7 CHAPTER

Discrete Probability Distributions

Random Variable

A **random variable** is a numerical outcome of a random process.

DEFINITION

Probability Distribution

A **probability distribution** is a model which describes a specific kind of random process.

DEFINITION

Discrete Random Variable

A **discrete random variable** is a random variable which has a countable number of possible outcomes.

When describing a discrete random variable, you should do the following.

1. State the variable.
2. List all of the possible values of the variable.
3. Determine the probabilities of these values.

DEFINITION

Random Phenomenon: Toss a die and observe the outcome of the toss.

1. *Identify the random variable*: X = the outcome of the toss of a die.
2. *Range of values*: Integers between 1 and 6, inclusive.

In this instance, $x_1 = 1, x_2 = 2, ..., x_6 = 6$. Although the value of the random variable is the same as the subscript in this case, there is usually no relationship between the two.

3. *Probability distribution*: The outcomes of the toss of a die and their probabilities are given below. The probabilities are deduced using the classical method and the assumption of a fair die.

Tossing a Die						
$X =$	1	2	3	4	5	6
$P(X=x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

Random Phenomenon: The number of defective integrated circuits received in a batch of 1000. Each outcome of the random variable is a numerical measure whose range of values is given below.

1. *Identify the random variable*: X = # of defective
2. *Range of values*: 0 → 1000 Inclusivly
3. *Probability distribution*: unkown

Random Phenomenon: The head nurse of the pediatric division of the Sisters of Mercy Hospital is trying to determine the capacity requirement for the nursery. She realizes that the number of babies born at the hospital each day is a random variable. She will have to develop a description of the randomness in order to develop her plan.

1. *Identify the random variable*: X = the number of babies born at Sisters of Mercy Hospital each day.
2. *Range of values*: Integers from 0 to some large positive number.
3. *Probability distribution*: Unknown, but could be estimated using the relative frequency idea from Section 6.1 in conjunction with historical data on hospital births at Sisters of Mercy.

Random Phenomenon: A local restaurant has an express policy that states that lunch will be served within 15 minutes of ordering, or it will be free. Obviously, the restaurant is keenly interested in not giving away its product and in delivering on its promise of a timely lunch. But the length of time to prepare each meal varies because of the difference in preparation times of each dish, the load on the kitchen, and the experience of the chefs and waitresses. Since time is measured on a continuous scale and the variability of meal preparation is not predictable, the time between ordering and receiving a meal is considered to be a continuous random variable.

1. *Identify the random variable*: X= how long
2. *Range of values*: 0 → a large #
3. *Probability density*: Unknown, but could be approximated using historical data.

Classify the following as either a discrete random variable or a continuous random variable.

a. The total points scored per football game for a local high school team. D

b. The daily price of a stock. C

c. The interest rate charged by local banks for 30-year mortgages. C

d. The number of times a backup of the computer network is performed in a month. D

e. The amount of sugar imported by the U.S. in a day. C

Discrete Probability Distribution

A **discrete probability distribution** consists of all possible values of the discrete random variable along with their associated probabilities.

Discrete probability distributions always have two characteristics.

1. The sum of all of the probabilities must equal 1.
2. The probability of any value must be between 0 and 1, inclusively.

DEFINITION

Consider the random phenomenon of tossing a coin three times and counting the number of heads. What is the probability distribution for the number of heads observed in three tosses of a coin?

Tossing a Coin		
Number of Heads, x	**$P(X = x)$**	**Simple Events**
0	$\frac{1}{8}$	TTT
1	$\frac{3}{8}$	HTT, THT, TTH
2	$\frac{3}{8}$	HHT, HTH, THH
3	$\frac{1}{8}$	HHH
Total	$\sum P(X = x_i) = 1.0$	

K. J. Johnson is a computer salesperson. During the last year he has kept records of his computer sales for the last 200 days.

Frequency Distribution					
Sales	0	1	2	3	4
Frequency	40	20	60	40	40

$\frac{40}{200} = .2$

He recognizes that his daily sales constitute a random process and he wishes to determine the probability distribution for daily sales. From the probability distribution he would like to determine the following.

a. The probability that he will sell at least 2 computers each day. 70%

b. The probability he will sell at most 2 computers each day. 60%

$x =$	0	1	2	3	4
$P(X = x)$	.2	.1	.3	.2	.2

An investor has decided that she will purchase a stock if there is at least a 50% chance that the price of the stock will be more than \$32 in thirty days. Assuming the price of the stock 30 days from now is described in the table below, should the investor purchase the stock?

