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Interaction of a self vibrating beam with chaotic external forces

Interaction entre les vibrations d'une poutre et un champ de forces aléatoires

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ABSTRACT

Measuring the complexity of physical systems has been traditionally a problem in numerous engineering applications. Lin [Entropy 10 (1) (2008) 1–5] showed that the structural complexity is related to other properties of a solid such as symmetry and its stability over time. In Ratsaby [Entropy 10 (1) (2008) 6–14] a model was introduced which defines the complexity of a solid structure not by a qualitative notion of entropy but by an algorithmic notion of description complexity. According to the model, a dynamic structure in a random surrounding acts as an interfering entity that deforms randomness. In the current Note we report on the results of an empirical study that analyzes the output response of a simulated elastic beam subjected to a field of external random forces input. The relationship between the complexity of the system and the stochasticity of the output is shown to support this model and is a first indication that solids act similar to algorithmic selection rules.

RÉSUMÉ

La mesure de la complexité des systèmes physiques intervient dans de nombreuses applications des sciences de l'ingénieur. Lin [Entropy 10 (1) (2008) 1–5] a montré que la complexité structurelle d'un système est liée aux propriétés ayant trait à ses symétries géométriques ainsi qu'à sa stabilité dans le temps. Le modèle de Ratsaby [Entropy 10 (1) (2008) 6–14] suggère d'évaluer la complexité des systèmes physiques par analogie à la mesure de la complexité des algorithmes. Selon ce modèle, un système physique sollicité par un environnement chaotique réagit comme une entité qui absorbe une partie du caractère aléatoire des sollicitations qui s'exercent sur celui-ci. Cette Note a pour objet de présenter la réponse d'un système vibratoire simple soumis à un champ de forces aléatoires. On montrera principalement que la relation obtenue entre la complexité du système et le caractère aléatoire du champ de déplacements qui en résulte, est analogue à celle qui prévaut pour les règles de sélection des algorithmes informatiques.

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1. Introduction

Measuring the complexity of dynamic structures has been traditionally a problem in the systems analysis literature. The world is full of dynamic complex systems that consist of simple components but which yet have a complex deterministic behavior. Examples of such systems are cellular automata, ecosystems, social systems, sensor networks (consisting of many simple processing units). A complex system is one whose dynamic structure is intricate and requires a long description. For dynamic systems the notion of complexity is usually attributed to the high level of unpredictability of their behavior. While a complex system does not necessarily have to be stochastic it still takes a considerable amount of analysis and computations to determine its behavior (the connection between randomness and complexity will be discussed later). In [1] an approximate entropy measure is used to distinguish between systems of different levels of complexity including deterministic and chaotic ones. Lin [2,3] introduced a new notion of entropy, called *static* entropy. His aim was to revise information theory in order to broaden the notion of entropy such that the naturally occurring phenomenon of increased symmetry of a system and the increased similarity of its components (observed in many spontaneous processes in nature) may be explained by information theoretic principles. He generalized the notion of entropy to dynamic (deterministic) structures and postulated that such processes are driven by an information minimization or symmetry maximization process. In [4] a model was introduced which defines the complexity of a solid structure not by a qualitative notion of entropy but by an algorithmic notion of description complexity. This model gives a precise quantitative definition of the information content of a given object. Based on concepts of randomness and complexity of finite binary strings it explains the relationship between algorithmic complexity of a solid to its information content and its stability. According to the model, dynamic structures are like points of interference in a random surrounding. Due to their complexity they resist randomness and become less stable over time. Dynamic structures cannot maintain their complexity indefinitely. Eventually, they undergo change in the direction of being less algorithmically complex which weakens their interference with the randomness of the surroundings and consequently become more stable. According to the model, a solid is represented by an algorithmic selection rule which interacts with external random force sequences that arise when random particles hit the structure. Through the inequality that relates stochasticity and chaoticity of random binary sequences it is shown in [4] that Lin's notion of stability corresponds to the stability of the frequency of 1s in the selected subsequence. This explains why more complex dynamic structures are less stable. Lin's third law is represented as the inevitable change that dynamic structure undergo towards conforming to the randomness in the surroundings. The current note presents first evidence in support of the theory of [4]. We report on recent results of experiments done in [5] that simulated a vibrating elastic solid subjected to a random external force. The solid studied is a one-dimensional vibrating elastic beam on which a random input force sequence is applied. The output response consists of the displacement of the beam observed at its other end over a finite time interval. The relationship between the complexity of the structure to the stochasticity of the output sequence is computed and is shown to agree with the model of [4].

