8 CHAPTER
Continuous Probability Distributions

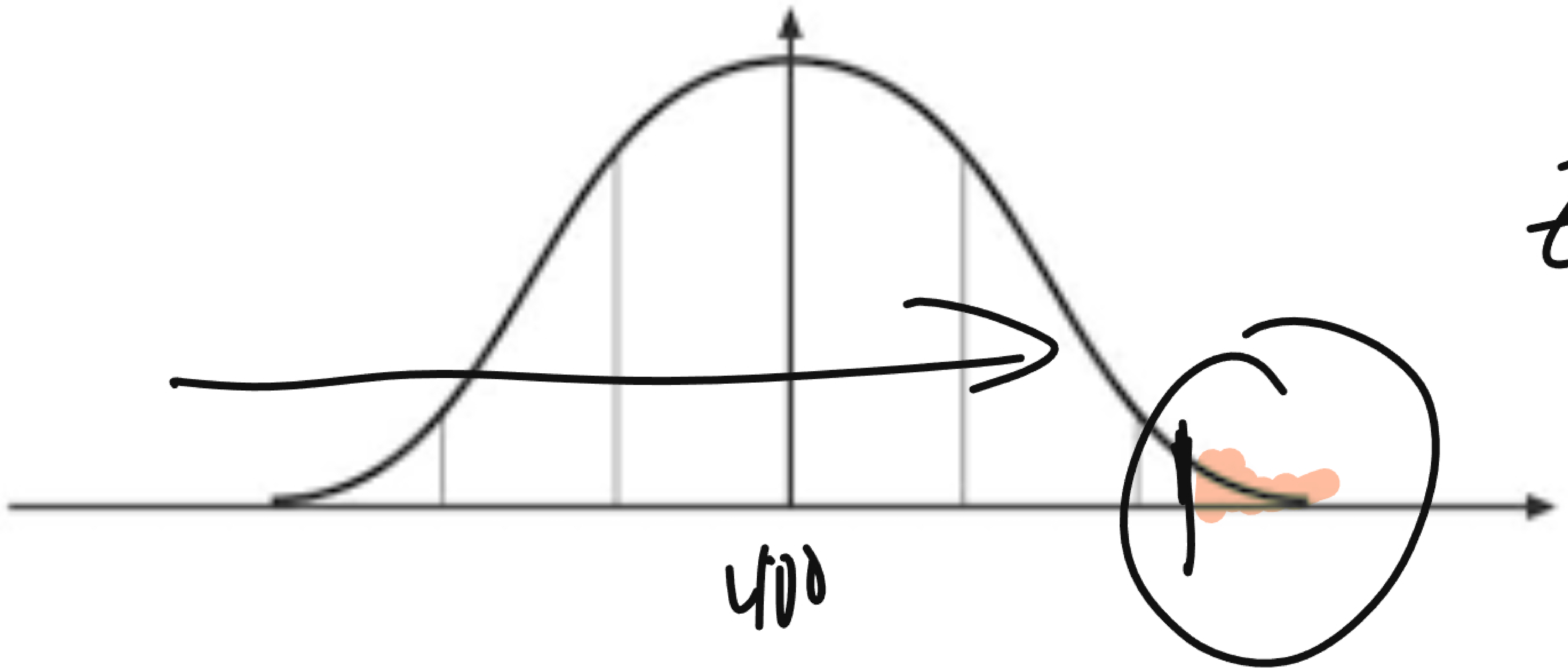
8.4 Applications of the Normal Distribution

8.6 Approximation to the Binomial Distribution

Suppose that a national testing service gives a test in which the results are normally distributed with a mean of 400 and a standard deviation of 100. If you score a 644 on the test, what fraction of the students taking the test exceeded your score?

$x = 644 \quad \mu = 400 \quad \sigma = 100$

$$z = \frac{x - \mu}{\sigma}$$



$$z = \frac{644 - 400}{100} = 2.44$$

$$1 - .9927 = \boxed{.0073}$$

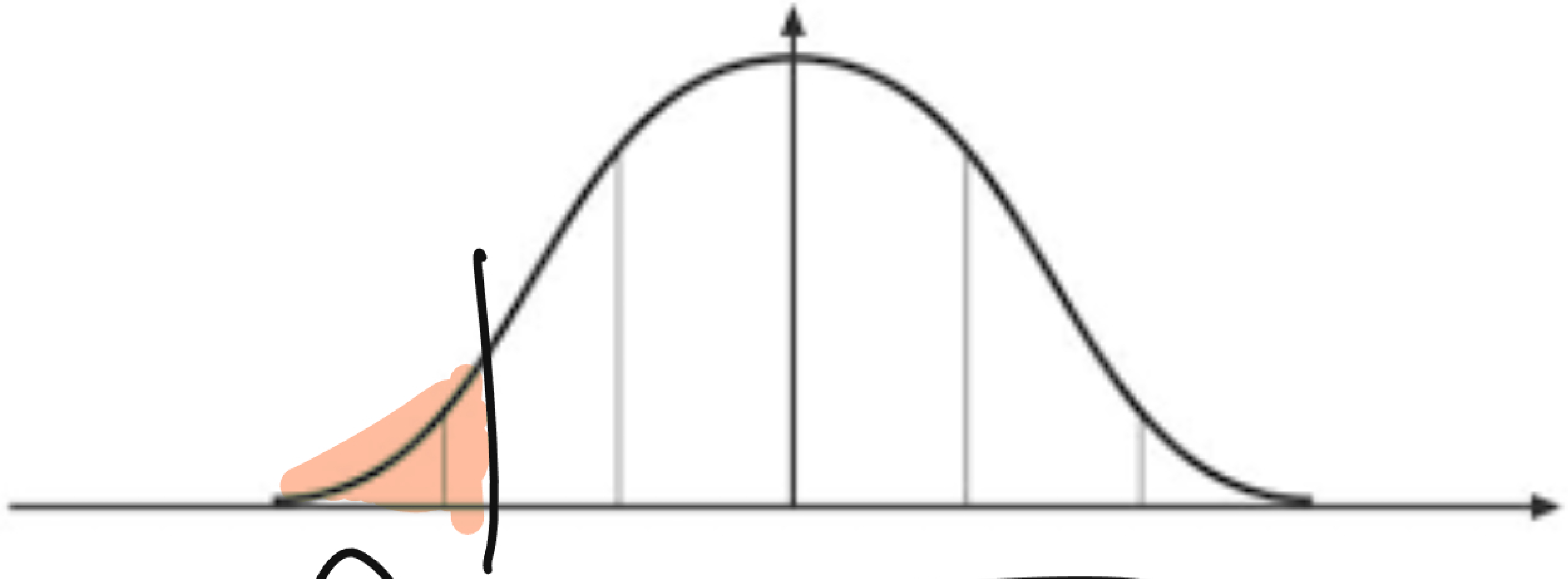
The Arc Electronic Company had an income of $200,000 last year. Suppose the mean income of firms in the industry for the year is $1,000,000 with a standard deviation of $500,000. If incomes for the industry are normally distributed, what proportion of the firms in the industry earned less than Arc?

