
HONI SOIT QUI MAL Y PENSE
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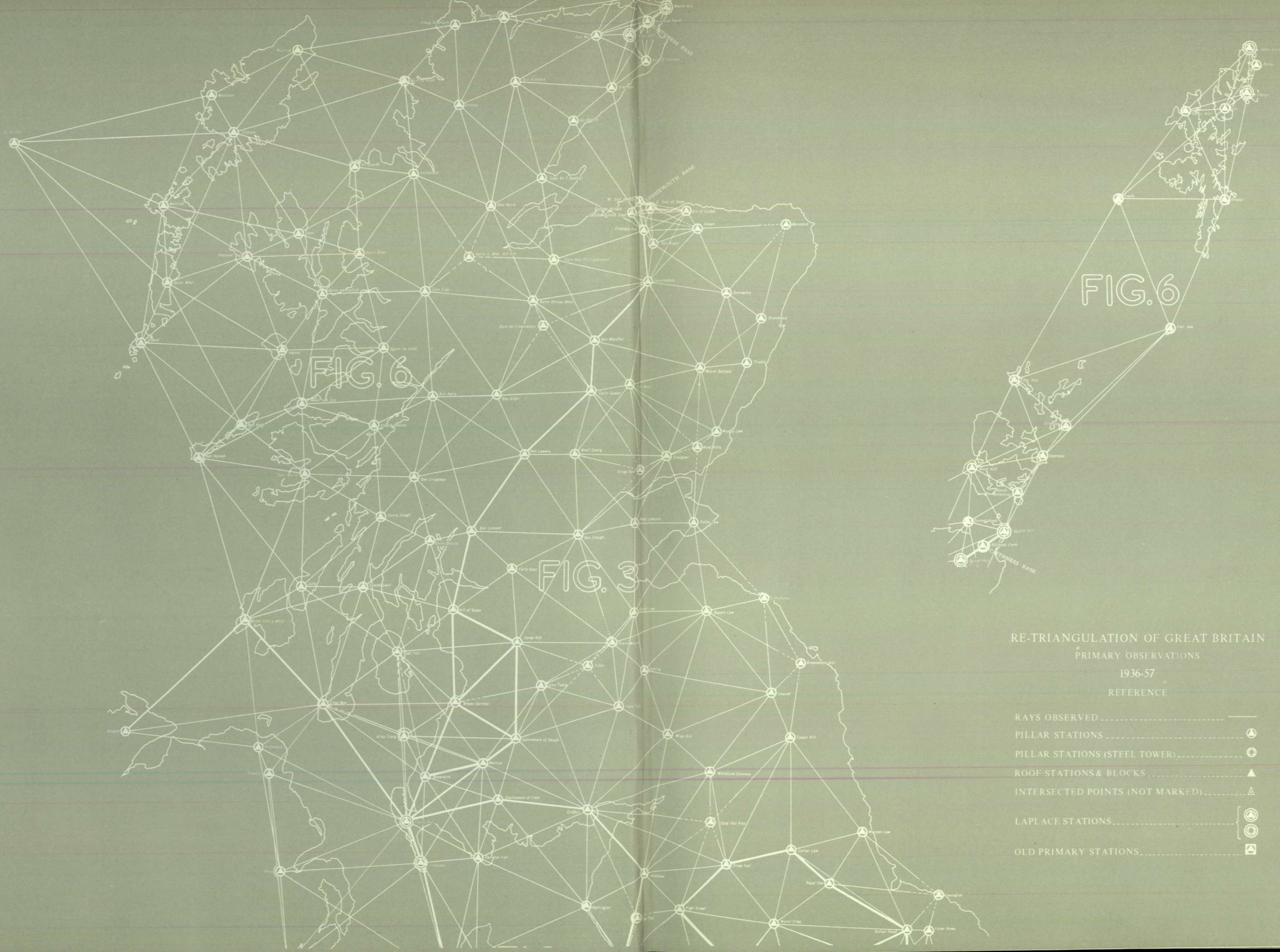
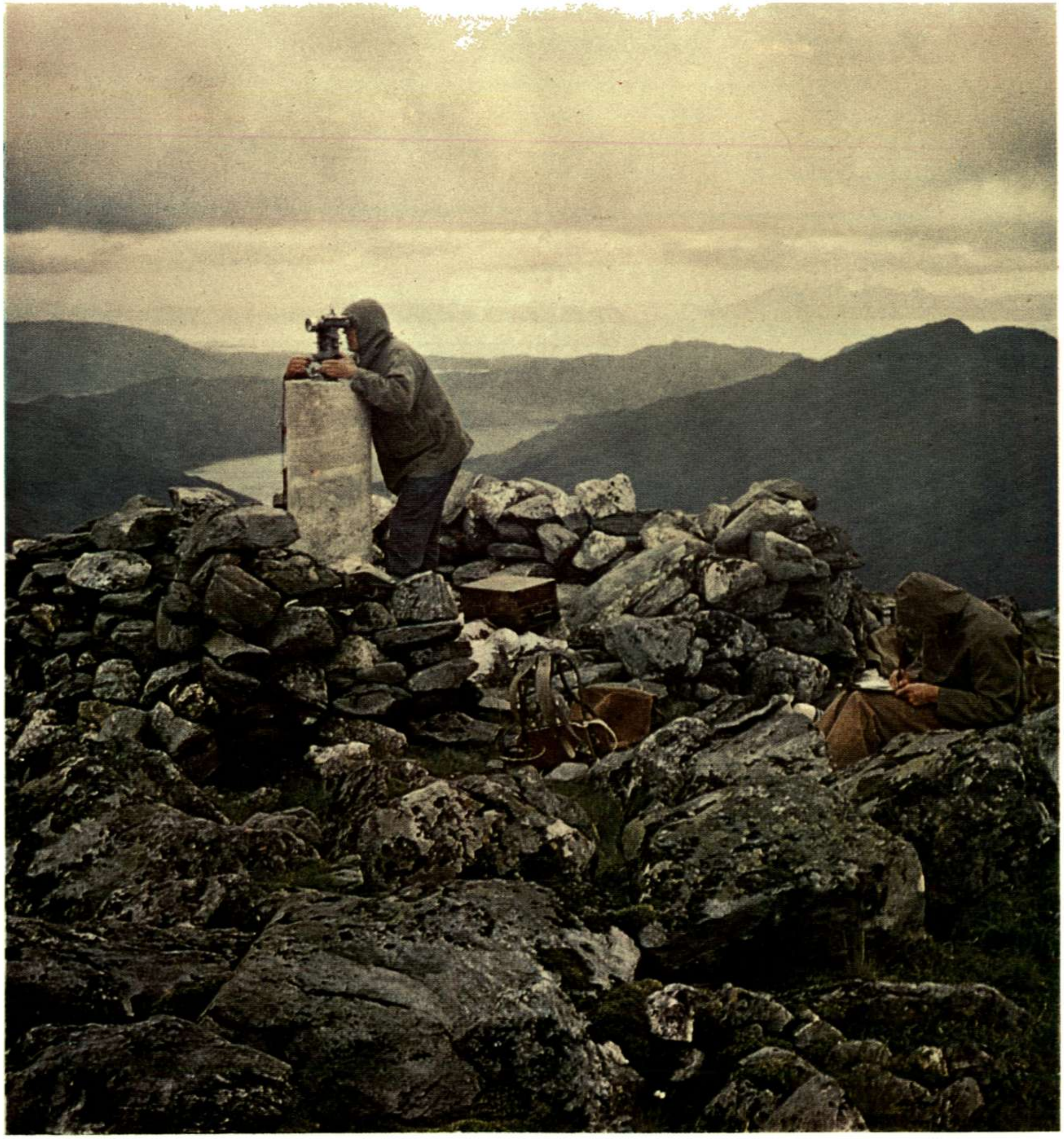


RE-TRIANGULATION OF GREAT BRITAIN
PRIMARY OBSERVATIONS
1936-57
REFERENCE

RAYS OBSERVED ______
PILLAR STATIONS ______ ◬
PILLAR STATIONS (STEEL TOWER) ______ ⊡
ROOF STATIONS & BLOCKS ______ ▲
INTERSECTED POINTS (NOT MARKED) ______ △
LAPLACE STATIONS ______ ◎
OLD PRIMARY STATIONS ______ ▣



Primary pillar Sgurr Na Ciche, Inverness-shire, looking west over the Sound of Sleat to Rhum, Eigg and Muck

ORDNANCE SURVEY

The History of the Retriangulation of Great Britain

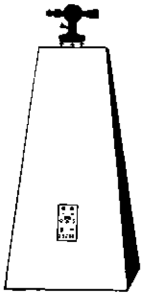
1935-1962

Written and compiled
by Officers of the Department
under the authority of the Director General
of the Ordnance Survey

LONDON: HER MAJESTY'S STATIONERY OFFICE
1967

Printed in England for Her Majesty's Stationery Office by J. W. Arrowsmith Ltd., Bristol

. . . If this fail,
The pillar'd firmament is rottenness,
And earth's base built on stubble.
COMUS



FOREWORD

The maps and plans of the Ordnance Survey have long been renowned. Their high quality has stemmed essentially from an accurate survey based upon a sound controlling framework, the Principal Triangulation of a century ago. The Retriangulation carried out between 1935 and 1962 provides, and will provide in the years to come, an equally sure foundation for the new maps and plans now being produced.

It is a privilege to have been associated with so great an enterprise, and I am grateful for this opportunity of paying tribute to the 'Ordnance Surveyor' upon whose skill and staunchness its success has depended. I am glad also to thank all those who in one way or another have contributed to the mammoth task of compiling and producing this History.

(R. C. A. EDGE)
Major-General
Director General, Ordnance Survey

Chessington, 1966

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PREFACE

Anyone closely concerned with triangulation in this country or indeed in any part of the world would acknowledge that the presence of Captain (later Colonel) Alexander Ross Clarke, Royal Engineers, in charge of Triangulation and Levelling at the Ordnance Survey in the 1850s was a most providential circumstance. For it was to him that it fell to compute and adjust the Principal Triangulation based on observations extending through half a century which had at last been completed to cover the country. In carrying out his task Clarke evolved principles of computation that have remained a model for geodesists. He also calculated the first of his well known series of figures of the Earth, one or other of which has been adopted for so many of the great land areas of the world. Clarke's *Account of the Observations and Calculations of the Principal Triangulation and of the Figure, Dimensions and Mean Specific Gravity of the Earth as derived therefrom* (see page xvii) which was published in 1858 is a geodetic classic, and, with his subsequent practical and theoretical work, established him as probably the most distinguished geodesist of his day.

When in 1935 after the passage of three-quarters of a century various circumstances—none of them reflecting adversely on Clarke's work—combined to make it necessary to embark on a new triangulation, it was no less fortunate that Major (later Brigadier) Martin Hotine, Royal Engineers, was in Clarke's old chair. At that time the Ordnance Survey faced a crisis. Decades of financial stringency culminating in the Geddes Axe of 1922 had progressively stripped it of resources with the inevitable result that the revision of the large scale plans had fallen further and further into arrears; until for large areas of the country where development had altered the face of the land these plans had become almost useless. Eventually a Departmental Committee under the chairmanship of Lord Davidson was set up to investigate. Its report, recognising the need for drastic action, resulted in governmental approval for an entirely new and greatly accelerated programme which entailed a wholesale expansion of the Ordnance Survey.

The first Chapter of this History explains how these events in turn led to the epoch-making decision to retriangulate the country—a decision which epitomised the revolutionary nature of the developments which marked this period. For if the revision of the large scale plans had suffered from lack of resources, still more was this the case for the Department's less obviously essential geodetic activities. Hotine thus found a situation in which the Ordnance Survey, having had its geodetic resources pruned to the irreducible minimum, was suddenly called upon to recreate in a few years a major triangulation which when last carried out had taken half a century. The existing staff and equipment of the Department were quite inadequate for this task. Apart from Hotine himself, few of the officers or surveyors had had experience of geodetic triangulation. Yet in less than four years, before war intervened to call a halt, the primary Retriangulation of England and Wales and half of Scotland had been completed. The Chapters of this History bear testimony to the nature of the effort that this achievement entailed, and to the qualities—moral, physical and professional—of the team of surveyors that was responsible. This team was trained by Hotine himself and he personally inspired them by his example.

It was he also who by his persistence and the exercise of his unrivalled talent for debate was mainly responsible for eliminating the innumerable obstacles which beset anyone who in this conservative land embarks on some operation unfamiliar to its inhabitants, more particularly an operation which depends so much for its success upon freedom of entry upon property.

In addition to his qualities of leadership Hotine's great intellectual gifts were indispensable. In the best traditions of his eminent predecessor he made full use of his mathematical powers to rationalise and bring up to date the methods used. In particular he was mainly responsible for evolving the National Grid which has proved of such inestimable benefit both for the purposes of computation and as a framework and referencing system for all the modern maps and plans of the Ordnance Survey.

So it is to Hotine more than to any other individual that we must be grateful that, when war ended, the Ordnance Survey was able to embark almost immediately upon the task of detail survey in those large areas of the country which had been covered by the Retriangulation. But he would be the last to deny due credit to the loyal team of observers, bookers, lightkeepers, tower builders, computers and others, both before and after the war, who brought the whole great undertaking to a successful conclusion. It is impossible to name all these and to mention any may seem invidious, but some there are who bore a special share of responsibility: A. R. Martin, G. F. Mullinger and the late A. C. Wilde who made most of the pre-war primary observations; H. J. W. Smith, R. J. Stone and B. Willis, who carried on after the war; W. Stuart and B. Watts, who were largely responsible for the administration of field parties; E. T. Bateman and R. G. Curtis who in succession were responsible for supervising computations and J. K. Holt who had a special share in evolving computational methods.

One important reason for which his successors have had cause to be grateful to Clarke is that he compiled a clear and detailed account of his work which was published in a lasting form within a few years of his completing it. Clearly we of the present generation should have been culpable if we had failed to produce a comparable record of the Retriangulation. But we have had to face difficulties which Clarke in his more peaceful and leisured times escaped. The intervention of World War II interrupted the primary triangulation and removed Hotine from the scene before it was complete. Fortunately he was able shortly before he left the Ordnance Survey to write an excellent narrative account of the work as far as he had taken it. This appeared in the *Empire Survey Review* in 1938 and this History has drawn largely upon it. But in the main the business of making available in sufficient detail for posterity all the work that had been done went by default until in 1955 a decision was made to compile and publish this History. But to make the decision was one thing and to implement it another. Gone were the days when, like Clarke, officers could be left in post for 27 years to concentrate almost exclusively on scientific matters. Officers during their much shorter tours nowadays have many preoccupations and it would have been well nigh impossible for such an officer, however well qualified, to have assembled all the scattered data and compiled a history such as this in the course of his normal duties. It was fortunate therefore that Her Majesty's Treasury, being persuaded of the importance of the task, agreed to an officer being placed on special duty for six months to carry it out. The officer selected was Major (now Lieutenant Colonel) J. Kelsey, Royal Engineers, and it is to him that we owe, in large measure, the compilation of this History.

Major Kelsey started work in February 1959 but unluckily was posted away in August 1959 before the first draft was complete, and it was not until 1963 that the body of the text could be handed over to the printers. During the intervening period a great deal of drafting, redrafting and editing took place in which a number of people had a share. The final work is therefore essentially a joint effort.

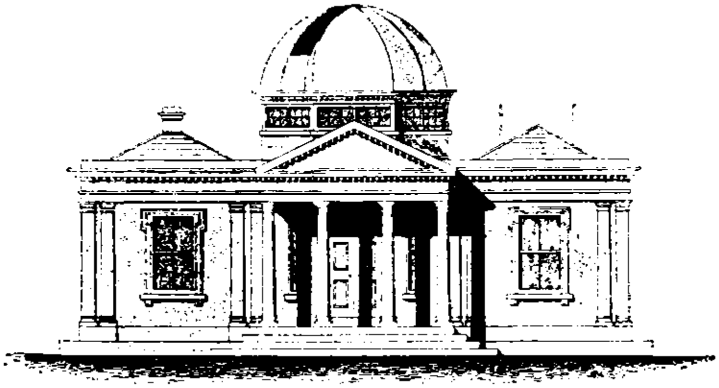
ORDNANCE TRIGONOMETRICAL SURVEY
OF
GREAT BRITAIN AND IRELAND

ACCOUNT
OF THE
OBSERVATIONS AND CALCULATIONS,
OF THE
PRINCIPAL TRIANGULATION;
AND OF THE
FIGURE, DIMENSIONS AND MEAN SPECIFIC GRAVITY,
OF THE
EARTH
AS DERIVED THEREFROM.

