Formula 5.1: Binomial Probability Formula

Let X denote the total number of successes in n Bernoulli trials with success probability p. Then the probability distribution of the random variable X is given by

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \ldots, n.$$

(nCr)

The random variable X is called a **binomial random variable** and is said to have the **binomial distribution** with parameters n and p.

Mortality According to tables provided by the National Center for Health Statistics in *Vital Statistics of the United States*, there is roughly an 80% chance that a person of age 20 years will be alive at age 65 years. Suppose that three people of age 20 years are selected at random. Find the probability that the number alive at age 65 years will be

$0 \to .008$
$1 \to .096$
$2 \to .384$
$3 \to .512$

a. exactly two. $.384 = P(x=2)$

b. at most one. $P(x \le 1) = .104$

c. at least one. $P(x \ge 1) = .992$

d. Determine the probability distribution of the number alive at age 65.

Formula 5.2: Mean and Standard Deviation of a Binomial Random Variable

$n = 3 \qquad p = .80$

The mean and standard deviation of a binomial random variable with parameters n and p are

$$\mu = np \quad \text{and} \quad \sigma = \sqrt{np(1-p)},$$

$$\mu = 3(.80) \qquad \sigma = \sqrt{(3)\underbrace{(.80)(.20)}_{+1}}$$

respectively. $\mu = 2.4$

$\sigma = 0.693$

6) (20 points) According to *JAVMA News*, a publication of the American Veterinary Medical Association, roughly 60% of U.S. households own one or more pets. Four U.S. households are selected at random.

a. Find the probability that of the four households the number that own one or more pets is exactly 3?

b. Find the probability that of the four households the number that own one or more pets is at least 3?

c. Find the probability that of the four households the number that own one or more pets is at most 3?

$P = .60$ $\quad n = 4$

$r = 0 \rightarrow .0256$

$= 1 \rightarrow .1536$

$= 2 \rightarrow .3456$

$= 3 \rightarrow .3456$

$= 4 \rightarrow .1296$

a) $P(x=3) = .3456$

b) $P(x \geq 3) = .4752$

c) $P(x \leq 3) = .8704$

Adjusted gross income	Frequency (1000s)	Event	Probability
Under $10K	26,268	*A*	.184
$10K–under $20K	22,778	*B*	.160
$20K–under $30K	18,610	*C*	.131
$30K–under $40K	14,554	*D*	.102
$40K–under $50K	11,087	*E*	.079
$50K–under $100K	30,926	*F*	.217
$100K & over	18,227	*G*	.128
	142,450		

d. If Event H shows an AGI between \$20K and \$100K and Event I show an AGI less than \$50K. What is the probability of H & I?

e. If Event H shows an AGI between \$20K and \$100K and Event I show an AGI less than \$50K. What is the probability of H or I?

$H \rightarrow 20 - 100$ $\quad H + I$

$I \rightarrow < 50$

$20 - 100$

$0 \longleftarrow 50$

$20 \leftrightarrow 50$

d. If Event H shows an AGI between \$20K and \$100K and Event I show an AGI less than \$50K. What is the probability of H & I?

e. If Event H shows an AGI between \$20K and \$100K and Event I show an AGI less than \$50K. What is the probability of H or I?

H → 20 - 100　　H or I

I → < 50

20 - 100

0 ← 50

0 ↔ 100

$d = \frac{.088}{0.158} = .557$ **Living arrangement** $e = \frac{0.088}{0.500} = 0.176$

		Alone L_1	With spouse L_2	With others L_3	$P(A_i)$
Age (yr)	15–24 A_1	0.006	0.012	0.157	0.175
	25–44 A_2	0.030	0.184	0.123	0.337
	45–64 A_3	0.047	0.216	0.067	0.330
	Over 64 A_4	0.046	0.088	0.024	0.158
	$P(L_j)$	0.129	0.500	0.371	1.000

3) (9 points) In the United States, telephone numbers consist of a three-digit area code followed by a seven-digit local number. Suppose neither the first digit of an area code nor the first digit of a local number can be a zero but that all other choices are acceptable.

a. How many different area codes are possible?
b. For a given area code, how many local telephone numbers are possible?
c. How many telephone numbers are possible?

a) $9 \cdot 10 \cdot 10 \rightarrow 900$

b) $9 \cdot 10 \cdot 10 \quad 10 \cdot 10 \cdot 10 \cdot 10 = 9{,}000{,}000$

c) $900 \cdot 9000000 = 8{,}100{,}000{,}000$

4) (6 points) Investment firms usually have a large selection of mutual funds from which an investor can choose. One such firm has 30 mutual funds. Suppose that you plan to invest in four of these mutual funds, one during each quarter of next year. In how many different ways can you make these four investments?

$nCr \rightarrow$ order doesn't matter

$nPr \rightarrow$ order matters

$n = 30$ $\quad r = 4$ $\quad {}_{30}P_4 = 657{,}720$

5) (15 points) An office has six incoming phone lines. The probability distribution of the number of busy lines are as follows:

y	0	1	2	3	4	5	6
$P(Y = y)$	0.052	0.154	0.232	0.240	0.174	0.105	0.043
	0	0.154	0.464	0.720	0.696	0.525	0.258

a. What is the probability of 4 lines being busy? 0.174

b. What is the probability of between 2 and 4 lines (inclusive) are busy? 0.646

c. On average how many lines are busy?

μ = multiply each collumn then add answers

$\mu = 2.817$