8 CHAPTER
Continuous Probability Distributions

Continuous Uniform Distribution

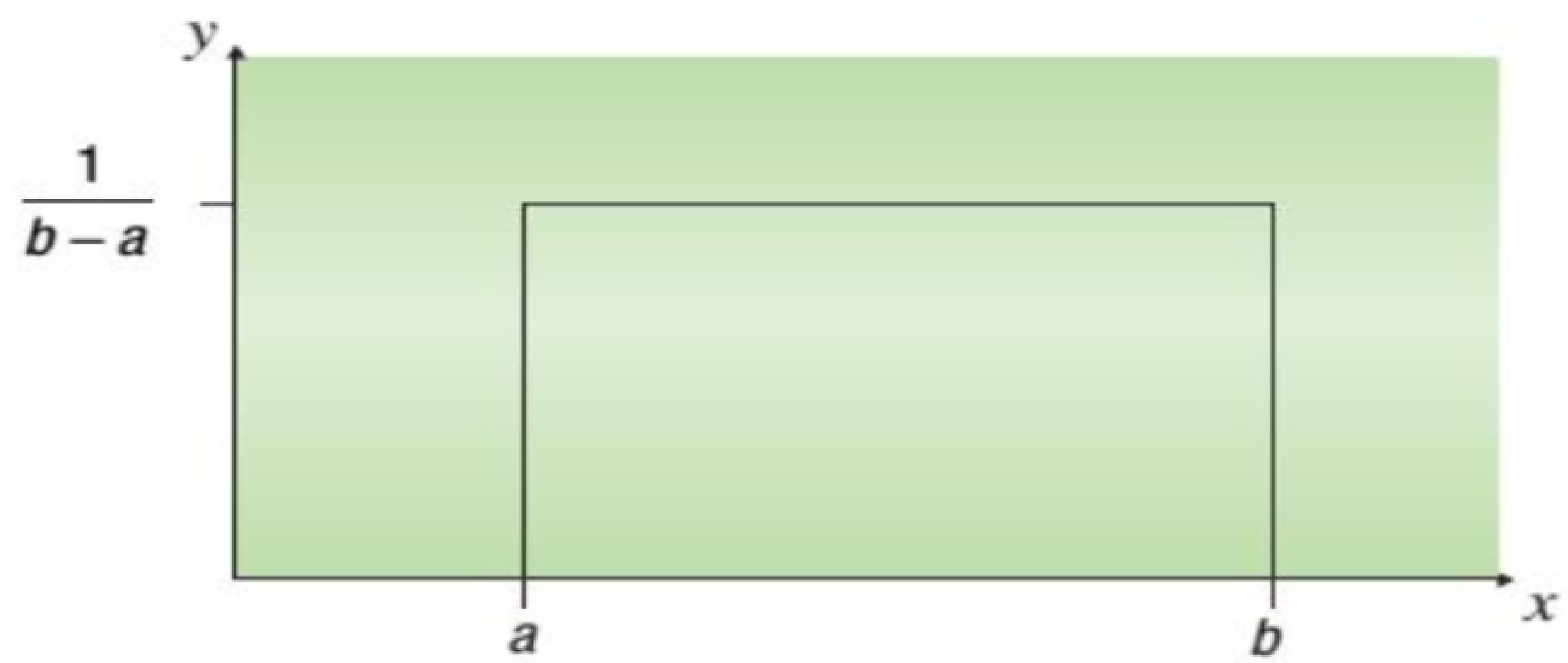


Figure 8.1.1

Uniform Probability Density Function

The **uniform probability density function** is given by

$$f(x) = \begin{cases} \dfrac{1}{b-a} & \text{for } a \le x \le b \\ 0 & \text{otherwise.} \end{cases}$$

The mean and standard deviation are given by the following expressions.

$$\mu = \frac{a+b}{2} \text{ and } \sigma = \frac{b-a}{\sqrt{12}}$$

FORMULA

olar Bear Frozen Foods manufactures frozen French fries for sale to grocery ore chains. The final package weight is thought to be a uniformly distributed ndom variable. Assume X, the weight of French fries, has a uniform stribution between 57 ounces and 63 ounces.

$\mu = \frac{63+57}{2} =$

$\sigma = \frac{63-57}{\sqrt{12}} =$

What is the mean weight for a package?

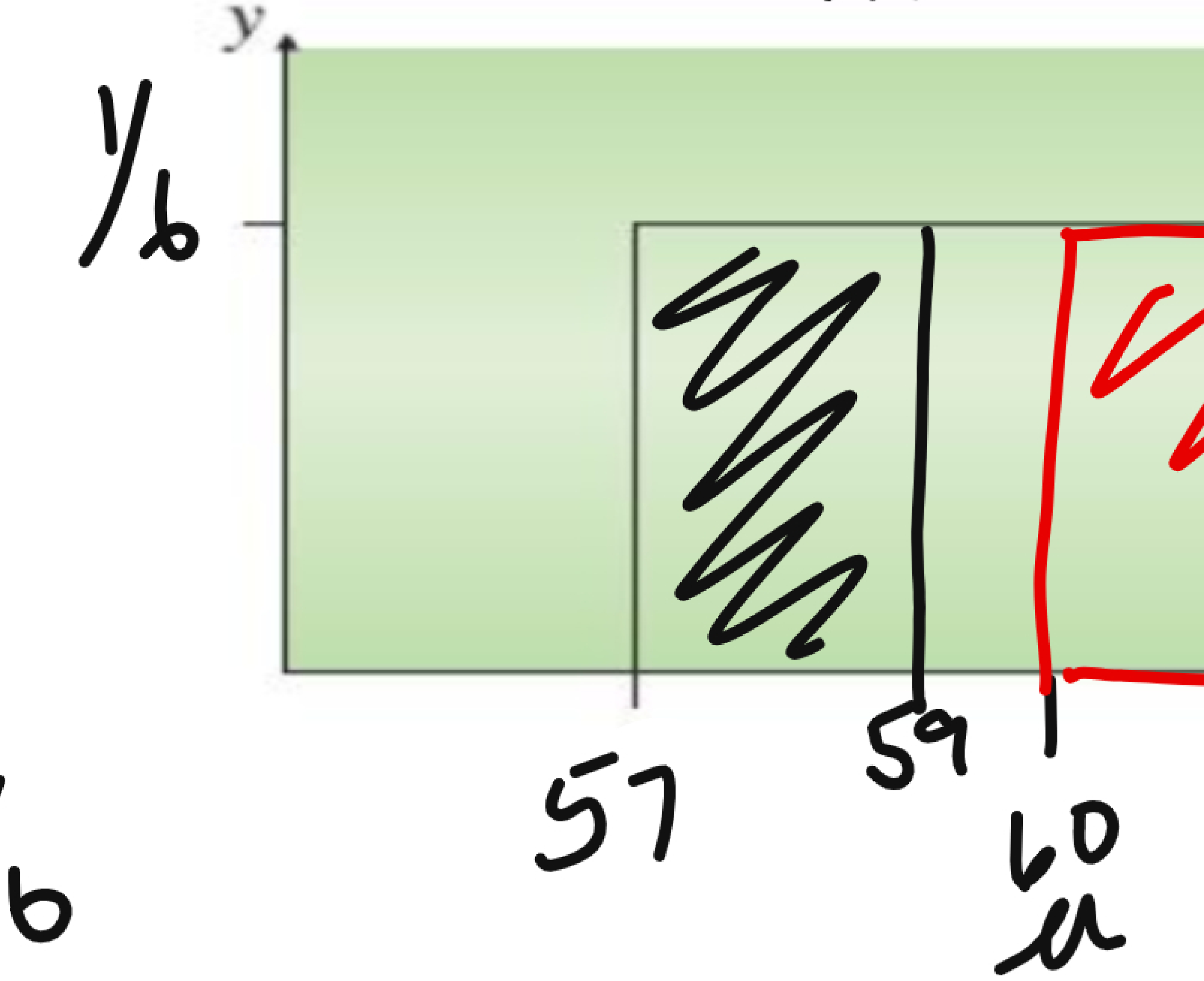
What is the standard deviation for the weight of a package?

What is the probability that a store will receive a package weighing less than 59 ounces? $2(1/6) = 1/3$

What is the probability that a package will contain between 60 and 63 ounces? $3(1/6) = 1/2$

What is the probability that a package will contain more than 62 ounces? $1/6$

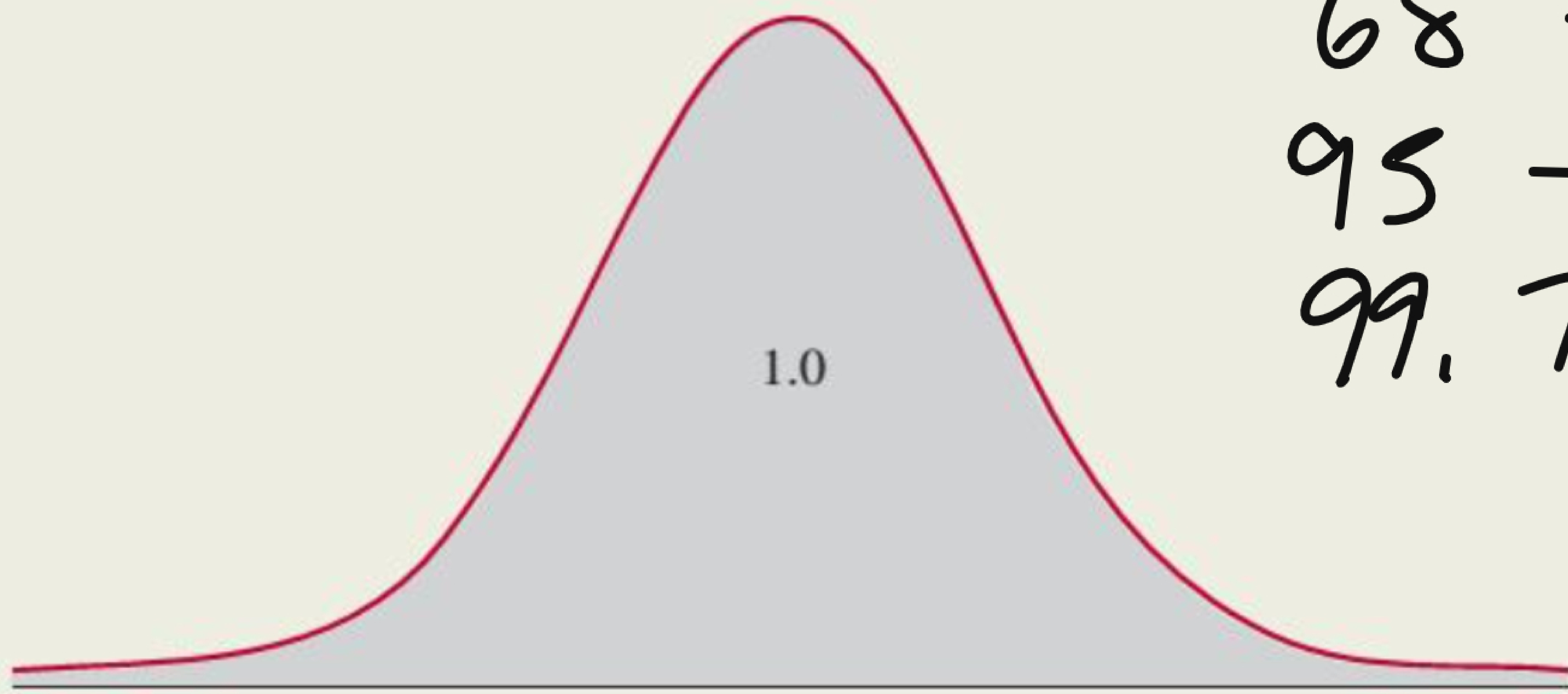
Find the probability that a package will contain exactly 60 ounces.