PUBLISHED BY ORDER OF THE MASTER-GENERAL AND BOARD OF ORDNANCE.

Drawn up by CAPTAIN ALEXANDER ROSS CLARKE, R.E. F.R.A.S. under the direction of

LT COLONEL H. JAMES, R.E. F.R.S. M.R.I.A., &c.
SUPERINTENDENT OF THE ORDNANCE SURVEY.



ASTRONOMICAL OBSERVATORY AT THE ORDNANCE SURVEY OFFICE, SOUTHAMPTON

LONDON
PRINTED BY GEORGE EDWARD EYRE, AND WILLIAM SPOTTISWOODE,
PRINTERS TO THE QUEEN'S MOST EXCELLENT MAJESTY.
1858.

The first drafts of Chapters 1 to 5 were prepared under the direction of Brigadier L. J. Harris. Major Kelsey himself wrote the basic drafts for Chapter 1, Chapter 2 paras 2.00 to 2.09, Chapter 3 paras 3.00 and 3.04 to 3.08, Chapter 4 paras 4.00 to 4.21, 4.26 and 4.27. He was also responsible for most of the initial detailed planning and organisation, and he co-ordinated and supervised the preparation of the diagrams and illustrations. Mr. R. J. Stone wrote the initial drafts of Chapter 2 paras 2.10 to 2.18, Chapter 3 para 3.01 and Chapter 7 (fieldwork). He also collected and arranged the photographic illustrations. Mr. J. K. Holt wrote the initial drafts for Chapter 2 paras 2.19 to 2.33, Chapter 4 paras 4.22 to 4.24, Chapter 5, and Chapter 7 (computations). He also supervised the preparation of the final draft. Mr. R. G. Curtis was responsible for the compilation of all the appendices. Figures and diagrams were drawn by Mr. V. H. Watts. Brigadier R. C. A. Edge wrote Chapter 6 as well as certain parts of Chapters 3 and 4 (paras 3.09, 3.10, 4.25 and 4.28 to 4.31). He also examined the final draft. Lieutenant Colonel P. J. Carmody, Royal Engineers, took over the task of general co-ordination from Major Kelsey and himself wrote the final draft of Chapter 7.

With such a multiplicity of authors whose contributions varied widely in style and content a general editor became a necessity. The services of Major General J. C. T. Willis (Retd) were secured for this important task.

A number of individuals assisted in the detailed examination of the draft at various stages. Colonel D. I. Burnett examined the first drafts of Chapters 1 to 5. Chapter 2 was also examined by Brigadier H. A. L. Shewell (Retd), Mr. H. H. Brazier and Mr. H. F. Rainsford, the two last named checking the mathematics in detail. Brigadier G. Bomford (Retd) drafted para 3.03 dealing with the Shoran connection with Norway and examined the drafts of Chapters 5 and 6. Dr. A. R. Robbins also examined Chapter 5. Others who examined and checked various parts of the text were Colonel M. H. Cobb (para 3.01), Dr. R. d'E. Atkinson (paras 3.05 to 3.10), Dr. O. Trovaag of the Geographical Survey of Norway (para 3.03), and M. Segons of the Institut Géographique National of France (para 3.02).

Many others, too numerous to mention, participated in the compilation and detailed checking of the large amount of numerical and mathematical data.

Acknowledgement and thanks are due to the following:

To the United States Air Force for permission to include the account of the Shoran connections with Norway, the Faeroes and Iceland (paras 3.03, 3.04).

To the Director of the Institut Géographique National of France for assistance in compiling the account of the cross-Channel connection, in particular the description of the 'Cercle Azimuthal Répétiteur' (para 3.02).

NOTE

REFERENCES

Paragraphs, formulae, tables and textual figures, have been numbered chapter by chapter. For example, paragraphs 2.00 to 2.33, formulae (2.1) to (2.19), Tables 2.1 and 2.2 and textual figures Fig. 2.1 to Fig. 2.12, are in Chapter Two; paragraphs 5.00 to 5.09, formulae (5.1) to (5.23), Tables 5.1 to 5.6, and textual figures Fig. 5.1 to Fig. 5.11 are in Chapter Five; and so on. Where necessary for the sake of clarity paragraphs have been sub-divided, and the sub-divisions numbered within the main paragraph, for example 3.100 to 3.103 in paragraph 3.10 of Chapter Three. Cross references to paragraphs are shown by giving the paragraph number prefixed by the symbol §. Note that the abbreviation Fig. has been used invariably for a textual figure, whereas Figure written in full indicates a primary triangulation figure, thus Figure 1 to Figure 7.

Diagrams have been numbered 1 to 20 without reference to chapter numbers. To facilitate reference to the diagrams the names of primary stations in the text are followed by their record numbers in parentheses, thus Holyhead (117). Diagram 2 shows the whole of the primary Retriangulation with the stations denoted by their record numbers; it also contains a key to their names arranged in numerical order.

Superscripted numbers in brackets have been used for references to the Bibliography, thus(1).

NOTATION

As far as possible internationally agreed or commonly used symbols and sign conventions have been adopted.

The lists of defined symbols in paragraphs 2.19 and 5.01 include those symbols most frequently used in Chapters Two and Five respectively. The lists are not exhaustive, and do not necessarily include symbols used in other chapters. However, quantities not defined in paragraphs 2.19 and 5.01 are invariably defined when used.

CHAPTER ONE

Introduction

1.00 Purpose of this Publication

Great Britain is one of the few countries in the world where two primary triangulations have been observed during the past two centuries. The first triangulation developed over a long period (1783–1853), mainly from projects which were initiated from time to time to solve particular scientific problems, and was described by Captain A. R. Clarke in his account of the Principal Triangulation published in 1858(1).

By 1935 it had become apparent that the old triangulation framework was in many respects inadequate for modern requirements, and it was decided to carry out a fresh triangulation covering the whole country. This Retriangulation, started in 1935, was continued, apart from the period of the Second World War (1939–45), until its virtual completion in 1962.

The object of this publication is to provide a history of this Retriangulation, and to place on record details of all observations and computations connected with it in such a manner that they will be available for posterity. The Retriangulation has now been substantially completed, and although certain geodetic work closely connected with it remains to be done, it has been thought best not to wait for this, but to complete the record while it is still possible to consult individuals who took part in the early stages of the work. Already many have left the Ordnance Survey and some have died or been incapacitated by sickness.

1.01 A Brief History of the Retriangulation

The Retriangulation was started in 1935, when the reconnaissance for the new primary triangulation was first put in hand. The work came to a standstill on the outbreak of war in September 1939, by which time the primary network covered the whole of England and Wales, and extended as far north as the Moray Firth in Scotland. Secondary triangulation, based on the new primary framework, was put in hand in 1938 in order to provide the control required for new large scale surveys on national sheet lines. The need for such surveys was discussed in two reports(2)(3) issued by the Davidson Committee in 1936 and 1938. At the end of the 1939–45 war the entire resources of the Triangulation Branch were devoted to secondary and lower order work, as the need for the new large scale surveys had by then become paramount. The primary observations were not resumed until 1949 and were completed in 1952 (see § 2.18). Observation of the last block of secondary and tertiary triangulation was completed in the spring of 1962.

THE OLD TRIANGULATION

1.02 The History of the Principal Triangulation

In 1783 Monsieur Cassini de Thury, the Director of the French Royal Observatory, drew the attention of the British Government to the need for an accurate definition of the distance between Dover and London. Monsieur Cassini had already obtained the distance between Paris and Calais and had observed across the Channel from Calais to Dover. He was anxious to extend this measurement to London to connect the two observatories of Paris and Greenwich, there being, in his view, a discrepancy between the accepted position of Greenwich Observatory, relative to that of Paris, of approximately 11″ of longitude and 15″ of latitude. The British Government referred his suggestion to the Royal Society, and, from the geodetic point of view, there is no doubt but that it had been made at a very opportune moment. Everyone connected with survey at that time received Cassini's approach with enthusiasm, and no one more so than Major-General William Roy, then Surveyor-General of Coasts and Engineer for Making and Directing Military Surveys in Great Britain. The resonance of Roy's title was belied by the extent of the resources at his disposal, which were in fact non-existent. Roy himself, however, had for long been pressing for the establishment of a National Survey. His enthusiasm was shortly to be rewarded, for the British Government sanctioned the work and commanded him to carry it out with the assistance of the Royal Society and a military staff.

Angular measurements were to be observed with the Great Circular Theodolite, commissioned by the Royal Society in 1784 and built by Jesse Ramsden, the finest instrument maker of his time. The horizontal circle of Ramsden's instrument was three feet in diameter, giving measurements of arc to tenths of a second. Jesse Ramsden was a man whose 'artist's genius disdained time restrictions'. His somewhat dilatory nature was later to prove a considerable thorn in the side of Roy, who in one of his letters commented 'On one occasion he attended at Buckingham Palace precisely as he supposed at the time named in the Royal mandate. The King remarked that he was punctual as to the day and hour, while late by a whole year'!

The first step in Roy's programme consisted of the measurement, in 1784, of a base on Hounslow Heath; a work carried out with glass tubes approximately 18 feet in length. King George III took an active interest in the proceedings, for on 21st August 'His Majesty deigned to honour the operation by his presence . . . entering minutely into the work of conducting it, which met with his gracious approbation'. Emulating the Royal Example '. . . the very worthy President of the Royal Society repeatedly visited the Heath and with that liberality of mind which distinguishes all his actions ordered his tents to be pitched near at hand, where his immediate guests and numerous visitors met with the most hospitable supply of every necessary and even elegant refreshment'. *O si sic omnes!* Between 1787 and 1788 London and Dover were connected by triangulation; a further connection was then observed across the Channel, with the co-operation of the French. General Roy died in 1790 soon after he had completed his account of these triangulation operations for the *Philosophical Transactions of the Royal Society.*

Much of the general interest in the formation of a National Survey died with Roy. It was, however, providential that the then Master-General of Ordnance, Charles Lennox, Duke of Richmond, was a man who shared Roy's ambitions and enthusiasm, and who had for long been a patron of local survey and cartography in Sussex. The Duke's knowledge and administrative powers were employed to unite the established practices of private, or civil, surveys on the one hand, and of military survey on the other. The technical standards and procedures, originally laid down by Roy in the quarter of a century prior to 1790, were adopted by the Duke when he ensured the continuation of Roy's triangulation in South East England, followed by the official establishment of the Trigonometrical Survey in 1791. Roy has often been called 'The Father of the Ordnance Survey'. If this be an apt description, then surely the Duke of Richmond can well claim the status of midwife. It is fitting that one of the pillars of the Retriangulation, within a few yards of Roy's birthplace, specially designed and maintained, is suitably inscribed to the memory of this great surveyor.

The new-born Ordnance Survey consisted originally of but three military officers, assisted by working parties of soldiers. With these slender resources the work was pushed steadily forward over the whole of Southern England between 1790 and 1798 and a further base was measured on Salisbury Plain. Between 1800 and 1809 the triangulation was extended to include Yorkshire, primarily in order to measure an arc of the meridian. The average length of the sides of the triangles was 35 miles, though some of them reached as much as 55 miles which is unduly long by modern standards.

The main reason for this extension of the triangulation was to obtain a more precise knowledge of the shape and dimensions of the Earth, but, as a by-product, the framework thus obtained was used to control the production of a One Inch to One Mile map. By 1824 this phase of the work was completed.

In the same year a Royal Commission, appointed to investigate problems of survey and land valuation in Ireland, recommended that the whole island should be surveyed at a scale of Six Inches to One Mile. This work was started in 1825 and absorbed the entire resources of the Ordnance Survey, including three companies of Sappers and Miners, destined to become, in later years, the Corps of Royal Engineers.

Little triangulation was therefore undertaken in Great Britain until 1838 when the need for closer control became imperative as a result of the decision to survey the North of England and South of Scotland at the scale of Six Inches to One Mile which had been originated in Ireland. The increased density of control points necessitated by the adoption of this larger scale naturally involved a corresponding increase in the work of the observers. Between 1838 and 1850 a vast number of secondary and tertiary points were observed. Almost every visible station was included in this formidable undertaking, and it was common for fifty or more points to be observed from one station. Indeed at the station erected above the cross of St. Paul's Cathedral, more than 1,600 points were observed within a period of several months. Verily there were giants in those days!

By 1853 Great Britain was covered by a number of triangulation stations which had been co-ordinated both for geodetic reasons, and for the control of local surveys. No comprehensive pattern or design had been established at the outset of the work, and the result was not unnaturally haphazard in the extreme. By a process of selection and rejection from this huge and somewhat amorphous mass of data, Clarke, then in charge of the Trigonometrical and Levelling Departments, virtually created what is now known as the Principal Triangulation of Great Britain (Diagram 1). He produced, from the observations taken between 1783 and 1853, an interlocking network of well conditioned triangles. This network was geometrically of great strength since it involved no fewer than 920 condition equations to find corrections to 1,554 observed directions, subsequently used to fix 218 points. The system was rigorously adjusted by the method of least squares in 21 separate, but not all entirely independent, figures, the corrections obtained from the solution of one figure being substituted in the condition equations of adjoining figures as a means of securing an overlap in the adjustment. The average triangular misclosure (regardless of sign) was 2·8″. The directly measured length of the Salisbury Plain Base was found to be greater than the length derived through the triangulation from the Lough Foyle Base by one part in 93,000. The scale of the triangulation was fixed by accepting a weighted mean of the two bases, and the position and azimuth were derived from the Royal Observatory at Greenwich (see § 3.060).

1.03 An Evaluation of the Principal Triangulation

It has been seen that the Principal Triangulation was essentially created by an office analysis of, and subsequent selection from, the available data. In spite of the early date of many of the observations and the primitive character of the instruments used, there is no doubt that the Principal Triangulation as derived by Clarke was of sufficient accuracy to justify its use to determine a figure of the Earth. In all probability it would also have been quite adequate as the basis of a secondary triangulation during the nineteenth century. Unfortunately, the old secondary triangulation, as will be seen later, was never analysed and adjusted in the same way as the Principal Triangulation, nor was it rigorously connected to it.

In the early part of the twentieth century the question arose as to whether Clarke's Principal Triangulation could reasonably be used as an extension to the European geodetic network, much of which had been completed in the latter part of the nineteenth century. Two investigations were therefore carried out at this time. A base was measured at Lossiemouth in 1909 to check the geodetic accuracy of the Principal Triangulation. This base, which was remote from the two original bases in Southern England and Northern Ireland, showed an agreement in scale of 1 : 60,000 with the triangulation. This was a very satisfactory check on the Principal Triangulation, establishing its accuracy over long distances. This did not, however, preclude the possibility of much greater local errors which might have cancelled out, and which could have led to inconsistencies between blocks of secondary triangulation based on the primary work.

In 1929, therefore, Figure 21 (Yorkshire) was re-adjusted, with the addition of a few more lines and five more stations, which had been omitted from the original adjustment, and with a re-arrangement of the fixed boundary conditions. This re-adjustment introduced a relative shift of no less than 7 seconds on certain lines. Admittedly this was a severe test since Figure 21 was the last of the original figures to be adjusted and was thus surrounded by previously adjusted work; but it did indicate quite conclusively that there were local errors which far exceeded the overall error of the framework. As will be seen later, these conclusions were supported by a comparison with the results of the new primary triangulation which indicated appreciable errors locally in the old work, but a remarkable degree of accuracy over longer distances, due probably to the geometrical strength of the network.

1.04 The Old Secondary and Tertiary Triangulations

The old triangulation in Great Britain was never designed as a comprehensive system on which control for large scale surveys could be based. The secondary and tertiary triangulations were observed purely to provide a basis for the large scale surveys covering areas of individual counties or groups of counties. Each of these limited areas had its own projection with its own origin. These areas are referred to as 'county units' in the following paragraphs. Each triangulation station was marked by a hole about 1 inch in diameter cut in a large stone and buried 12–18 inches beneath the surface. These stones, called 'freestones', were generally of a type of rock not found in the locality. The descriptions were generally very poor and varied in quality from a dimensioned plan to a statement such as 'Mr. Brown who lives in the cottage at the foot of the hill knows the position of the station'. After a lapse of 100 years or more and the consequent demise of Mr. Brown this naturally complicated the task of finding such stations. With increased development in England the destruction rate in secondary and tertiary triangulation stations was high, although stations on hill tops could normally be recovered if the site had not been built upon.

