7.4 The Binomial Distribution

Binomial Experiment

A **binomial experiment** is a random experiment which satisfies all of the following conditions.

1. There are only two outcomes on each trial of the experiment. (One of the outcomes is usually referred to as a *success*, and the other as a *failure*.)
2. The experiment consists of n identical trials as described in condition 1.
3. The probability of success on any one trial is denoted by p and does not change from trial to trial. (**Note:** The probability of a failure is $1 - p$ and also does not change from trial to trial.)
4. The trials are independent.
5. The binomial random variable is the count of the number of successes in n trials.

DEFINITION

Binomial Probability Distribution Function

The **binomial probability distribution function** is

$$P(X = x) = {}_nC_x p^x (1-p)^{n-x}$$

where ${}_nC_x$ represents the number of possible combinations of n objects taken x at a time (without replacement) and is given by

$${}_nC_x = \frac{n!}{x!(n-x)!} \text{ where } n! = n(n-1)(n-2)\cdots 2\cdot 1 \quad \text{and } 0! = 1;$$

n = the number of trials,

p = the probability of a success, and

x = the number of successes in n trials.

FORMULA

Toss a coin 4 times and record the number of heads. Is the number of heads in 4 tosses a binomial random variable?

Heads, Not Heads

# of Heads

$n = 4$

$P = 1/2$

Events
TTTT
HTTT, THTT, TTHT, TTTH
HHTT, HTHT HTTH, THHT, THTH, TTHH
THHH, HTHH, HHTH, HHHT
HHHH

Heads	Probability
0	1/16
1	4/16
2	6/16
3	4/16
4	1/16

Roll a single six-sided die 4 times and record the number of sixes observed. Does the number of sixes rolled in 4 tosses of a die meet the conditions required of a binomial random variable? Construct the probability distribution for this experiment.

success -> 6

failure -> not 6

$p = .167$

$n = 4$

x	
0	P(X=0) = 0.48148
1	P(X=1) = 0.38611
2	P(X=2) = 0.11611
3	P(X=3) = 0.01552
4	P(X=4) = 0.00078

$P(x \le 2)$

P(X≤2) = 0.98370

Expected Value of a Binomial Random Variable

The **expected value** of a binomial random variable can be computed using the expression

$$\mu = E(X) = np,$$

where n and p are the parameters of the binomial distribution.

FORMULA

Variance and Standard Deviation of a Binomial Random Variable

To find the **variance** of a binomial random variable, use the expression

$$\sigma^2 = V(X) = np(1 - p).$$

Therefore, the **standard deviation** of a binomial random variable is given by

$$\sigma = \sqrt{V(X)} = \sqrt{np(1-p)}.$$

FORMULA

The US Land Management Office regularly holds a lottery for the lease of government lands. Your company has won the rights to 12 leases. Historically, about 10% of these lands possess sufficient oil reserves for profitable operation. Construct the distribution for the number of leases that will be profitable. What is the probability that at least one of the leases will be profitable?

$p = 10\% = .1$ probability of success

$n = 12$ # of trials

$X \geq 1$ $P(X \geq 1) = 0.71757$

$\mu = np = 12(.1) = 1.2$

$\sigma = \sqrt{np(1-p)}$

$= \sqrt{12(.1)(.9)} = 1.039230485$

"Would you say you eat to live or live to eat?" was asked to each person in a sample of 1001 adults in a Gallup Poll taken in April 1996. Seventy-four percent of the respondents answered eat to live, 23% answered live to eat, and 3% had no opinion. Assuming these percents are accurate, find the probability, in 12 randomly chosen adults, that the number who would answer "eat to live" is:

a. exactly 7. $P(X=7) = 0.11434$

b. no more than 10. $P(X \leq 10) = 0.85935$

c. at most 11. $P(X \leq 11) = 0.97304$

d. at least 3. $P(X \geq 3) = 0.99995$

$n = 12$ $p = .74$

Continuous Uniform Distribution

$P =$

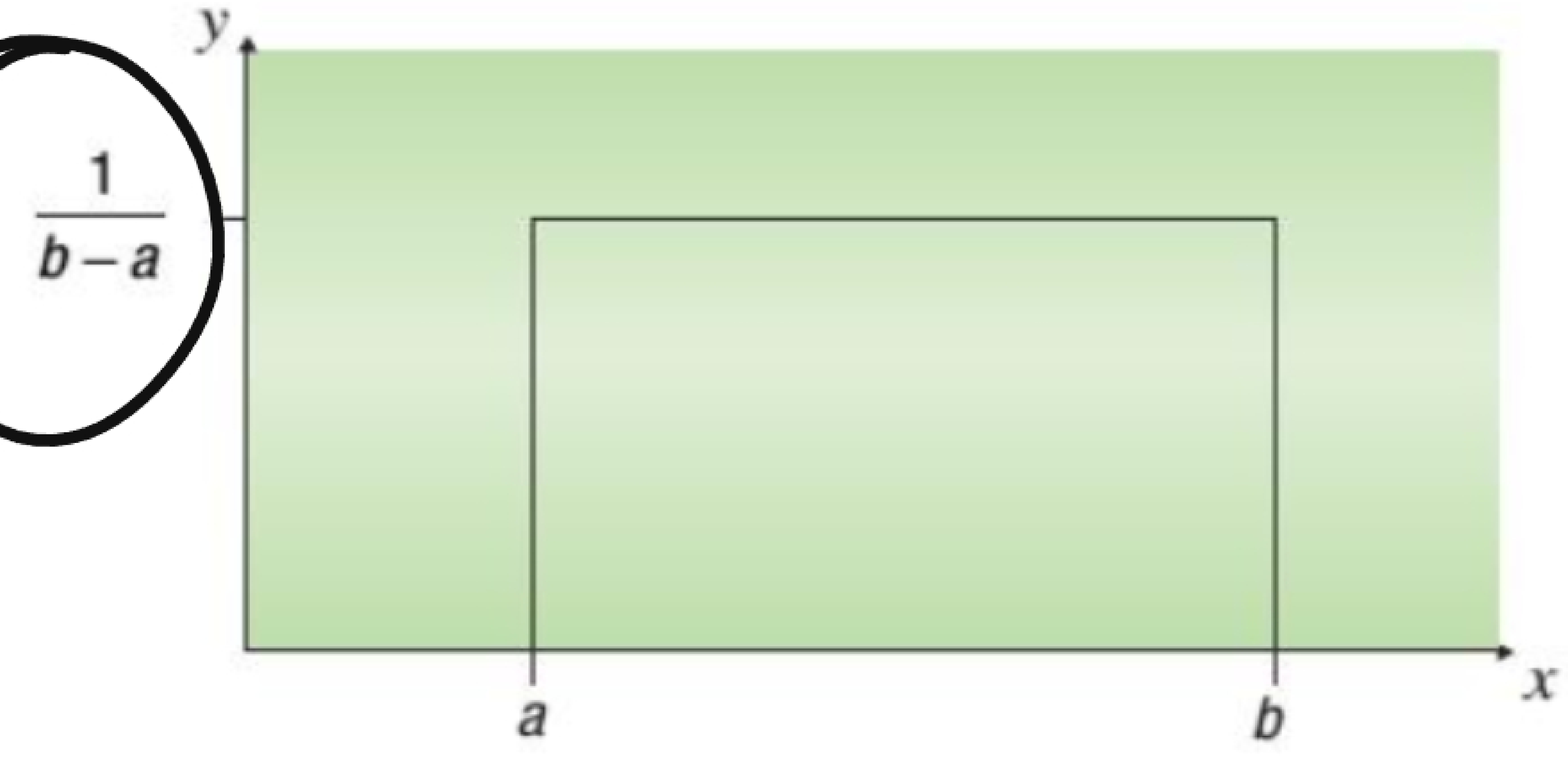


Figure 8.1.1

$\mu = \frac{a+b}{2}$

$\sigma = \frac{b-a}{\sqrt{12}}$

Uniform Probability Density Function

The **uniform probability density function** is given by

$$f(x) = \begin{cases} \dfrac{1}{b-a} & \text{for } a \le x \le b \\ 0 & \text{otherwise.} \end{cases}$$

The mean and standard deviation are given by the following expressions.