Properties of the Normal Distribution

The total area under any normal curve equals one.

68 - 1σ
95 - 2σ
99.7 - 3σ



Within a fixed number of standard deviations from the mean, all normal distributions contain the same fraction of their probability.

The probability of a normal random variable being in some interval corresponds to the area under the curve bounded by the specified interval.

PROPERTIES

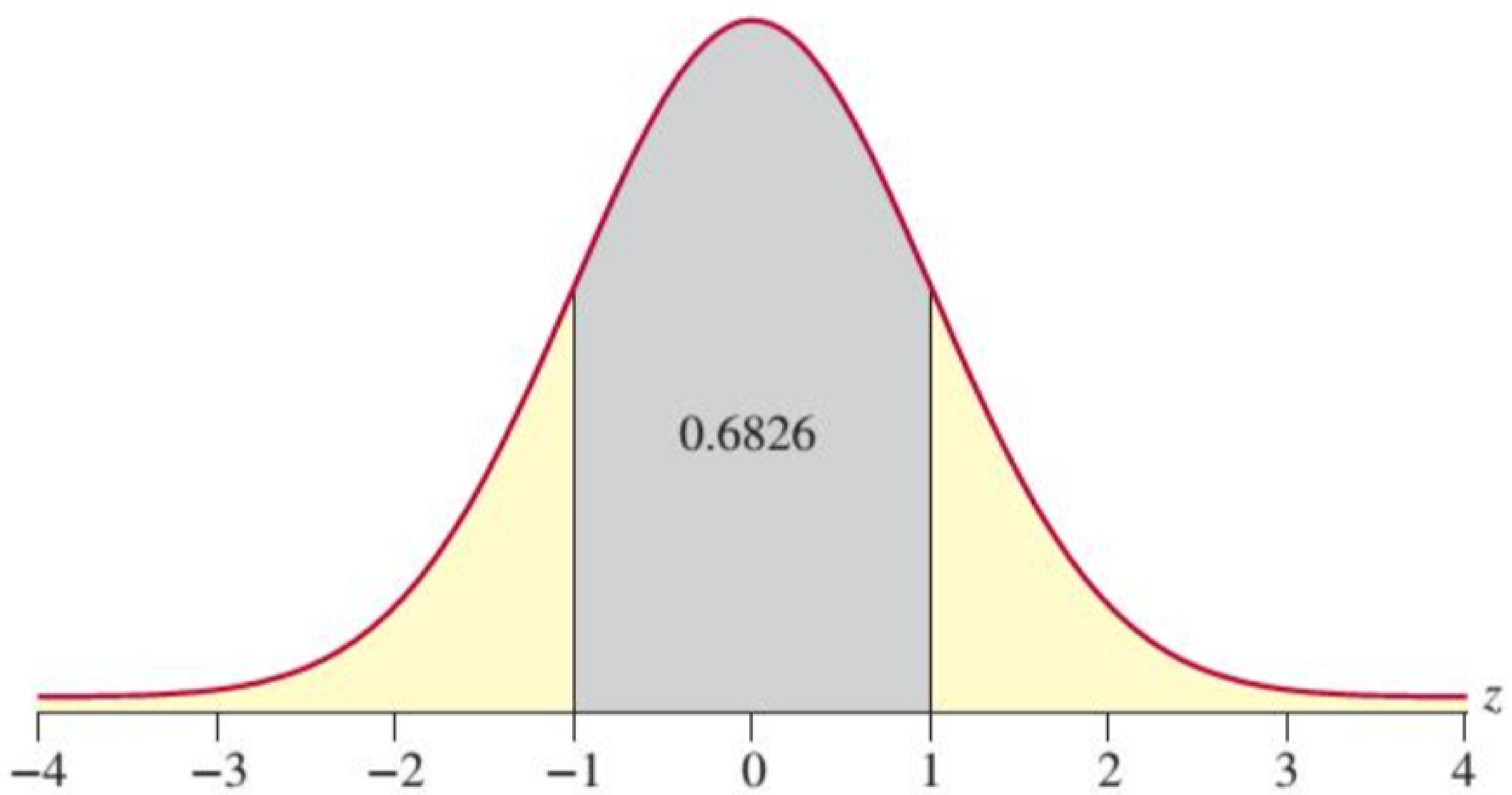


