0)^3}{3} = \frac{8}{3}$$

EXAMPLE 3 Evaluate $\int_0^3 (x^3 - 6x)\,dx$.

$$= \frac{x^4}{4} - 3x^2 \Big|_0^3$$

$$= \left(\frac{(3)^4}{4} - 3(3)^2\right) - \left(\cancel{\frac{(0)^4}{4} - 3(0)^2}\right)$$

$$= \frac{81}{4} - 27 = \boxed{-\frac{27}{4}}$$

$$\frac{108}{4}$$

$$\int_0^3 (x^3 - 6x)\,dx = \lim_{n\to\infty} \sum_{i=1}^n f(x_i)\,\Delta x = \lim_{n\to\infty} \sum_{i=1}^n f\left(\frac{3i}{n}\right)\frac{3}{n}$$

$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^n \left[\left(\frac{3i}{n}\right)^3 - 6\left(\frac{3i}{n}\right)\right]$$

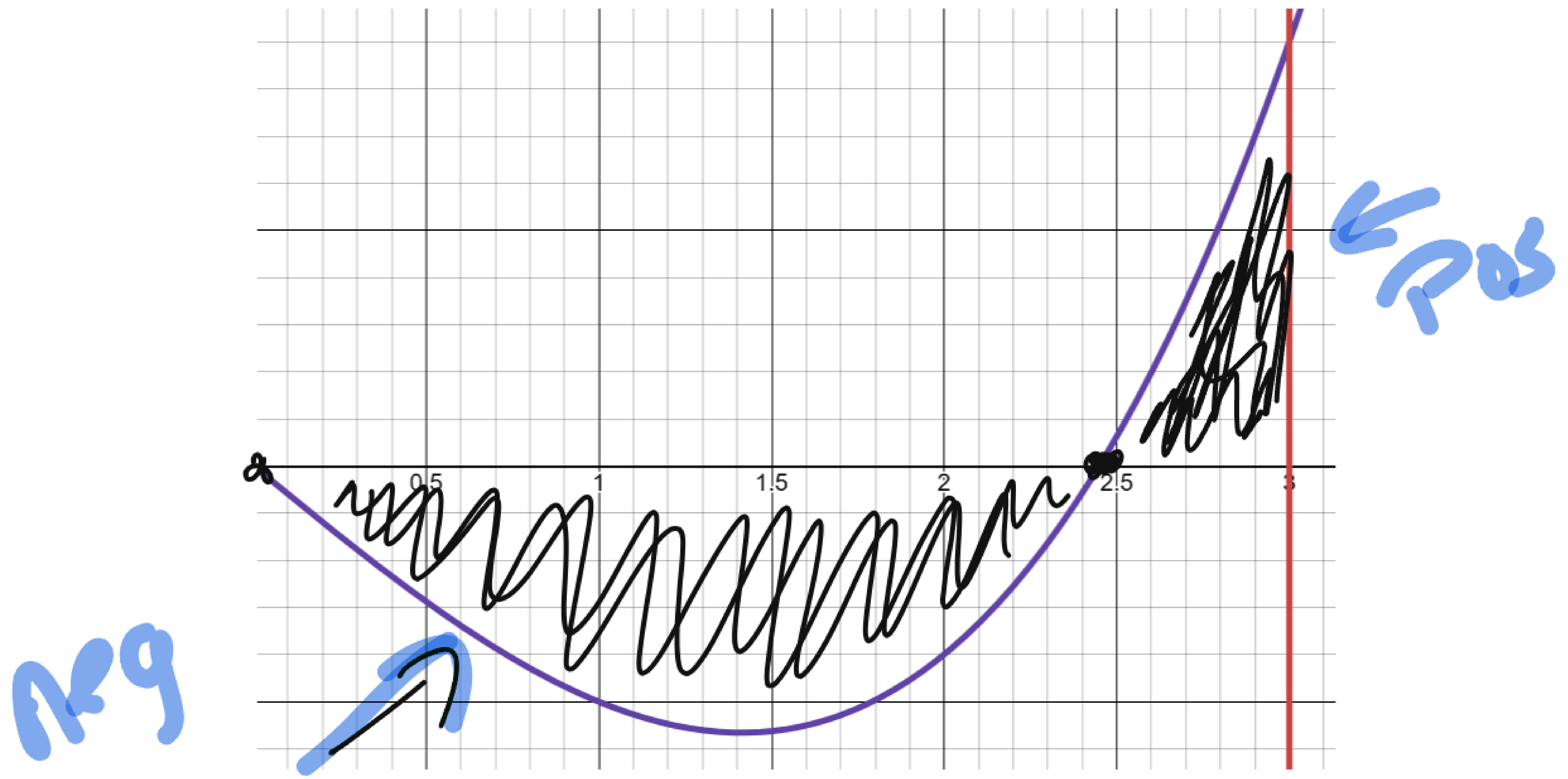
$$= \lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^n \left[\frac{27}{n^3} i^3 - \frac{18}{n} i\right]$$