$x = 200{,}000$

$\mu = 1{,}000{,}000$

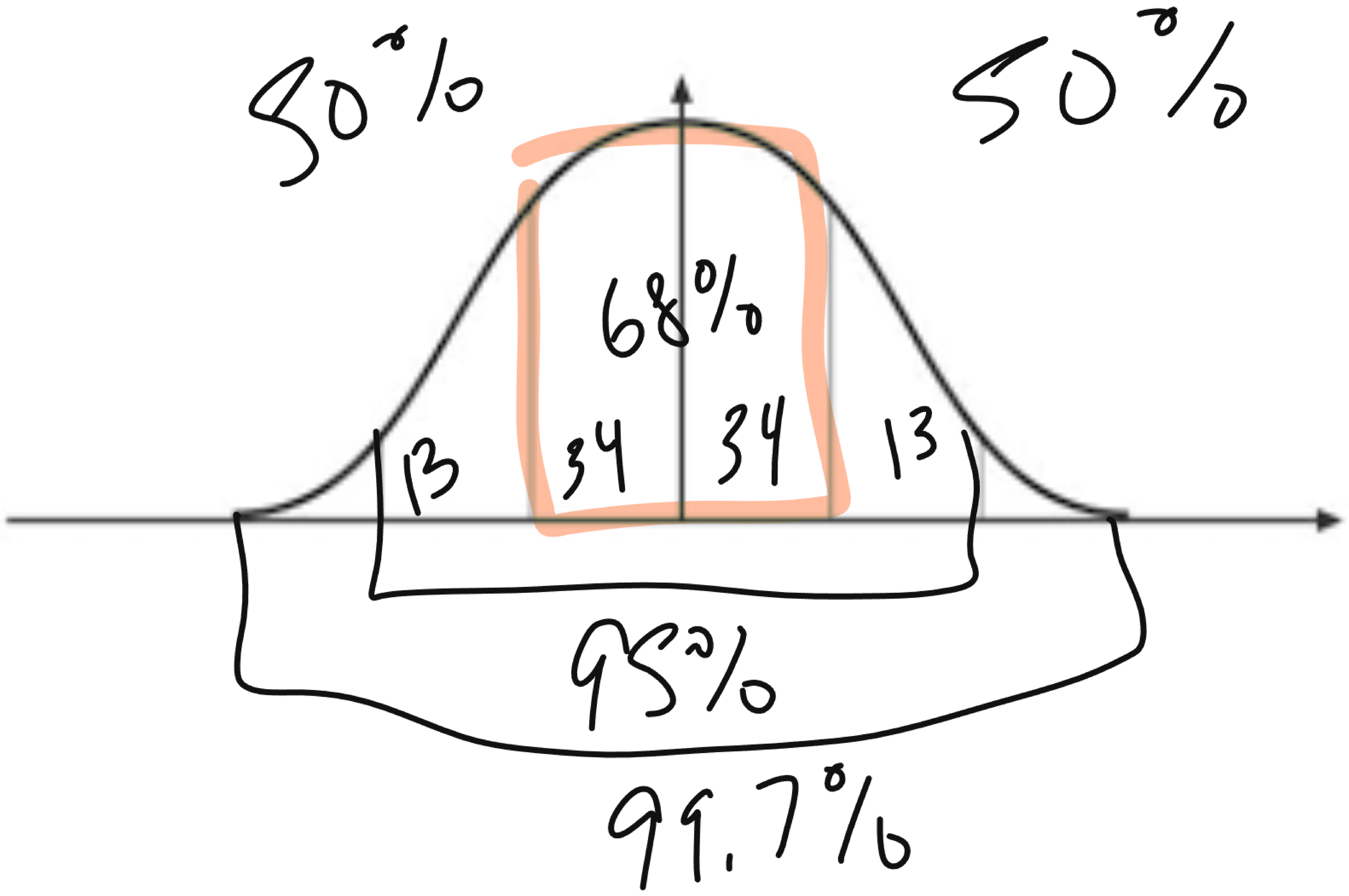
$\sigma = 500{,}000$

$$z = \frac{200000 - 1000000}{500000} = -1.60$$



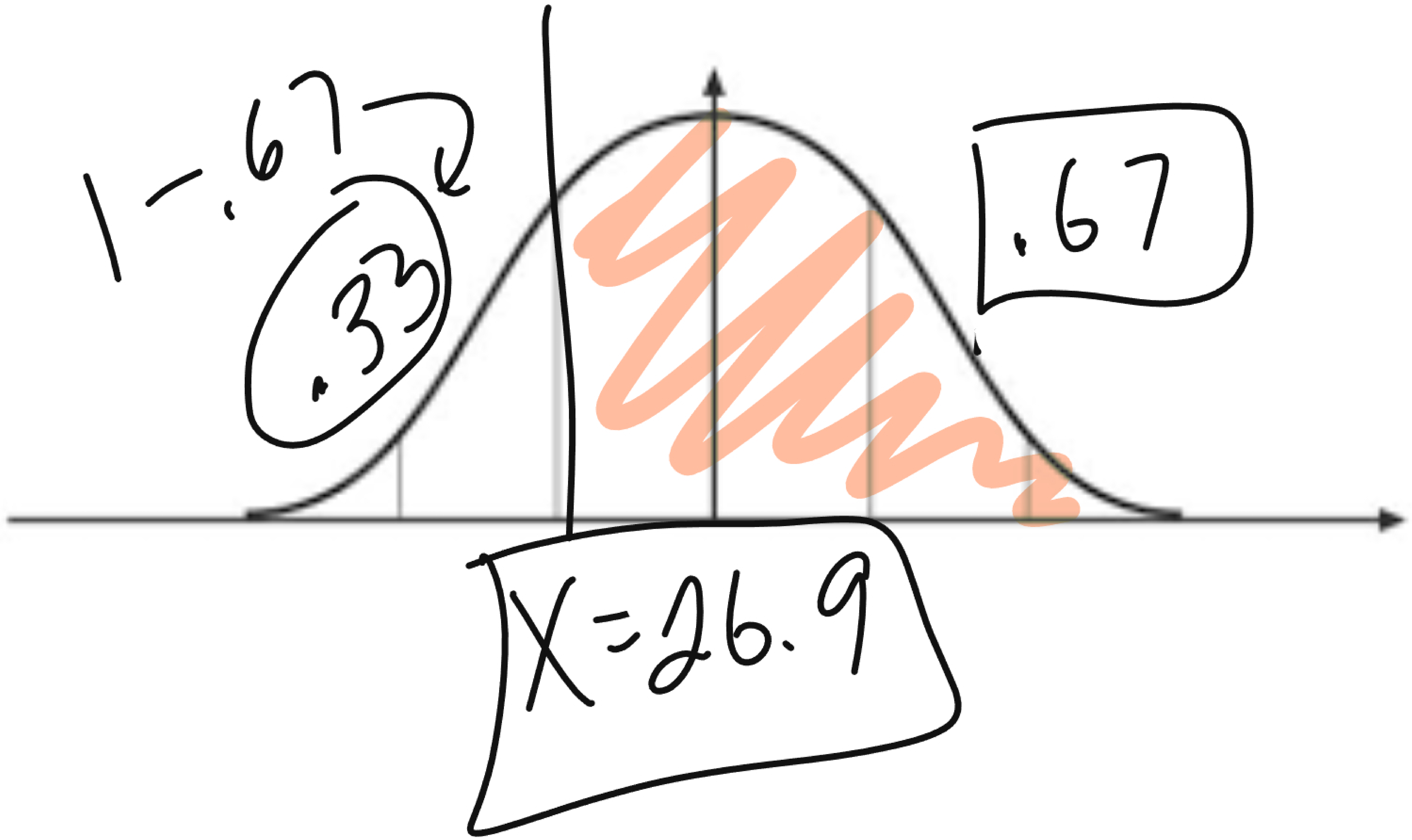
$\boxed{0.0548}$

5.48%


50%
50%
68%
13
34
34
13
95%
99.7%

If a normal distribution has a mean of 28.0 and a standard deviation of 2.5, what is the value of the random variable X that has an area to its right equal to 0.6700?

