$\boxed{1}$ **Definition** The **tangent line** to the curve $y = f(x)$ at the point $P(a, f(a))$ is the line through P with slope

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

provided that this limit exists.

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EXAMPLE 1 Find an equation of the tangent line to the parabola $y = x^2$ at the point $P(1, 1)$.

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$y = x^2$ $\quad (1, 1)$

$a \quad f(a)$

$f(x) = x^2$

$a = 1$

$f(a) = 1$

$$\lim_{x \to 1} \frac{x^2 - 1}{\cancel{x - 1}} \qquad \cancel{(x-1)}(x+1)$$

$$\lim_{x \to 1} x + 1 \to 2 = m$$

$y - 1 = 2(x - 1)$

$\boxed{y = 2x - 1}$

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$x - a = h$ $\qquad$ $x = a + h$

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EXAMPLE 2 Find an equation of the tangent line to the hyperbola $y = 3/x$ at the point (3, 1).

$$\begin{matrix} a & f(a) \end{matrix} \qquad \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} \qquad f(x) = \frac{3}{x}$$

$$f(3+h) = \frac{3}{3+h}$$

$$\lim_{h \to 0} \frac{\boxed{\frac{3}{3+h} - 1}}{h} \qquad \frac{3}{3+h} - \frac{3+h}{3+h}$$

$$\frac{\frac{-h}{3+h}}{h} = \frac{-\cancel{h}}{3+h} \cdot \frac{1}{\cancel{h}} = \frac{-1}{3+h}$$

$$\lim_{h \to 0} \frac{-1}{3+h} \to \frac{-1}{3}$$

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$\boxed{3}$ **Definition** The **instantaneous velocity** of an object with position function $f(t)$ at time $t = a$ is

$$v(a) = \lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$$

provided that this limit exists.

EXAMPLE 3 Suppose that a ball is dropped from the upper observation deck of the CN Tower, 450 m above the ground.

(a) What is the velocity of the ball after 5 seconds?

(b) How fast is the ball traveling when it hits the ground?

$s = \underline{4.9} t^2$

$9.8 = g$

$$V(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

$$\lim_{h \to 0} \frac{4.9(a+h)^2 - 4.9a^2}{h}$$

$4.9(a^2 + 2ah + h^2) - 4.9a^2$

$4.9a^2 + 9.8ah + 4.9h^2 - 4.9a^2$

$9.8ah + 4.9h^2$

$h(9.8a + 4.9h)$

$$\lim_{h \to 0} \frac{\cancel{h}(9.8a + 4.9h)}{\cancel{h}} = \boxed{9.8a}$$

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4 Definition The **derivative of a function f at a number a**, denoted by $f'(a)$, is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

if this limit exists.

$$\boxed{f'(x) = y' = \frac{dy}{dx}} = \frac{df}{dx} = \frac{d}{dx} f(x) = Df(x) = D_x f(x)$$

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EXAMPLE 4 Use Definition 4 to find the derivative of the function $f(x) = x^2 - 8x + 9$ at the numbers (a) 2 and (b) a.

$$f'(a) = \lim_{h \to 0} \frac{[(a+h)^2 - 8(a+h) + 9] - (a^2 - 8a + 9)}{h}$$

$$f'(a) = \lim_{h \to 0} \frac{h(2a - 8 + h)}{h} = \boxed{2a - 8}$$

$$f'(2) = 2(2) - 8 = \boxed{-4}$$

EXAMPLE 3 If $f(x) = \sqrt{x}$, find the derivative of f. State the domain of f'.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{(\sqrt{x+h} + \sqrt{x})}{(\sqrt{x+h} + \sqrt{x})}$$

$$f'(x) = \lim_{h \to 0} \frac{\cancel{(x}+\cancel{h}) \cancel{- x}}{\cancel{h}(\sqrt{x+h} + \sqrt{x})} = \lim_{h \to 0} \frac{1}{\underbrace{\sqrt{x+h} + \sqrt{x}}_{\sqrt{x} + \sqrt{x}}} = \boxed{\frac{1}{2\sqrt{x}}}$$