$$= \lim_{n\to\infty} \left[\frac{81}{n^4} \sum_{i=1}^n i^3 - \frac{54}{n^2} \sum_{i=1}^n i\right]$$

$$= \lim_{n\to\infty} \left\{\frac{81}{n^4}\left[\frac{n(n+1)}{2}\right]^2 - \frac{54}{n^2}\,\frac{n(n+1)}{2}\right\}$$

$$= \lim_{n\to\infty} \left[\frac{81}{4}\left(1 + \frac{1}{n}\right)^2 - 27\left(1 + \frac{1}{n}\right)\right]$$

$$= \frac{81}{4} - 27 = -\frac{27}{4} = -6.75$$


0.5
1
1.5
2
2.5
3
Pos
Neg

$$\int_1^3 e^x \, dx$$

$$e^x \Big]_1^3 = \boxed{e^3 - e}$$

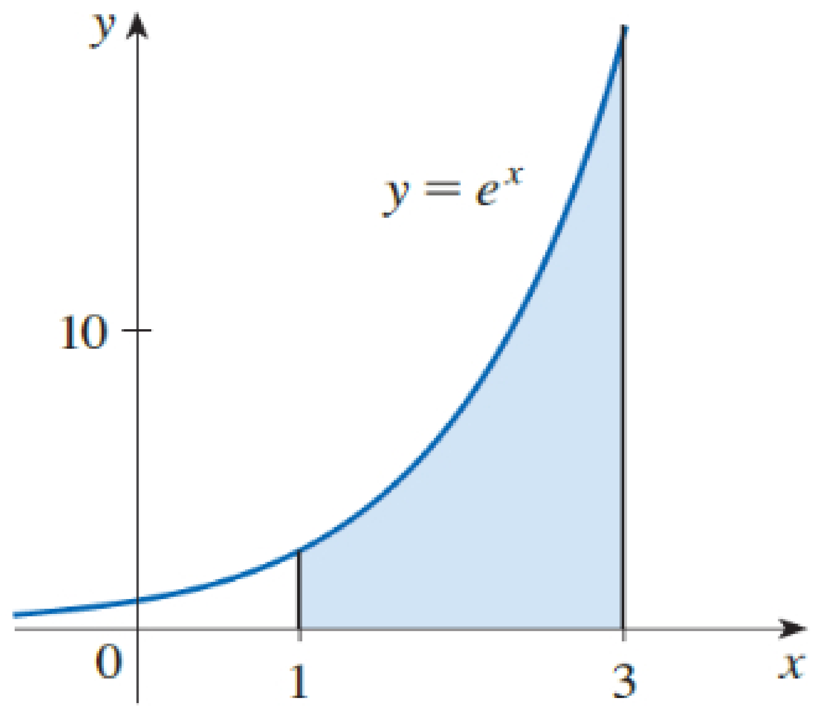


FIGURE 8

$$\int_0^3 e^x \, dx = e^x \Big]_0^3 = e^3 - e^0 = \boxed{e^3 - 1}$$

Properties of the Integral

1. $\int_a^b c\,dx = c(b-a)$, where c is any constant

2. $\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$

3. $\int_a^b c f(x)\,dx = c\int_a^b f(x)\,dx$, where c is any constant

4. $\int_a^b [f(x) - g(x)]\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$

$$\int_{-1}^{2} (4x^2 + x + 2)\, dx = \frac{4x^3}{3} + \frac{x^2}{2} + 2x \Big|_{-1}^{2}$$

$$\left[\frac{4(2)^3}{3} + \frac{(2)^2}{2} + 2(2)\right] - \left[\frac{4(-1)^3}{3} + \frac{(-1)^2}{2} + 2(-1)\right]$$

$$\frac{39}{2}$$

The Fundamental Theorem of Calculus, Part 1 If f is continuous on $[a, b]$, then the function g defined by

$$g(x) = \int_a^x f(t)\,dt \qquad a \leqslant x \leqslant b$$

FTC I

is continuous on $[a, b]$ and differentiable on (a, b), and $g'(x) = f(x)$.