$X = ?$ $\mu = 28.0$ $\sigma = 2.5$ $z = -0.44$



$$z = \frac{X - \mu}{\sigma}$$

$$-0.44 = \frac{X - 28}{2.5}$$

$$2.5(-0.44) = X - 28$$

$$-1.1 = X - 28$$

$$+28 \qquad +28$$

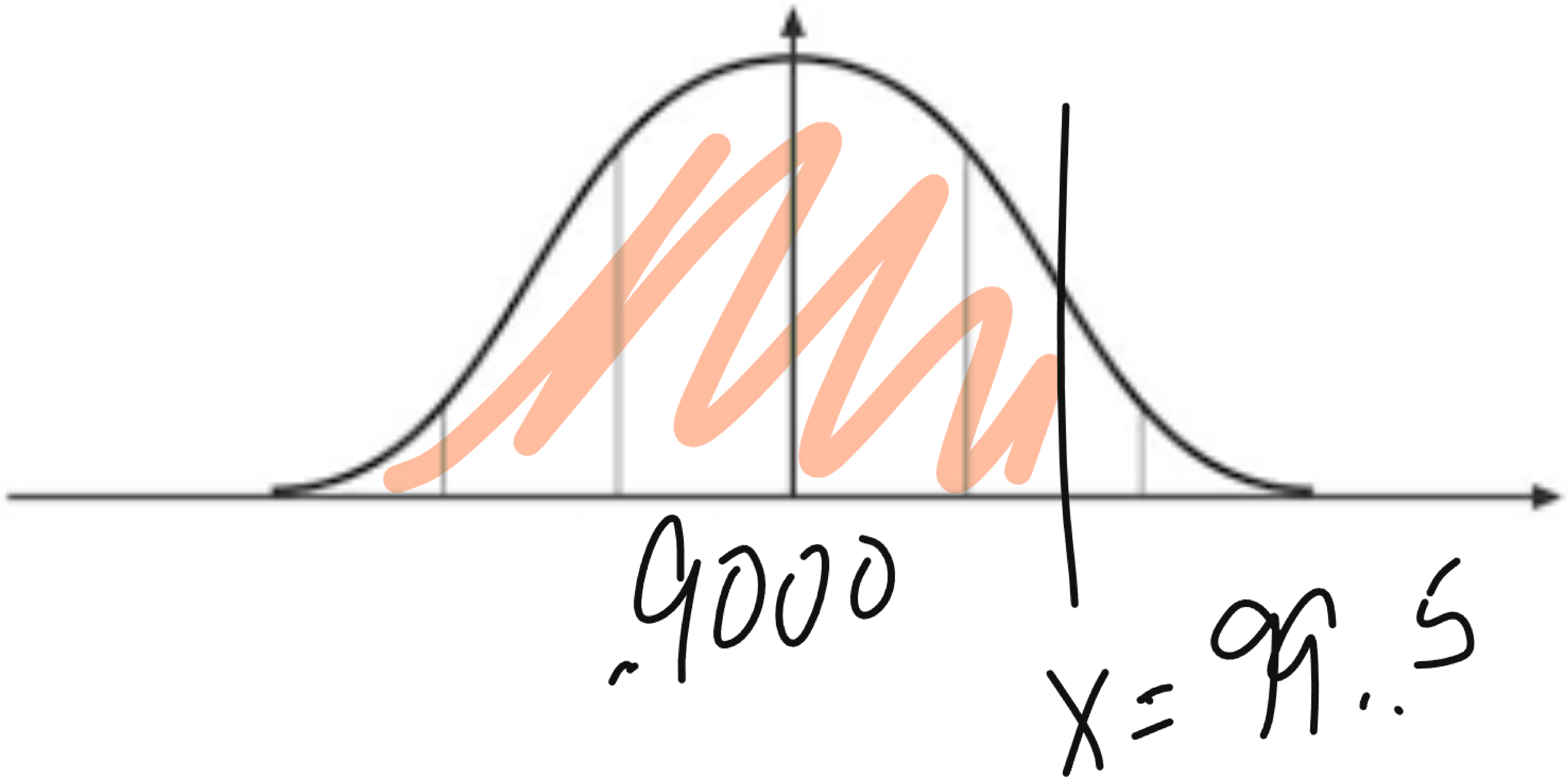
$$26.9 = X$$

	0	1	2	3	4	5	6	7	8	9
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

$$z = -0.44$$

The body temperatures of adults are normally distributed with a mean of 98.60 °F and a standard deviation of 0.73 °F. What temperature represents the 90th percentile?

.9000



$x = ?$

$\mu = 98.6$

$\sigma = 0.73$

$z = 1.28$

$$1.28 = \frac{x - 98.6}{0.73}$$

$$0.9344 = x - 98.6$$