Figure 8.2.1

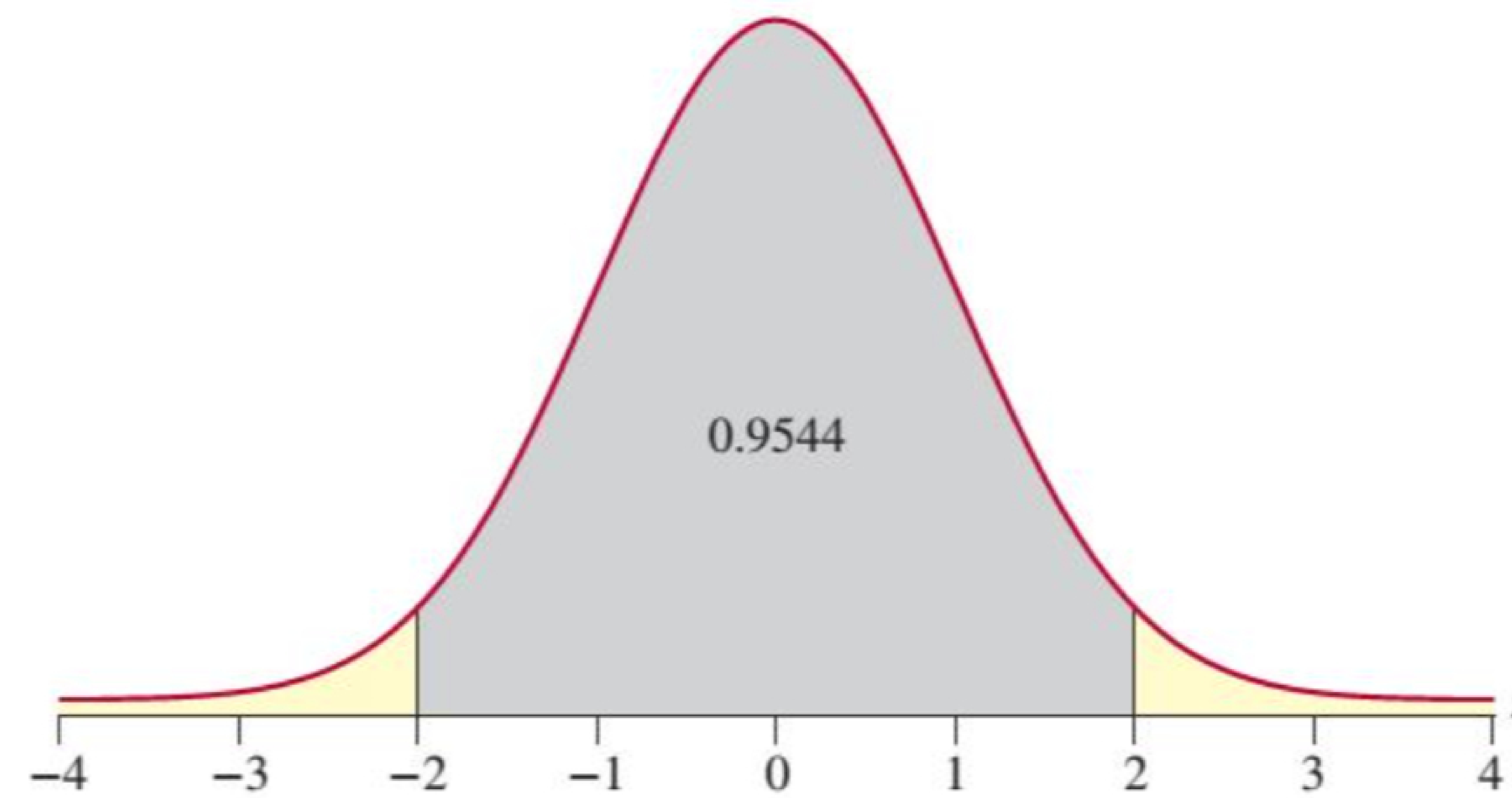


Figure 8.2.2

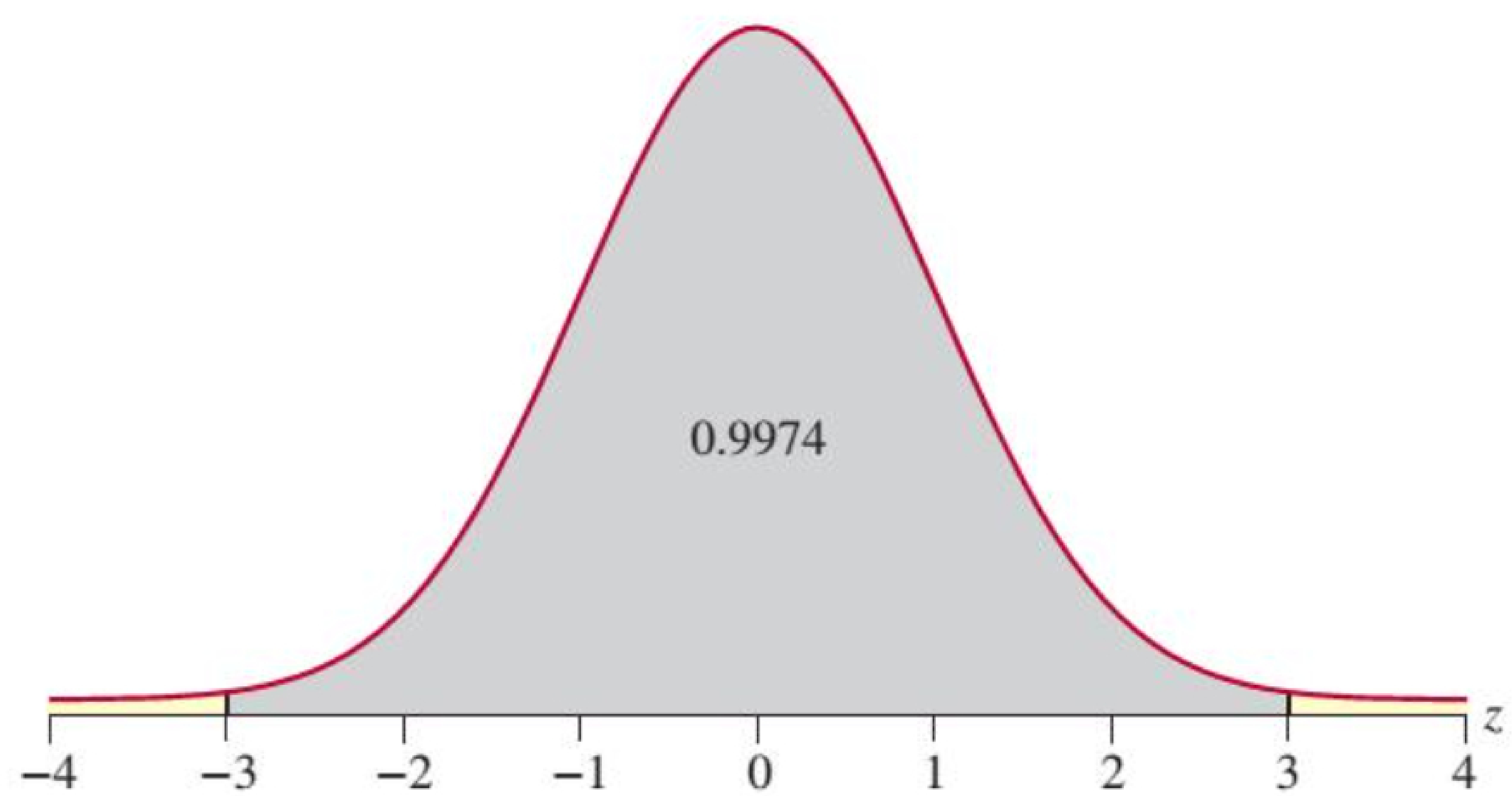


Figure 8.2.3

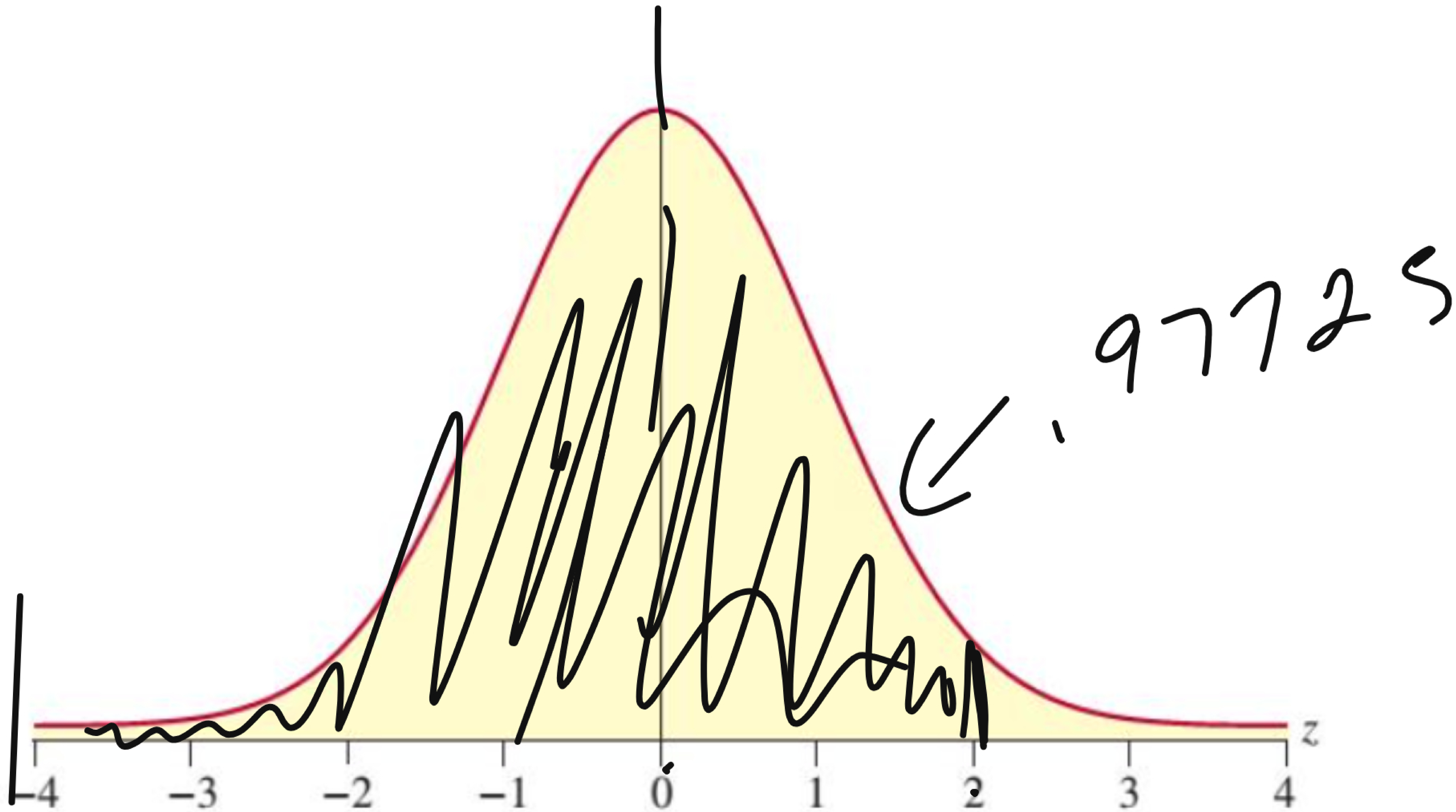


Figure 8.3.1

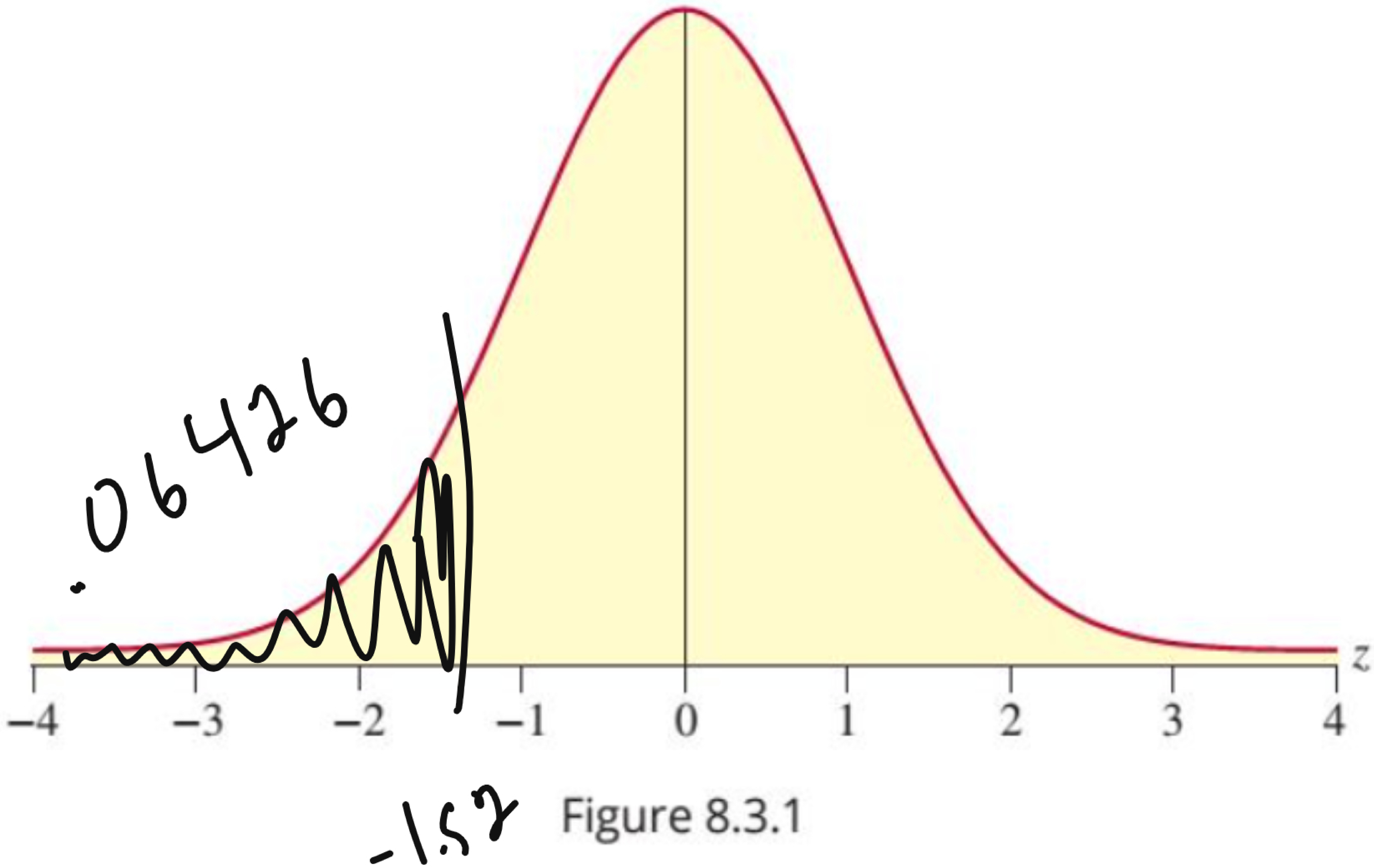


Figure 8.3.1