2. Description of the problem

Consider an elastic beam having a length L (for instance, a bridge). It has some finite descriptive complexity consisting of all the information contained in the engineering design documents. These documents can be put into a single computer file, i.e., a finite binary string z. This binary string has an algorithmic complexity which is defined as the length of the shortest computer program that can generate it. This is the Kolmogorov complexity $K(z)$ of z (see [6]). Now consider a *random* input force sequence applied at one of the two ends of the bridge, for instance, suppose there is a person jumping up and down sporadically on the bridge at its entrance (position 0). Denote by x the binary sequence representing this up/down symbols over some fixed time-interval $[0, T]$. Intuitively, being that x is random makes its complexity $K(x)$ maximal and hence close to its actual length $\ell(x)$ since there is no redundancy in the patterns of x that can be used to compress it significantly below its length. Now consider an observer which measures the displacements on the beam at its other end (position L). He records this over the time interval $[0, T]$ and compares it to a fixed threshold thereby producing a binary output sequence y consisting of up/down symbols that represent the displacement of the beam at position L. This sequence has a finite algorithmic complexity $K(y)$. In this note we report on recent results [5] that show that for such a physical system, an estimate of the system complexity $K(z)$, the output complexity $K(y)$ and its level of randomness are related according to the model of [4] which represents a solid as an algorithmic selection rule.

3. Algorithmic complexity and randomness

3.1. Algorithmic complexity

Kolmogorov [6] proposed to measure the conditional complexity of a finite object x given a finite object y by the length of the shortest binary sequence π (a program for computing x) which consists of 0s and 1s and which reconstructs x given y. Formally, this is defined as

$$K(x|y) = \min\left\{\ell(\pi) : \phi(\pi, y) = x\right\} \tag{1}$$

where $\ell(\pi)$ is the length of the sequence π, ϕ is a universal partial recursive function which acts as a description method, i.e., when provided with input (π, y) it gives a specification for x. The word universal means that the function ϕ can emulate any Turing machine (hence any partial recursive function). One can view ϕ as a universal computer that can interpret any programming language and accept any valid program π. The Kolmogorov complexity of x given y as defined in (1) is the length of the shortest program that generates x on this computer given y as input. The special case of y being the empty binary sequence gives the unconditional Kolmogorov complexity $K(x)$. This has been later extended by [7] to the prefix-complexity which requires π to be coded in a prefix-free format.

3.2. Algorithmic randomness

The notion of randomness of finite objects (binary sequences) aims to explain the intuitive idea that a sequence, whether finite or infinite, should be measured as being more unpredictable if it possess fewer regularities (patterns). There is no formal definition of randomness but there are three main properties that a random binary string of length n must intuitively satisfy. The first property is the so-called *stochasticity* or frequency stability of the sequence which means that any binary word of length $k \leqslant n$ must have the same frequency limit (equal to 2^{-k}). This is basically the notion of normality that Borel introduced and is related to the degree of unpredictability of the sequence. The second property is *chaoticity* or disorderliness of the sequence. A sequence is less chaotic (less complex) if it has a short description, i.e., if the minimal length of a program that generates the sequence is short. The third property is *typicalness*. A random sequence is a typical representative of the class Ω of all binary sequences. It has no specific features distinguishing it from the rest of the population. An infinite binary sequence is typical if each small subset E of Ω does not contain it (the correct definition of a 'small' set was given by Martin-Löf [8]). The research area on algorithmic randomness studies the relationship between complexity and stochasticity of finite and infinite binary sequences. Algorithmic randomness was first considered by von Mises in 1919 who defined an infinite binary sequence α of zeros and ones as random if it is unbiased, i.e. if the frequency of zeros goes to $1/2$, and every subsequence of α that we can extract using an admissible selection rule (see definition below) is also not biased. Kolmogorov and Loveland [9] proposed a more permissive definition of an admissible selection rule as any (partial) computable process which, having read any n bits of an infinite binary sequence α, picks a bit that has not been read yet, decides whether it should be selected or not, and then reads its value. When subsequences selected by such a selection rule pass the unbiasedness test they are called Kolmogorov–Loveland stochastic (KL-stochastic for short). Martin-Löf [8] introduced a notion of randomness which is now considered by many as the most satisfactory notion of algorithmic randomness. His definition says precisely which infinite binary sequences are random and which are not. The definition is probabilistically convincing in that it requires each random sequence to pass every algorithmically implementable statistical test of randomness. From the work of [10] Martin-Löf's randomness can be characterized in terms of Kolmogorov complexity (1) of α. An infinite binary sequence $\alpha = \{\alpha_i\}_{i=1}^{\infty}$ is Martin Löf random if and only if there is a constant c such that for all n, $K(\alpha_1, \ldots, \alpha_n) \geqslant n - c$ where K is the prefix Kolmogorov complexity.