The observations were never rigorously adjusted and were in some cases computed on the local county Cassini origins, which were used for the projection of the Six-inch and 1/2500 survey. As a result, adjoining blocks of secondary triangulation were out of sympathy with each other and there were irregular and indefinite discrepancies along boundaries of adjacent county units amounting to as much as 50 feet. Many of the old records are lost, but it seems certain that no serious attempt was made prior to 1920 to compute any secondary figure in sympathy with the primary network. Furthermore, both the method of calculation of the secondary figures and also the method used for transforming the secondary points from geographical co-ordinates to local county rectangular co-ordinates, though obscure, appear to be unsatisfactory by modern standards. Use had to be made of a somewhat random collection of observations. Neither time nor money permitted recourse to an elaborate adjustment of secondary observations by least squares. The methods used were adequate for the immediate purpose, but perhaps too little thought was given to the possibility of further extensions, and to the need for careful maintenance of records.

Little can be said about the tertiary triangulation; it was computed without adjustment by rectangular co-ordinates on local county projections. There were in consequence considerable discrepancies between adjacent blocks and particularly along boundaries of county units. Points were co-ordinated by two or more triangles and the mean value accepted. With such methods errors were bound to increase when a triangulation was carried forward over several miles.

1.05 Reasons for the Decision to Observe a New Triangulation

For all practical purposes there was no consistent national triangulation of Great Britain, but only a large number of semi-independent triangulations which were not in sympathy with the primary stations of the Principal Triangulation. It was impossible to re-adjust these lower order triangulations to bring them into sympathy with the sound framework of the Principal Triangulation. This was due partly to the fact that too few of the old stations could be recovered with sufficient certainty to connect these detached triangulations to the primary work by a limited amount of re-observation, and partly to the fact that the original observations, undertaken solely for the purpose of providing rapid control for 1/2500 mapping of county units, were not sufficiently accurate to cover larger areas.

At the inception of the 1/2500 survey in 1854 it was thought that these county units would be permanent. Although the alteration of administrative boundaries, which became very common in the twentieth century in Great Britain, has not justified this expectation, the decision to adopt independent surveys of limited areas followed contemporary surveying practice, notably in the

French surveys of 'Communes', which to a large extent provided the model for the British 1/2500 series. It is also likely that the angular distortions of the then fashionable Cassini projection gave rise to a reluctance to consider the single projections of larger areas. If a single Cassini projection belt had been used to cover the whole of Great Britain, the maximum angular distortion between an angle computed from co-ordinates and the corresponding angle measured on the ground, would have been more than 4 minutes. Such a distortion would have been quite intolerable even for minor instrumental surveys.

So long as the county unit remained the survey unit of this country, no inconvenience resulted from the existence of these independent surveys. Some inconvenience was, however, felt even before the original survey had been completed in 1892, since development across the boundaries of the county units could not be illustrated on a single plan. To overcome this difficulty various attempts were made to extend large-scale plans across county unit boundaries by recomputing the triangulation system of one county unit and adjusting it to that of the adjoining county unit, and by replotting the detail survey to this adjusted control. Such filled plans were slightly inconsistent with plans of the same locality plotted on the adjoining projection, but did at least overcome the understandable reluctance of the general public to paying for a plan which might be nine-tenths blank paper. In some cases an entire county survey was transferred in this manner, but even where data for connecting the two triangulations existed, the fact remained that these minor triangulations were not intended for such extensions and were not sufficiently accurate for that purpose. Consequently, these expedients usually resulted in serious inaccuracies in the position of points of detail on the plans, inaccuracies which were accentuated at the next revision. Furthermore the discontinuity was in many cases merely moved elsewhere. Largely as a result of such defects in the basic control, the fabric of the 1/2500 survey was by 1934 showing signs of collapse. In certain areas of rapidly expanding post 1914–18 war development, especially in Northern London, the need for re-survey as opposed to revision had become apparent. Attempts to patch up the existing triangulation as a control for such new surveys served merely to underline the rapidly increasing inadequacy of the existing framework; an inadequacy which was further emphasised by the need for reliable control in mining areas especially liable to subsidence. In short, the secondary and tertiary triangulations of Great Britain had outlived their usefulness.

For these reasons it was decided in 1935 to carry out an entirely fresh secondary triangulation. The decision to observe a new primary network also, rested less on considerations of accuracy than on the fact that too few of the old primary station centres could be recovered with certainty, and that they were generally too far apart to provide a framework for the rapid and economical execution of a modern self-consistent secondary triangulation. The cost of a primary triangulation on 30-mile sides was in any case but a small fraction of the cost of secondary work on 4-mile sides, and it was considered unsound to incur the considerable expense of fresh secondary work without an assurance that the relatively inexpensive primary foundation was secure at all points. For these reasons it was also decided to observe a new primary triangulation as an essential basis for the completion of a new secondary triangulation.

CHAPTER TWO

The Primary Retriangulation

GENERAL

2.00 Layout of the Primary Retriangulation

The primary or first order triangulation was designed as a broad network of triangles with an average side length of 20 to 30 miles. A main chain extends from the South of England to Central Scotland in three overlapping figures. This main chain is never less than 50 miles wide and forms a strong framework from which the remaining figures extend to cover the whole country. In all there are seven figures as illustrated at Diagram 4.

Figure 1. Southern England
Figure 2. Central and Northern England } constituting the main chain
Figure 3. South and Central Scotland
Figure 4. West England and Wales
Figure 5. South-east England and East Anglia
Figure 6. North Scotland
Figure 7. Isle of Man

For various reasons additional primary stations were added to these main figures, viz:

3 stations in the Spurn Head extension of Figure 2
3 stations in various parts of Figure 5
3 stations in Figure 6.
See § 2.18 and § 2.33 for details.

The overall intention was to provide a strong network of triangles extending over the whole country rather than the more conventional series of chains controlled in scale and azimuth by bases and Laplace stations.

The reasons for the departure from normal geodetic practice were given in articles written by Major (now Brigadier) M. Hotine, R.E., which were published in the *Empire Survey Review* in 1938(4). It was considered preferable in a small country like Great Britain to treat the adjustment of triangulation from a strictly geometrical point of view, since a strong network of triangles, uninfluenced by the interaction of linear and angular measures and unchanged by astronomical measurements, would ensure geometrical consistency of shape over the whole area. It was claimed at the time (a claim which has since been justified by the results of the lower order triangulations) that a strong network of triangles would spread the accumulation of error inherent in any triangulation, thinly and uniformly over the whole area covered. An additional advantage was that in such a network no one ray was essential. Under the climatic conditions prevailing in Great Britain, the ability to dispense with an individual ray, which could not be observed when the station was occupied, could well save the expense of re-occupation. Furthermore, the observing programme was rendered more flexible by such an element of elasticity.

Ideally the whole of this network would have been adjusted in one comprehensive operation, but in 1936 it was impracticable to do so with the computing machines then available. In addition, co-ordinate values of the primary stations in England and Wales were urgently required to control the lower order triangulations, which were being observed concurrently with the primary. It was for these reasons that the country was divided into a small number of figures, overlaps being provided between such figures to avoid discontinuity across the figure boundaries. (See § 2.243).

A similar geometrical solution had been used by Clarke in the Principal Triangulation although in this case the figures were much smaller and consequently more numerous. As a result the later figures (notably Figure 21) were uncomfortably hemmed in by previously fixed conditions. This situation does not arise in the Retriangulation where, in general, each figure is restrained on one edge only.

2.01 Scale and Orientation

The method of fixing the scale and orientation of the network was determined by the need to avoid disturbing the graticule of the existing large scale plans, which were based on the old triangulation. Figures 1 and 2, forming part of the central chain, were computed and adjusted in the first place. Figure 1 was computed and adjusted independently. Figure 2 was then adjusted to the northern edge of Figure 1. The scale, azimuth and position of the combined Figures 1 and 2 were then adjusted to give the best mean fit at 11 points of the old Principal Triangulation which were coincident with stations of the Retriangulation (see § 2.27). In the adjustment no account was taken of the 1937 and 1938 base measurements for reasons given in § 4.00. This was partly because it was necessary to fit the new triangulation to the old, and partly because, as stated above, it was considered that a dense continuous net adjusted by purely geometric means would be more reliable than a system of chains adjusted to a few observed bases and azimuths. In fact no astronomical azimuth control existed at that time.

2.02 Connections to other Countries

Connections were made to the primary triangulations of the following countries:

France: in 1951 in co-operation with the Institut Géographique National.
Ireland: in 1952 in co-operation with the Survey Departments of Eire and Northern Ireland.

The United States Air Force made connections between Iceland, The Faeroes, Scotland and Norway in 1953 and 1954 by measuring trilateration nets by Shoran, as part of a geodetic tie between North America and Europe.

FIELDWORK

2.03 Introduction

A full account of the procedures used in the fieldwork of the Retriangulation (i.e. the reconnaissance, the erection of station marks, and the observations) has been given by Brigadier Hotine(4). These procedures continued to be used with only very slight modifications throughout the whole of the primary Retriangulation; the following paragraphs are based largely on Brigadier Hotine's original account.

2.04 Reconnaissance

2.040 THE 'PAPER' SCHEME

It was possible to draw up a paper scheme for most of the primary Retriangulation by examination of large-scale topographic maps for possible obstructions to the proposed rays, making due allowance for curvature and refraction. The fact that certain of the proposed lines had been observed in the old Principal Triangulation was, of course, of material assistance. Arrangements were made for the paper scheme to be verified and amended as necessary on the ground by special reconnaissance parties. Figures 1 and 2 were verified in 1935, Figures 3 and 4 in 1936, and Figure 5 in 1937.

The reconnaissance parties checking the paper scheme were supplied with approximate bearings, normally in relation to a close reference object wherever possible, and with computed vertical angles along the proposed ray. They were equipped with small Tavistock theodolites and quarter-inch to one mile planetable sheets, on which rays of the paper scheme were inked in as they were verified. In clear weather, which was seldom experienced, the work consisted merely of setting the theodolite in altitude and azimuth along each ray and sighting the other end. In less clear weather the same procedure was adopted at both ends of the ray in order to establish the absence of obstruction at least half way along the ray. In cases of doubt, the ray was verified by lighting the far terminal with a beacon lamp or heliograph.

2.041 REPORTS

In addition to verifying the rays on the paper scheme, reconnaissance parties were also required to provide full information, by means of a sketch on a reconnaissance report form, as to visibility in all other directions, partly to assist the later secondary reconnaissance and partly to provide data for choosing alternative or additional stations should these be necessary for the primary work. Reconnaissance reports also contained:

(*a*) Information relating to ownership.
(*b*) A description of the site and sufficient information to enable a decision to be made as to the nature of the mark and the extent of station preparation likely to be necessary.
(*c*) A description of the means of access.
(*d*) Details of temporary marks left to guide the station preparation parties.
(*e*) Corrected bearings to other stations.
(*f*) A full description of the point actually selected.

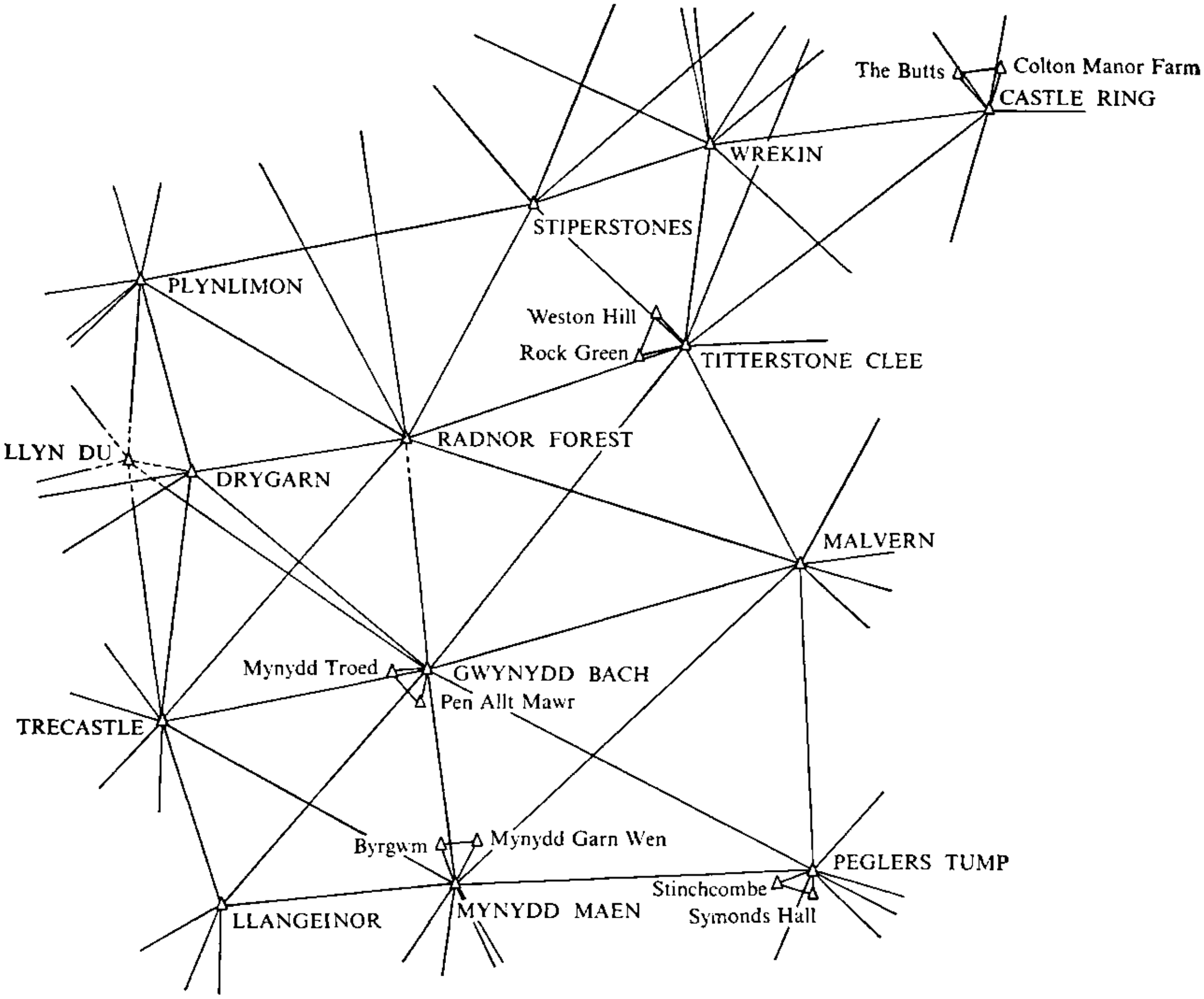


FIG. 2.1. **Substitute stations**

Whenever possible, the new point selected coincided with an old station of the Principal Triangulation or was related to an old station by measurement; however, such other considerations as the necessity for avoiding extensive clearance of trees, ancient monuments, possible future road widening, or grazing rays, frequently entailed the choice of entirely fresh sites. Grazing rays were defined as rays passing within 20 feet of intervening ground or building, etc. The reconnaissance parties made special reports on any rays where a graze was unavoidable, in order that the question of omitting such rays might be fully considered. In a country as densely populated as Great Britain there is by no means a free choice in the selection of stations, particularly where these are to be marked in a conspicuous and readily accessible manner. Apart from the actual land owner, who very rarely raised any objection, it was frequently necessary to consult a variety of other interests before the station could finally be located and constructed.

Reconnaissance parties were required to show on their diagrams all cross-connecting rays which were open, whether these were included in the paper scheme or not, although for various reasons not all of these rays might later be selected for observation. This procedure produced a sufficiently strong network, even allowing for the later omission of lines which could not be observed for any reason. Although many of the lines on the paper scheme were found to be obstructed, it was never necessary to site additional stations. It was frequently necessary, however, to alter the siting of a station, either locally in detail or to an entirely different feature, to obtain better connections. This was usually done on the initiative of the reconnaissance party, which provided detailed reconnaissance reports for such fresh stations and which was in any case required to report on possible stations on buildings or towers whose potentialities could only be appreciated in the field.