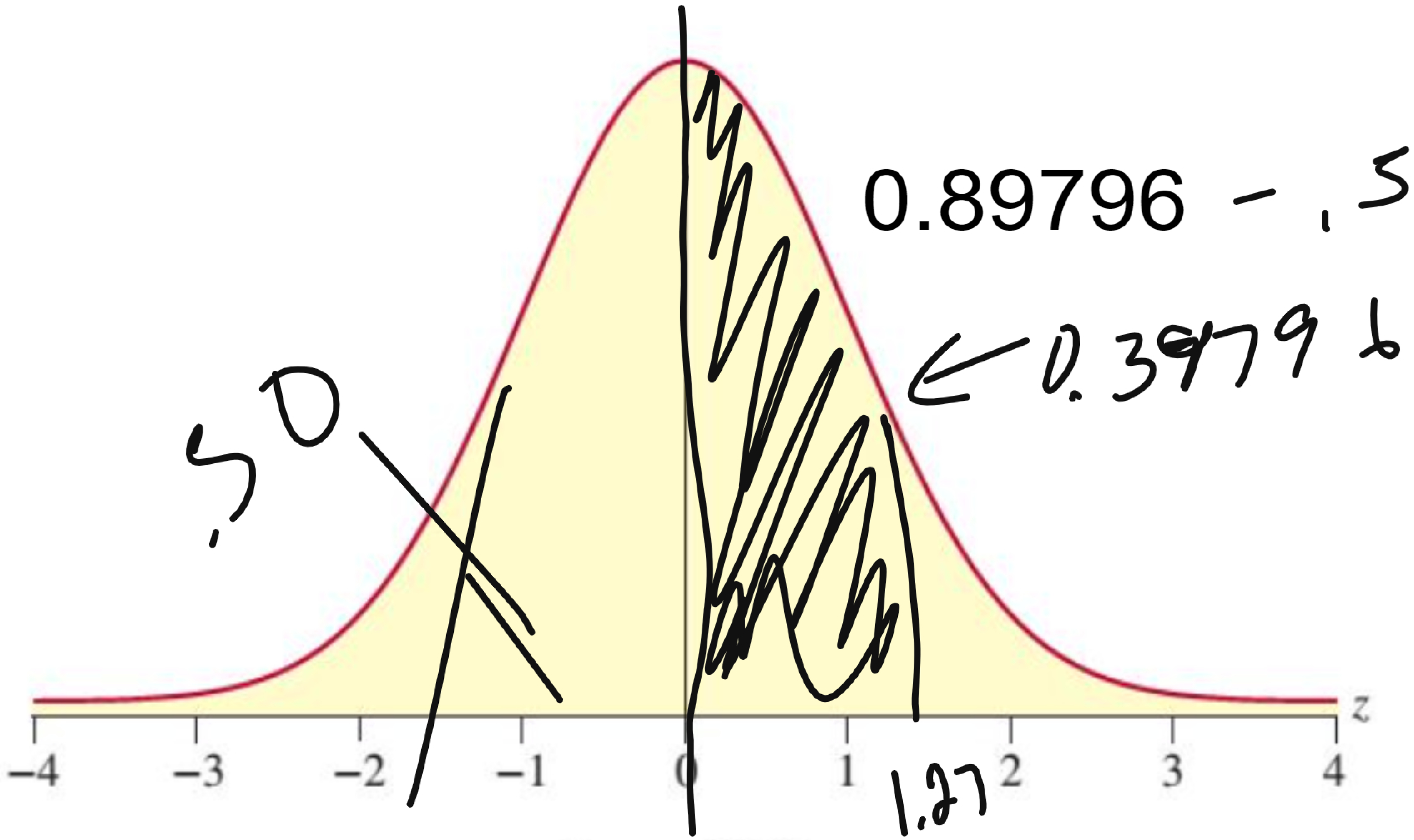


Figure 8.3.1

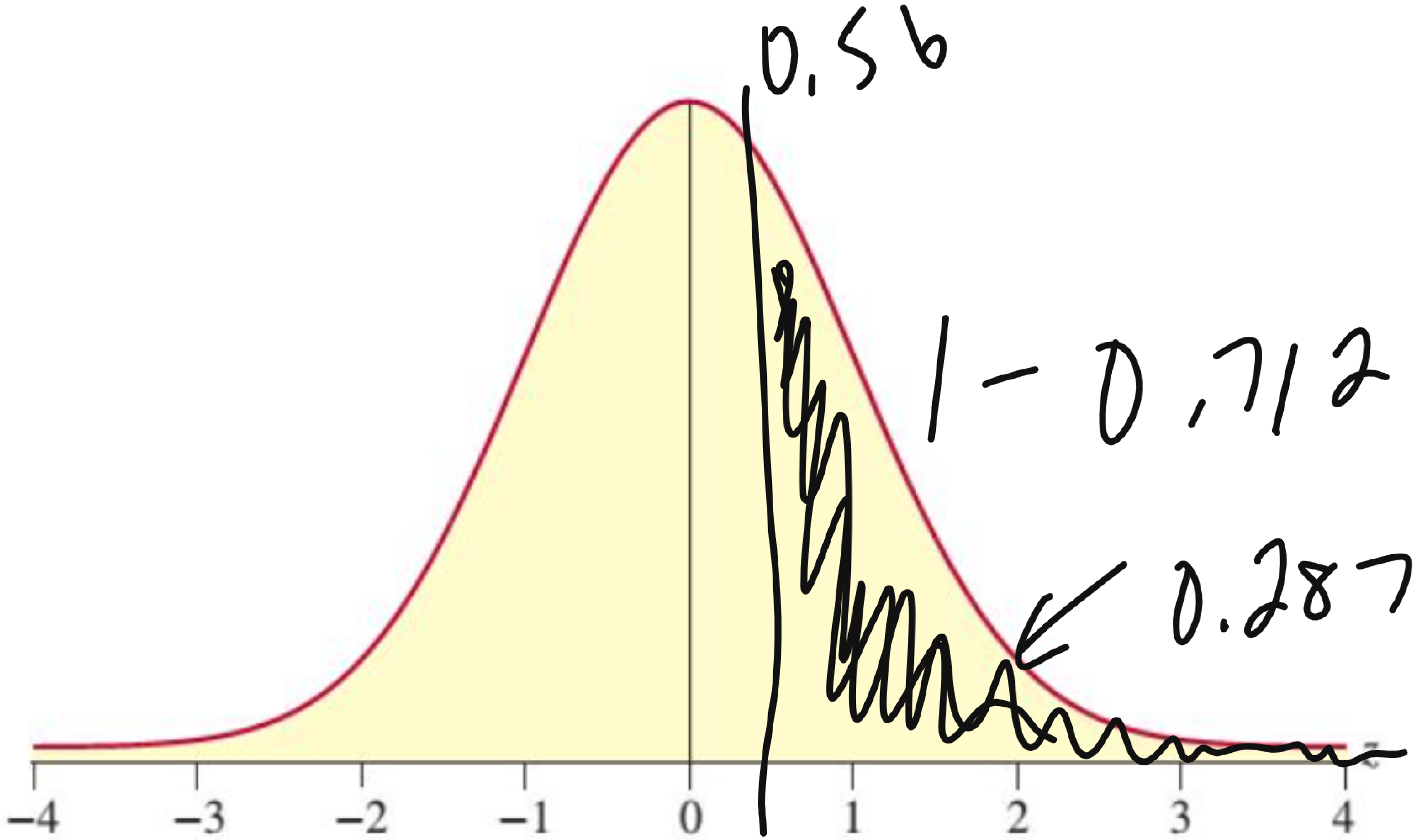
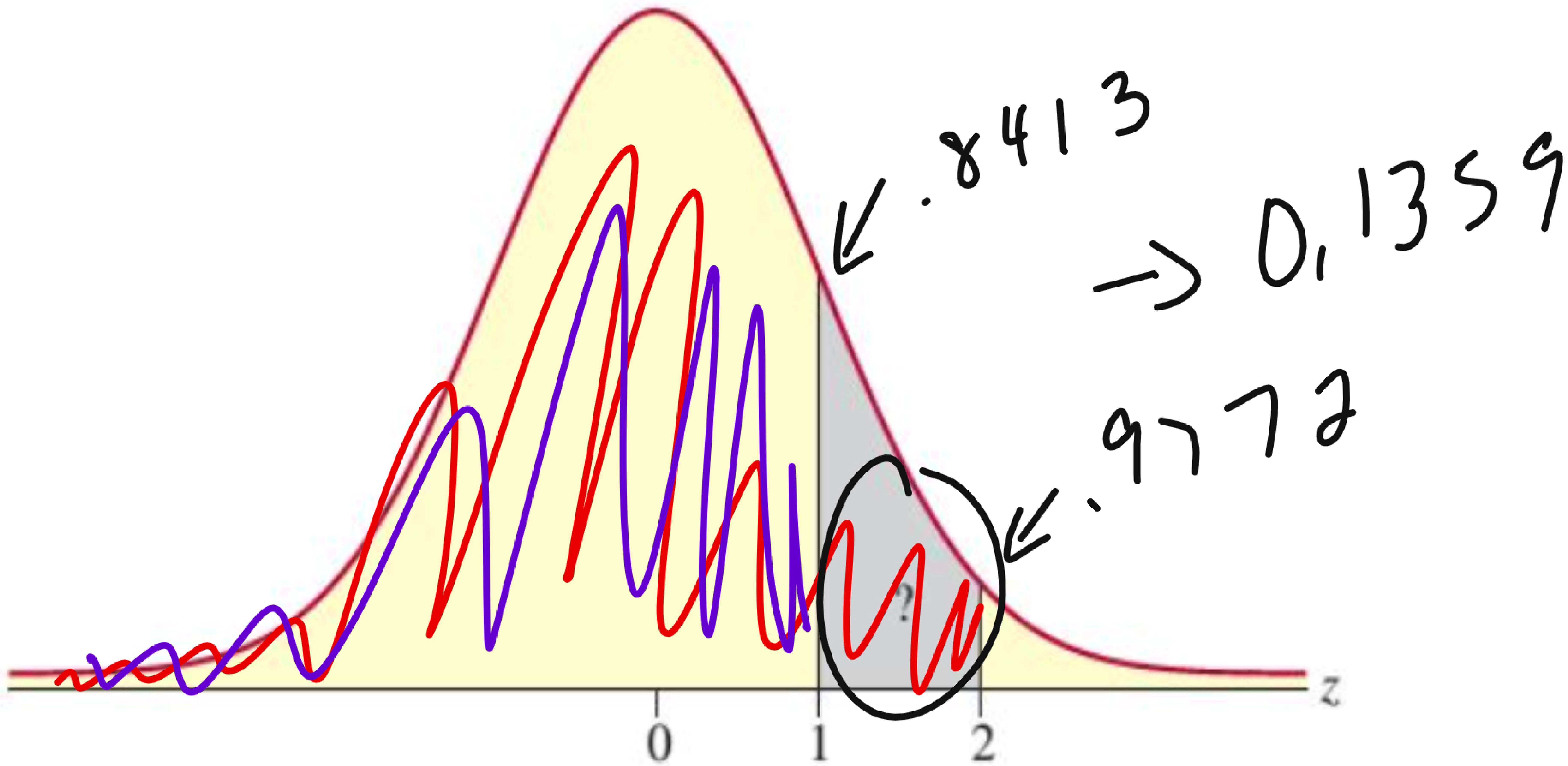
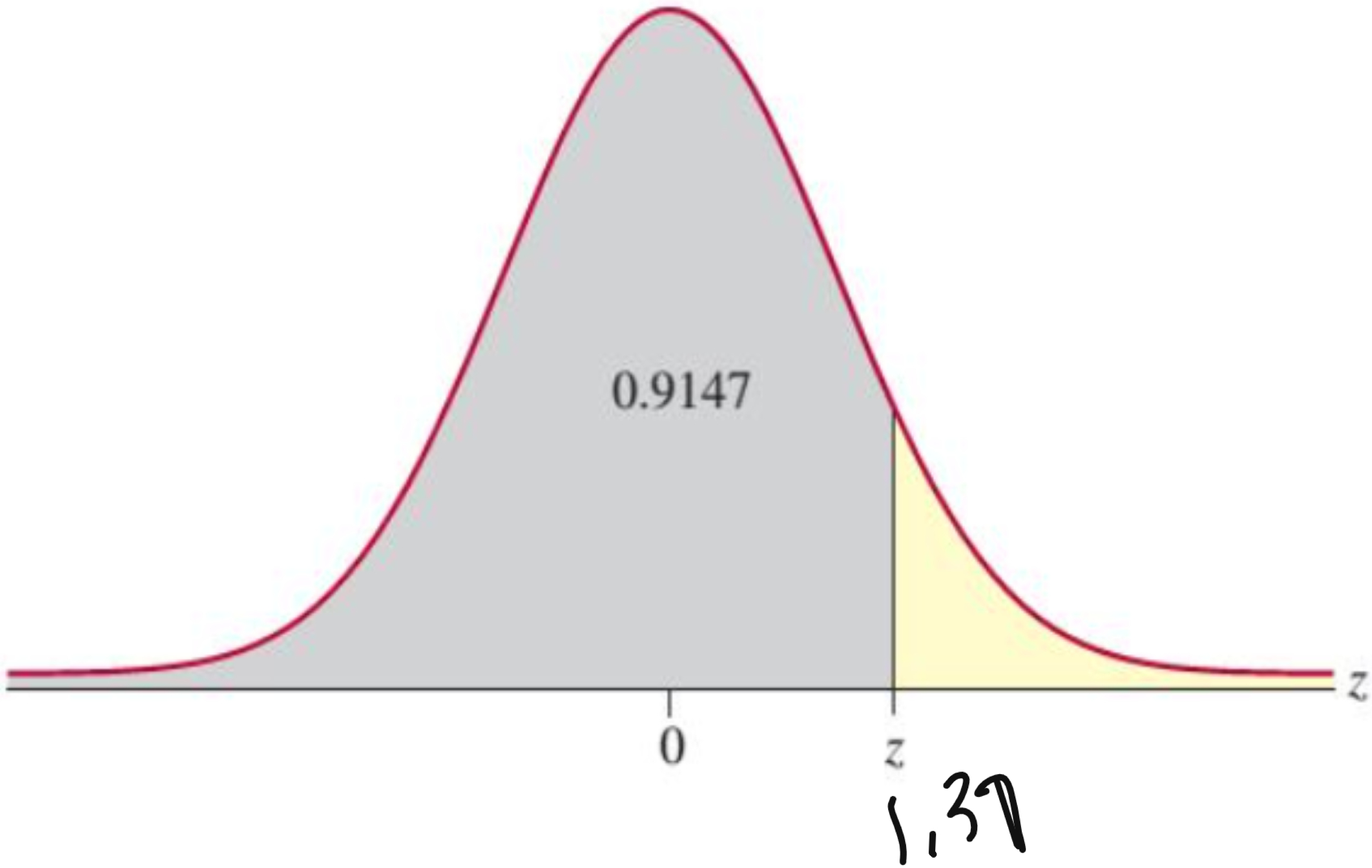
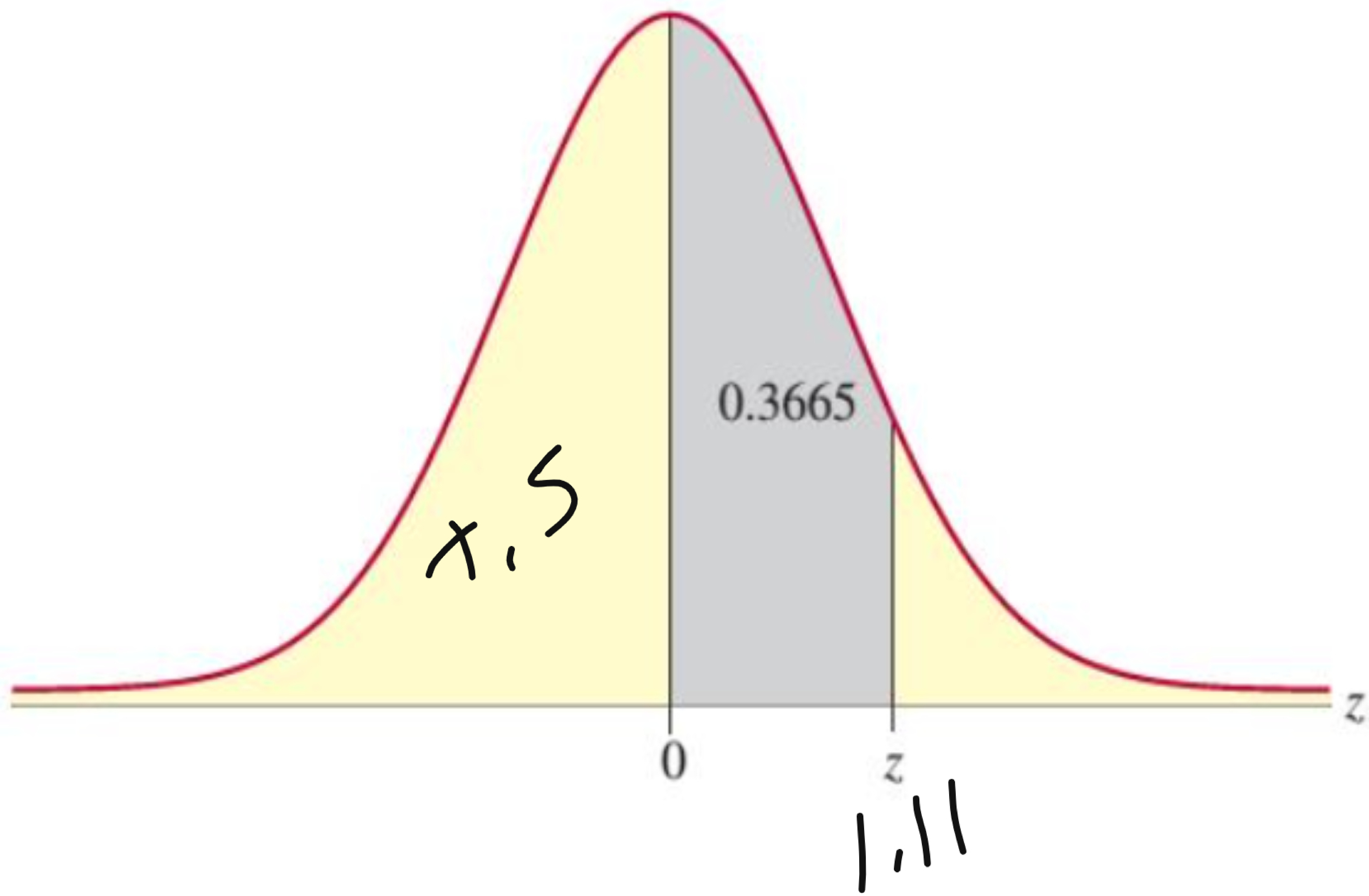