3.3. Selection rule

In this section we describe the notion of a selection rule. As mentioned in the previous section this is a principal concept used as part of tests of randomness of sequences. Let Ξ be the space of all finite binary sequences and denote by Ξ_n the set of all finite binary sequences of length n. An admissible *selection rule* R is defined based on three partial recursive functions f, g and h on Ξ. Let $x = x_1, \ldots, x_n$. The process of selection is recursive. It begins with an empty sequence $\emptyset$. The function f is responsible for selecting possible candidate bits of x as elements of the subsequence to be formed. The function g examines the value of these bits and decides whether to include them in the subsequence. Thus f does so according to the following definition: $f(\emptyset) = i_1$, and if at the current time k a subsequence has already been selected which consists of elements $x_{i_1}, \ldots, x_{i_k}$ then f computes the index of the next element to be examined according to element $f(x_{i_1}, \ldots, x_{i_k}) = i$ where $i \notin \{i_1, \ldots, i_k\}$, i.e., the next element to be examined must not be one which has already been selected (notice that maybe $i < i_j$, $1 \leqslant j \leqslant k$, i.e., the selection rule can go backwards on x). Next, the two-valued function g selects this element x_i to be the next element of the constructed subsequence of x if and only if $g(x_{i_1}, \ldots, x_{i_k}) = 1$. The role of the two-valued function h is to decide when this process must be terminated. This subsequence selection process terminates if $h(x_{i_1}, \ldots, x_{i_k}) = 1$ or $f(x_{i_1}, \ldots, x_{i_k}) > n$. Let $R(x)$ denote the selected subsequence. By $K(R|n)$ we mean the length of the shortest program computing the values of f, g and h given n.

3.4. Randomness deficiency

Kolmogorov introduced a notion of randomness deficiency $\delta(x|n)$ of a finite sequence $x \in \Xi_n$ as follows:

$$\delta(x|n) = n - K(x|n)$$

where $K(x|n)$ is the Kolmogorov complexity of x not accounting for its length n, i.e., it is a measure of complexity of the information that codes only the specific pattern of 0s and 1s in x without the bits that encode the length of x (which is $\log n$ bits). Randomness deficiency measures the opposite of chaoticity of a sequence. The more regular the sequence the less complex (chaotic) and the higher its deficiency.

3.5. Biasness

From the previous sections we know that there are two principal measures related to the information content in a finite sequence x, stochasticity (unpredictability) and chaoticity (complexity). An infinitely long binary sequence is regarded random if it satisfies the principle of stability of the frequency of 1s for any of its subsequences that are obtained by an admissible selection rule [11,9].

Kolmogorov [6] showed that the stochasticity of a finite binary sequence x may be precisely expressed by the deviation of the frequency of ones from some $0 < p < 1$, for any subsequence of x selected by an admissible selection rule R of finite complexity $K(R|n)$. The chaoticity of x is the opposite of its randomness deficiency, i.e., it is large if its Kolmogorov complexity is close to its length n. The works of [6,12] relate this chaoticity to stochasticity. In [12] it is shown that chaoticity implies stochasticity. This can be seen from the following relationship (with $p = 1/2$):

$$\left|\nu\big(R(x)\big) - 1/2\right| \leqslant c\sqrt{\frac{\delta(x|n) + K(R|n) + 2\log K(R|n)}{\ell(R(x))}} \tag{2}$$

where for a binary sequence s, we denote by $\nu(s) = \frac{\#(s)}{\ell(s)}$ the frequency of 1s in s where $\#(s)$ denotes the number of 1s in s, and $\ell(R(x))$ is the length of the subsequence selected by R, $c > 0$ is some absolute constant.