In the flatter and more highly developed areas, such as East Anglia, a paper scheme was of little or no use, and reconnaissance had to be carried out entirely in the field by occupying and recording visibility from several trial stations until a satisfactory scheme had been evolved. It was found economical in such cases to divide the reconnaissance into two parts; a preliminary reconnaissance, in which likely areas were selected and possible connections obtained; and a final reconnaissance in which the selected stations were sited, reconnoitred and reported on in detail. Stations on buildings, etc. where the standard type of triangulation pillar could not be constructed, were usually marked permanently during this final reconnaissance. It would not, of course, have been economical to have taken two such 'bites at the cherry' in undeveloped country, where some such centrally organised method as was adopted for the East African Arc(5) would be necessary, but the method answered the purpose in an area where roads were abundant and cross-country journeys presented little difficulty.

2.042 RECONNAISSANCE IN AREAS LIABLE TO SUBSIDENCE

Land subsidence in mining areas is frequently accompanied by lateral movement, which, apart from destroying the permanent value of a station, may cause serious inaccuracy between successive occupations of the station, either for extensions of the primary network or for connections to the secondary net. The stability of every primary station was accordingly considered in consultation with colliery surveyors and with the Geological Survey Department. If there was no alternative to the occupation of a station likely to subside, two substitute stations were constructed on safe ground adjacent to the suspect station, sited so as to form a well-conditioned triangle with the main station. These substitute stations were observed at the same time as the main station, as a means of establishing the amount of any future movement of the main station. The substitute stations were intended as secondary stations and were sited accordingly. Examples of these substitute stations are given in Fig. 2.1. Castle Ring (60) near Birmingham, for instance, is an essential primary station in a coal mining area, the safety of which was assured throughout the 1936 observing season but which was likely to be subject to lateral movement later. Titterstone Clee (62) on the Welsh Border, was likely to be disturbed sometime after its establishment in 1936 owing to the combined effect of stone quarrying and the peculiar local geological structure. In many areas it was found subsequently that this system of substitute stations was not completely reliable, since the substitute stations were themselves liable to be undermined in later mining operations. Primary stations on buildings, such as York Minster (22) and Lincoln Minster (80), have invariably been provided with substitutes, since experience showed that such stations are especially liable to loss by structural alteration or re-leading of the roof.

2.043 CLEARANCE OF RAYS

To avoid delaying observations, and also to avoid meticulous written descriptions for the benefit of station marking parties, reconnaissance parties were required to clear any trees or undergrowth necessary to put through the scheme. This is not as simple a matter in Great Britain as it is in the Tropics. Permission must first be obtained and occasionally compensation agreed, while it is also necessary to avoid any outcry against spoiling the beauties of the English countryside, of which trees form so important a part. Sites requiring extensive clearing were always reported to headquarters before any action was taken and were fully examined for possible alternatives, e.g. for consideration as to whether a particular ray might be omitted without seriously weakening the network; and for balancing the cost and other disadvantages of clearing against the possible cost of using a steel tower (see § 2.07). Similar detailed consideration was necessary for sites scheduled under the 1931 Ancient Monuments Act or other Antiquities, even though these may be no more imposing than the sites of prehistoric entrenched camps or burial-grounds, which unfortunately were almost always placed on hill-tops, but which are safeguarded against excavation or defacement without prior consultation. A due balance between the preservation of the past and the needs of the present and future is naturally a subject into which, occasionally, violent personal prejudices may enter. It constituted one of the most difficult administrative problems which had to be faced, and surveyors in other countries may account themselves fortunate in being without it.

2.05 Records

As soon as the reconnaissance reports were received at headquarters for a particular station, two files—a 'Field' file and an 'Office' file—were opened for the station and registered; both remain in commission as long as the station exists. The field file contains a copy of the reconnaissance report, an abstract of the rough bearings and vertical angles to surrounding stations, any maps necessary to locate the station, and any special instructions for station marking, observing or beaconing parties, who may occupy the station subsequently. Arrangements were made for the field file to be in the possession of all such field parties when they occupied the station. From time to time any information which might be of use to their successors, was added, even though only of such temporary value as comments on the available lodgings or caveats affecting the farmer and his livestock. In addition to copies of the reconnaissance report and a rough abstract, which may be required if the field file goes astray, the office file contains copies of all reports and correspondence concerning the station. The system of filing on a station basis was found most convenient for rapid reference and for disseminating information at the required time and place. It ensures that such apparently trivial matters as the wishes of a land-owner regarding the use of a particular route to the station are not forgotten; it has helped, in so far as any paper system is of value for the purpose, in maintaining a high level of co-operation between various surveying parties among themselves and with local interests. A complete historical record of all work at the station is also assured, and there is little doubt that many of these files will make very interesting reading in years to come. Matters of general policy were dealt with on the normal departmental files, but where these had a special bearing on a particular station, copies of the relevant minutes, correspondence and decisions, were also included in office files as a permanent part of the manuscript records of the Retriangulation.

As a further means of ensuring co-operation between the various parties and of disseminating information, a *Bulletin* was published weekly during the field season and given a wide circulation. This contained a statement of the location of parties, a summary of reports and of progress made during the week, general administrative and technical instructions, and any interesting, or even amusing, anecdotes having a bearing upon the work. Even in so highly developed a country as Great Britain it is difficult to know what is happening on the next hill but one, and some grain of comfort may perhaps be imparted to an observer who has spent a week in the clouds by the knowledge that others have been in the same or a worse predicament; while healthy competition to get into the 'Stop Press' with completion of a difficult station never has any harmful effect. The most careful planning and organisation will never eliminate unforeseen situations, which may be eased by the knowledge that a particular station has, or has not yet, been constructed, or that a reconnaissance party is working in the neighbourhood and can lend a hand in emergency. Successful triangulation, even more than other surveying operations, requires initiative from all personnel employed on it, yet initiative might do more harm than good if it were not based on adequate information and an occasional glimpse of the whole picture. The *Bulletin* was, however, widely read by other branches of the Department and by many outside interests, who kept in touch with the work and who would otherwise have had to have been informed by interview or correspondence. In addition, the *Bulletin* has provided a valuable continuous record, similar to the 'War Diary' of a military unit, and a complete set of the *Bulletins* is retained as a permanent part of the Retriangulation records. Any one who has had to use an old triangulation, however carefully its recorded values and descriptions may have been preserved, knows that a study of its field history is very often necessary.

To assist the general direction of the entire triangulation, by enabling immediate decisions to be taken without the necessity for reference to a multiplicity of files and reports, large mounted wall-maps were continuously maintained to show the state of progress of the various operations and the distribution of field parties. The main diagram was a cellulose-sprayed quarter-inch to one mile map of the whole country on two walls. On this, primary stations were marked by hollow inked triangles which could be erased if the station was not finally selected. These station symbols were expanded to show various stages in station preparation.

Primary rays in the preliminary paper scheme were shown by pecked lines, which were filled in when the field reconnaissance definitely proved that they were open. These were drawn over half way along the ray with a thicker line when observed from either end. Separate skeleton observing diagrams in which certain of the reconnoitred lines were omitted for different reasons, were prepared and copies issued to all field parties concerned. In the same way separate diagrams for figural adjustment were prepared from the wall diagram for issue to computers, or to check the selection of conditions.

2.06 Station Marking

2.060 LOSS OF STATION MARKS OF THE PRINCIPAL TRIANGULATION

Station marks of the old Principal Triangulation were usually buried, and only a few of the old primary and secondary stations were provided with some form of mark above ground level which normally consisted of a cairn, and then only when the station was on the top of a high mountain. Subsequent use of the station normally required the use of a special tool, known as a 'searcher', to probe for the buried tile or other mark before attempting to expose it by digging. Owing to rapid change in topographic detail since the Principal Triangulation, and to the impossibility in any case of providing adequate descriptions on bare rounded hills, this operation of searching was frequently protracted and often entirely unsuccessful. It is inevitable in a rapidly developing country that some triangulation stations must be destroyed, but the damage can be repaired cheaply by siting and fixing new stations, provided that information is obtained early enough to effect these repairs before too many stations in the vicinity have been destroyed. Far too many of the old buried stations had, however, been dug up or built over through ignorance. Legally, these marks were, and still are, safeguarded from interference by the Survey Act of 1841, and ignorance of the Law, we are told, is no defence. It would, nevertheless, require the pen of an A. P. Herbert to describe an action against the constructor of a Super-Cinema for destroying a triangulation station during the erection of a Wonder Organ; or against an archaeologist for throwing away a rococo specimen of tiling which he had excavated with care and scientific precision; or against the Police of a certain County Borough for gingerly removing a 'Type A Socket' in the belief that it was a hitherto undiscovered Zeppelin bomb. Yet these three cases have a solid foundation in fact.

2.061 DESIGN OF THE TRIANGULATION PILLAR

For these reasons all primary stations of the new triangulation were marked in a solid, permanent, and obvious manner, which in the majority of cases took the form of a small concrete or stone pillar, illustrated in Fig. 2.2. The design, which is discussed below, affords ready access to the station for beaconing or observing. The pillar consists of a truncated pyramid, square in section, rising 4 feet above ground level. It is normally made of concrete and is cast *in situ*. Into the top of the pillar is set a brass fitting called a 'spider' incorporating three grooves 120° apart. The spider ensures that instruments can be automatically centred over the intersection of the three grooves when the feet of the tribrach of the instruments are placed in the grooves. Three lengths of bent brass rod are inset in the top of the pillar forming loops to which the theodolite is lashed by cord. The centre of the spider carries a screw plug, which may be removed by means of a special tool in order to insert any suitable type of opaque beacon in the central pipe of the pillar. This plug carries a centred smaller plug, which may be raised by unscrewing and which is threaded to take the ordinary military heliograph, or electric beacon lamp designed for the purpose. For duplex helio the smaller plug is completely removed and reversed. Its underside is drilled to take the stem of the duplex mirror, and the adjustable helio mirror is set up alongside the pillar on a tripod. A hollow tube runs down the centre of the pillar, to enable the spider to be accurately plumbed over a brass bolt set in the base of the pillar with the aid of sighting tubes set at right angles. This brass bolt, known as the 'upper' or 'pillar' bolt, is in turn centred over another brass bolt called the 'lower mark', which is set beneath, and independently of, the foundations of the pillar. The purpose of this lower mark is to provide a means of locating the station should the pillar be destroyed. Into

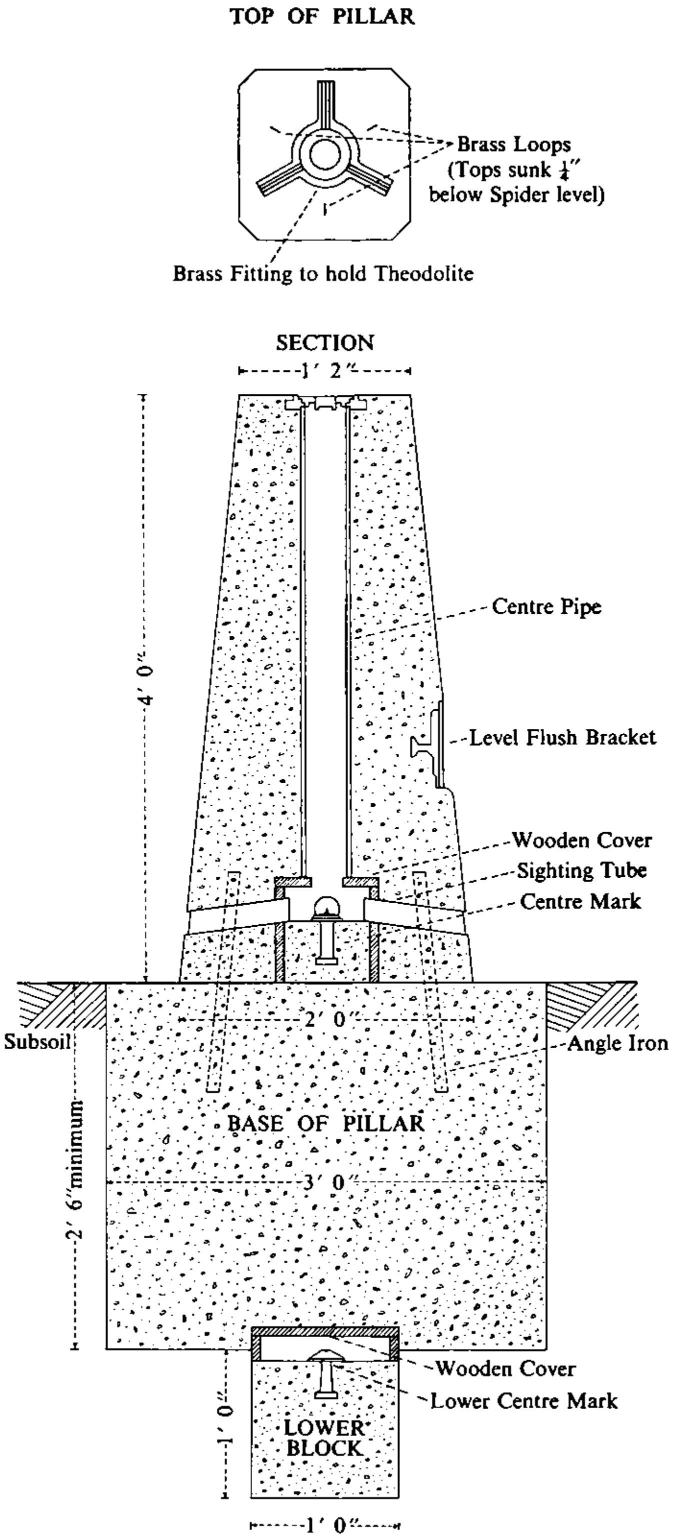


FIG. 2.2. Design of standard triangulation pillar

the side of the pillar is inserted a type of bench mark called a 'flush bracket'. The published height of the station refers to the tip of the arrow on this flush bracket which also gives it an unique and visible reference number, since all such flush brackets are numbered.

2.062 CONSTRUCTION OF THE PILLAR

The lower buried mark, consisting of a brass bolt set in concrete, is first inserted at a sufficient depth below ground level to be independent of the pillar foundations. This depth naturally varies with the soil; on boggy ground, which is sometimes encountered on hill-tops, it was sometimes necessary to excavate as much as 15 feet before reaching rock or firm soil on which to emplace the lower mark. In such a case a correspondingly deep pillar foundation is necessary, whereas on outcropping solid rock, a bolt is simply cemented in a hole drilled in the rock.

The lower mark and its concrete setting is covered with a small wooden box (which eventually disintegrates) to prevent adherence to the pillar base, and so to prevent disturbance of the lower mark in case the pillar should be moved. Concreting of the pillar base is commenced immediately over and around the box covering the lower mark, without interposing any loose earth or stones, which might weaken the pillar foundation, and is carried up to ground-level, where it is left rough to set. Four angle-iron reinforcing bars are set in the base to project well up into the corners of the pillar, as a means of preventing fracture between the base and the pillar. The pillar bolt is also set in the base; it is arranged vertically over the lower mark by means of a plummet and marked board resting between marked pegs, previously fixed in correct relation to the lower mark.

The pillar bolt is next covered with a small wooden box, which is provided with side holes (to take the inner ends of the four sighting and drainage pipes) and a top hole (to take the lower end of the galvanized pipe running down the centre of the pillar). See also § 2.065(b). Wooden shuttering, which may later be taken apart and used again, is next erected on the pillar base. This shuttering has four side holes to take the outer ends of the four sighting pipes, which may now be inserted, and a wedge fillet to which the level flush bracket in one side of the pillar may be wired in a vertical position. It also carries wooden corner fillets to provide an automatic chamfering to the edges of the pillar. The centre pipe, which serves as further reinforcement, is set in position and plumbed, the plumbing being continually checked during concreting. A good 4 : 2 : 1 mix of concrete, containing sharp well-washed sand and crushed stone as aggregate, is then poured into the shuttering and rammed. Before the concrete sets, the brass spider, complete with holding-down bolts, is set over the centre pipe and is carefully plumbed over the pillar bolt from a special temporary fitting to the spider, by sighting in both directions through the lower sighting tubes. Concreting is then carried up to the level of the top of the spider with an allowance of about ⅜ in. for later settlement of the concrete. After a day to set, during and after which the green concrete may need protection from frost by a liberal covering of sacks, the shuttering is removed and the pillar is faced with cement plaster, to prevent possible disintegration by ice forming in cavities. Three of the sighting tube openings are plugged with paper and lightly cemented over to conceal their presence from visitors, who are often apparently unable to resist the temptation to stuff any 'foreign body' that comes to hand into the tubes. The fourth tube must be exposed to allow condensation to drain out of the pillar.