Figure 8.3.1


.8413
→ 0,1359
.9772
?
0
1
2
z


0.9147
0
z
z
1,37


0.3665
0
z
z
1.11

$1.27 \le z \le 3$

$P(1.27 \le z < 3)$

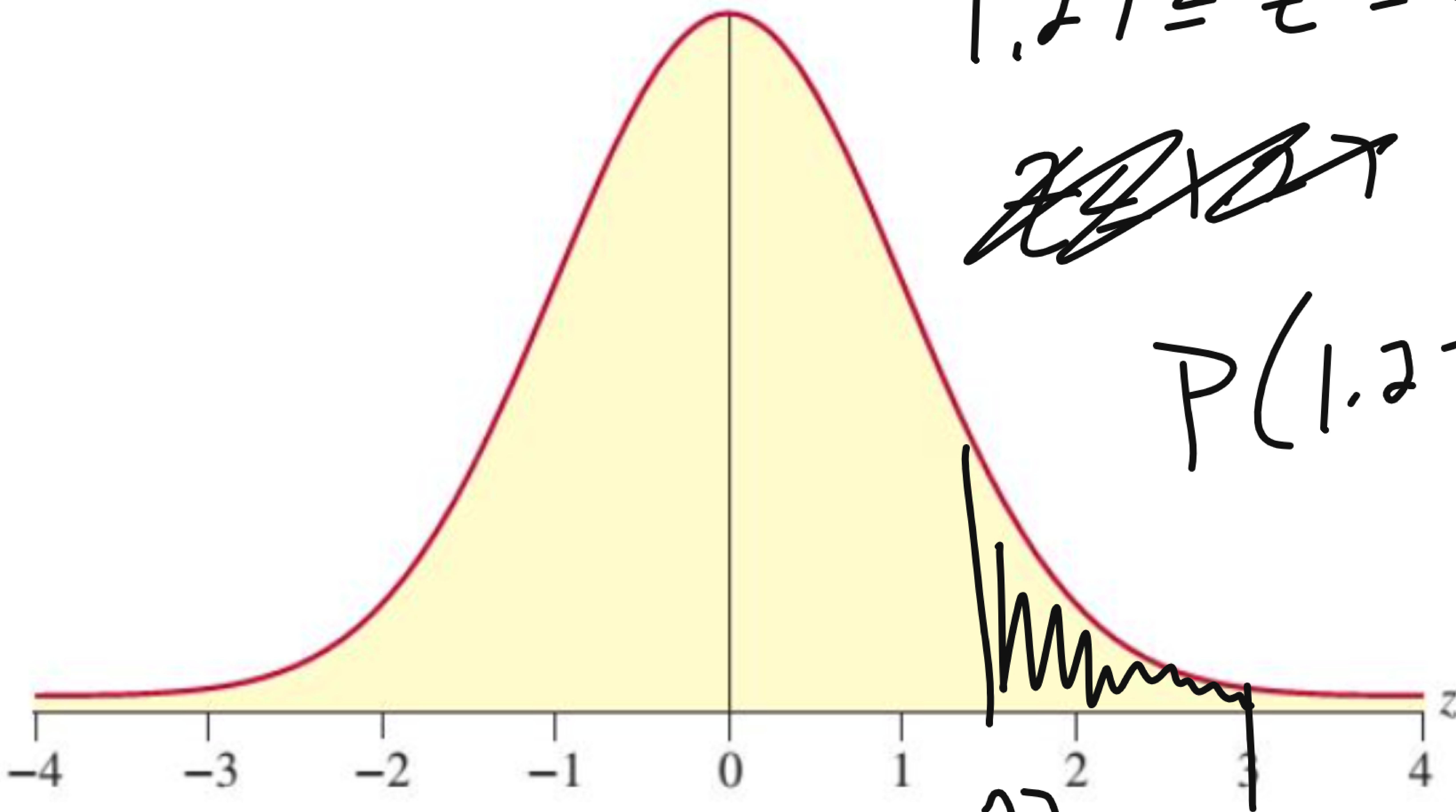


Figure 8.3.1

Standardizing a Normal Random Variable

The following formula can transform any normal random variable into a standard normal random variable, z.

$$z = \frac{x - \mu}{\sigma}$$

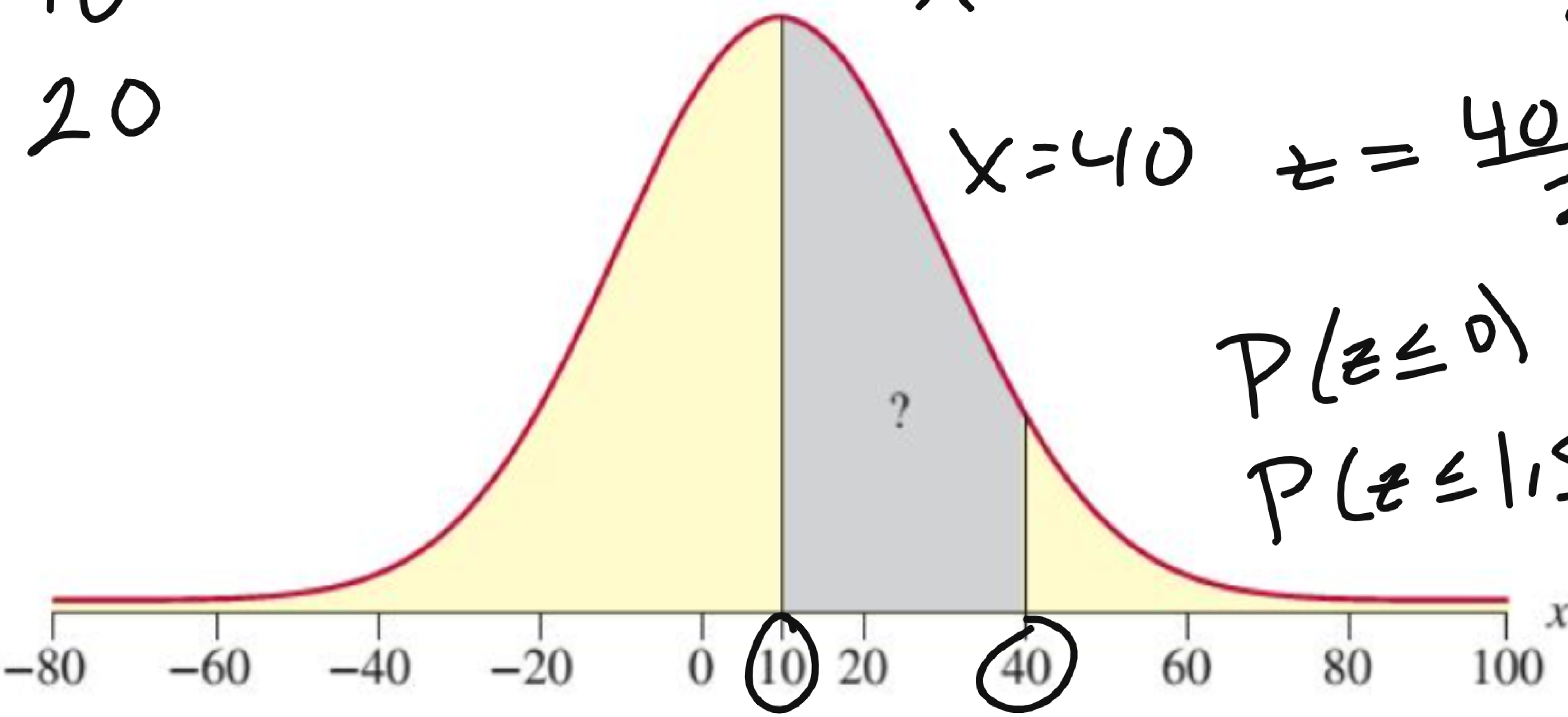
where x is a normal random variable with mean μ and standard deviation σ.

FORMULA

probability that a normal random variable with a mean of 10 and a standard
n of 20 will lie between 10 and 40.

. 10

20



$X = 10 \quad Z = \frac{10-10}{20} = 0$

$X = 40 \quad z = \frac{40-10}{20} = 1.5$

$P(z \le 0) = 0.5000$

$P(z \le 1.5) = 0.9332$

$P(0 \le z \le 1.5) = .4332$

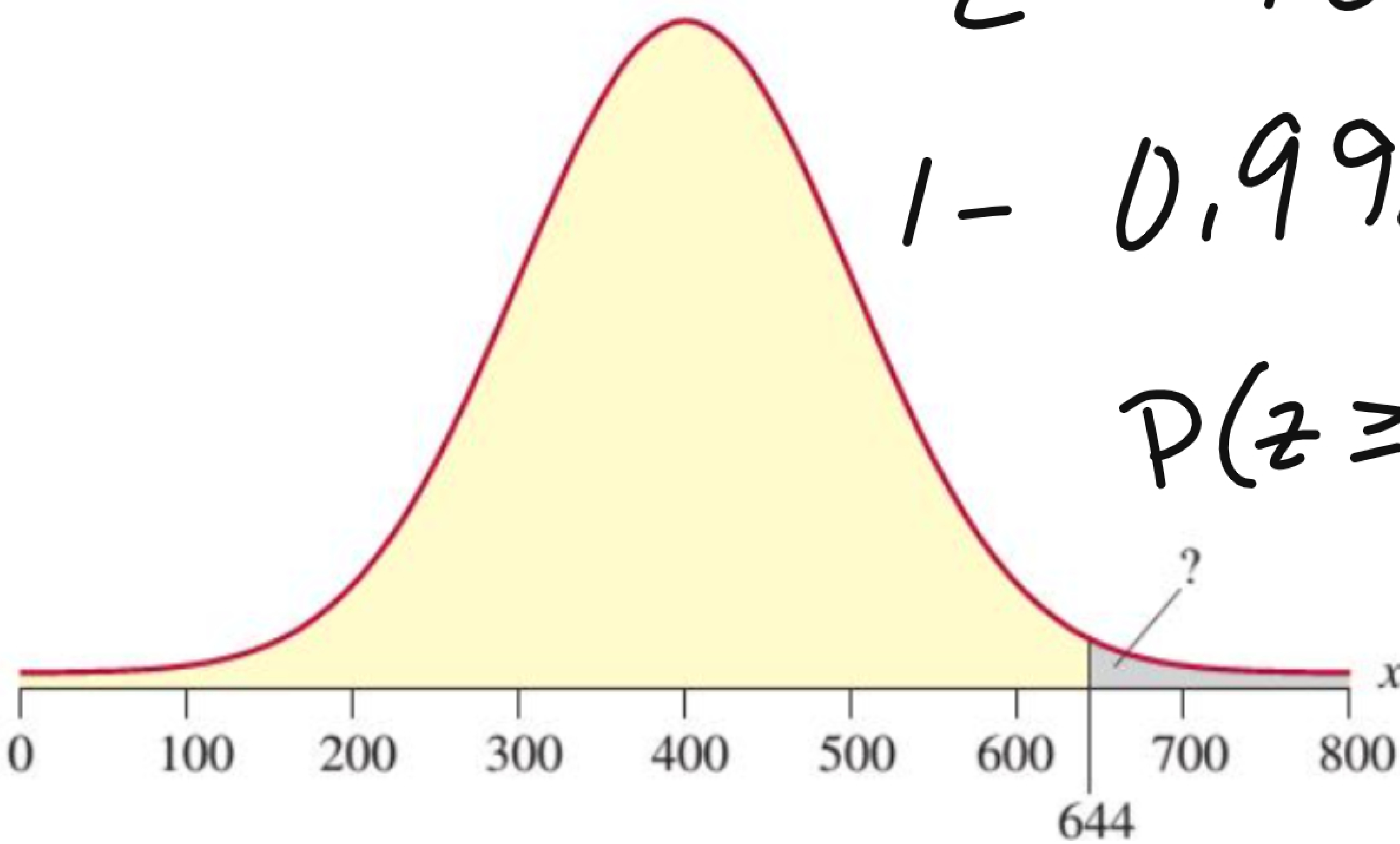
Suppose that a national testing service gives a test in which the results are normally distributed with a mean of 400 and a standard deviation of 100. If you score a 644 on the test, what fraction of the students taking the test exceeded your score?