From this we see that as the chaoticity of x grows (randomness deficiency decreases) the stochasticity of the selected subsequence grows (the bias from 1/2 decreases). The information content of the selection rule, namely $K(R|n)$, has a direct effect on this relationship: the lower $K(R|n)$ the stronger the stability (smaller deviation of the frequency of 1s from 1/2). In [13] the other direction which shows that stochasticity implies chaoticity is proved.

4. Aim

In this note we report on recent experimental results that indicate that the fundamental notion of randomness and its relationship to complexity (as discussed in the previous section) underlie the behavior of real physical systems. Besides offering a new way of interpreting the interaction of a general physical system and external stimulus the result that we present directly support the ideas of [4] (discussed above) and indirectly raise the possibility that, at least in part, some laws of physics could be the result of a subtle interplay between random sequences of actions and complex dynamic structures.

Our focus is on a system composed of a vibrating elastic solid (described by the classical equations of solid mechanics) and its interaction with a random input force. We show that as a result of this interaction the solid deforms the randomness of the input sequence and produces an output sequence whose stochastic and algorithmic properties follow those of an output subsequence selected by a selection rule of a finite complexity. Based on computer simulations of a large number of solids, we discuss the results of experimental work [5] that seem to indicate that the complexity of the system inversely affects the stochasticity of the solid's displacement (the observed output) in a manner that agrees with the theory (2).

We now describe the system that was simulated.

4.1. The solid

The solid consists of an elastic homogeneous and one-dimensional beam of length L. Let us denote by x the position on the beam so that $0 \leqslant x \leqslant L$ and by $\vec{x}$ the unit vector on the x-axis. Denote by $\vec{f} = f(x,t)\vec{x}$ a force applied at time t on position x in the direction of $\vec{x}$. We define by $\vec{u} = u(x,t)\vec{x}$ the displacement at time t on x. The classical equation which describes the field of displacements u at a specific position and time when a force f is applied is as follows:

$$\left(\frac{\partial^2 u}{\partial t^2} - \frac{E}{\rho}\frac{\partial^2 u}{\partial x^2}\right)(x,t) = f(x,t) \quad (0 < x < L, t > 0) \tag{3}$$

where E is Young's modulus (the ratio of stress to corresponding strain when the beam behaves elastically), and ρ is the mass density. We impose the following boundary conditions:

$$u(0,t) = u(L,t) = 0, \quad \forall t > 0 \tag{4}$$

i.e., the beam is fixed at its two ends so the only displacements is due to internal elasticity stresses of the material. Let $u_0(x), u_1(x)$ be two given functions that satisfy $u_0(0) = u_0(L) = 0$. As initial conditions we set the following,

$$u(x,0) = u_0(x), \quad 0 < x < L \tag{5}$$

$$\frac{\partial u}{\partial t}(x,0) = u_1(x), \quad 0 < x < L \tag{6}$$

Eqs. (3)–(6) represent the model that describes the deformations of the elastic solid. Using standard numerical approximation we divide the length L into $N+2$ equally spaced discrete points x_j, $j = 1,2,\ldots,N+1$ starting at $x_1 = L/(N+1)$ and similarly discretize the time interval $[0,T]$ into $B+1$ time instants t_i, $i = 0,1,\ldots,B$ starting at $t_0 = 0$. Denoting by

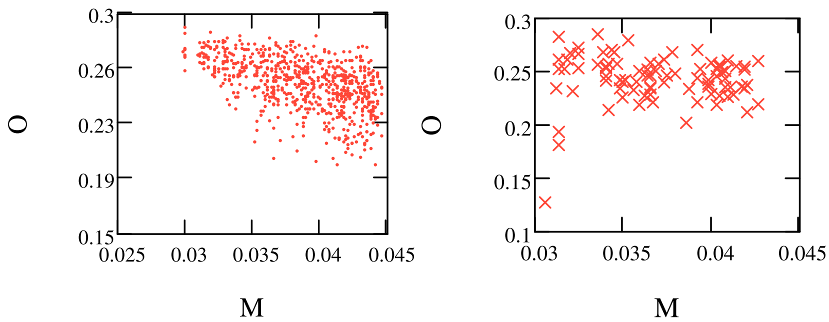

Fig. 1. Output's complexity O versus the complexity M, (a) with random force input, (b) with no input.

