Objectives

Define, identify, and graph quadratic functions.

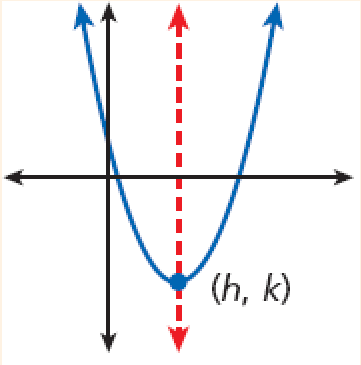
Identify and use maximums and minimums of quadratic functions to solve problems.

Vocabulary

axis of symmetry
standard form
minimum value
maximum value

This shows that parabolas are symmetric curves. The **axis of symmetry** is the line through the vertex of a parabola that divides the parabola into two congruent halves.

Axis of Symmetry | **Quadratic Functions**

WORDS	ALGEBRA	GRAPH
The axis of symmetry is a vertical line through the vertex of the function's graph.	The quadratic function $f(x) = a(x - h)^2 + k$ has the axis of symmetry $x = h$.	

Example 1: Identifying the Axis of Symmetry

Identify the axis of symmetry for the graph of $f(x) = -\frac{1}{2}(x+5)^2 - 8$ **.**

Rewrite the function to find the value of *h.*

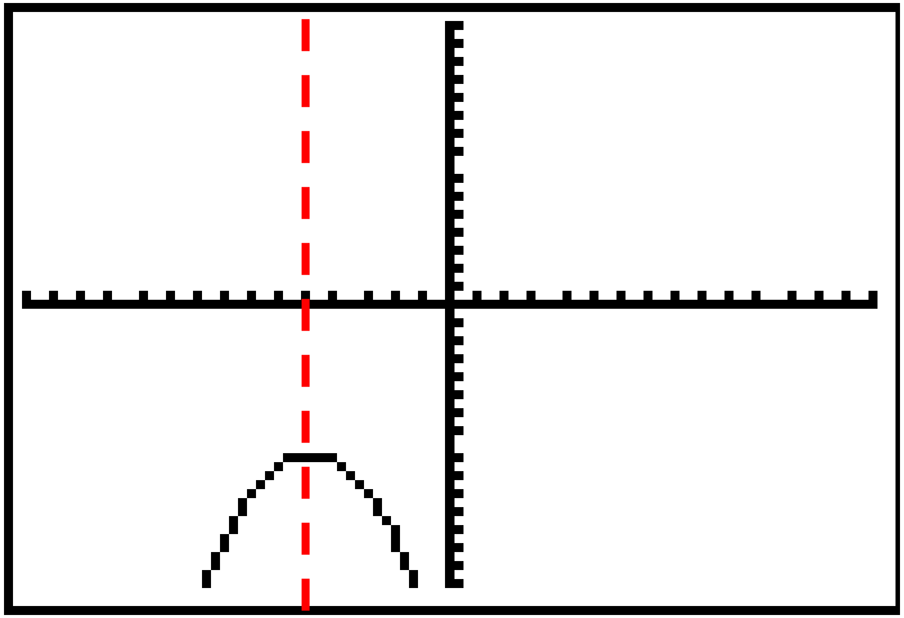
$$f(x) = -\frac{1}{2}\left[x-(-5)\right]^2 - 8$$

Because $h = -5$, the axis of symmetry is the vertical line $x = -5$.

Example 1 Continued

Check

Analyze the graph on a graphing calculator. The parabola is symmetric about the vertical line $x = -5$.



Check It Out! Example1

Identify the axis of symmetry for the graph of

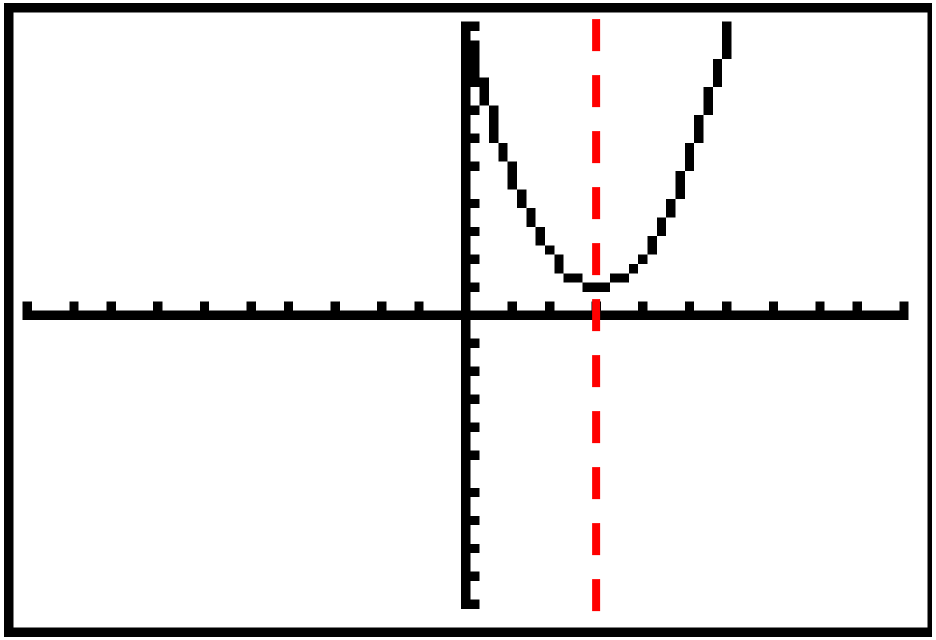
Rewrite the function to find the value of *h.*

$f(x) = [x - (3)]^2 + 1$

Because $h = 3$, the axis of symmetry is the vertical line $x = 3$.