$$z = \frac{644-400}{100} = 2.44$$

$$1 - 0.9927$$

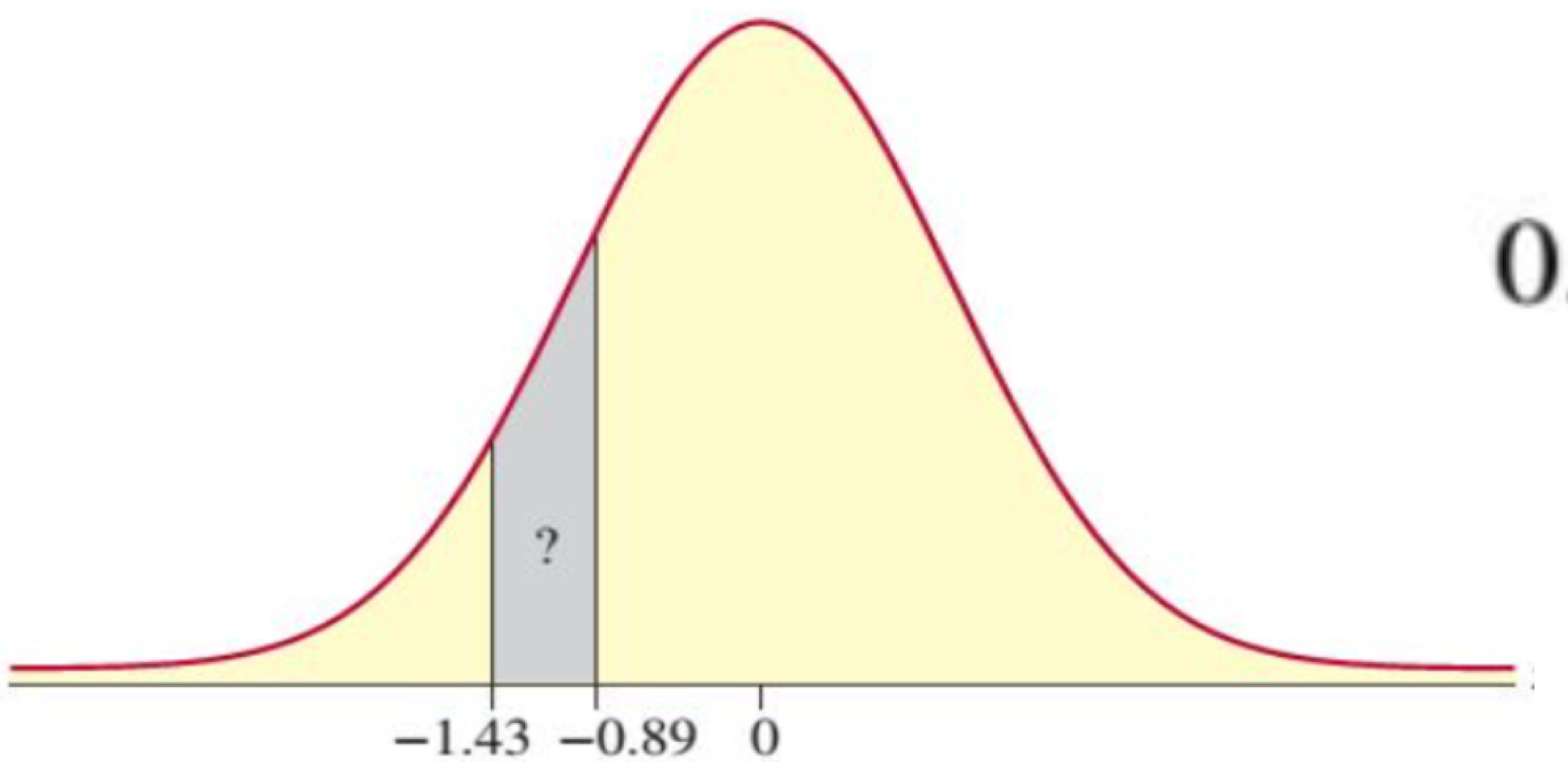
$$P(z \geq 2.44) = .0073$$

$$.73\%$$



Suppose that for 132 shoppers making a purchase at a clothing store, the total each shopper will spend follows a normal distribution with a mean of \$234 and a standard deviation of \$94. What is the probability that the next purchase total will be between \$100 and \$150?

$$P(100 < x < 150) = P\left(\frac{100-234}{94} < z < \frac{150-234}{94}\right) = P(-1.43 < z < -0.89)$$



$$0.1867 - 0.0764 = 0.1103$$

The body temperatures of adults are normally distributed with a mean of 98.60 °F and a standard deviation of 0.73 °F. What temperature represents the 90th percentile?

$\mu = 98.6 \quad \sigma = 0.73$

$z = 1.28 = \dfrac{x - 98.6}{0.73}$ (0.73)

$0.9344 = x - 98.6$

$+98.6 \quad +98.6$

$99.53 = x$

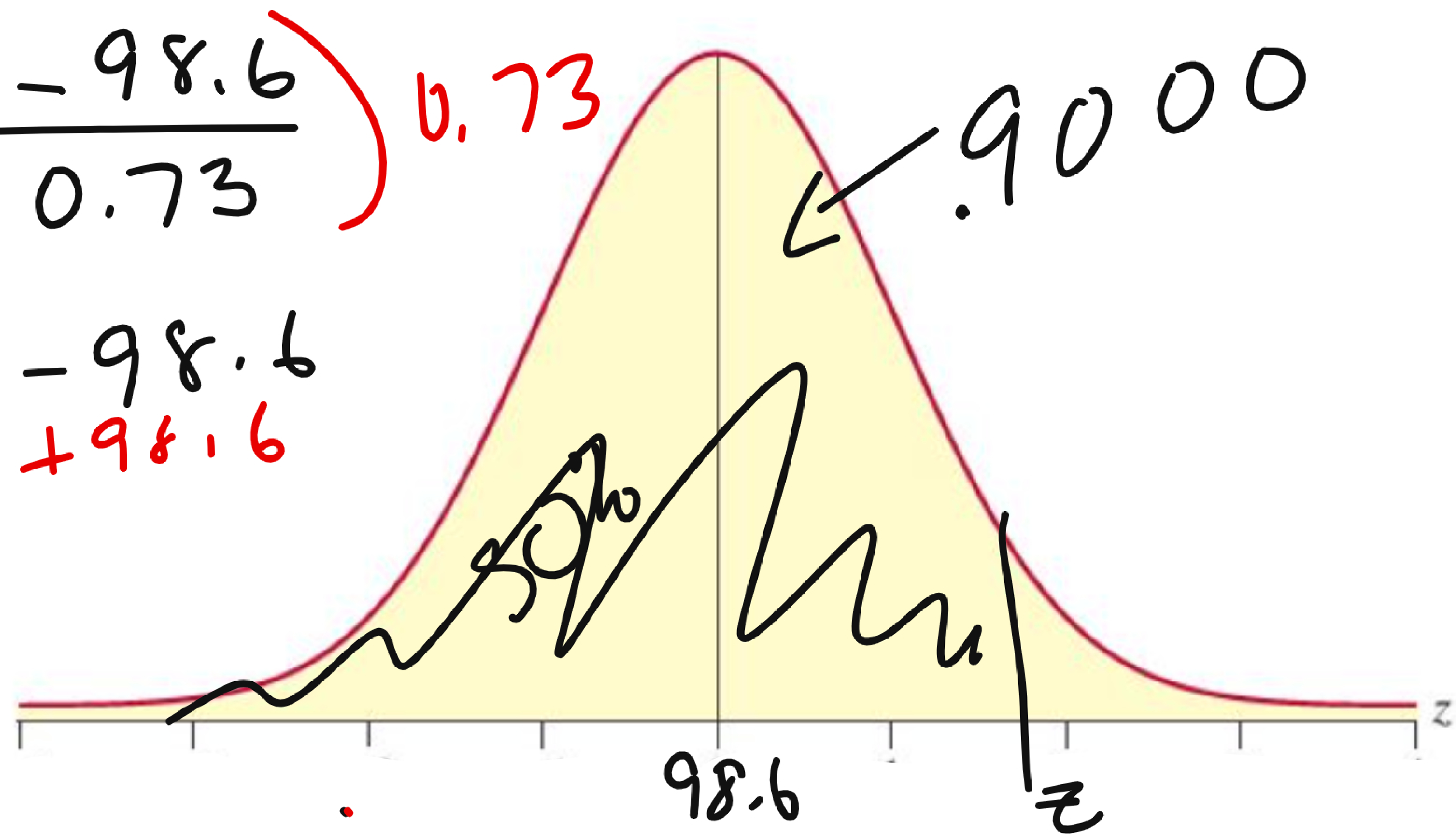


Figure 8.3.1

A statistics teacher believes that the final exam grades for her business statistics class have a normal distribution with a mean of 82 and a standard deviation of 8.

$z = 1.28$

$x = (1.28 \times 8) + 82$

a. Find the score which separates the top 10% of the scores from the lowest 90% of the scores. 92.24

b. The teacher plans to give all students who score in the top 10% of scores an A. Will a student who scored a 90 on the exam receive an A? Explain.