Pillars were erected, in the period prior to 1939, by specially trained constructors working individually to a set programme, although it was necessary in the first place to make special arrangements to prepare enough of them ahead of observing. These men made their own arrangements for purchase of materials, hire of local labour, and for transport, which varied in particular cases from lorries or pack-animals to man-power. Sometimes it was necessary to transport the only available pack-ponies nearer to the station by lorry; sometimes the final stage was by rope and bucket. Once the Survey was badly let down by a constructor who ceased putting in adequate foundations as soon as he ceased to be under direct supervision, but usually it was possible, although not easy, to obtain an excellent type of man with the necessary qualities of honesty, initiative, and determination. It was cheaper to employ individual men in this manner than to maintain and move a party of sufficient strength and equipment to dispense with local hiring, although the constructor was sometimes provided with a motor-van in the less populated areas; and cheaper also than either single or block contracts, although the constructor would frequently arrange for a contractor to supply and transport materials if this should be more advantageous than hiring his own transport. However, owing to the shortage and cost of casual labour in the post-war period, and other difficulties, it was necessary to modify the organisation and to form the constructors into groups. For further details see Chapter 7, § 7.03.

2.063 PERMISSION TO ERECT PILLARS

The Survey Act of 1841 conveyed statutory authority for the establishment of permanent 'marks, stones or posts' without consent, acknowledgment, or ground-rent, but with agreed or arbitrated compensation for damage. Needless to say, however, these wide powers could not be used arbitrarily, and more reliance was placed on peaceful persuasion, leading to a free consent and willing co-operation, than on force. Over 6,000 pillars and 10,000 marks of other types were established during the Retriangulation and only in 14 cases was it necessary to issue compulsory orders under the Survey Act. Land-owners generally were very ready to appreciate the advantages to themselves in particular of facilitating the National Survey, and of the merits of permanent accessible reference points as opposed to the continual disturbance occasioned by buried marks. Before assenting, some took legal advice, which was invariably in favour of granting the Survey full facilities; but as a result thorny questions were sometimes raised such as the abrogation of 'squatter's rights', or provision for removal of the pillar in case the site should later be required for building purposes. Some even required an assurance that the pillar would provide an excellent scratching post for cattle. An unlimited supply of patience and of both the written and spoken word might have been required, but sooner or later a free permission was generally obtained, and with it a generous measure of assistance later for the surveying parties. Odd, or even irksome, conditions were sometimes imposed, but were met if it was within the power of the Survey to do so. The owners of grouse moors and deer forests usually and naturally required construction to be left until after the shooting or stalking season. One very well-known land-owner of ancient and honourable lineage in Scotland gave his consent to the construction of a pillar 'on this barely accessible spot', but only on condition that he should be given an opportunity of seeing it done, 'if it is done'. It was done. Others required the pillar to be toned to match the colour of local stone, while some required the entire pillar to be constructed of local stone, 'which will be provided free', in order to avoid attracting pilgrimages of hikers. Another imposed the condition that the pillar should be constructed during the ensuing week, since he was about to hand over the area to the National Trust and he could not 'answer for them'. In this, however, he showed needless apprehension. The National Trust own many beautiful hill-sites throughout the country—which are to be preserved unspoiled for ever—and were accordingly consulted in principle at an early stage. In spite of the fact that the Trust and its local Committees (in the words of one of their Secretaries) 'exist to be shot at', and that someone in this crowded country may always be relied on to voice loudly an extreme view, the National Trust at once took the preservation of triangulation stations under their wing, and actively co-operated by providing National Trust emblems for incorporation in the pillars. The Council for the Preservation of Rural England was also consulted and supplied a design for construction in rough-dressed local stone, wherever it was possible and desirable to construct the pillar in this manner. The Survey was sometimes able to repay such help in kind. For example, one local National Trust Committee already had a pillar supporting a brass topograph and not unreasonably did not very much like the idea of another 'wart' alongside it on perhaps the most famous beauty-spot in the South of England. They relied largely on public subscription for maintaining the site, but had frequently been robbed of the entire contents of the collecting-box and even of the box itself. The topograph was removed, re-designed, and incorporated in the triangulation pillar with an Ordnance Survey guarantee of accuracy, the original having been a mere 40 miles out in longitude. The pillar was also provided with a plinth for children to stand on. A steel collecting-box and explanatory inscription were fitted snugly in reinforced concrete beyond the reach of even the highest class amateur cracksman; and, possibly because of the added security and for other reasons which any salesman will readily understand, public subscriptions surpassed all records and hopes.

These highlights are perhaps emphasised by the inevitable shadows. In one case a well-known novelist protested angrily against the dismantling of a cairn (originally built by the Ordnance Survey itself nearly 100 years ago, but which had apparently in course of time acquired a local sanctity) and after lengthy correspondence was only appeased by a promise that another cairn to replace the old one would be erected near the pillar. Another and perhaps darker shadow was a protest from another Government Department. The Office of Works, the official guardian of ancient monuments, took alarm at the idea of archaeological sites being 'desecrated' by concrete pillars and countered the Survey Act of 1841 by quoting another more recent enactment which forbade interference with all 'scheduled' sites without its permission. When two Acts of Parliament come into conflict there is material for a first-class debate, and much correspondence ensued, some of it being acrimonious. However, after several experts had expressed their views on the matter, including Mr. (now Sir) Mortimer Wheeler, a scheme was devised which was acceptable to the Ministry of Works, whereby the Archaeological Officer of the Ordnance Survey studied the reconnaissance reports of each proposed station. He either agreed to the proposed site or asked for it to be moved as required, and informed the Ministry of Works of his decision. The scheme remains in force and has enabled archaeological considerations to be given due weight in the selection of sites for triangulation stations.

Pillars have been erected at all primary stations where the site is structurally suitable or can reasonably be made so. Although no great harm would have resulted from the occasional substitution of some other form of permanent mark in isolated cases, it was felt that any weakening in this respect would inevitably cause a landslide, particularly in the secondary triangulation. If, for instance, a general exemption had been granted to archaeological sites, then the entire triangulation of Wiltshire for example would have remained on an impermanent and inaccurate basis, necessitating frequent repair and re-observations. In the case of one primary ground station, Peglers Tump (88), a low, but visible, mark stone was inserted, since the station had necessarily to be sited on top of a tumulus containing an excavated chamber; but this station was safeguarded against loss by the provision of substitute pillars.

2.064 ROOF STATIONS

Stations on buildings were marked by means of a large brass bolt let into the roof and provided with four witness marks consisting of smaller brass bolts on other parts of the building, such as the parapets, which would not usually be affected by structural repairs to the roof itself. See also Chapter 7, § 7.032(b).

Needless to say, a building was not selected unless the only alternative would be a high steel tower, and only very solidly constructed permanent buildings were utilized. It was frequently necessary in such cases as at Lincoln Minster (80), to make special arrangements for independent support of the instrument, and for staging designed to distribute the weight of the observer away from the instrument support. Such items of station preparation were usually carried out by local contractors working to a specification drawn up after full consideration of details provided in the reconnaissance report.

2.065 MODIFICATION TO THE ORIGINAL PILLAR DESIGN

When at a late stage the secondary triangulation had progressed to the Highlands of Scotland it was necessary to design a new and lighter pillar, cylindrical in shape, in order to avoid excessive transport costs. (See § 7.031.) Apart from this only two minor modifications were made to the original very successful design.

(*a*) To prevent unauthorised persons removing the main centre plug, two hexagon-headed 'Allen' screws were fitted to secure the centre plug to the spider. Also a split pin was fitted to the small centre plug which could only be extracted by removing the main centre plug. The modification was carried out between 1947 and 1950.

(*b*) The metal centre pipe was originally intended as a support for the pole of a metal opaque beacon which was to be left permanently on the pillar. However, in the event, these opaque beacons were not used. Consequently, in pillars built after 1950, the metal centre pipes were replaced by cardboard tubes, which are very much lighter to carry and are sufficiently robust to support the weight of the concrete until it sets.

2.066 MAINTENANCE OF PRIMARY RETRIANGULATION PILLARS

Between 1947 and 1950 the majority of pillars built at primary stations from 1935–1939 were inspected. It was found that 5% needed extensive repairs, and 12% needed minor repairs to prevent further deterioration. The remainder were in good condition. Of those needing extensive or minor repairs the main causes of damage were:

Vandalism	43%
Effect of weather	26%
Faulty construction of foundations	21%
Miscellaneous	10%

Pillars in densely inhabited areas or near holiday centres and beauty spots are more subject to vandalism; pillars on high exposed sites are most affected by weather.

In 1951 a system of inspection was instituted, whereby all pillars are inspected and repaired once every 10 years; certain pillars at sites frequented by the public and therefore more liable to damage by vandalism are inspected more frequently. (See also § 7.04.)

2.07 Steel Observing Towers

In the flat and enclosed areas of East Anglia visibility between stations is very restricted. By reducing the ruling side length from about 25 miles to 10 miles and by using all suitable high buildings, it was hoped to produce a satisfactory primary triangulation. However, after extensive reconnaissance it was found that an adequate triangulation could not be established without the frequent use of temporary steel towers. As a result, light portable steel 'Bilby' towers were purchased from the United States of America in 1938.

Reconnaissance had to be carried out before the towers were delivered, so reconnaissance parties operating in areas where towers were likely to be used were supplied with full details of the towers, both as regards the available sectional heights and the base area required for towers of different heights. The reconnaissance party was required to establish exactly what height of tower would be necessary to clear local obstructions and also give a sufficient command for the required length of ray. This was done by direct measurement of the heights of trees, etc. both in the immediate vicinity of the station and on any intermediate feature likely to obstruct the ray, combined with calculation of intervisibility between the proposed stations after making due allowance for curvature and refraction. In many cases, however, the tree tops were simply observed from surrounding stations, sometimes after being flagged, in order to establish intervisibility. Owing to the expense of transporting and erecting steel towers, they were, in the early stages of the Retriangulation, avoided whenever possible even at the expense of undertaking a certain amount of clearing in order to use a station at ground level. To some extent the strength of the network suffered in consequence, and subsequently steel towers were used as required by the demands of the triangulation. In an attempt to avoid re-erecting the tower for the later secondary triangulation, observations to the latter were made while the tower was erected for primary observations; reconnaissance parties were therefore instructed to complete the secondary reconnaissance over a sufficient area to establish a ring of ground level secondary stations around the primary. Subsequently, difficulty was sometimes experienced in extending the secondary work from the scheme surrounding the primary steel-tower stations, and the steel towers had to be re-erected over the primary stations to complete the secondary observations.

Full information regarding the Bilby steel tower is published in *Special Publication No. 158* of the United States Coast and Geodetic Survey(6). The system of erection, etc. developed in America was followed with a few minor modifications. Shallow concrete footings were cast at a depth of about 5 feet below ground level to support the wooden bearers at the base of the steel members. By this means it was found that the tower could be more accurately centred over a previously fixed station mark, and that thereafter it was less likely to be thrown off centre by uneven subsidence of the footings. It was found that towers which had been in position for over a month, and exposed occasionally to high winds, had not moved off centre by more than a tenth of an inch; however the centring of the tower was invariably checked before observations were made. As a precautionary measure, the outer tower was provided with steel wire guys attached at two-thirds the height above ground, tensioned by turnbuckles and anchored to large screw, or angle iron, pickets. The guy attached to the tower leg carrying the ladder rungs has a split cradle to allow an uninterrupted climb. Some consideration was given to the problem of protecting the towers from lightning, but it appeared that any feasible form of lightning conductor could not be effective enough to divert the discharge from the steel framework. Personnel were accordingly warned to stand clear of towers during storms but to leave beacon lamps alight.

Two tower erection parties were employed in East Anglia in 1938, each party consisting of eight men, including a lorry driver and a pillar constructor. The latter was required because where possible the tower was erected before the concrete pillar, it being more convenient to erect the pillar beneath the centre of the tower than vice versa. Each party had a 6-ton six-wheel Bedford-Unipower lorry, which would take either two towers, or one tower and the team. A trained team working on previously prepared foundations could unload and erect a 103-foot tower in 10 hours, and dismantle it in rather less time, although work was slowed down considerably by high winds or rain.

2.08 Instruments

2.080 THEODOLITES

The theodolite used in the Retriangulation was the Geodetic Tavistock theodolite manufactured by Messrs. Cooke, Troughton & Simms, of London and York. This theodolite had been used successfully in East Africa on the arc of the 30th meridian, so it was natural that the same instrument should be selected for the Retriangulation.

Nevertheless, the instruments were subjected to extensive laboratory tests before field observations started in order to:

(*a*) Ensure that the theodolites were not liable to errors due to axis strain, which had been noted at that time in other types of geodetic instruments.
(*b*) Calculate graduation errors of the horizontal plate.
(*c*) Give the observers extensive practice in their use, and show the capacity of the instruments under ideal conditions.

These tests were fully described by Brigadier Hotine in the *Empire Survey Review*, and are reproduced as Appendix 11. They showed that the Geodetic Tavistock theodolite was sensibly free of error due to axis strain and had a probable graduation error of less than 0·1 seconds, but probably their most important result was the confidence in the theodolites that they engendered in the observers, who in consequence were less likely to blame the instruments for those large errors which occasionally arise under field conditions.

As far as can now be established, 12 of these instruments were used between 1936 and 1939. It is of interest that 6 are still in use, and have been used for such operations as the transfer of geodetic levels by trigonometrical methods, where the larger vertical circle of this early type of instrument is of great advantage. Of the remainder, 3 were buried near Dunkirk before the 19th Field Survey Company, R.E. was evacuated in 1940. Attempts were made to recover them in 1944, but without success. One instrument was destroyed when the Ordnance Survey Office at Southampton was bombed in 1940, and one was damaged rather less gloriously by being dropped from a steel tower in 1951. The other instrument is displayed in the Record Room of the Ordnance Survey at Chessington.

In 1946 and 1947 a further 10 geodetic theodolites were purchased from Messrs. Cooke, Troughton & Simms who had modified the design. The main differences were: smaller horizontal and vertical circles, a quick-release clamp for the upper plate, and a generally lighter and more compact design. The essential differences are tabulated below:

	Original Model	*Later Models* (V.500)
Focal Length	10·1 in.	7·3 in.
Diameter of horizontal circle	5·5 in.	5 in.
Diameter of vertical circle	3·5 in.	2·75 in.
Weight	32 lb.	27 lb.
Casing	Nickel-iron	Gunmetal

However, the old type of instrument continued to be used for observations from the majority of primary stations. This was partly to ensure that the angular measurements throughout the whole of the work might be taken under the same conditions, and partly to meet the preference of the observers for the older instruments which, though heavier, were more robust and remained stable in the high winds which often prevail in the Highlands of Scotland when visibility is good.

2.081 BEACON LAMPS

The beacon or signal lamps used in the Retriangulation were also produced by Messrs. Cooke, Troughton & Simms. During the period 1936 and 1939 lamps of various alternative designs were used, and one type has been fully described by Brigadier Hotine(4). For work in the Highlands of Scotland a charging van was equipped with an L.32 'Pioneer' petrol-electric charging unit, and was located in a mobile central pool for the supply of recharged batteries. In the more thickly populated parts of the country, however, the lightkeepers themselves arranged for a supply of fresh batteries from motor garages or battery service stations by hiring a replacement until their own accumulator had been recharged. In order to provide a still more intense light in very bad visibility, it was sometimes found desirable to 'boost' the light by applying more than 6 volts; sometimes as much as 12 volts was necessary. This considerably shortens the life of the bulb, but on occasion this may be more economical than a protracted delay in observing a difficult ray. It was found that the life of the bulb could be prolonged by first warming up for 5 to 10 minutes on the normal voltage and then stepping up 2 volts at a time without allowing the lamp to go out as the voltage was increased. This was done by using a split lead, one arm of which was connected to the higher voltage before disconnecting the other arm. The latter must, however, be disconnected almost immediately in order to avoid short circuiting the extra cell. Such procedure was harmful to the battery as well as to the bulb, but was considered expedient if the observations were to be obtained.