VGA monitors manufactured by TSI Electronics have life spans which have a normal distribution with an average life span of 15,000 hours and a standard deviation of 2000 hours. If a VGA monitor is selected at random, find the following probabilities.

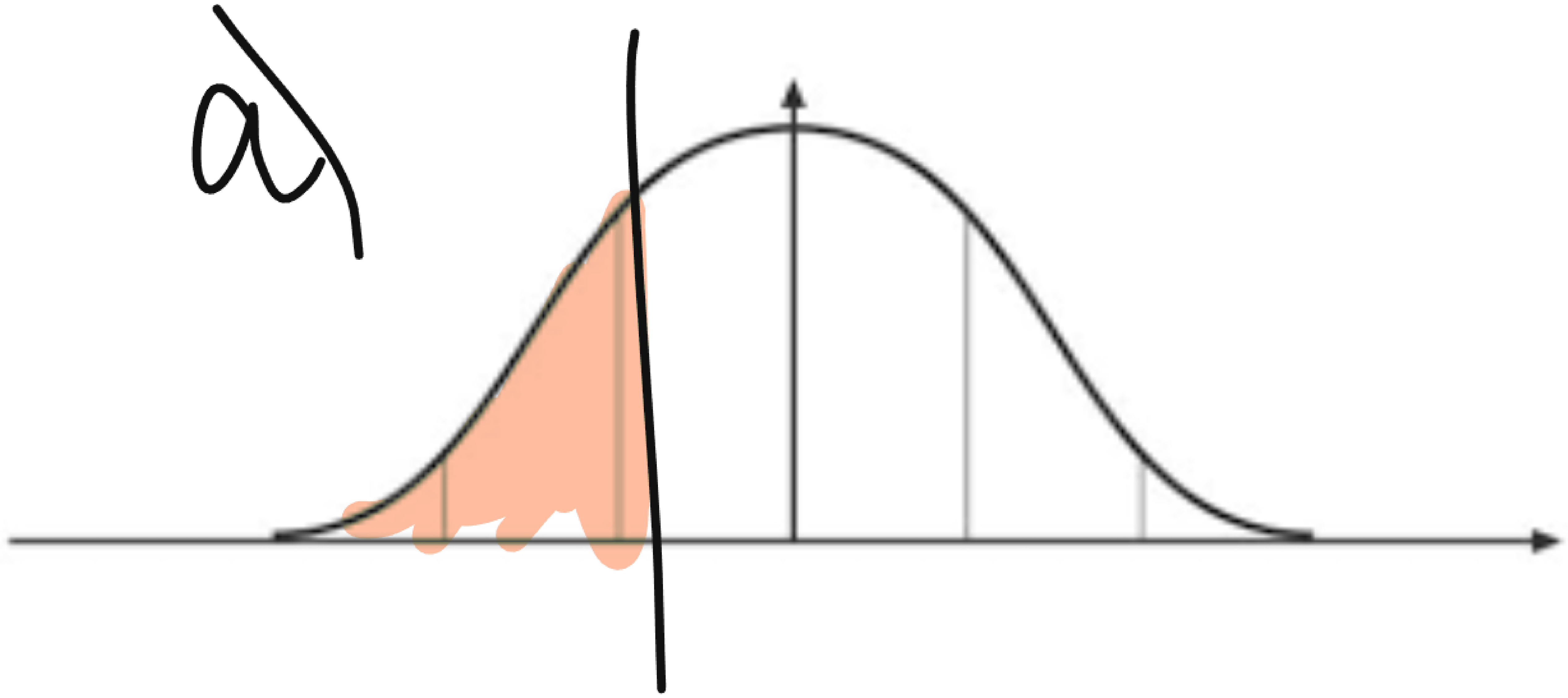
$X = 12000$
$\mu = 15000$
$\sigma = 2000$
$Z = -1.5$

a. The probability that the life span of the monitor will be less than 12,000 hours. .0668 6.68%

b. The probability that the life span of the monitor will be more than 18,000 hours.

c. The probability that the life span of the monitor will be between 13,000 hours and 17,000 hours.

a)



Example 8.6.1

a. Assuming $n = 20$ and $p = 0.5$, use a normal random variable (Y) to approximate the probability that a binomial random variable (X) is 5 or less.