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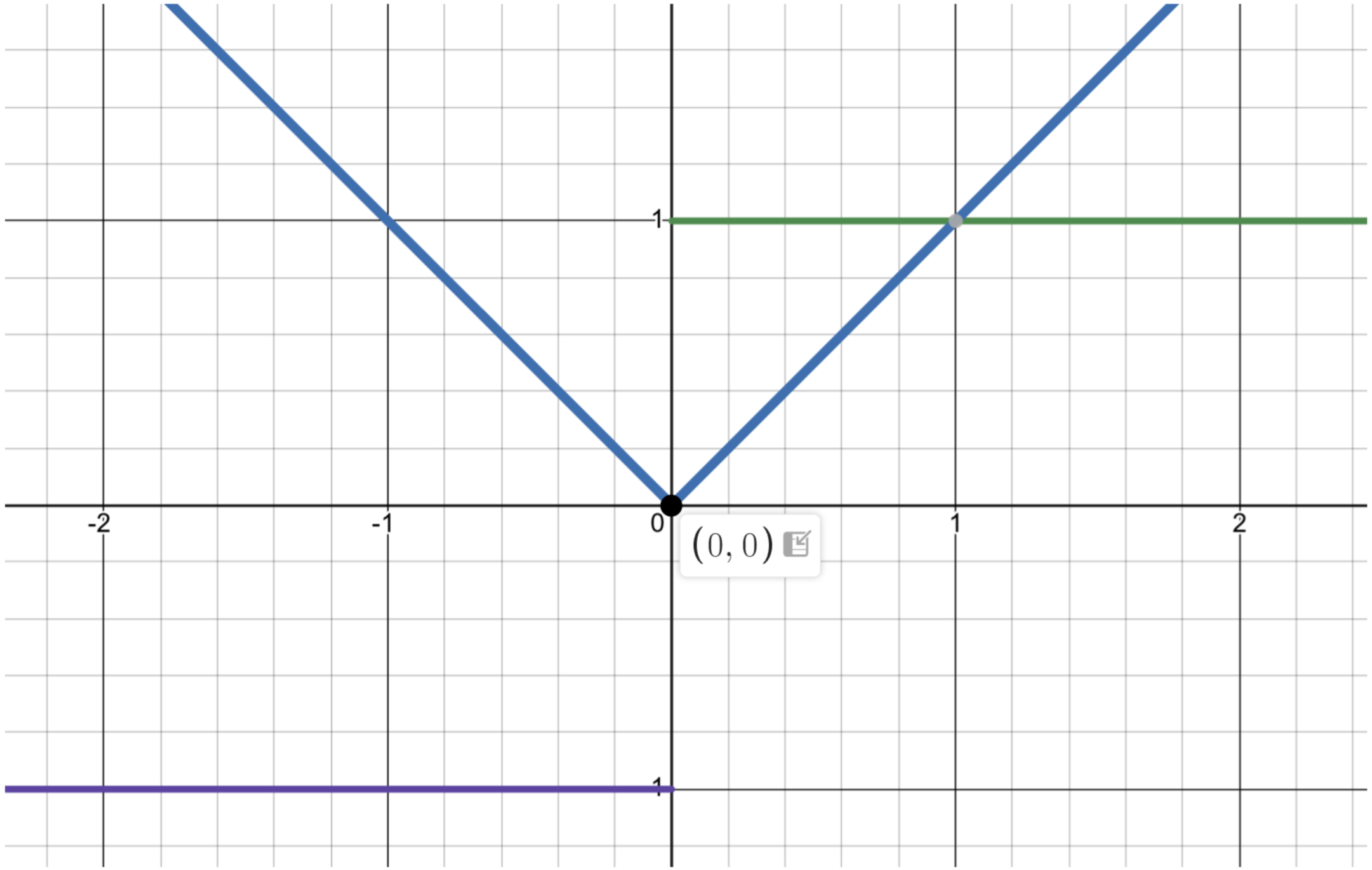
EXAMPLE 5 Where is the function $f(x) = |x|$ differentiable?



$(-\infty, 0) \cup$
$(0, \infty)$

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7

be

t *a*

(a) A corner

(b) A discontinuity

(c) A vertical tangent

4 **Theorem** If f is differentiable at a, then f is continuous at a.

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$$\cancel{4u^2}+\cancel{3u}+\cancel{4uh}-h-\cancel{1}$$

$$-5h$$

$$\frac{\dfrac{-5h}{(4u+4h-1)(4u-1)}}{h} \rightarrow \frac{-5\cancel{h}}{(4u+4h-1)(4u-1)} \cdot \frac{1}{\cancel{h}}$$

$$\lim_{h\to 0} \frac{-5}{(4u+4h-1)(4u-1)} = \boxed{\frac{-5}{(4u-1)^2}}$$

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The cost of producing x ounces of gold from a new gold mine is $C = f(x)$ dollars.

(a) What is the meaning of the derivative $f'(x)$? What are its units?

(b) What does the statement $f'(800) = 17$ mean?

