10 CHAPTER Estimation: Single Samples

Estimator

An **estimator** is a strategy or rule that is used to estimate a population parameter. If the rule is applied to a specific set of data, the result is an **estimate**.

DEFINITION

Table 10.1.1 - Point Estimators

Point Estimator	Parameter Being Estimated	Point Estimate
$\bar{x}$	μ	$\bar{x} = 12.7$
$\hat{p}$	p	$\hat{p} = 0.37$
s	σ	$s = 6.4$

$100(1-\alpha)\%$ Confidence Interval for the Population Mean, σ Known

If σ is known and the sample is drawn from a normal population or $n > 30$, a **$100(1-\alpha)\%$ confidence interval for the population mean** is given by

$$\bar{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}}.$$

FORMULA

$$\bar{x} \pm z_{a/2}\frac{\sigma}{\sqrt{n}}$$

Table 10.2.1 - Critical Values of z

Confidence $(1-\alpha)$	$z_{\alpha/2}$
0.80	1.28
0.90	1.645
0.95	1.96
0.99	2.575

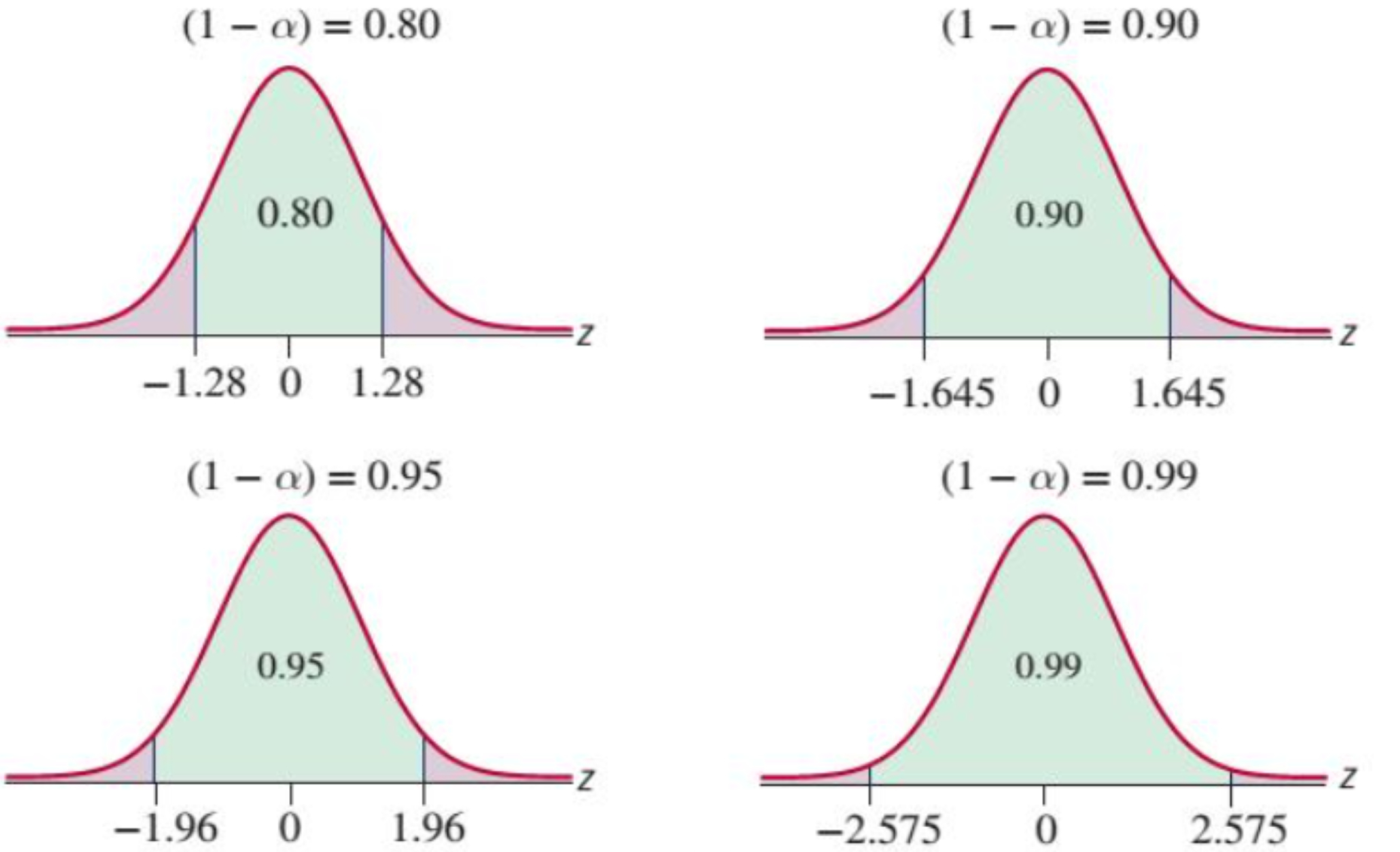

$(1 - \alpha) = 0.80$
0.80
z
−1.28
0
1.28
$(1 - \alpha) = 0.90$
0.90
z
−1.645
0
1.645
$(1 - \alpha) = 0.95$
0.95
z
−1.96
0
1.96
$(1 - \alpha) = 0.99$
0.99
z
−2.575
0
2.575

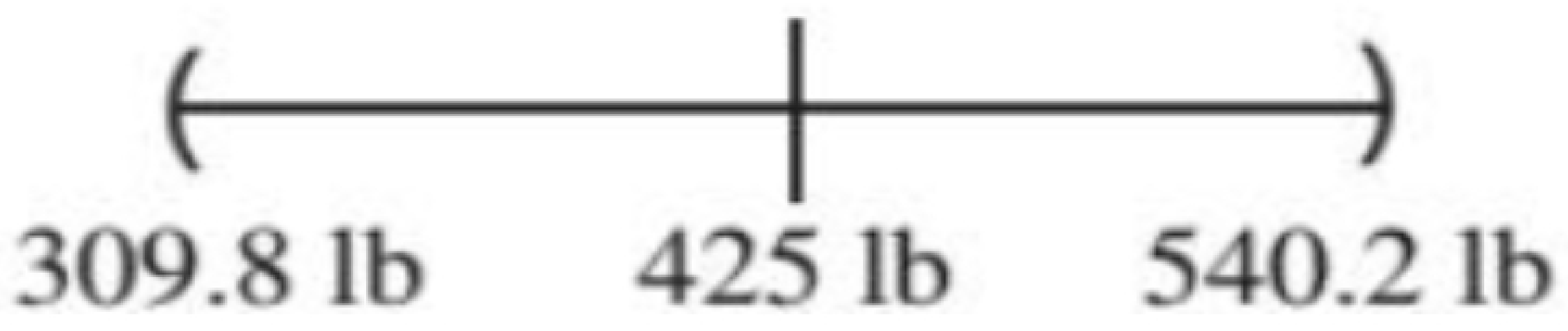
Figure 10.2.4

A random sample of 100 car engines has a mean weight of 425 pounds. Construct 80%, 90%, 95%, and 99% confidence intervals for the population mean if the standard deviation of the population is 900.