Check It Out! Example1 Continued

Check Analyze the graph on a graphing calculator. The parabola is symmetric about the vertical line $x = 3$.



These properties can be generalized to help you graph quadratic functions.

Properties of a Parabola

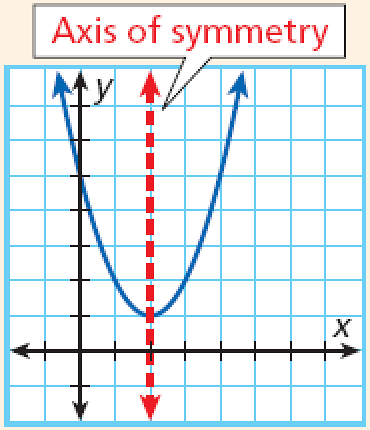
For $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$, the parabola has these properties:

The parabola **opens** upward if $a > 0$ and downward if $a < 0$.

The **axis of symmetry** is the vertical line $x = -\frac{b}{2a}$.

The **vertex** is the point $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

The ***y*-intercept** is c.



Helpful Hint

When *a* is positive, the parabola is happy (U).
When the *a* negative, the parabola is sad (∩).

Example 2A: Graphing Quadratic Functions in Standard Form

Consider the function $f(x) = 2x^2 - 4x + 5$.

a. Find the axis of symmetry.

The axis of symmetry is given by $x = -\frac{b}{2a}$.

$x = -\frac{(-4)}{2(2)} = 1$ *Substitute –4 for b and 2 for a.*

The axis of symmetry is the line $x = 1$.

Example 2A: Graphing Quadratic Functions in Standard Form

Consider the function $f(x) = 2x^2 – 4x + 5$.

b. Find the vertex.

The vertex lies on the axis of symmetry, so the *x*-coordinate is 1. The *y*-coordinate is the value of the function at this *x*-value, or $f(1)$.

$$f(1) = 2(1)^2 – 4(1) + 5 = 3$$

The vertex is (1, 3).

Example 2B: Graphing Quadratic Functions in Standard Form

Consider the function $f(x) = -x^2 - 2x + 3$.

a. Find the axis of symmetry.

The axis of symmetry is given by $x = -\frac{b}{2a}$.

$$x = -\frac{(-2)}{2(-1)} = -1$$

Substitute –2 for b and –1 for a.

The axis of symmetry is the line $x = -1$.

Example 2B: Graphing Quadratic Functions in Standard Form

Consider the function $f(x) = -x^2 - 2x + 3$.

b. Find the vertex.

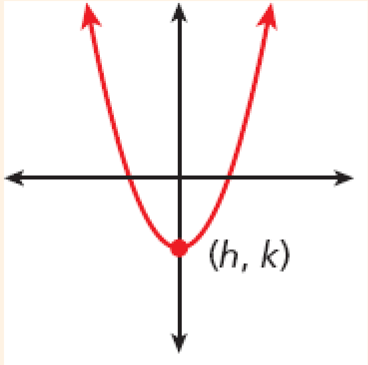
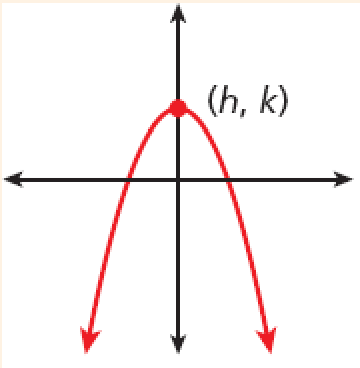
The vertex lies on the axis of symmetry, so the *x*-coordinate is –1. The *y*-coordinate is the value of the function at this *x*-value, or *f*(–1).

$f(-1) = -(-1)^2 - 2(-1) + 3 = 4$

The vertex is (–1, 4).

Substituting any real value of *x* into a quadratic equation results in a real number. Therefore, the domain of any quadratic function is all real numbers. The range of a quadratic function depends on its vertex and the direction that the parabola opens.

Minimum and Maximum Values

OPENS UPWARD	OPENS DOWNWARD
When a parabola opens upward, the y-value of the vertex is the **minimum value**.	When a parabola opens downward, the y-value of the vertex is the **maximum value**.
D: $\{x \mid x \in \mathbb{R}\}$ R: $\{y \mid y \geq k\}$	D: $\{x \mid x \in \mathbb{R}\}$ R: $\{y \mid y \leq k\}$
	
The domain is all real numbers, $\mathbb{R}$. The range is all values greater than or equal to the minimum.	The domain is all real numbers, $\mathbb{R}$. The range is all values less than or equal to the maximum.