(c) Do you think the values of $f'(x)$ will increase or decrease in the short term? What about the long term? Explain.

$$y = \text{fixed cost} + (\text{material cost})(x)$$

$$y = b + mx$$

Let $H(t)$ be the daily cost (in dollars) to heat an office building when the outside temperature is t degrees Fahrenheit.

(a) What is the meaning of $H'(58)$? What are its units?

(b) Would you expect $H'(58)$ to be positive or negative? Explain.

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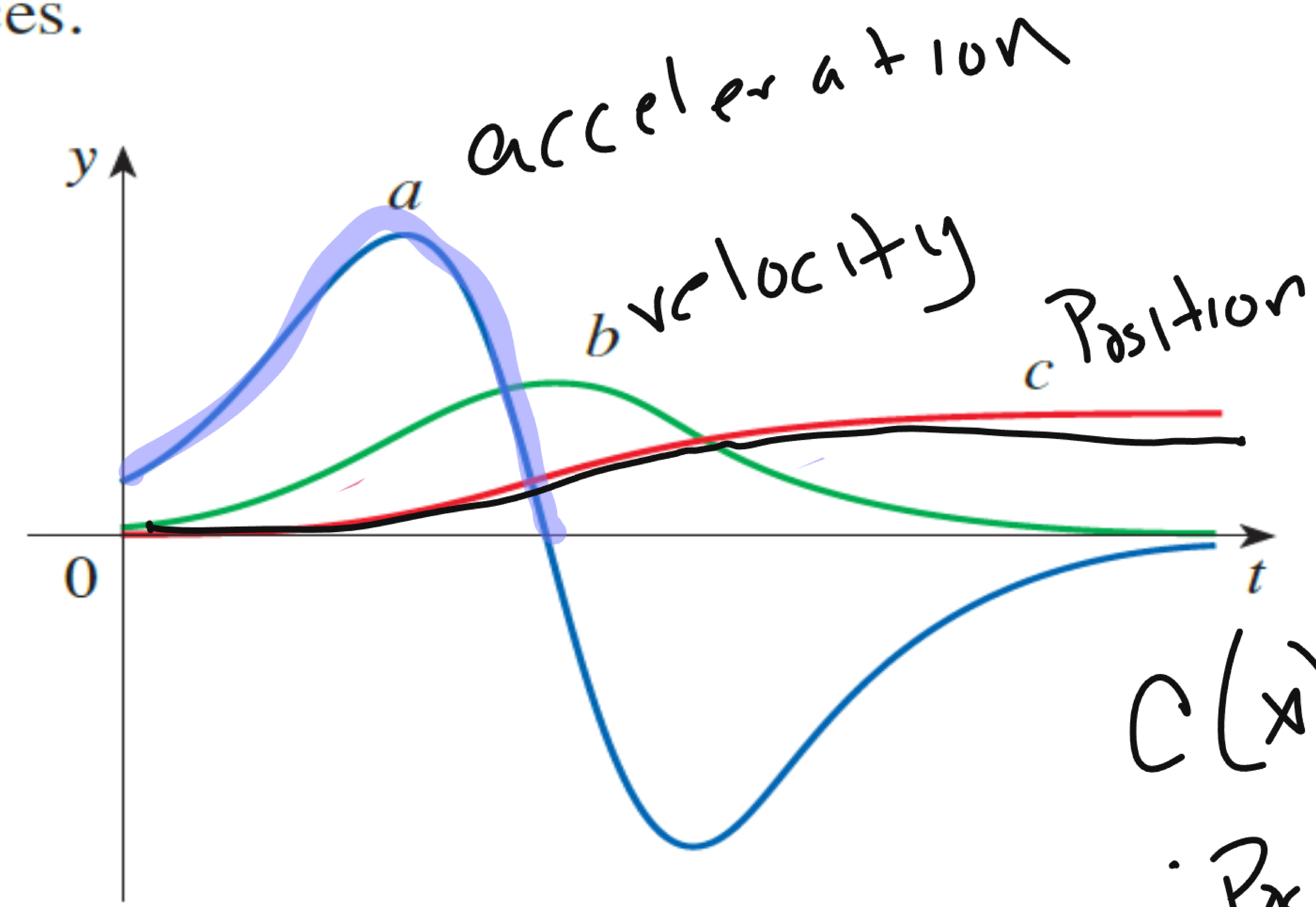
The figure shows the graphs of three functions. One is the position function of a car, one is the velocity of the car, and one is its acceleration. Identify each curve, and explain your choices.



Position

Velocity $= s'(x)$

acceleration $= s''(x)$
$= v'(x)$

$c(x) : b'(x) = a(x)$

Pos