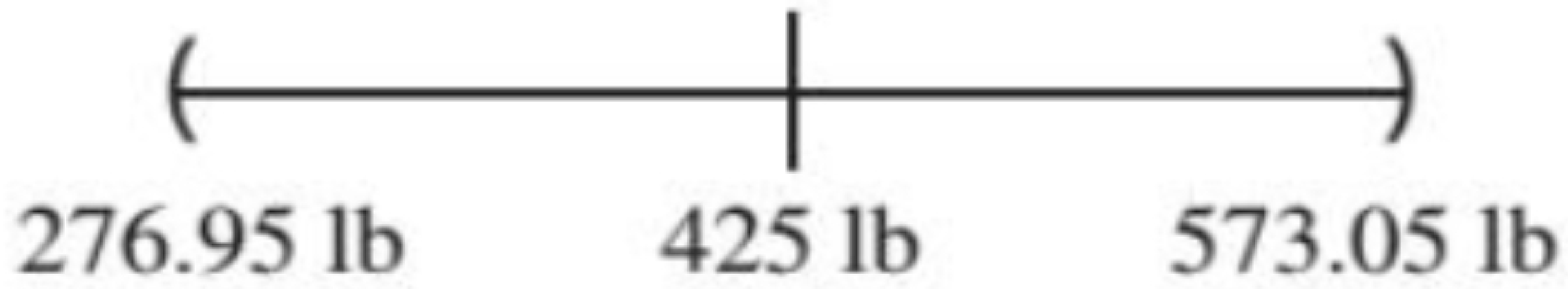
$n = 100$ $\quad \bar{x} = 425$

$\sigma = 900$

80% $425 \pm 1.28 \cdot \frac{900}{\sqrt{100}}$

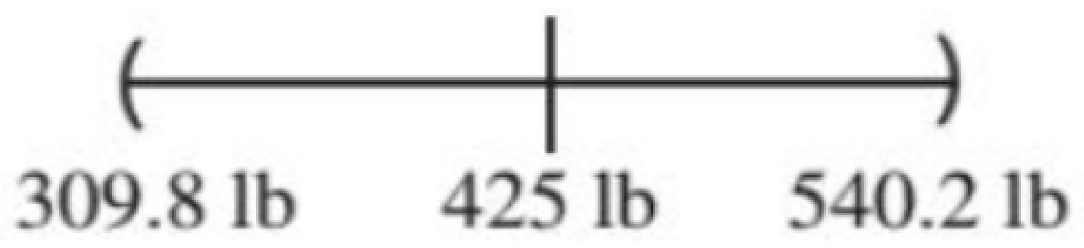


90% $425 \pm 1.645 \cdot \frac{900}{\sqrt{100}}$



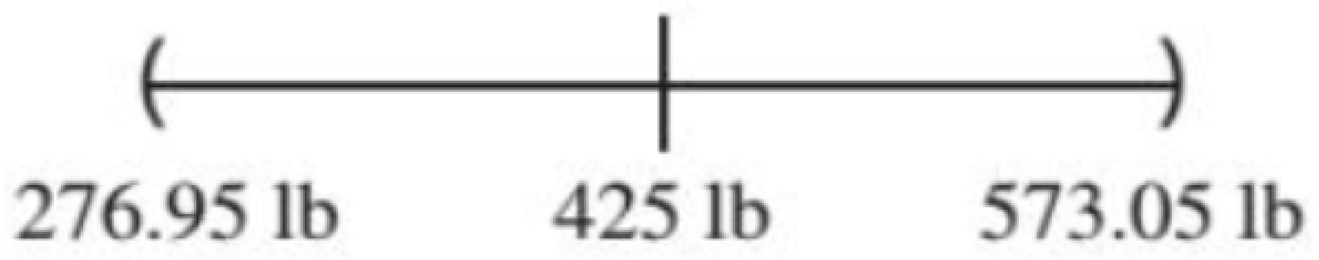
80% Confidence Interval

$$425 \pm 1.28 \cdot \frac{900}{\sqrt{100}} \text{ or } 309.8 \text{ to } 540.2$$



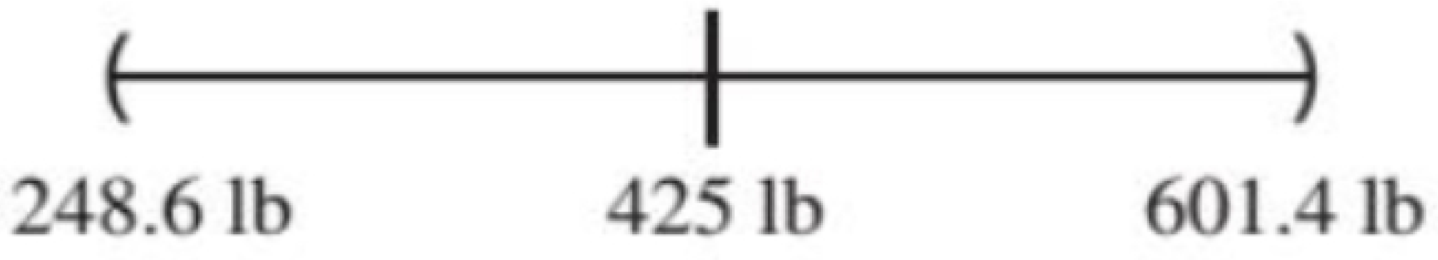
90% Confidence Interval

$$425 \pm 1.645 \cdot \frac{900}{\sqrt{100}} \text{ or } 276.95 \text{ to } 573.05$$