2.082 BEACONING

Observing hours were usually from two hours before sunset to five hours after sunset, the vast majority of observations having been obtained at night. High-power electric lamps could, however, be seen in daylight at considerable distances in clear cloudy weather, and this form of illumination was accordingly used at times for the daylight period. All lightkeepers were also provided with heliographs for use on the rare occasions when sufficiently continuous sunlight made it worth while to set them. For a few long rays across the hazy industrial districts, 10-inch heliographs were used.

The method of emplacing lighting gear depended naturally on the type of station. Centred beacon lamps were attached to the spider of concrete pillars as previously described. On roof stations a tripod was used with a three-footscrew levelling tribrach to which the lamp or helio was attached, centring being effected by plumb-line. An additional centred beacon lamp for the use of a second observer was arranged in such cases by setting a fully extended telescopic tripod over a short tripod, the tall tripod being of course set up and plumbed first. A few large and heavy telescopic tripods were obtained which could be set to a height of as much as 10 feet and which were designed to take either beaconing gear or the Geodetic Tavistock theodolite; the object of these special tripods being to clear battlements, etc. or to provide a sound instrument stand on sloping roofs or other awkward emplacements. They were also useful, however, for double-beacon emplacements when the lower tripod must be fully extended to clear obstructions, and when there was insufficient room to emplace an eccentric beacon.

The emplacement of beacons on Bilby steel triangulation towers was studied with especial care. In the interests of economy, large areas of secondary work around these towers were observed concurrently with the primary observations in order to avoid re-erection of the tower in a later season. The tower had to be freed as quickly as possible if an extensive programme was to be carried through with an economically small number of towers, and for both reasons it was frequently necessary for beacon lamps for several observers to be emplaced on a single tower, possibly at the same time as it was being occupied by another observer. Two standard spiders were bolted to the centring plates of the tower; one, at the top of the outer tower, being used exclusively for beaconing; and the other, on the inner tower, being used for either theodolite or beacon emplacement. In addition, a reverse plug was supplied which may be screwed to the under-side of the inner-tower spider for the attachment of an extra beacon lamp upside down.

In addition to the foregoing arrangements for additional centred beacons, it was frequently necessary to set eccentric lights where more than one observer was working in the area. To avoid centring errors these were set instrumentally on the line to a 'leading light', shown by the observer requiring such an eccentric light, and were never set from reconnaissance bearings. They were usually tripod emplacements set on plumb-lines in the case of pillar stations, or roof stations where there was sufficient room. In the case of Bilby towers, special brackets were made to clamp eccentric lighting gear to the angle-iron hand-rail surrounding the observer's platform. This was too close for the near focus of theodolite telescopes, so that alignment had to be effected by first setting a ground peg on the line to the leading light and then setting up over the peg in order to sight back to the station centre.

Alignment of the beam, in the case of both helios and lamps, was effected by means of the semaphore board, first introduced by McCaw on the African Arc of Meridian and described in detail in the *Empire Survey Review*(7). Lightkeepers were usually equipped with one of the old 'tracing' instruments used on the original detail survey and, since these instruments had no vertical circles (although the line of collimation could be set level), it had been necessary to simplify the method of setting the semaphore board in height. This was done by first setting it at the same level as the instrument and then moving it up or down a measured distance to allow for the elevation or depression to the distant station, and for the difference in height between the instrument and beacon. The setting was subsequently still further simplified by marking a scale on the board in order to set it in both altitude and azimuth concurrently. The board was clamped to a guyed ranging rod for ground and roof stations, and was carried on a specially constructed adjustable arm attached to the hand-rails of Bilby towers. The upper outer-tower beacon on Bilby towers could not readily be provided with a semaphore board so that this beacon was usually reserved for the short secondary lines, where accurate alignment of the beam was of less consequence.

2.09 Procedure for Observing and Booking

The procedure for observing and booking was also given in detail by Hotine(4). These detailed instructions are reproduced in full at Appendix 12, but the main points are summarised below:

1. The circles are to be illuminated electrically for all observations, whether by day or night.
2. The instrument is always to be swung right on Face Left and left on Face Right. The same rule applies also to the slow motion screws.
3. The steadiest and most reliable light should be chosen as R.O.
4. Observations will be by continuous rounds, commencing on the R.O. on Face Left, changing face after intersecting the last beacon, intersecting the latter first on Face Right and closing on the R.O.
5. A light which is temporarily obscured, may be filled in at any time during a single face round, provided that certain precautions are taken.
6. Directions on both faces are to be measured once to all lights on each of sixteen zeros for primary rays. At stations where exceptional delay or difficulty is experienced, observers will forward the results of the first eight zeros to H.Q. and await instructions.
7. Vertical angles are not required on primary rays.
8. All observations are to be booked in ink on squared paper. Mistakes in booking are to be lightly crossed through but not erased; under no circumstances is one figure to be superimposed on another.

It is emphasised that the above is but a brief summary of the full instructions to observers.

The accurate measurement of angles in a triangulation depends to a large extent on a number of apparently trivial precautions, many of which are self-evident to a trained surveyor but which are, for that very reason, liable to be overlooked during the stress and strain of rapid work in the field. The same meticulous care is necessary in recording the observations if these are to be fully understood by a different staff of computers, not only at the time the observations were taken but also possibly 100 years later. For these reasons the instructions issued to observers were in great detail and were rigidly enforced.

They were designed to fit the particular instrument used—the Geodetic Tavistock. The robust construction of this instrument rendered certain commonly accepted practices of the time not entirely essential. If the instrument has been rigidly emplaced, it is, for instance, possible to swing in either or both directions and even to change face between successive pointings on the same beacon and yet to reproduce the readings within the errors of pointing and reading. The instrument will remain for hours on a concrete pillar without appreciable change in level, by night and by day, provided that it is shielded from direct rays of the sun. Nevertheless some of the usual precautions were included in the instructions, partly in order to form meticulous habits on the part of the observer and partly to eliminate the minor errors which might arise from neglecting them even with an instrument of this type.

OBSERVATIONS

2.10 General Organisation of Observing Parties

Primary observations were usually carried out by two or more observing parties, each consisting of one observer and a number of lightkeepers. The parties worked to a carefully prepared programme, which detailed the order in which stations would be observed, and the moves of the various sections into which the parties were divided. So far as was possible, allowance was made for the time necessary to occupy difficult stations. In the event, each season's programme required modification at one time or another, and the observers were given a certain amount of freedom to make minor alterations on the spot. Major programme changes were authorised by the officer in charge of triangulation, in consultation with the senior observer.

In order to ensure smooth working, when two observing parties were likely to be working near each other, the senior observer was always nominated in advance, and could, when necessary, assume overall charge of operations. In the early years, when organisation and procedure had not yet assumed their definitive forms, the officer in charge was very frequently present in the field, supervising operations and taking observations.

As the supervisory staff and the field parties gained experience, the organisation was changed. Variations in topography and in methods of communication also dictated modifications of the original plan. A brief description has therefore been included, giving the organisation of the observing parties and an outline account of each year's work. Appendix 13 gives a diary of the fieldwork of the Primary Retriangulation. The progress of observations year by year is also given at Diagram 3. For reference purposes in the text, the station number is given in brackets after the station name.

Lack of space has prevented the station names being included on the diagrams, but the location of stations may be determined by reference to the list of stations given on Diagram 2.

2.11 Primary Observations, 1936

2.110 ORGANISATION

At the beginning of April 1936, the personnel who were to form the observing parties assembled at the Southampton Office, having been employed for a month on building the pillars from which they would later observe. They consisted of regular soldiers from the Survey Battalion R.E. All had received some basic training in trigonometrical survey, either in the Survey Battalion R.E., or in 19th Field Survey Company R.E., but relatively few had had much practical experience; most of the non-commissioned officers had been employed on various survey operations abroad, but again only a few had had experience of geodetic triangulation.

Observations during this first season therefore had to be carried out with comparatively untrained personnel, and for this reason three observing parties were arranged to work independently in the south, centre, and north, of Figures 1 and 2 (see Diagram 4) with the object of avoiding eccentric beacons. Each observer had a number of lightkeepers under his immediate orders, with a call on a central pool for exceptionally difficult stations. A detailed programme of moves, which could be set in motion by simple code signals, was laid down for each observer and lightkeeper; this was so arranged that, with a reasonable margin for non-uniform progress, the initial stations of the central and northern observers should be clear before the observer to the south required them. The programme was also arranged to reduce moves to a minimum, and to allow extra time for the occupation of difficult stations. It was necessary to vary the programme during the season, to allow for the more rapid progress of the southern party in the better weather prevailing in their area; and furthermore the chain was considerably broadened.

2.111 STRENGTH OF OBSERVING PARTIES

For the first primary season, the strength of an observing party was approximately twelve men; an observer, two senior assistants, and nine lightkeepers. Of the two senior men who accompanied the observer, one booked the observations and was at the same time trained to observe when opportunity occurred, and the second acted as a general factotum, in particular supervising the activities of the lightkeepers. Each lightkeeper worked on his own, the observer exercising control by light signals, telegrams, and occasionally, visits by his chief assistants.

2.112 PROGRESS

It was a considerable achievement that this first programme went very nearly as planned. Apart from the credit which must obviously be given to the three observers, and to their immediate assistants, the efforts of these first lightkeepers must not be overlooked. Official Ordnance Survey vehicles had only just begun to appear; there were in fact, only nine small motor-vans between the three observing parties, which when divided into their working units, totalled no less than 28 well-separated independent sections. A number of lightkeepers used their own private transport, the Department, of course, meeting the running costs. In the event nearly every known form of powered (and unpowered) wheeled transport was used, including a three-wheeled vehicle with a van body, and a motor-cycle and sidecar, in which it was sometimes necessary to carry a very large passenger. It was only with considerable difficulty that this passenger could be inserted in the sidecar; but he emerged at least once with considerable celerity—when a telegraph pole intervened between machine and sidecar. Neither should the cyclists be forgotten: it was their painful lot to cycle many miles wearing 'Everest' carriers to which were strapped one, and occasionally two, full-sized 6-volt accumulators which required charging. Their longer moves—which of course entailed shifting their camping kit—were executed by hired lorry and by train.

By the middle of October, the main chain in England had been completed, and extended from Coringdon (11) in Dorset and Dunnose (10) in the Isle of Wight to Wisp Hill (317) and Tosson Hill (95) in Roxburghshire and Northumberland respectively: covering the Welsh Marches on the western side and extending well into Lincolnshire on the east. Fifty-six primary stations were occupied as observing stations, and 447 directions were observed.

2.12 Primary Observations, 1937

2.120 GENERAL

During 1937 it was intended to carry the main chain northwards from the Border, over the Grampians to the Great Glen and the coast of Aberdeenshire, thereby completing Figure 3, and if time permitted, to extend westwards and complete the primary Retriangulation of Wales and South-west England (Figure 4). Whilst the field parties took their well-earned leave, planning and preparation for the following season were carried out in considerable detail during the winter of 1936–37.

2.121 CHANGE IN ORGANISATION

It was realised that, in Scotland, the ruling factor would be difficulty of access to mountain stations, combined with the fact that no reliance could be placed on hiring local labour in most districts. This necessitated strong lightkeeping parties, moved as infrequently as possible, and given time to occupy the more difficult stations well in advance. In these circumstances a number of independent observers each with their own lightkeepers would have required too many men and much waste effort in re-occupying stations. A centralised organisation was accordingly adopted, with three observers (known as eastern, central and western) moving more or less abreast on a carefully dovetailed programme. The observers were served by strong lightkeeping sections operating up to three beacons from each station—one centred and two eccentric.

To save unnecessary movement, lightkeeping parties remained in position until the observers had passed right through them, when the lightkeepers 'leap-frogged' well ahead again. The programme contained full details as to which lights were to be eccentric, and as to the order in which leading lights were to be shown, so that the whole organisation might be kept moving by the simplest of code signals without the necessity for transmitting long messages by morse or for passing messages by hand to parties at difficult stations.

For the 1937 season therefore the number of surveyors on the primary observations was increased to 69, divided into 29 sections as follows:

3 observing sections of three men each.
20 lightkeeping sections, the strength of which ranged from one to three men.
4 battery charging and distribution sections—one man to each.
2 vans in reserve—one man to each.

Transport, both official and private, was increased in numbers to deal with the much larger party and consisted of 12 official motor-vans, 12 private cars, and two motor-cycle combinations.

This left only four lightkeeping sections without transport, but their problems were now eased, as on long moves they were transported by the reserve vehicles, and their batteries were both collected and delivered—to the *foot* of the mountain of course!

One other significant change was that, whereas in 1936 almost the entire party consisted of serving Royal Engineers, in 1937 it was necessary to reinforce their numbers by the addition of 19 temporary civil assistants, most of whom were comparatively new to Ordnance Survey work. They, like their military brethren, had not yet experienced the pleasures of lightkeeping in the Highlands of Scotland.

2.122 PREPARATORY WORK

A detailed schedule of loading transport and assembling personnel was completed, and all sections were standing by ready to move off from Southampton on 5th April, at which date weather reports from geodetic levelling parties working in the south of Scotland indicated that only a few minor roads were obstructed by snowdrifts and, except at heights above 2,000 feet, snow had either cleared or was patchy. The order to move off was given and all sections left Southampton that day. In the *Bulletin* published that week the Officer in charge of Triangulation quoted the order issued by Wellington to Lieutenant-Colonel Colby, R.E., on 24th June 1826, which set in motion the Principal Triangulation of Ireland. This order authorised a party of 40 artillerymen as guards for the surveyors, who were required to 'behave themselves in all Respects according to Law'.

2.123 PROGRESS

The move north was made, and on 7th April all sections were in position on their stations, ready to start observations on the evening of the 8th. The speed of this move was remarkable, even allowing for the comparatively good roads to the Border from Southampton. During a little over 48 hours sections had averaged 350 miles by road and had climbed hills up to an elevation of 3,200 feet,* mostly in pouring rain and carrying a minimum of a hundredweight of equipment: this was in addition to setting up camps and the usual 'household' tasks. As can be deduced, all were imbued by a spirit of enthusiasm and determination: even the senior observer at Whitelyne Common (93), who reported—'... Beyond a wet bed, wet clothes, wet everything, there is nothing to report ...'.

Meanwhile the western and eastern observers were experiencing similar weather at Criffel (96) and Tosson Hill (95) respectively and it was not until the evening of the 24th April that all three observers completed their stations, and moved to start the second stage.

Conditions improved somewhat in the next fortnight and the parties moved rapidly north across the Lowlands and by the 11th May it was considered that the half-way mark was in sight. However, the latter half of the programme promised to be more arduous, as the Grampians lay ahead, but on the other hand the industrial belt had been completed. Towards the end of May the central observer moved from Earls Seat (327) to Meall Dearg (305); but the west and east observers were held up by bad weather at Hill of Stake (319), and Lumsdaine (324), near Berwick-on-Tweed. Final observations at these two stations were therefore left for the time being and the observers were moved to their next scheduled stations, Ben Lomond (336) and Ben Cleugh (307) to prevent dislocation of the programme.

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* Sca Fell (92).

Excellent visibility made life easier for all sections at the start of their attack on this mountain range, and at first, all went well: but the whole elaborate organisation broke down at the beginning of June, owing to the existence of a few stations—notably Ben Macdhui (302)—which persistently formed clouds when other, lower, stations were clear. In these circumstances a rigid adherence to the original programme would have entailed delaying the whole party because one observer happened to be held up; and this would have meant that Figure 3 could not be completed during a short field season. The whole party was accordingly reorganised on the spot to deal with the cloud-formers. One observer was left behind on Ben Lawers (315) with reduced lightkeeping sections and all arrangements were made for him to move rapidly to Carn Gower (332), another difficult station. The other two observers outflanked Ben Macdhui (302) to the east, occupying the lower stations during a spell of particularly bad weather, while a fourth observing section was organised to occupy Ben Macdhui (302) itself.