Stock Prices								
x	30.0	30.5	31.0	31.5	32.0	32.5	33.0	
$P(X = x)$	0.05	0.10	0.20	0.25	0.20	0.15	0.05	$\sum P(X = x_i) = 1.0$

.20

Expected Value

The **expected value** of the random variable X is the mean of the random variable X. It is denoted $E(X)$ and is given by computing the following expression

$$\mu = E(X) = \sum \left[x \cdot p(x) \right]$$

where $p(x) = P(X = x)$.

DEFINITION

sum

μ

Table 7.2.1 - Calculating K.J. Johnson's Expected Number of Sales per Day		
x	$p(x)$	$x \cdot p(x)$
0	0.2	0
1	0.1	0.1
2	0.3	0.6
3	0.2	0.6
4	0.2	0.8
Total		$E(X) = 2.1$

Investment Alternatives			
Option A		Option B	
Profit (Dollars)	Probability	Profit (Dollars)	Probability
–2000	0.2	–3000	0.2
0	0.1	–1000	0.1
1000	0.3	2000	0.2
2000	0.3	3000	0.3
4000	0.1	4000	0.2

Mean (μ) = **900.0000**

Standard Deviation (σ) = **1757.8396**

Variance (σ^2) = **3090000.0000**

Mean (μ) = **1400.0000**

Standard Deviation (σ) = **2576.8197**

Variance (σ^2) = **6640000.0000**

Variance and Standard Deviation of a Discrete Random Variable

The **variance of a discrete random variable** X is given by the following formula.

$$\sigma^2 = V(X) = \sum\left[(x-\mu)^2 p(x)\right]$$

The **standard deviation of a discrete random variable** X is therefore,

$$\sigma = \sqrt{V(x)} = \sqrt{\sum (x-\mu)^2 p(x)}.$$

DEFINITION

Investment Alternatives			
Option A		**Option B**	
Profit (Dollars)	Probability	Profit (Dollars)	Probability
–2000	0.2	–3000	0.2
0	0.1	–1000	0.1
1000	0.3	2000	0.2
2000	0.3	3000	0.3
4000	0.1	4000	0.2
$E(X_A) = \sum[x_A \cdot p(x_A)]$ $= (-2000)0.2 + (0)0.1$ $+ (1000)0.3 + (2000)0.3$ $+ (4000)0.1$ $= \$900$		$E(X_B) = \sum[x_B \cdot p(x_B)]$ $= (-3000)0.2 + (-1000)0.1$ $+ (2000)0.2 + (3000)0.3$ $+ (4000)0.2$ $= \$1400$	

Roll a die

$10

Tossing a Die						
X =	1	2	3	4	5	6
P(X = x)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
Win/Loss	-20	0	0	10	20	30

$$-\frac{20}{6}+\frac{0}{6}+\frac{0}{6}+\frac{10}{6}+\frac{20}{6}+\frac{30}{6}=\frac{40}{6}=6.67$$

Red on Roulette Wheel

\$1

red	$18/38$	\$1	$= 18/38$
other	$20/38$	\$ -1	$= -20/38$

$+ \quad = -2/38 = -0.053$

Determine whether or not the following distribution is a probability distribution. If the distribution is not a probability distribution, give the characteristic which is not satisfied by the distribution.

x	1	2	3
$P(X = x)$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$

A regional hospital is considering the purchase of a helicopter to transport critical patients. The relative frequency of X, the number of times the helicopter is used to transport critical patients each month, is derived for a similarly sized hospital and is given in the following probability distribution.

x	0	1	2	3	4	5	6
$p(x)$	0.15	0.20	0.34	0.19	0.06	0.05	0.01

a. Find the average number of times the helicopter is used to transport critical patients each month.

b. Find the variance of the number of times the helicopter is used to transport critical patients.

c. Find the standard deviation of the number of times the helicopter is used to transport critical patients.

d. Find the probability that the helicopter will not be used at all during a month to transport critical patients.

e. Find the probability that the helicopter will be used at least once to transport critical patients.

f. Find the probability that the helicopter will be used at most twice to transport critical patients.

g. Find the probability that the helicopter will be used more than three times to transport critical patients.

Mean (μ) = **2.0000**

Standard Deviation (σ) = **1.3565**

Variance (σ^2) = **1.8400**

0.15

0.85

0.69

0.12