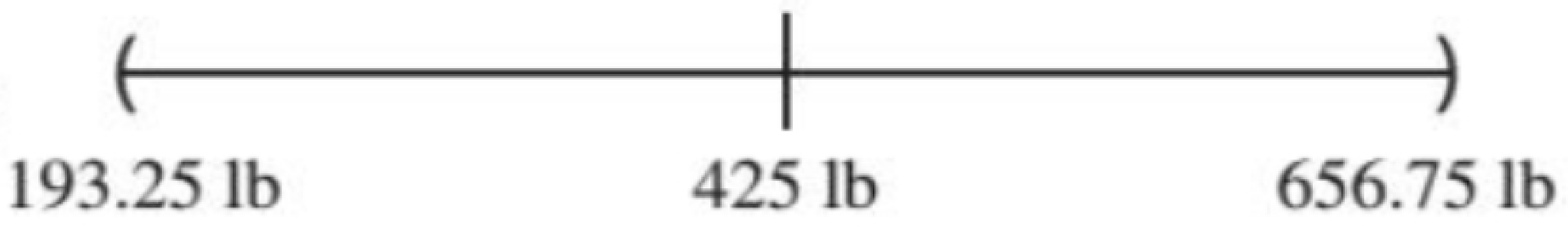
95% Confidence Interval

$$425 \pm 1.96 \cdot \frac{900}{\sqrt{100}} \text{ or } 248.6 \text{ to } 601.4$$



99% Confidence Interval

$$425 \pm 2.575 \cdot \frac{900}{\sqrt{100}} \text{ or } 193.25 \text{ to } 656.75$$



An analyst is interested in investigating the average durability of the bladder within the soccer balls that his company manufactures. Suppose a random sample of 100 soccer balls is selected, and the balls are put through a pressure test to determine the PSI at which the bladder will burst. It is known from past experiences that the population standard deviation of the pressure at which a ball bursts is 13.25 PSI. The sample mean PSI necessary to pop a soccer ball is found to be 147.58 PSI. Calculate a 95 percent confidence interval for the population mean PSI necessary to pop a soccer ball.

$n = 100$

$\sigma = 13.25$

$\bar{x} = 147.58$

$95\% \rightarrow z_{\alpha/2} = 1.96$

$$\mu = \bar{x} \pm z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$= 144.983$

$= 150.177$

Student's *t*-Distribution

Provided the population from which the sample is drawn is normally distributed or $n > 30$, the distribution of the quantity

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

where s is the standard deviation of the sample, has a **Student's *t*-distribution**.