This fourth section at first set up camp in the glen at the foot of the mountain (to quote the contemporary *Bulletin*) '. . . in delightful sylvan scenery surrounded by hand-fed deer and Highland dancers of the gentler sex in training for the Braemar games. The section, which has a three-hour climb, can only appreciate these delights through closed eyelids, and would prefer them to be less audible . . .'. These influences, and the necessity to be at the pillar whenever the cloud lifted, decided the section to live at the top of the mountain and they were given a lightkeeping section who made a daily supply-run with food and fuel for men and lamps. The two men concerned started their vigil on the night of the 4th June, and to their great credit observations to and from the mountain were completed by the night of the 13th/14th June. During this period they also found time to rescue a girl climber who had become lost in the snow and mist.

Shortly afterwards the other three observers completed the remaining difficult stations, and all parties were now on the 'downhill-run' across Strathspey and Strathbogie and the main chain observations for 1937 were nearly complete. About this time, possibly as an encouragement, the *Bulletin* quoted the following:

A HUNDRED YEARS AGO

> Extracts from the diary of one of Colby's assistants, Lieutenant Dawson, describing the work of station location and preparation during the early stages of the Scottish Triangulation in 1819. Observations are now proceeding in the same area.
>
> 'Friday, 23rd July: Captain Colby took me and a fresh party of the soldiers on a station hunt, to explore the country to the westwards and northwards of west. Our first halting place was to be Grantoun, at a distance of twenty-four miles, and Captain Colby having, according to his usual practice, ascertained the general direction by means of a pocket compass and a map, the whole party set off, as if on a steeplechase, running down the mountainside at full speed over Cromdale, a mountain about the same height as Corrie Habbie (2,200 feet) crossing several beautiful glens, wading the streams which flowed through them, regardless of all difficulties which were not absolutely insurmountable on foot. . . . The distance travelled by us that day was calculated at thirty-nine miles.
>
> Saturday, 24th July: Started at nine o'clock, I was dreadfully stiff and tired from the previous day's scramble, and with difficulty reached Pitmain (thirteen miles) to dinner . . . Garviemoor Inn, distant eighteen miles was to be our next stage, and I really thought it was more than I could accomplish that day, but Captain Colby said it was not. It was his intention, however, to leave the beaten road immediately, and crossing a rough boggy tract of country to the northward, to gain the summit of Cairn Derig a mountain about 3,500 feet high and about ten miles distant, and having built a large pile of stones upon it, to proceed again across the country to Garviemoor. I kept pace with him throughout the remainder of the day, and arrived at the Inn at half-past eleven o'clock at night, much more fresh than at the end of our first stage the day before. . . . The distance travelled that day was forty miles.'

As the parties completed their revised programmes they congregated at Turriff in Aberdeenshire on 21st June to reorganise for further work, and moved off again on 23rd June.

The three observing parties now took up separate tasks as follows:

1. The observation of Figure 4, to extend the main chain into South-West England, and Wales.
2. Observations at a few stations to complete Figure 3.
3. Observations at Liddington Castle (35), which had previously only been intersected, to enable the Ridgeway Base to be connected to the Retriangulation.

As the parties would now be operating for the most part separately the centralised form of organisation was abandoned for the time being. The fourth observing party had been disbanded on the completion of Ben Macdhui (302).

The senior observer therefore took one party to South-West England and commenced observing there at the beginning of July, working west from the western edge of the main chain.

The other two observers concentrated on the completion of Figure 3, one working around the base extension at Lossiemouth (350, 351, etc.), and the other occupying stations on the sides of the chain that had so far not been visited by an observer. This latter party made excellent progress and at the start completed three stations in three nights. But for the desirability of obtaining extra observations at the Lossiemouth Base terminals (350 and 351) and at Bin of Cullen (349) this record would probably have been equalled by the party there. As in the 1910 base extension(8) observations, however, the possibility of abnormal refraction along the base and along certain rays out of the Bin of Cullen (349) had called for more protracted observations. Two rays out of Bin of Cullen (349) passed close to the side of a large memorial cairn, and in addition to screening and cooling the cairn, it was thought advisable to take balanced observations between day and night, and on different days. A similar problem was encountered at Findlays Seat (340).

Progress continued to be good and by the middle of July the observations around the Lossiemouth Base had been completed and the party from there were moving into South-West Scotland to occupy some 'intersected' stations. Meanwhile the party on the eastern edge of Figure 3 were still maintaining good progress, after their flying start previously mentioned, and had taken Warden Law (142), a station in the smoky industrial area south of Newcastle-on-Tyne, in their stride. However, they now crossed over to the west side of Figure 3, and were brought to an abrupt halt on the notorious Black Combe (2). No doubt the following, included in the *Bulletin* at the end of a week in which no observations were possible, provided some consolation:

'BLACK COMBE'

(The poem was written in 1813 by Wordsworth, and the 'Geographic Labourer' referred to was no less than Major-General Mudge.)

* * * * *

'Written with a slate pencil on a stone, on the side of the Mountain of Black Combe:

Stay, bold Adventurer; rest awhile thy limbs
On this commodious Seat! for much remains
Of hard ascent before thou reach the top
Of this huge Eminence—from blackness named,
And, to far-travelled storms of sea and land,
A favourite spot of tournament and war!

Know . . .
That on the summit whither thou art bound
A Geographic Labourer pitched his tent,
With books supplied and instruments of art,
To measure height and distance; lonely task,
Week after week pursued!

. . . Once, while there he plied his studious work,
Within the canvas Dwelling, colours, lines,
And the whole surface of the outspread map,
Became invisible . . . total gloom
In which he sat alone, with unclosed eyes
Upon the blinded mountain's silent top!'

* * * * *

'With the more modern instruments, it is hoped that the period of time indicated in the last line of the second verse will be somewhat reduced.'

But, once away from this station the party again made good progress and, having also completed the observations at Liddington Castle (35), gathered at Southampton in preparation for a programme in Wales (Figure 4). This was at the end of July, at which time the party in South-West Scotland were struggling to complete Merrick (301). The third party in South-West England were forging ahead in better weather.

The party in Wales made a flying start, yet again, and rapidly completed Stiperstones (64), moving quickly on to Radnor Forest (71). Thence their movements west across South Wales brought their lightkeepers in contact with members of the South-West England party and eccentric lights were once again necessary—this time for rays across the Bristol Channel.

By the middle of August the work in South-West Scotland was completed and this party also moved into Figure 4—working south from North Wales. A modified form of central organisation was therefore again necessary as there were now two parties in Wales and one in South-West England.

Progress continued steadily, without unusual incidents, in both Wales and the West Country until early October when the primary observing programme for the 1937 season was completed.

2.124 ABNORMAL LATERAL REFRACTION

It was necessary for the observer in South Wales to re-occupy Radnor Forest (71) (through no fault of his own) as there were one or two very large misclosures around that station which pointed to weakness in the Radnor Forest (71)–Gwynydd Bach (72) ray. As there was no abnormal range in the observations it was thought that the trouble was due to abnormal lateral refraction. The re-observations confirmed this theory and an analysis showed that this lateral refraction varied on different nights. (Similar trouble had been experienced on the rays Wingreen (17) to Bradley Knoll (14) in England, and Findlays Seat (340) to Corryhabbie (342) in Scotland.) Finally, it was decided to omit the direction from the adjustment of Figure 4.

2.125 ABNORMAL VERTICAL REFRACTION

Another mention of abnormal refraction occurs in the *Bulletin* during the latter part of the 1937 season, when observations were being taken across the Bristol Channel. Both observers had been instructed to intersect Rat Island Lighthouse (Int. 3) on Lundy Island whenever possible, and the observer in the West Country endeavoured to do so from Trevose Head (173) in Cornwall —a ray about 50 miles long. In the event, it proved impossible. Information from the local coastguards pointed to the fact that the light was visible only on rare occasions, possibly three or four times a year. It was computed that for the light of the lighthouse to be visible from Trevose Head (173), it would have to be at an altitude of 350 feet, or more than 160 feet higher than it actually is. Therefore on the rare occasions that it has been seen, the coefficient of refraction which is normally accepted in Great Britain as 0·08 would have to be approximately 0·17. Such conditions may be similar to those which obtain in a desert mirage.

During the season 91 primary stations were occupied as observing stations, and observations were made on 642 directions, made up as follows:

	Stations	*Directions*
Figure 3 (Scotland)	47	335
Figure 4 (Wales & South-West England)	44	307

(These figures do not include the occupation of substitute stations. See § 2.042)

The year's output accordingly showed an increase of 50% on 1936. The weather generally was better, although not outstandingly good, but stations in the 1937 programme were usually more inaccessible.

2.13 Primary Observations, 1938

2.130 STEEL TOWERS

The main feature of the 1938 season's work in Figure 5, which covered the Eastern Counties of England, was the need to erect steel towers at no fewer than 34 of the primary stations (see § 2.07) in order to secure the requisite sights. An attempt was also made to complete the secondary work around these tower stations while the primary towers were in position, and this usually required the erection of further towers on secondary stations before getting down to ground level. Nine Bilby steel towers were available in 1938. This was an insufficient number for comfortable working of the East Anglian programme, including the considerable volume of secondary work, but since it was unlikely that a greater number could be utilised on later secondary work, it was not considered economical to purchase more. The situation was eased to some extent by the generous assistance of the Geodetic Survey of Denmark who kindly loaned two more towers of similar design.

2.131 ORGANISATION

The type of organisation required for the observation of Figure 5 was greatly influenced by the number of steel towers available and by the likely variation in the progress due to delays at tower stations.

Two independent observing parties were formed, each comprising:

One main observing section:	the senior observer and nine lightkeepers
One subsidiary observing section:	the assistant observer and six lightkeepers
One tower erection section:	six erectors, one lorry driver and one pillar constructor.

The senior observer controlled the whole party, but the sections operated separately.

The detailed planning of the programme was for the first time delegated to the senior observer in the field and only very general instructions were issued from headquarters, in just sufficient detail to settle the moves of the towers and to ensure co-ordination between the two parties. Each senior observer settled his own programme on a day to day basis depending on the weather, although a proportion of his lightkeepers were usually ready in position for the primary lines. The second observer in each party was employed on 'mopping-up' primary rays, or on secondary work, whilst his senior did either primary or secondary work, depending on the visibility. If the weather was comparatively calm, the senior occupied a steel tower; in windy and particularly in gusty weather, he concentrated on observations into steel towers. Such a very flexible organisation required good road communications, which of course existed in the East Anglian area, and highly trained personnel. Its adoption enabled normal progress to be achieved on the primary observations despite the extensive use of steel towers, and in addition large areas of secondary work were completed.

The equinoctial gales normally experienced in September were likely to interfere seriously with observations taken late in the season, and therefore every effort was to be made to complete tower stations by the end of August. For that reason all parties were to be moved on as soon as observations were completed at steel towers, including those to and from adjacent ground stations. It was hoped that this would enable that part of the network containing steel towers to be pushed ahead, even though it entailed filling in certain lines between ground stations at a later date.

2.132 PROGRESS

By the middle of April 1938, sufficient steel towers had been erected for a start to be made on observations, and the two parties started work in the south of the East Anglian figure: one party to the east of the Chipping Barnet Church Tower (185)–Leith Hill Tower (50) ray, and the other to the west. Although visibility was good on the whole, the observers were considerably delayed at the start by strong winds: however they now had a useful reserve of work in the secondary observations, and it was frequently possible to observe from other stations into the primary steel tower, even during windy weather.

Progress suffered a temporary setback in May, with a reversion to wintry weather, with rain, sleet, snow and high winds, but this in turn gave way to a good spell and before long the observers were pressing hard on the heels of their steel-tower teams, who were now achieving record erection (and dismantling) times which have never since been equalled in Great Britain. To quote an example, one of many, the steel tower at Walpole St. Peters (427) in Norfolk was unloaded, erected, and occupied by a lightkeeper, in 7½ hours, the 'footings' having been prepared in advance, of course.

By the middle of July, such good progress had been made that a number of men were released for base-measurement duties at Lossiemouth in Scotland, and at the end of the month there was little primary work left to do. By the 31st August the last primary observations had been taken and the secondary triangulation around the steel towers was also largely completed.

All observations were completed by the 17th September 1938, and consisted of:

Primary stations occupied (including 35 steel-tower stations)	87
Primary directions observed on 16 zeros	634
Secondary stations occupied (including 47 steel-tower stations, and the re-occupation of certain primary stations for secondary work)	302
Secondary directions observed on eight zeros	2,199

The main Primary Retriangulation of England and Wales was completed.

2.14 Primary Observations, 1939

In 1939, the main triangulation effort was switched to secondary work, and there was no full-scale primary observation programme in the months immediately preceding the outbreak of war. A small extension was observed on the east of the main chain in England to provide control for minor triangulation which was required for an experiment in and around the city of Kingston-upon-Hull.

The figure observed consisted of a centre-point quadrilateral based on the main chain stations Cave Wold (131) and Acre (132) and extending east to the Spurn Point–Withernsea area to two steel-tower stations, Tunstall (451) and Dimlington (452). Stone Creek (450), at the centre of the figure, was another steel-tower station.

The whole figure was considered as a block of secondary triangulation, and the secondary and tertiary observations were taken concurrently with the primary. Owing to very bad weather in April and May the task was a protracted one. The following work was done in 1939:

Stations occupied (E. Yorks) including four steel-tower stations	5
Directions observed on 16 zeros	23

2.15 Primary Observations, 1949

No primary observations were taken from 1940 to 1945 because of the Second World War. In the immediate post-war period until 1949 all resources of the Triangulation Branch were concentrated on the secondary and tertiary triangulation which was urgently required to control large-scale surveys of the main industrial areas of Great Britain. By 1949, the immediate requirement for lower order triangulation had been satisfied and work on the primary recommenced with the extension of the main chain to the Shetland Islands. Starting in the south at Hill of Stake (319), Sliabh Gaoil (303) and Beinn Bheula (330), the chain varied slightly in width until it left the mainland. Over the Orkney and Shetland Islands the chain narrowed to a single triangle following almost exactly the same pattern as the Principal Triangulation.

2.150 PREPARATION AND PROGRAMME

As it was now over 10 years since any primary observations had been taken in Great Britain, an intensive programme of training was carried through in the winter of 1948/49. A detailed plan was produced for the forthcoming field season. As the stations to be occupied were generally on mountains or islands that were difficult of access, it was essential to allow sufficient time for long moves between stations, especially for the sea-journeys in the extreme north.

Two independent observing parties were planned, each with two observers. As the chain was narrow the observing parties were to keep well apart, each taking roughly half the chain and working northwards. The dividing line was Ben Hutig (378)–Bad Mor (376)–Hill of Yarrows (391) (i.e. just south of the Caithness Base).

2.151 ORGANISATION

Strong sections would be required, as the South party had to occupy a large number of difficult stations in the remote parts of Inverness, Ross and Cromarty, and Sutherland, and the North party would frequently have sections inoperative whilst in transit between island stations. A total strength of 44 men, 22 to each party was therefore allocated, and this proved to be the bare minimum in the case of the South party—their observing section in particular being very hard pressed on more than one occasion. To alleviate the physical burden on the observers, the two observers either occupied alternate stations or else 'double-banked' each other on the more difficult mountains, each observing on alternate nights. A similar procedure was followed for lightkeeping sections by allocating two sections to the more difficult stations. It is interesting to note that all four of the 1949 observers had been primary lightkeepers in the pre-war seasons.

The 22 men in each party were divided up as follows:

Two observing sections: each $\left\{\begin{array}{l}\text{one observer}\\ \text{one booker}\end{array}\right.$
Nine lightkeeping sections: each two surveyors

The observing sections also operated the mobile charging plant. Each section had its own vehicle, which was now official and generally consisted of an ex-War Department 15 cwt. van. The North party also had a proportion of 10 cwt. vans, since nothing larger could be transported to some of the smaller islands; indeed at times such transport was hazardous even for the 10 cwts.!

Another major difference was that all personnel were now civilians, with the exception of the Officer in charge of Triangulation.

2.152 PROGRESS

Observations started eventually on the 11th May—the senior observer in the South party falling waist-deep into a bog, which by now was apparently becoming traditional for senior observers on their first night 'up'. The same evening in the North the observing party had barely reached the top of the hill when a fog-bank rolled in off the North Sea—another experience which was to prove typical.

However, weather on the whole was good at the start, and both parties began by completing two stations in the first week.