$X \leq$

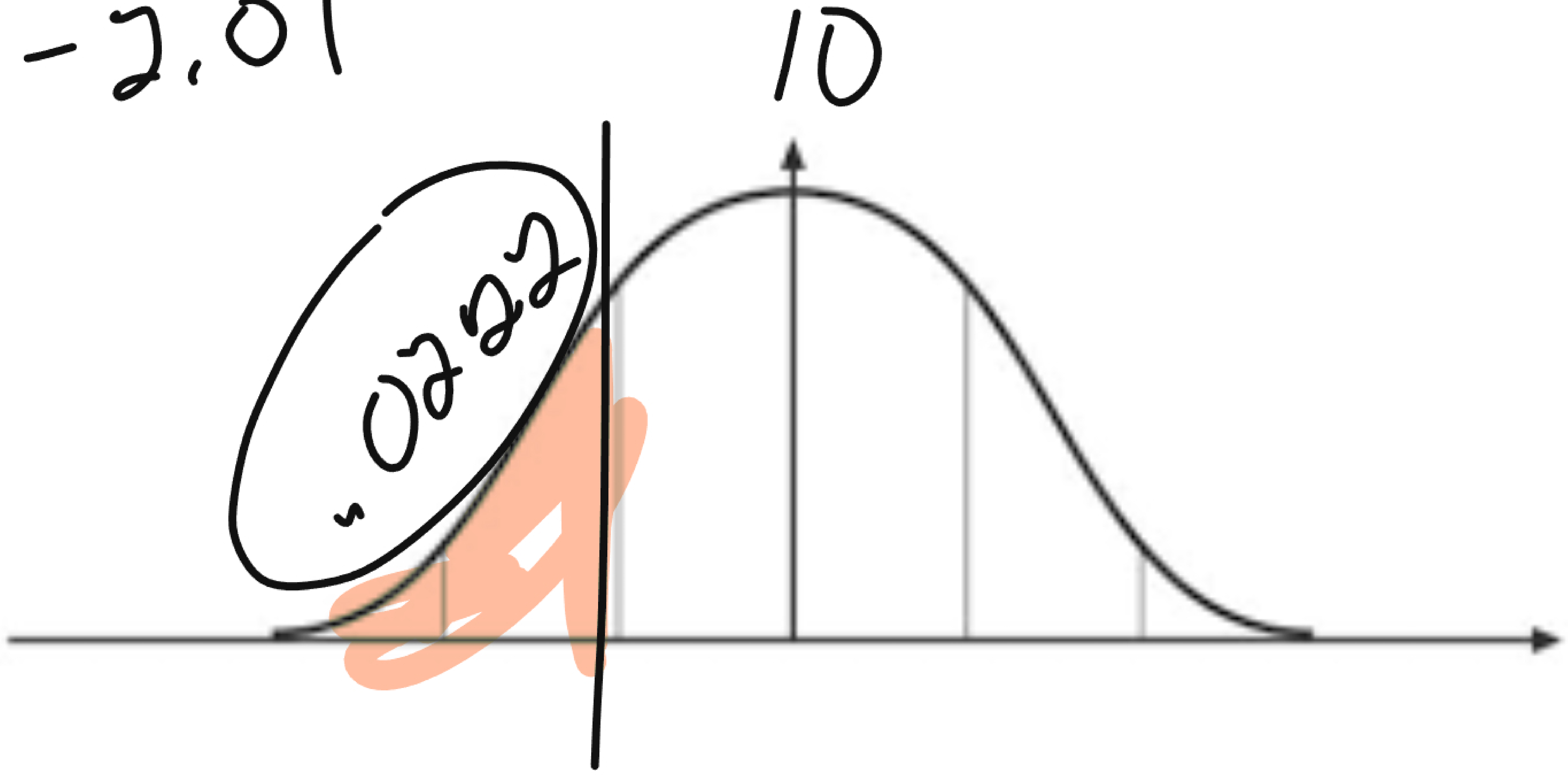
$x = 5.5$

$X = 5.5$

$\mu = 10$

$\sigma = 2.236$

$z = -2.01$



$$\mu = E(X) = np = 20(0.5) = 10$$

$$\sigma = \sqrt{V(X)} = \sqrt{np(1-p)} = \sqrt{20(.5)(1-.5)}$$
$$= 2.236$$

$n = 20$

$p = 0.5$

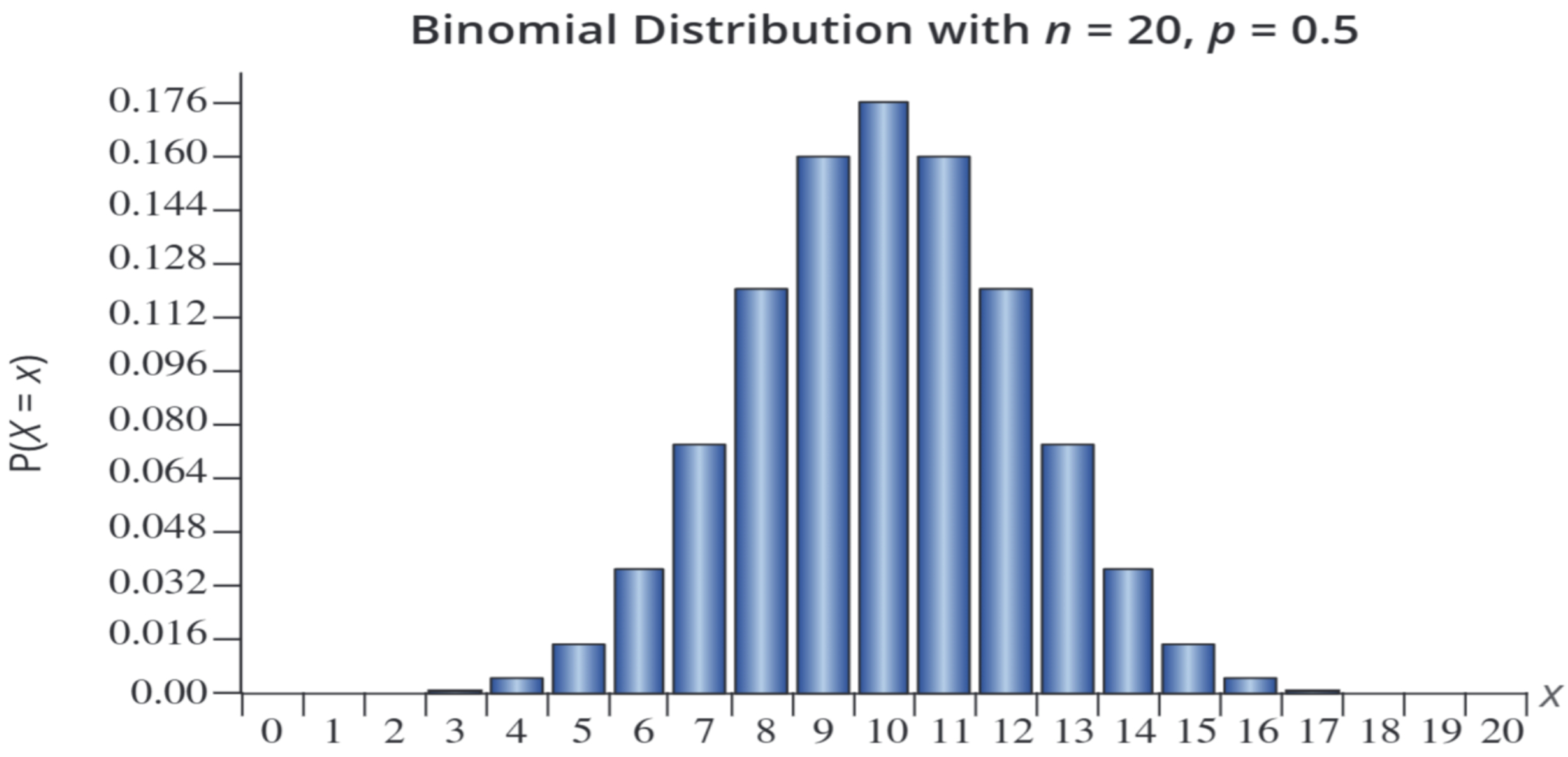

Binomial Distribution with n = 20, p = 0.5
P(X = x)
0.176
0.160
0.144
0.128
0.112
0.096
0.080
0.064
0.048
0.032
0.016
0.00
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
x

Figure 8.6.1

An advertising agency hired on behalf of Tech's development office conducted an ad campaign aimed at making alumni aware of their new capital campaign. Upon completion of the new campaign, the agency claimed that 20% of alumni in the state were aware of the new campaign. To validate the claim of the agency, the development office surveyed 1000 alumni in the state and found that 150 were aware of the campaign. Assuming that the ad agency's claim is true, what is the probability that no more than 150 of the alumni in the random sample were aware of the new campaign?