FORMULA

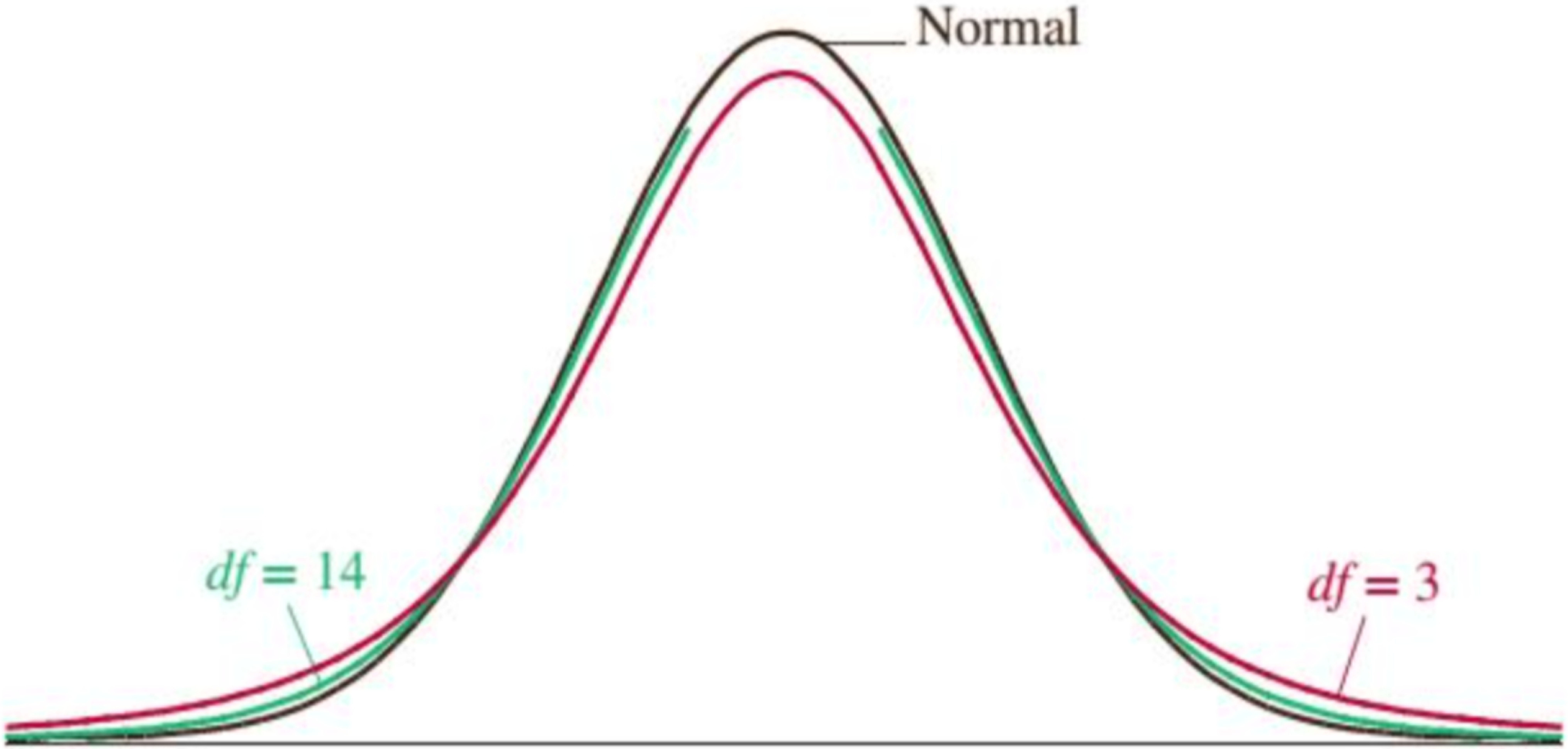


Figure 10.2.5

$100(1 - \alpha)\%$ Confidence Interval for the Population Mean, σ Unknown

If σ is unknown and the sample is drawn from a normal population or $n > 30$, a $100(1 - \alpha)\%$ **confidence interval for the population mean** is given by

$$\boxed{\bar{x} \pm t_{\alpha/2,\, n-1} \frac{s}{\sqrt{n}}}$$

where $t_{\alpha/2,\, n-1}$ is the critical value for a t-distribution with $n - 1$ degrees of freedom which captures an area of $\frac{\alpha}{2}$ in the right tail of the distribution. For this confidence interval $E = t_{\alpha/2,\, n-1} \frac{s}{\sqrt{n}}$.

FORMULA

A random sample of hours worked in a week by student interns is drawn from a normal population with unknown mean and variance. The seven data values sampled are as follows.

25, 19, 37, 29, 40, 28, 31

$= 23.33335864$

$= 36.4664136$

Construct a 95% confidence interval for the population mean.

$n = 7$ $\quad \bar{x} = 29.9$ $\quad \cancel{\sigma}, s = 7.1$

95% $\quad \cancel{z} \quad t = 2.447$

$D.f. = n - 1 = 6$

A manufacturing company is interested in the amount of time it takes to complete a certain stage of the production process. The project manager randomly samples 10 products as they come from the production line and notes the time of completion. The average completion time is 23.45 minutes with a sample standard deviation of 4.32 minutes. Based on this sample, construct a 95% confidence interval for the average completion time for that stage in the production process. Assume that the population distribution of the completion times is approximately normal.