$z = -0.84$

c. Find the score which separates the lowest 20% of the scores from the highest 80% of the scores. 75.28

d. The teacher plans to give all students who score in the lowest 10% of scores an F. Will a student who scored a 65 on the exam receive an F? Explain.

$x = (-1.28)(8) + 82$

$x = 71.76$

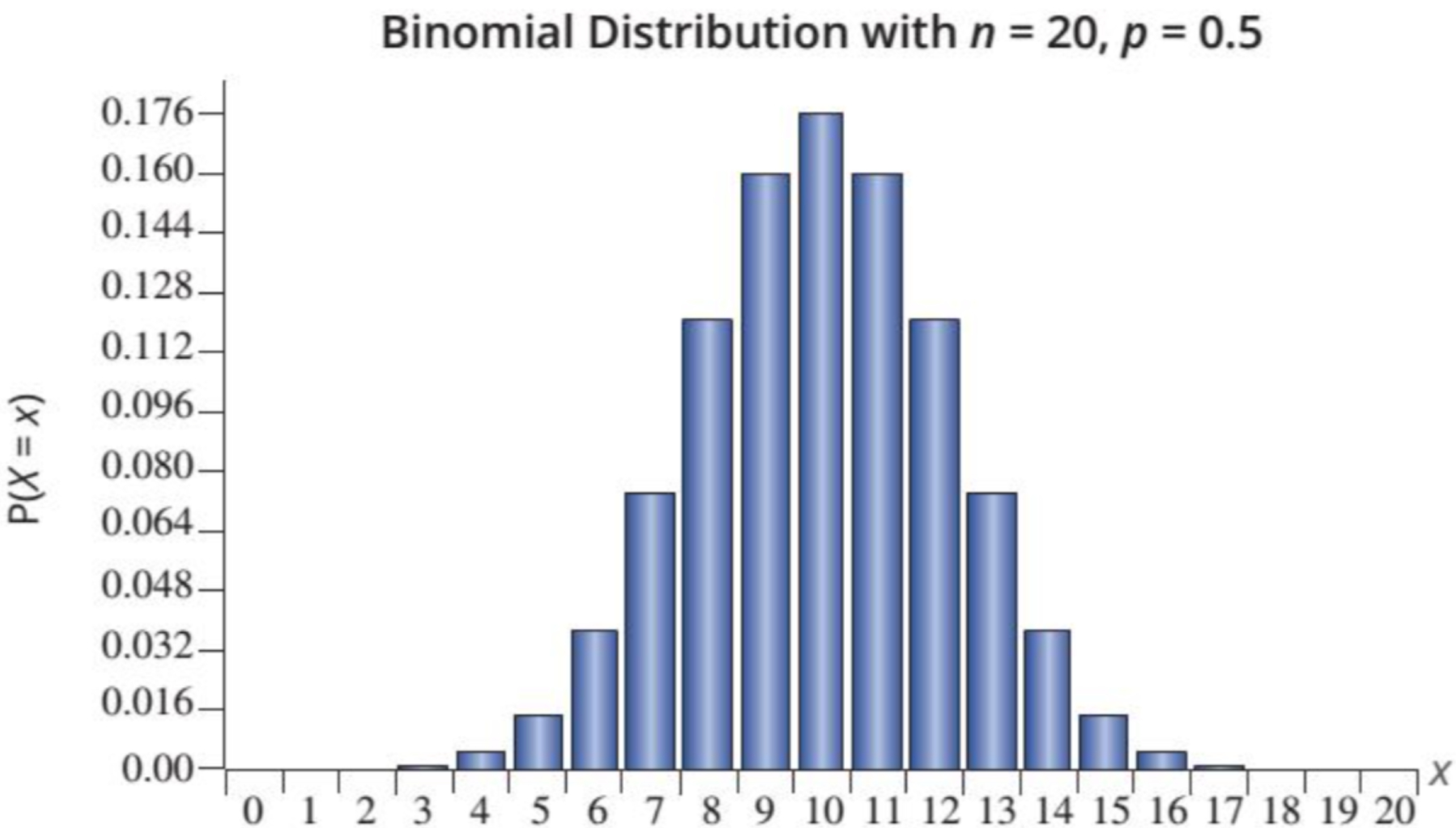
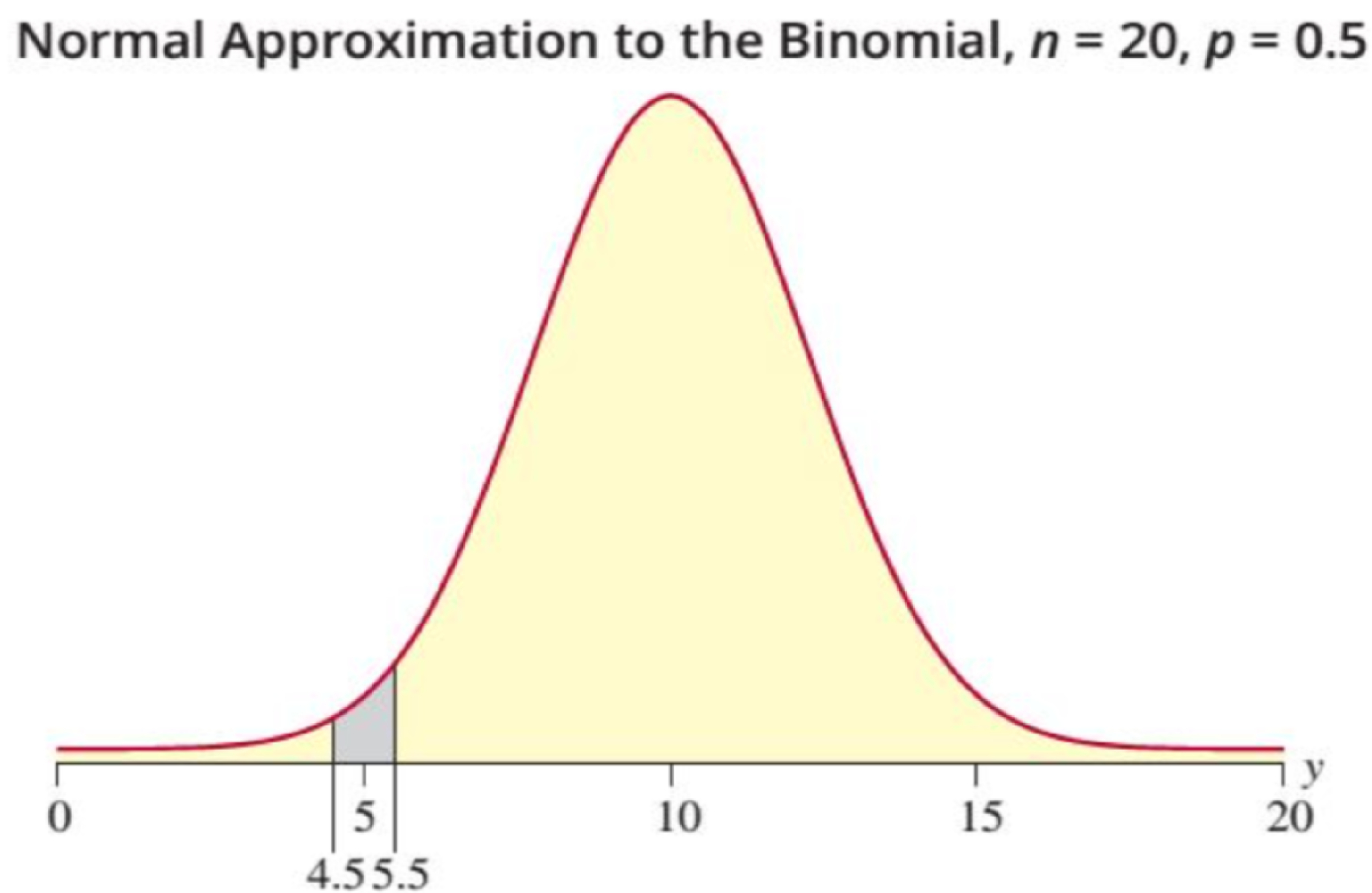

Binomial Distribution with n = 20, p = 0.5
P(X = x)
0.176
0.160
0.144
0.128
0.112
0.096
0.080
0.064
0.048
0.032
0.016
0.00
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
X

Figure 8.6.1


Normal Approximation to the Binomial, n = 20, p = 0.5
0
5
10
15
20
y
4.5
5.5
Figure 8.6.3

$\mu = np$ $\qquad \sigma = \sqrt{np(1-p)}$

$\mu = (20)(0.5) = 10 \qquad \sigma = \sqrt{(20)(.5)(.5)} = 2.24$

a) $P(x \le 5.5)$ $\qquad z = \frac{5.5 - 10}{2.24} = -2.01$

$= 0.0222$

b) $P(x \ge 3.5)$ $\qquad z = \frac{3.5 - 10}{2.24} = -2.90$

$1 - .00187$

$.99813$

c) $P(x = 5) = P(4.5 \le x \le 5.5)$ $\qquad -2.46 \le z \le -2.01$

$.0069 \qquad 0.0222$

0.0153

An advertising agency hired on behalf of Tech's development office conducted an ad campaign aimed at making alumni aware of their new capital campaign. Upon completion of the new campaign, the agency claimed that 20% of alumni in the state were aware of the new campaign. To validate the claim of the agency, the development office surveyed 1000 alumni in the state and found that 150 were aware of the campaign. Assuming that the ad agency's claim is true, what is the probability that no more than 150 of the alumni in the random sample were aware of the new campaign?

$n = 1000$ $p = 0.20$ $x \leq 150.5$

$\mu = 200$ $\sigma = 12.65$

$$z = \frac{150.5 - 200}{12.65} = -3.91$$

uppose a virus is believed to infect two percent of the population. If a sample f 3000 randomly selected subjects are tested, answer the following questions.

Vhat is the probability that between 40 and 80 (inclusive) of the subjects n the sample will be infected?

$\mu = 60$ $\sigma = 7.67$

$P(39.5 \le X \le 80.5)$

$= 0.99242$

$z = \frac{39.5 - 60}{7.67} = -2.67 \rightarrow .00379$

$z = \frac{80.5 - 60}{7.67} = 2.67 \rightarrow .99621$