$\tilde{u}(j,n)$, $1 \leqslant j \leqslant N$, $1 \leqslant n \leqslant B$ the discrete approximation of $u(x_j,t_n)$, where u is the solution of (3)–(6) and denoting by $\tilde{f}(j,n) = f(x_j,t_n)$ the force applied at these discrete points then we have the corresponding set of difference equations

$$\tilde{u}(j,n+1) = 2\tilde{u}(j,n) - \tilde{u}(j,n-1) + (\Delta t)^2 \tilde{f}(j,n) + \cdots + \left(\frac{\Delta t}{\Delta x}\right)^2 \frac{E}{\rho}\big[\tilde{u}(j+1,n) - 2\tilde{u}(j,n) + \tilde{u}(j-1,n)\big] \tag{7}$$

$$\tilde{u}(0,n) = \tilde{u}(N+1,n) = 0, \quad \tilde{u}(j,0) = u_0(x_j) \quad \text{and} \quad \tilde{u}(j,1) = u_0(x_j) + (\Delta t)u_1(x_j) \tag{8}$$

5. Results

In this section we report on the results of [5] which simulated the response of a solid described by the equations of the previous section. The solid is put into a vibrational state and is henceforth called a *system* which is then subjected to an input random force sequence. Let us now describe this in details: a system is a one-dimensional elastic beam whose length is divided into 31 positions, $0, 1, \ldots, 30$. Let n denotes discrete time. A vibrating force sequence $\tilde{f}(15,n)$ is applied at position 15 of the beam while for all remaining positions the applied force is of zero magnitude. The non-zero force sequence $\tilde{f}(15,n)$ makes the solid vibrate *a priori* hence we call the system a *vibrating solid*. This force sequence consists of a series of ternary values $-1, 0, +1$. The length of the sequence is 200 and the symbols are obtained sequentially by a repeated series of random draws using the random variable $\mathcal{F}$ with the following probability distribution: let $0 < p \leqslant 1$, then $\mathcal{F}$ takes the value 0 with probability $1 - p$, the value $+1$ with probability $p/2$, and -1 with probability $p/2$. The complexity of the sequence is controlled by the choice of p. Different values of p were used for different trials and used as the parameter of the distribution of $\mathcal{F}$. An external input force sequence $\tilde{I}(1,n)$ is applied at position 1 consisting of 200 randomly drawn binary values $+1$ and -1 each with probability $1/2$. The output of the system consists of the displacement of the solid at five positions represented by the real-valued functions $\tilde{u}(N-5,n), \ldots, \tilde{u}(N-1,n)$, $1 \leqslant n \leqslant 200$. Their values are discretized from real a to ternary $V(a)$ using the following rule: given $a \in \mathbb{R}$ then $V(a) = +1$, 0 and -1 if $a > \tau$, $|a| \leqslant \tau$ and $a < -\tau$, respectively, with $\tau = 0.1$. The five ternary sequences are appended together to form a single ternary output sequence of length 1000 (henceforth called the *output sequence*). The subsequence of this output which consists only of $+1$ and -1 values is selected (this is called the *output subsequence*). As an estimate of the complexity $K(x)$ of a sequence x we follow [14] (who used the theory of algorithmic complexity as a reference for developing practical distance measures for applications of pattern recognition) and use a compression algorithm (Gzip, which is a variation of the algorithm of [15]) to compress x. The length of the resulting compressed version of x is used as an approximation of $K(x)$. While algorithmic complexity theory holds only for the non-computable function K it has been shown [14] that compressor functions C (which include most existing algorithms such as Gzip) that are computable and satisfy (at least asymptotically with increasing sequence length) the properties of idempotency $C(xx) = C(x)$, symmetry $C(xy) = C(yx)$, monotonicity $C(xy) \geqslant C(x)$ and distributivity $C(xz) + C(y) \leqslant C(xz) + C(yz)$ up to an $O(\log n)$-error additive term, yield good approximations of K. Henceforth, by system complexity we mean the length of the compressed version of the vibrating force sequence. The output complexity is the length of the compressed version of the ternary output sequence. We now summarize and discuss the results obtained by [5] (for the specific experimental and statistical details we refer the reader to that paper). The experiments consisted of several hundreds of simulation trials where in each trial the response of a system (vibrating solid) to an externally applied input force sequence was computed. The numerical equations (7)–(8) represented the solid's model. As a choice of parameter values the following were used, $L = 20$, $T = 70$, $E = 0.7$, $\rho = 0.4$, $N = 30$, $B = 200$. Let M denote the ratio of the compressed length divided by the uncompressed length of the system and let O denote this ratio for the output sequence. A large M (or O) means that the compressed length is larger hence the complexity of the system (or output sequence) is larger. We sometime simply refer to M and O as the system and output complexity, respectively. Fig. 1 displays two sets of trials. In each trial of set (a) a random input force sequence was applied at position 1 (as described above). In each trial of set (b) *no* input sequence was applied. As is seen, the resulting behavior is clearly different in each of the two sets of trials. With an input present, as the complexity M increases there appears to be a decreasing trend in the value of O and an increase in the spread, i.e., the range of possible values of O. With no input, both O and its spread of values are basically constant with respect to M.