$P = 0.20$

$n = 1000$

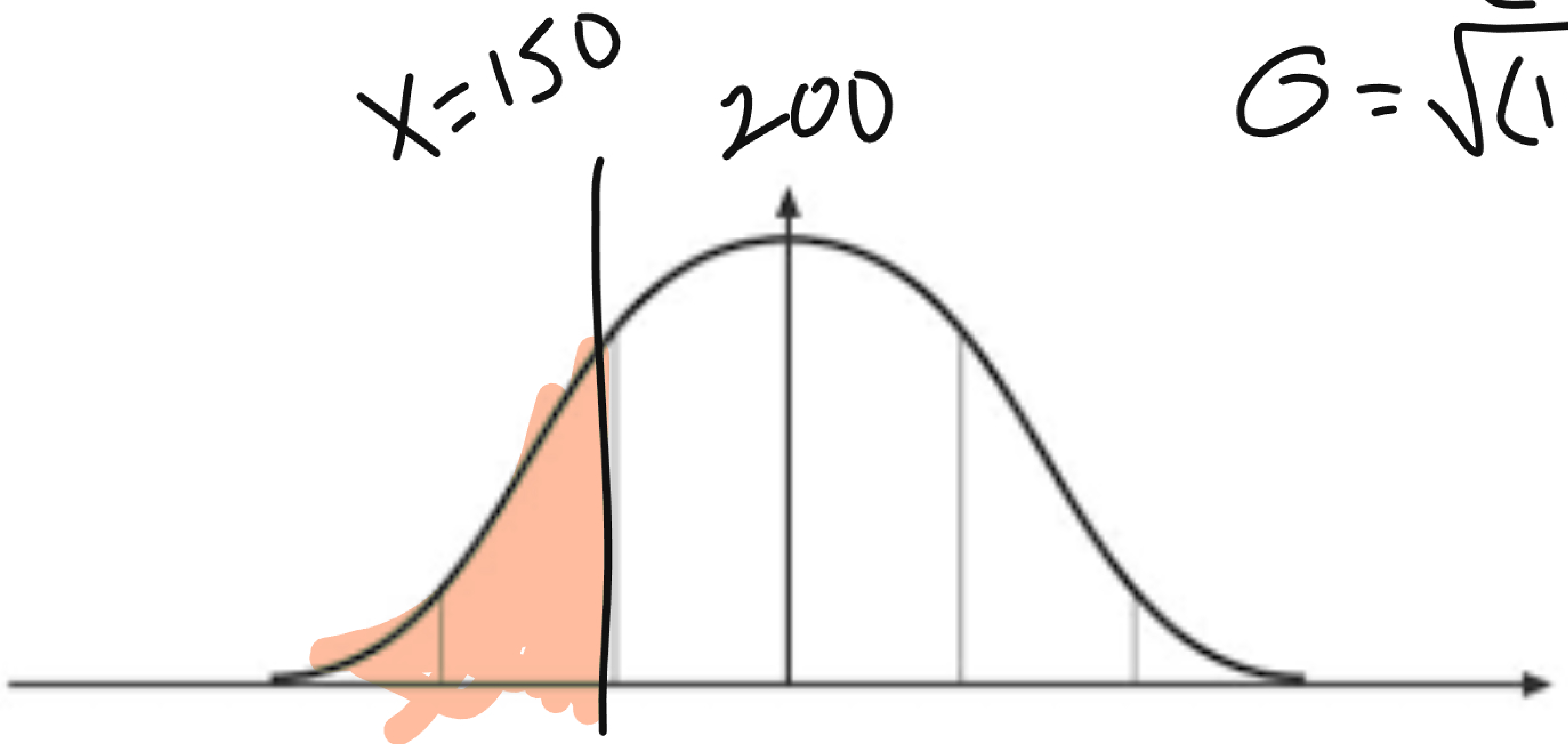
$X = 150.5$

$\mu = (.20)(1000) = 200$

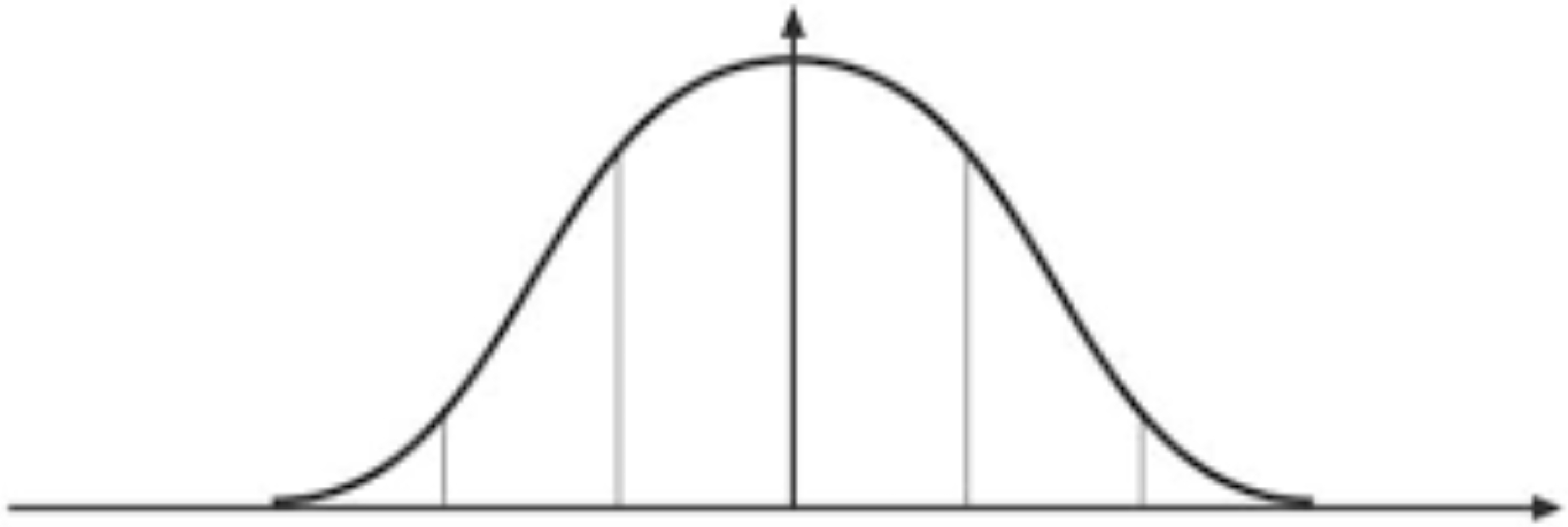
$\sigma = \sqrt{(1000)(.20)(1-.20)} = 12.649$