$$\mu = \frac{a+b}{2} \text{ and } \sigma = \frac{b-a}{\sqrt{12}}$$

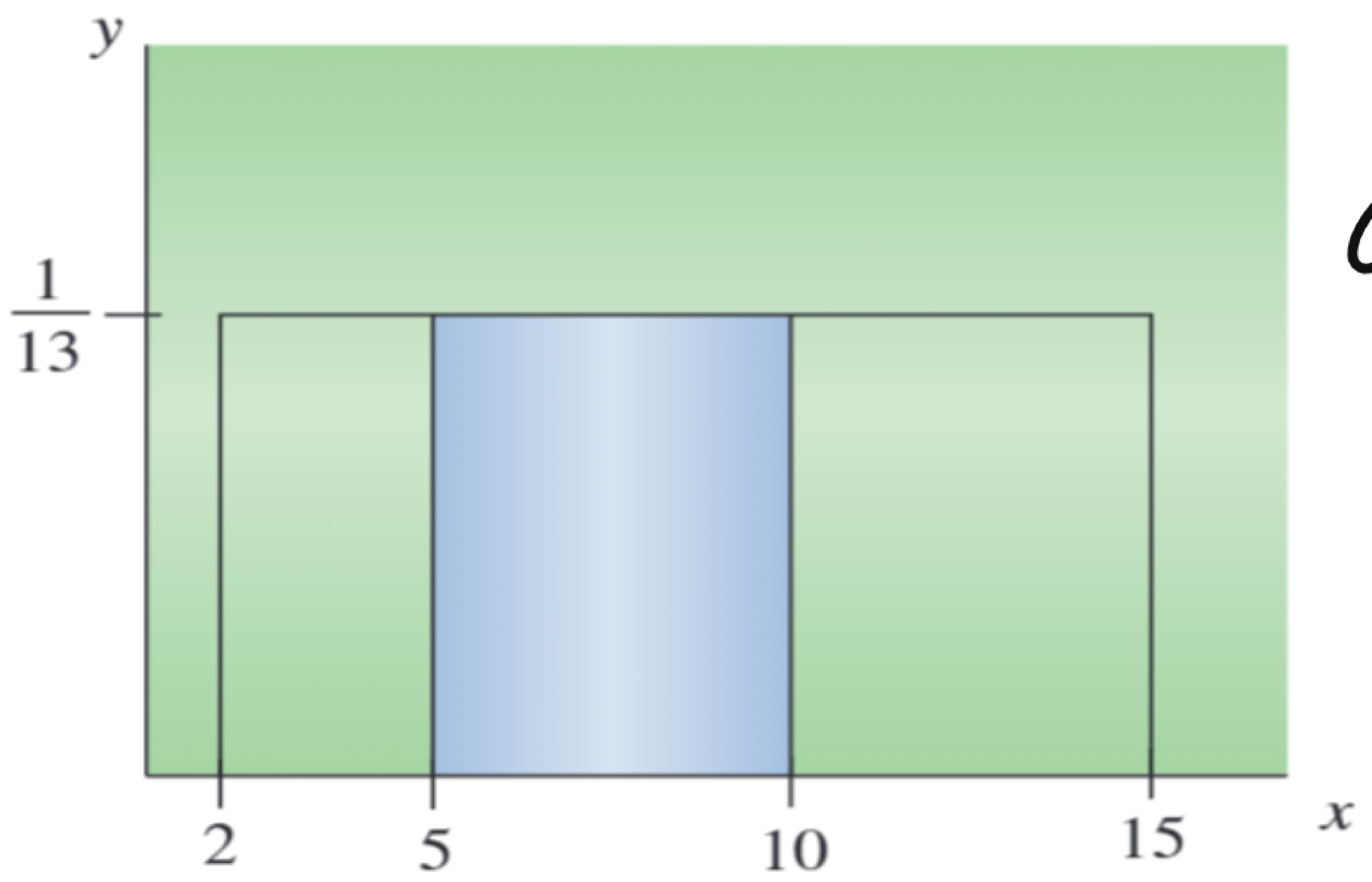
FORMULA

The fire department records how long it takes each of its trucks to reach the scene of a fire. Suppose that distribution of arrival times is uniform with the minimum time being 2 minutes and the maximum 15 minutes. Let X equal the time it takes a truck to arrive at the scene of a fire after being called.

a. What is the probability that a truck will reach a fire scene within 5 to 10 minutes?

b. What is the expected time until arrival of the fire truck?

c. What is the standard deviation of truck arrival times?



a) $P(5 \le X \le 10) = (1/13)(10-5)$

$= 5/13$

b) $E(X) = \mu = \frac{15+2}{2} = 8.5$

c) $\sigma = \frac{15-2}{\sqrt{12}} = 3.75$

A particular employee arrives to work sometime between 8:00 AM and 8:30 AM. Based on past experience the company has determined that the employee is equally likely to arrive at any time between 8:00 AM and 8:30 AM.

$a = 8$

$b = 8.5$

$H = \frac{1}{.5} = 2$

$\mu = \frac{8.5 + 8}{2} = 8.25$ 8:15 AM

$\sigma = \frac{8.5 - 8}{\sqrt{12}} = .144 \approx 8.6 \text{ min}$

$P \leq$ 8:10 am $\rightarrow$ $.167 (2) = .333$

$\underbrace{8{:}10 - 8{:}00} = 10 \text{ min}$

$.167$