$t = 2.262$ D.F. $= 9$ 95% two tail

$$\mu = 23.45 \pm 2.262\left(\frac{4.32}{\sqrt{10}}\right) = 20.35987287$$

$$= 26.54012713$$

Finding a Confidence Interval for the Population Mean

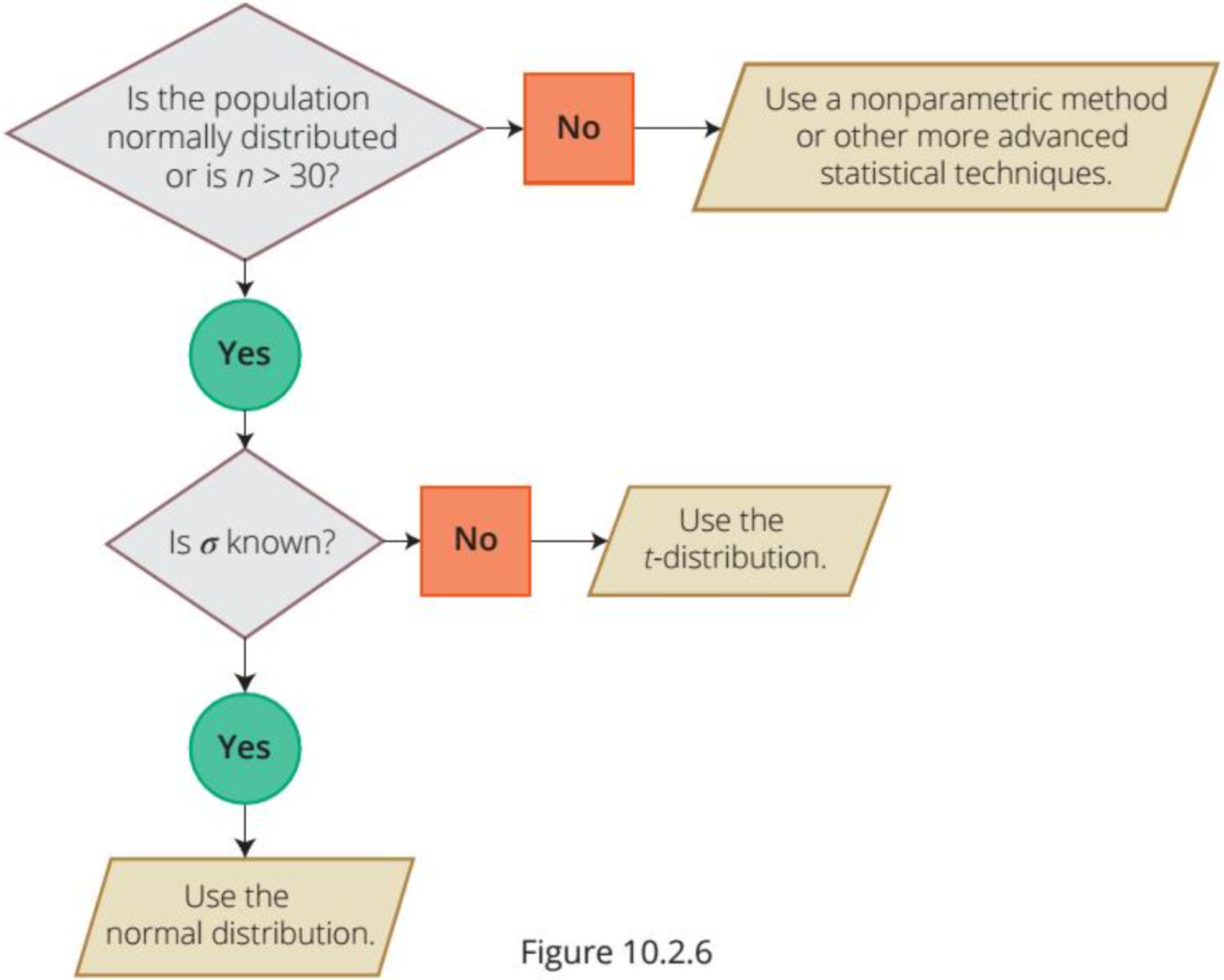


Figure 10.2.6

Sample Size Determination for Estimating a Population Mean

$$n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2$$

$$n = \left(\frac{1.96(15)}{1.5}\right)^2$$

FORMULA

Consider a population having a standard deviation of 15. We want to estimate the mean of the population. How large of a sample is needed to construct a 95% confidence interval for the mean of this population if the margin of error is equal to 1.5?

$$\left(\frac{1.96(15)}{1.5}\right)^2 = 384.16$$

An airline's maintenance manager desires to estimate the average time (in hours) required to replace a jet engine in a Boeing 767. How large a sample would be necessary if the manager wishes to be 95% confident of estimating the mean to within one-quarter of an hour ($E = 0.25$). Assume a preliminary sample of size $n = 31$ has an average replacement time of 16.7 hours with a standard deviation of 4.3 hours.

$$n = \left(\frac{1.96(4.3)}{.25}\right)^2 = 1137$$

percentage of the bill left as a tip for 25 randomly selected bills. The average tip was 18.3% of the bill with a standard deviation of 2.7%.

$= 16.78962$

a. Construct an interval to estimate the true average tip (as bill) with 99% confidence.