The Fundamental Theorem of Calculus, Part 2 If f is continuous on $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

FTC II

where F is any antiderivative of f, that is, a function F such that $F' = f$.

$$\int_0^{\pi} \cos x \, dx = \sin x \Big|_0^{\pi} \quad \sin \pi - \sin 0$$

$$0 - 0 = 0$$

$$\int_0^{\pi/2} \cos x \, dx = \sin x \Big|_0^{\pi/2} = \sin \frac{\pi}{2} - \sin 0$$

$$1 - 0 = 1$$

$$\int_{\pi/2}^{\pi} \cos x \, dx = \sin x \Big|_{\pi/2}^{\pi} = \sin \pi - \sin \pi/2$$

$$0 - 1 = -1$$

1 Table of Indefinite Integrals

$$\int cf(x)\,dx = c\int f(x)\,dx \qquad \int [f(x)+g(x)]\,dx = \int f(x)\,dx + \int g(x)\,dx$$

$$\int k\,dx = kx + C$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1) \qquad \int \frac{1}{x}\,dx = \ln|x| + C$$

$$\int e^x\,dx = e^x + C \qquad \int b^x\,dx = \frac{b^x}{\ln b} + C$$

$$\int \sin x\,dx = -\cos x + C \qquad \int \cos x\,dx = \sin x + C$$

$$\int \sec^2 x\,dx = \tan x + C \qquad \int \csc^2 x\,dx = -\cot x + C$$

$$\int \sec x \tan x\,dx = \sec x + C \qquad \int \csc x \cot x\,dx = -\csc x + C$$

$$\int \frac{1}{x^2+1}\,dx = \tan^{-1}x + C \qquad \boxed{\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}x + C}$$

$$\int \sinh x\,dx = \cosh x + C \qquad \int \cosh x\,dx = \sinh x + C$$

$$\int \frac{e^{\arcsin x}}{\sqrt{1-x^2}}\,dx$$

$\sin^{-1}(x)$

$\arcsin(x)$

$u = \arcsin x$

$$du = \frac{1}{\sqrt{1-x^2}}\,dx$$

Evaluate $\int \frac{\cos\theta}{\sin^2\theta}\,d\theta$. $\rightarrow$ $\int \cot\theta \csc\theta \, d\theta$

$= \boxed{\csc\theta + C}$

$$\underbrace{\frac{\cos\theta}{\sin\theta}}_{\cot\theta} \cdot \underbrace{\frac{1}{\sin\theta}}_{\csc\theta}$$

$$\int 2x\sqrt{1+x^2}\,dx$$

u-substitution

$$u = 1 + x^2$$

$$du = 2x\,dx$$

$$\int \sqrt{u}\,du$$

$$\int u^{1/2}\,du = \frac{u^{3/2}}{3/2} = \frac{2}{3}u^{3/2} = \boxed{\frac{2}{3}(1+x^2)^{3/2} + C}$$

$$f(x) = \frac{2}{3}(1+x^2)^{3/2}$$

$$(2x)(1+x^2)^{1/2}$$

$$2x\sqrt{\boxed{1+x^2}}$$

$$\frac{2}{3}y^{3/2} = \frac{\cancel{2}}{\cancel{3}}\frac{\cancel{3}}{\cancel{2}}y^{1/2}$$

$$\int x^3 \cos(\underline{x^4+2})\,dx$$

$$u = x^4 + 2$$

$$\frac{du}{4} = \frac{4x^3}{4}\,dx$$

$$\frac{1}{4}du = x^3 dx$$

$$\int \cos(u)\,\boxed{x^3 dx}$$

$$\int \cos(u)\,\frac{1}{4}du$$

$$\frac{1}{4}\int \cos u\,du = \frac{1}{4}\sin u = \boxed{\frac{1}{4}\sin(x^4+2) + C}$$

e $\int \sqrt{2x + 1}\, dx$

$u = 2x + 1$

$\frac{du}{2} = \frac{2}{2}\, dx \rightarrow \frac{1}{2} du = dx$

$\int \sqrt{u} \left(\frac{1}{2} du\right)$

$\frac{1}{2} \int u^{1/2}\, du \rightarrow \frac{1}{\cancel{2}}\left(\frac{\cancel{2}}{3} u^{3/2}\right) = \frac{1}{3} u^{3/2}$

$= \boxed{\frac{1}{3}(2x+1)^{3/2} + C}$

$$\int e^{5x}\,\boxed{dx}$$

$u = 5x$

$du = 5\,dx \rightarrow \frac{1}{5}du = dx$

$$\int e^{u}\,\frac{1}{5}\,du = \frac{1}{5}e^{u} = \frac{1}{5}e^{5x}$$

$$\int e^{4x} = \frac{1}{4}e^{4x}$$

$$\int \tan x \, dx$$

$u = \cos x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

$$\int \frac{\boxed{\sin x \, dx}}{\cos x}$$

$$-\int \frac{1}{u} du = -\ln|u|$$

$$= -\ln|\cos x| + C$$

$$\boxed{\int \tan x = -\ln|\cos x| + C}$$

$$\int \frac{e^{\arcsin x}}{\sqrt{1-x^2}}\,dx$$

$$u = \arcsin x$$

$$du = \frac{1}{\sqrt{1-x^2}}\,dx$$

$$\int e^{\arcsin x} \cdot \frac{1}{\sqrt{1-x^2}}\,dx$$

$$\int e^u\,du = e^u = \boxed{e^{\arcsin x} + C}$$