$Z = \dfrac{150.5 - 200}{12.649} = -3.91$

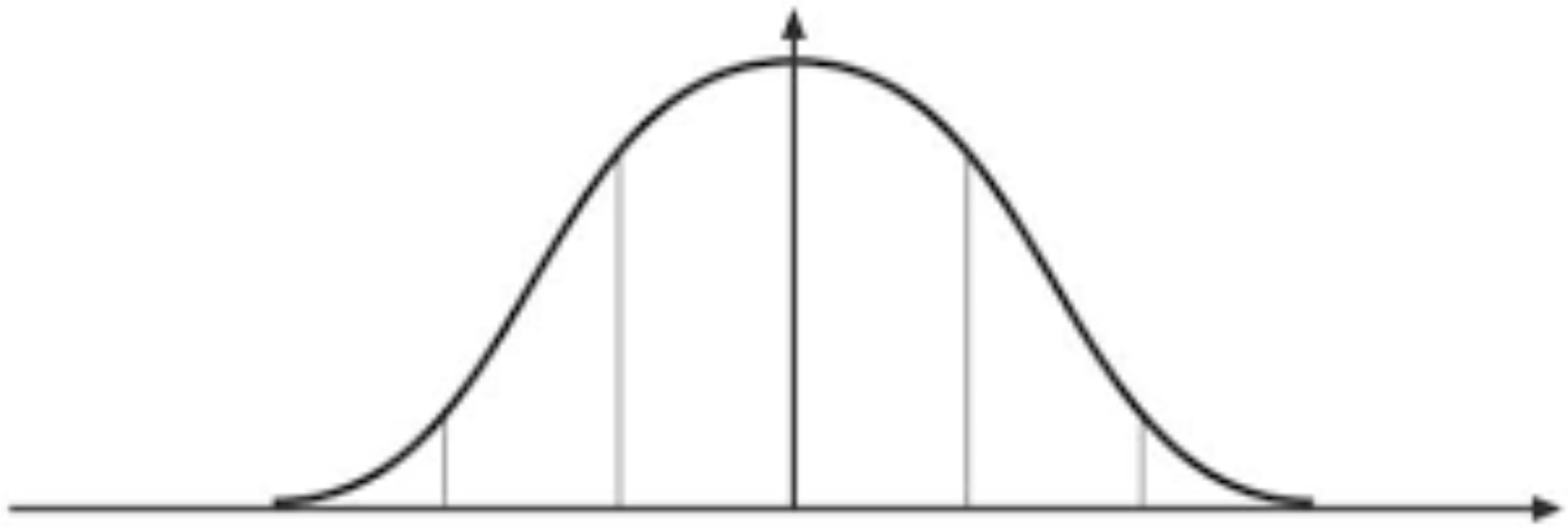
$.00001$



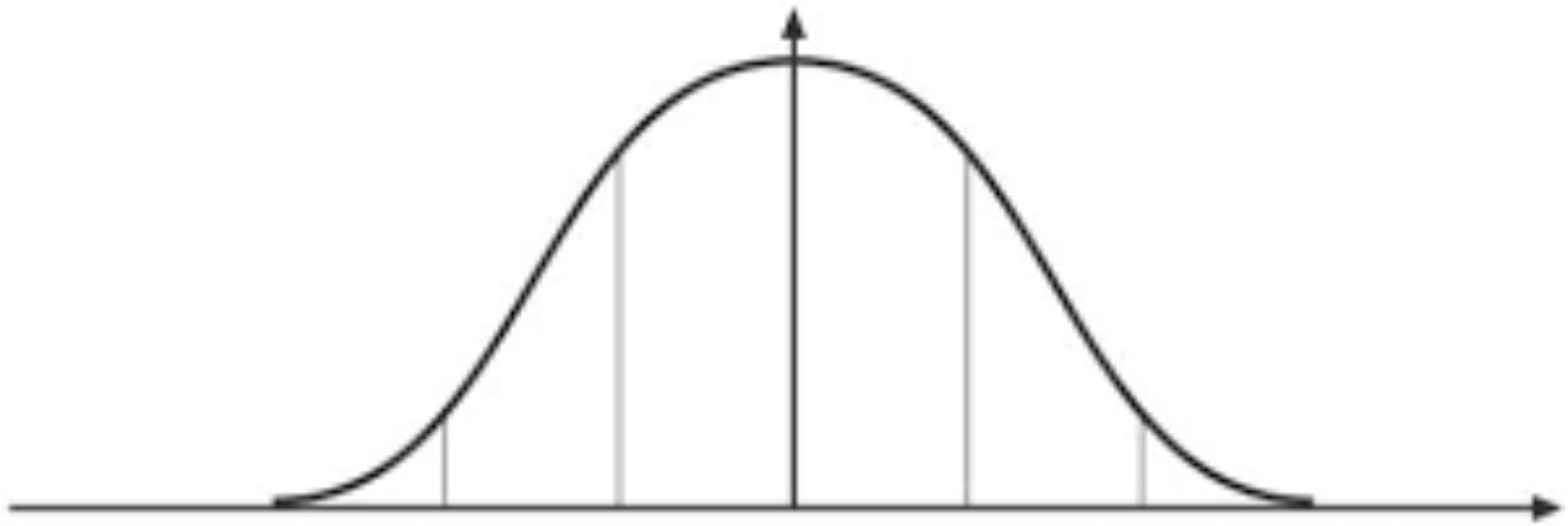
A popular restaurant near Tech's campus accepts 200 reservations on Saturdays, often the day of a home Tech football game. Given that many of the reservations are made weeks in advance of game day, the restaurant expects that about eight percent will be no-shows. What is the probability that the restaurant will have no more than 20 no-shows on the next Saturday of a football weekend?



A small commuter airline is concerned about reservation no-shows and, correspondingly, how much they should overbook flights to compensate. Assume their commuter planes will hold 15 people. Industry research indicates that 20% of the people making a reservation will not show up for a flight.



A cell phone manufacturer has developed a new type of battery for its phones. Extensive testing indicates that the population battery life (in days) obtained by all batteries of this new type is normally distributed with a mean of 700 days and a standard deviation of 100 days. The manufacturer wishes to offer a guarantee providing a discount on batteries if the original battery purchased does not exceed the days stated in the guarantee. What should the guaranteed battery life be (in days) if the manufacturer desires that no more than 5% of the batteries will fail to meet the guaranteed number of days?



A manufacturing plant utilizes 3000 electric light bulbs that have a length of life that is normally distributed with a mean of 500 hours and a standard deviation of 50 hours. To minimize the number of bulbs that burn out during operation hours, all the bulbs are replaced after a given period of operation. How often should the bulbs be replaced if we want not more than 2% of the bulbs to burn out between replacement periods?

Suppose a virus is believed to infect two percent of the population. If a sample of 3000 randomly selected subjects are tested, answer the following questions.

a. Find the expected number of subjects sampled that will be infected.

b. Find the standard deviation of the number of subjects sampled that will be infected.

c. What is the probability that fewer than 30 of the subjects in the sample will be infected?

d. What is the probability that between 40 and 80 (inclusive) of the subjects in the sample will be infected?

e. Find the probability that at least 70 of the subjects in the sample will be infected.