$n = 25$ $\bar{x} = 18.3$ $\cancel{\sigma =}$ or

$= 19.81038$

99% C.I. $s = 2.7$

$t = 2.797$ D.F. $= 24$

$$\mu = 18.3 \pm 2.797\left(\frac{2.7}{\sqrt{25}}\right)$$

Estimate the fraction of defective transistors in a lot containing 100,000 transistors. Suppose a sample of size 800 is drawn from the lot, and 5 transistors were found to be defective.

$$\frac{5}{800} = .00625$$

100(1 − α)% Confidence Interval for the Population Proportion

If the sample size is sufficiently large, i.e., $np \geq 10$ and $n\left(1-p\right) \geq 10$, the 100(1 − α)% **confidence interval for the population proportion** is given by the expression

$$\hat{p} \pm z_{\alpha/2}\sigma_{\hat{p}},$$

where $z_{\alpha/2}$ is the distance from the point estimate to the end of the interval in standard deviation units, and $\sigma_{\hat{p}}$ is the standard deviation of $\hat{p}$. For this confidence interval $E = z_{\alpha/2}\sigma_{\hat{p}}$ where

$$\sigma_{\hat{p}} \approx \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}.$$

FORMULA

Suppose a sample of 410 randomly selected radio listeners revealed that 48 listened to WXQI. Find a 95% confidence interval for the proportion of radio listeners that listen to WXQI.

$$\hat{p} = \frac{48}{410} = .117 \qquad z = 1.96$$

$$\sigma = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{(.117)(.883)}{410}} = 0.016$$

$$C.I. = .117 \pm 1.96(0.016) \qquad = 0.08564$$

$$= \qquad\qquad = 0.14836$$

Sample Size Determination for Estimating a Population Proportion

The sample size necessary to estimate the population proportion to within a particular error with a certain level of confidence is given by

$$n \approx \frac{z_{\alpha/2}^2 \hat{p}\left(1-\hat{p}\right)}{E^2},$$

where $\hat{p}$ is the estimate of the population proportion obtained from the pilot study.

$$n = \frac{1.96^2(.5)(.5)}{.03^2} = 1067.111111$$

In the HarrisX/Forbes poll, among the 689 respondents who identify as political independents, Trump leads 42%-40%, while Harris maintains a significant lead in such key demographics as suburban women (52%-40%), African-American men (57%-30%), African-American women (77%-13%), and white respondents with a college degree (49%-46%).

In the *Gallup Poll Monthly*, it was reported that 31% of the people surveyed in a recent poll claimed that vegetables were their least favorite food. Surprisingly, only 14% responded with liver, and 10% of those surveyed did not submit a response because they claimed that they liked everything. The poll was based upon a sample of 1001 people. Assume that a random sample of Americans was chosen, and construct a 90% confidence interval for the percentage of all Americans who say that vegetables are their least favorite food.

$\hat{p} = .31$

$\sigma_{\hat{p}} = .015$

$n = 1001$

90% = 1.645

C.I. → 0.285325 — 0.334675

The State Bureau of Standards must inspect gasoline station pumps on a regular basis to be sure they are operating properly. A recent survey of a randomly selected group of 61 pumps produced a sample mean of 9.75 gallons dispensed for a pump reading ten gallons. If the sample had a standard deviation of 1.12 gallons, find the 80% confidence interval for the mean amount of gas dispensed when a gas pump reads ten gallons.

$\bar{x} = 9.75$

$s = 1.12$

$n = 61$

$80\% = 1.282$

$$9.75 \pm 1.282\left(\frac{1.12}{\sqrt{61}}\right)$$

$= 9.566159526$

$= 9.933840474$

According to a study conducted by the American Stock Exchange, 87% of 500 young Americans surveyed said that they can't count on Social Security as a source of income when they retire. Construct a 90% confidence interval for the proportion of young Americans who feel they can't count on Social Security as a source of income when they retire. (Assume the sample was randomly selected.)

$\hat{p} = .87$

$n = 500$

$\sigma_{\hat{p}} = .015$

$\sqrt{\dfrac{(.87)(1-.87)}{500}}$

$90\% \Rightarrow 1.645$

$CI \to .87 \pm 1.645(.015)$

$= 0.845325$

$= 0.894675$

A hot dog vendor is evaluating a downtown location by counting the number of people who walk past the prospective location on a particular day during lunch time (i.e. 11:00 AM to 2:00 PM). A preliminary study has indicated a standard deviation of about 30 people per lunch period. How many lunch periods will be needed to estimate the average number of people who walk past the prospective location during the lunch period to within 9 people with 90% confidence?

$$\left(\frac{1.645 \cdot 30}{9}\right)^2 \quad = 30.06694444$$