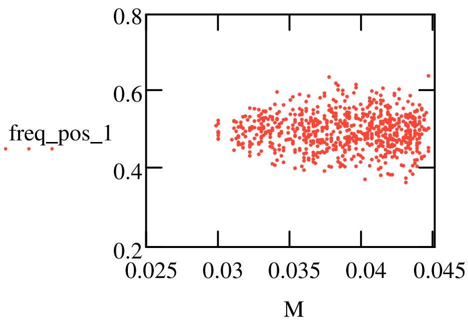

Fig. 2. Output frequency of 1s versus M.

To verify the above results a statistical analysis was performed. Linear regression yielded the following estimate for O: $\hat{O}(M) = 0.341 - 2.284M$. This confirms that with an increase in the system's complexity there is a decrease in the complexity of the output sequence. The negative correlation between these two complexities suggests that the system deforms the input randomness thereby producing a less complex output sequence.

When no input is present the output complexity is almost unaffected by the system's complexity. There is hardly any correlation between them as observed via a linear regression estimate for O which yields a slope of -0.12 with respect to M (by absolute value this is almost two orders of magnitude less than the slope when there is an input present). Thus the level of chaoticity in the output sequence is *not* simply due to the vibrating force but is caused by the system's interaction with the random input sequence.

Linear regression on the spread S of values of O with respect to M yields the following estimate: $\hat{S}(M) = -0.044 + 2.245M$. Thus as the system becomes more complex there is higher variability in the complexity of the output sequence. These results agree with the model introduced in [4] which says that a solid effectively acts as a selection rule picking bits from the input sequence and producing a possibly less random output. This is evident in the significant decrease in the output complexity and increase in its spread of values which indicates that the possibility of a less-complex output sequence increases as the system's complexity rises. To test the level in which the system deforms the randomness of the input sequence an experiment was done to measure the stochasticity of the output subsequence. In Fig. 2 we plot the frequency of 1s in the output subsequence (this is the number of 1s divided by the number of non-zero symbols in the output sequence). As can be seen, with an increase in the system complexity there appears to be an increase in the spread of possible frequency-of-1 values. To verify this we measured the spread W of possible values of frequency of 1s in the output subsequence as a function of the compressed system length divided by the length of the output (binary) subsequence (henceforth denoted by M'). Linear regression on W yields an estimate $\hat{W}(M') = \sqrt{-0.214 + 0.95M'}$. This result indicates that as M' increases there is an increase in the spread of possible values of frequency of 1s in the output subsequence at a rate which agrees with the rate of $O(\sqrt{K(R|n)/\ell(R(x))})$ as predicted by (2). We note again that our compressed lengths are based on the computable compressor algorithm (Gzip) while (2) applies to the non-computable Kolmogorov complexity K hence we are not validating the theory but using it only as a reference.

6. Conclusions

The results described in previous section imply that a system based on classical equations of mechanics consisting of a vibrating solid subjected to external random input-force acts similar to an algorithmic selection rule of a finite complexity. It produces an output sequence whose stochastic and chaotic properties are effected by the system's complexity in a manner which follows basic principles of algorithmic randomness. The results are only a first step in the direction of testing the model of [4]. The solid studied here has a complexity that depends not just on the intrinsic parameters E, ρ of the differential equations but also on the pattern of the vibrational force. Further work is needed to examine the response of different kinds of solid structures. A more general question is how to measure the descriptional complexity of a dynamic system that is described based on a set of differential equations.

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