By the 21st May the two observers in the South party had reached Ben Nevis (323) and, without realising it, were embarking on a long vigil. Conditions were extremely uncomfortable: for the first week the cloud did not lift at all and the temperature averaged 26°F. Snow was 5 to 6 feet deep on the summit and the snow line, after fresh falls, was down to the 3,500 feet level. For the first few days both observers enthusiastically went up each night and took turns to observe, as the high winds at such low temperatures made it difficult to observe more than a few zeros without a break to get the circulation going again. Eventually, the physical strain of operating for 13 hours out of every 24—climbing and descending for 5 hours and standing-by to observe for 8—proved too much and the observers spent alternate nights on the summit. During the second week, observing was occasionally possible as the cloud broke at rare intervals: the temperature also rose to 34°F at times, but quickly fell below freezing again with very strong winds and snow-showers. The section on duty spent most of their time huddled in a minute tent around a small primus stove. It was not until near the end of their stay that the party discovered that the tent was, in reality, pitched on an overhanging cornice of snow with nothing solid beneath! Eventually, observations were completed on the evening of the 11th June after 22 nights.

In the meanwhile, the North party, being blessed not only with much lower hills but also with better weather, were now at the seventh stage of their programme; one observer being in the Orkneys at Ward Hill (466)—a mere 1,400 feet—while the other observer was already off on the long move to Fair Isle (458), via Lerwick in Shetland. Their luck with the weather continued to hold and progress was rapid, despite delays in the moves between the islands when it was necessary on each occasion to wait for the mail steamer. Small boats were available but their owners were reluctant to make inter-island trips owing to the strong cross-currents and the tide-races, which were often a dramatic sight.

By the 18th June one observer was on Westray Island and observing from Fitty Hill (460). The ray to Foula (461) which at a distance of 67 miles was the longest ray to date in the Retriangulation, was completed without much loss of time and by the 24th the section was back in Kirkwall waiting for the next boat to Shetland. Two days later on the evening of the 26th June the second observer was waiting to observe at Fair Isle (458), with his instrument set up on the pillar.

The usual cloud lay on the summit, and after a while the observer walked down the hill to see what the visibility was like beneath the cloud. As he came out of the mist he found that he was in the middle of a colony of nesting skuas, the pirates of northern waters. According to their usual habit when disturbed, the birds flew away a short distance and then approached at high speed, over covering hummocks of peat, and tried to graze the top of his head with their feet. As can be imagined this is an alarming experience as the arctic skua is a powerful bird (with a 3-foot wing span), and the observer beat the air wildly with his arms—so much so that he dislocated his right shoulder joint. Emergency arrangements were made to replace him but the injured observer managed to continue with one hand and his booker's assistance, and after a while they put the shoulder back by their united efforts, aided by the centre-pole of their tent!

The pace for the North party now quickened considerably and thanks to good weather, amenable boat-owners, and some hectic moves by land and sea, Fair Isle (458), Foula (461) and Brassa (456) were all completed about 10 days after the skua incident.

Ben Alder (335) was meanwhile engaging all the efforts of the South party. Although considerably lower than Ben Nevis (323), their previous station, difficulty of access was the major factor. The only alternative to a 3½-mile walk over hummocky peat from the de-bussing point to the camp site at the foot of the mountain was a slightly shorter passage by rowing up the loch. However, despite these difficulties, the section made good progress and seven days later the observer telegraphed to Headquarters: 'Ben Alder completed—rowing heavy kit to dam side.' This early finish was largely due to a commendable effort by two lightkeepers at Carn an Fhreiceadain (331) whose batteries suddenly failed them on the morning of the 17th June at 0115 hours: they raced down the mountain and back up again relighting with fresh batteries at 0235 hours: this enabled the observer to complete observations at the station, Ben Alder (335), including the ray to the notorious Ben Macdhui (302) (which was temporarily out of cloud).

But the South party's troubles were by no means at an end and Ben Wyvis (379), Anteallach (389), Carn Eige (386) and Conival (384) had yet to be tackled. Thanks to the most strenuous and unremitting efforts by the whole party, however, excellent progress was made, a spell of fine weather making life easier. By the 11th July the South party reported that they had cracked their last 'hard nut'. Simultaneously the weather broke.

The final stages were on the lower hills in Caithness and these were completed by 17th July.

The same fine weather spell had also accelerated progress in the Shetlands and the two observers there moved rapidly north, taking alternate stations, and the last two—Saxavord (463) and Balta (455)—being observed on the same night. By the 24th July all primary observations for the 1949 season were completed.

Thirty-five primary stations had been occupied, and 194 directions observed on 16 zeros.

2.153 RESULTS

When the observations were processed and analysed it was found that the average misclosure (regardless of sign) of the triangles north of a line Foula–Brassa (461–456), in Shetland, was 2″·84. As mentioned above this area had been observed at speed, generally with the 16 zeros on each direction being observed in one night; all observations concerned were completed in the one fine spell of weather. All rays crossed stretches of sea for much of their length. It was therefore considered that lateral refraction was possibly the cause of these misclosures and a decision was made to re-observe the triangles concerned in the Shetlands during 1950, spreading the observations on each direction over at least two nights.

2.16 Primary Observations, 1950

2.160 ORGANISATION

It was decided that July would be the best month to attempt the re-observations. Owing to the commitments of the secondary and tertiary programme, only a skeleton primary party could be spared to undertake this work and the total strength was ten men comprising:

1 Observer
1 Assistant Observer
8 Lightkeepers

All the above operated separately as one-man parties, the lightkeepers booking in turn when the observers reached their stations. The senior observer had been in charge of the North party in 1949.

Official transport was kept to a minimum as the majority of moves could be made almost entirely by water.

2.161 PROGRESS

The party arrived at Lerwick on the 4th July and weather was good at the start, the senior observer occupying Brassa (456), and the second observer, Foula (461). Brassa (456), the lower station, was completed very quickly and by the 8th the senior observer was climbing Ronas Hill (462) (for the first of many times) in heavy rain and low cloud. In the following week the wind gradually increased to gale force at sea level and the hill remained in cloud. At Foula (461) the position was the same and the two observers resigned themselves to waiting, hoping, and getting wet.

Another week passed by and the weather became even worse, the whole of the Fair Isle and Shetland area now being in the grip of a deep depression. Eventually, the requisite two nights' observations were completed from both Foula (461) and Ronas Hill (462) during the evening of the 28th July, after continuous occupation of 22 and 21 nights, respectively.

Progress now speeded up again and both Yell (467) and Fetlar (459) were completed in the following week. It seemed that the programme was virtually completed, but the weather was gradually breaking up again and although Saxavord (463) was completed fairly quickly, Balta (455), the last station and an uninhabited island, proved more difficult and could not be occupied for several days due to gales and high seas.

Eventually, however, Balta (455) was completed, and all that remained was a last visit to Saxavord (463) for one outstanding direction—to Ronas Hill (462).

The party arrived back in Aberdeen on the 20th August, having taken about 50 days overall to observe the 30 directions at seven primary stations. As a result of the re-observations in 1950 the triangle closures in Shetland were now entirely satisfactory, having been reduced from an average of 2″8 in 1949 to 0″7 in 1950. The mean observed directions differed by less than 1″0 except at Yell (467), where the average difference (ignoring sign) was 2″1. From an examination of the 1949 triangle misclosures it was apparent that the trouble lay in the ray Yell–Ronas Hill (467–462). This direction had been altered by 3″7 in the 1950 re-observations. The reason for this change was not immediately apparent; the station was observed on both occasions by the same observer whose work was of a high standard. One suggested cause was that in 1949 the observations to all but one station had been completed in one evening and that abnormal lateral refraction may have occurred. The ray passed over the sea for one third of its length of 12 miles, although the clearance was about 1,000 feet. There was also the possibility that trouble may have been caused under these conditions by the slight graze near the pillar at Yell (467).

2.17 Primary Observations, 1951

2.170 GENERAL

The winter of 1950/51 was again one of intensive training, planning, and preparation for the following season, when the largest primary observing operation of all was to be mounted. It was intended that the whole of the remaining primary network in the west and north of Great Britain should be completed. Primary observations were to be commenced on the North Wales, Westmorland and Cumberland coasts, and were to be extended across the Irish Sea to the Isle of Man and thence to South-West Scotland. The net to be observed then extended north and west, starting in the east from the western edge of the main chain stations occupied in 1949, and covering the western Highlands and the Inner and Outer Hebrides, the total area of land and sea to be covered being approximately 3,500 square miles.

2.171 ORGANISATION AND PROGRAMMES

Planning was greatly facilitated by the fact that the senior observer had carried out a reconnaissance of every station during the preceding season, and three of the four observers had taken part in the 1949 programme.

The general plan adopted was similar to that employed in previous years. Broadly speaking, there were to be two main observing parties each containing two observers. Lightkeeping sections consisted of two men and one vehicle, but they and the observing sections were frequently combined into stronger units where this seemed advisable. Owing to the complexity of the net, one main programme was compiled in the utmost detail and every move was studied and accurately timed, in advance.

As the majority of the rays passed over water it was decided that observations on each direction would be spread over three nights.

2.172 STRENGTH

The total strength of the party was 46 men, divided up as follows:

4 observing sections each consisting of 1 Observer, 1 Assistant Observer and 1 Booker with two vehicles, one of which could be used as a mobile charging plant
17 Lightkeeping sections—each of two men with one vehicle

2.173 PROGRESS

Observing began on the evening of the 26th April with the observers at Holyhead (117), Llaneilian (116), South Barrule (469) and Snaefell (468), weather conditions being poor. Cloud was frequently down on the Isle of Man stations and visibility was generally bad owing to drifting cloud and fog-banks in the Irish Sea. Temperatures were low for the time of the year and observing was frequently interrupted by rain and sleet storms. However, good progress was made, the observer at Holyhead (117) finishing in the minimum of three nights. The Isle of Man stations were next to be finished, but it was the 12th May before Llaneilian (116) was completed, the direction to the notorious Black Combe (2) causing most of the delay.

As the parties moved northwards into Scotland, the weather suddenly broke, and gales and rain, which were to become so much a part of life for all parties, set in. On the night following his arrival at Merrick (301), the observer saw his tent blown to shreds and the party was forced to seek refuge in a nearby barn.

By the beginning of June the bad weather was seriously delaying progress and it was decided that only two nights' observations would be taken on each direction for the remainder of the programme. Parties were now beginning to occupy stations in the Inner Hebrides and the Western Highlands. Of the first island stations, Ailsa Craig (479) and Goat Fell (309) were cleared fairly quickly, but the observer on Jura (392) spent the first six nights in cloud with only very rare breaks. On the seventh night the cloud lifted from the top and all lights were visible, but the heavy dew not only continuously coated all external instrument lenses (a fairly common occurrence on Scottish mountains) but eventually found its way inside the horizontal micrometer system, obscuring the prisms. Determined not to be beaten the observer set to, with the aid of a pocket torch held by his booker, and stripped this part of the instrument, cleaned out the condensation, and completed observations in the early hours of the morning. To round off a good night's work he then descended the mountain, struck camp, engaged a ferry for the short trip across the water to Port Askaig, and just managed to catch the mail-boat for the mainland at 8.50 a.m.

'Munros' (Scottish peaks of over 3,000 feet) were now becoming the order of the day and the first lightkeepers on Sgurr na Ciche (371) (one of the stiffest of them all) reported that the pillar had been severely damaged by lightning. Fortunately it could still be used for the emplacement of a beacon lamp whilst emergency repairs were being made.

A general improvement in the weather now enabled some island stations and also Ben Cruachan (314) to be cleared reasonably quickly, although one observer was being delayed on Heaval (475) in Barra—a cloud-former, in spite of its low altitude. The senior observer therefore decided to leave Tiree where he had just completed Ben Hynish (368), and push on alone to Askival (374) on the Isle of Rhum. On arrival at the island he set up camp and left for the top at about 9 p.m. with only one assistant and a considerable load of equipment. To quote from his weekly report:

> '. . . It was decided to take the advice of the Factor, and try the approach from the south. Something must have gone wrong with our navigation, or else this "easy way" that the locals talk about is non-existent. After scaling precipitous cliffs and rock faces we arrived at the pillar at three-quarters of an hour after midnight, slightly scared and very tired. A grand sight greeted us on arrival: in the cool quiet night there were eight bright lights shining at us, and sundry lighthouses. . . .'

It was some 10 days, however, before he was able to complete the station.

At this time two observers were 'double-banking' at Meall nan Con (393), a comparatively easy station. Nevertheless, progress was slow as the weather was persistently wet. The senior of the two therefore decided to move on alone to the next station, Ben Nevis (323), to be in readiness for the next break in the weather. This proved to be a wise move, for three days later the weather cleared and not only was Meall nan Con (393) completed but a substantial number of observations were taken at Ben Nevis (323). On the same evening (Friday, 13th July), Askival (374) was completed and good progress was made on Sgurr na Ciche (371). Both Ben Nevis (323) and Sgurr na Ciche (371) were finished shortly after this, and two observers made long moves—one to Marrival (477) in North Uist, and the other to Clisham (472), in the Isle of Lewis, one of the highest hills in the Outer Hebrides. Meanwhile, the other two observers occupied stations in Skye.

Advantage was taken of this lull in the programme to interchange some lightkeeping sections from difficult to easy stations, and vice versa. In the course of these moves the section at Askival (374) interchanged with the party at Meall nan Con (393). On their arrival at the easier mainland station, they checked their stores and found that one essential item—a shovel—was missing. They therefore sent a telegram to their reliefs at Askival (374), 'HAVE YOU GOT OUR SHOVEL'. It speaks volumes for the ebullient spirits of the lightkeepers that despite the appalling weather and the difficult stations they were on, the following telegram (privately paid for, of course) came back at once:

> 'NO STOP HAVE NOT GOT YOUR SHOVEL BUT HAVE PICTURE OF GENERAL GORDON READING TIMES OUTSIDE HIS TENT AT KHARTOUM'

Almost incessant rain and cloud on the hills now made life increasingly difficult. In particular Clisham (472) was continuously in cloud for days on end. The station was completed after two weeks occupation, having been clear for only two hours during that time.

Because the programme was now running so far behind schedule it was found impossible to occupy Plat Reidh as this station was in the middle of a carefully preserved deer forest and the stalking season had begun. In the event it was possible to by-pass the station without weakening the net.

By the beginning of September the parties were on the final stations of their original programme, but it had been found that some triangle misclosures were excessively large, and a programme of re-observations had to be started at once, calling for the re-occupation of five primary stations. Three were difficult mountains—Carn Eige (386), Sgurr na Ciche (371) and Ben More (Mull) (377)—and two were easy—Ben Hynish (Tiree) (368) and Beinn Tart a' Mhill (Islay) (383). It was also necessary to take additional observations in the Caithness Base Area to co-ordinate a new station Hillhead Farm (478) which had been sited halfway along the base.

Half the personnel were sent to the Mull area and the remainder were occupied with the re-observations in the Central Highlands. Owing to the steadily deteriorating weather and heavy falls of snow on the higher hills it was impossible to complete these re-observations in 1951.

Observations in Caithness were finished by the middle of November and the last sections returned to headquarters on the 24th November, after seven months of the worst primary observing weather experienced in the Retriangulation.

Fifty-two primary stations had been occupied and 306 directions observed on 16 zeros.

Also in 1951, the primary Retriangulation in South-East England (Figure 5) was strengthened in preparation for the cross-channel connection to France. This involved the establishment of a new station, Frittenfield (480). At the same time extra observations were taken to obtain a better co-ordination of Paddlesworth (190).

2.18 Other Primary Observations

In only three weeks (April–May) of 1952 the primary re-observations left over from the 1951 season were completed, in very good weather conditions. Thus the observation of the main Primary Retriangulation was completed in seven field seasons. Subsequently individual additional stations were connected as follows:

1953 Herstmonceux (481) ⎫
1954 Greenwich Observatory (482) ⎭ In connection with the astronomical programme (see Chapter 5).
1955 North Tolsta (484) Used as a terminal in the Shoran connection to Iceland (see Chapter 3).
1957 St Kilda (486) This was required by the Director of Military Survey on behalf of the Air Ministry in connection with the establishment of a guided weapons firing range.

COMPUTATIONS

2.19 Notation

The symbols used are listed below. Specific values of some quantities are indicated in the text by suffixes, e.g. φ_2 = latitude of point 2, C_1 = convergence at point 1, etc. Where double suffixes with an intervening stop are used, they indicate either a quantity measured in the direction first suffix to second suffix, e.g. $A_{1.2}$ = azimuth from point 1 to point 2; or a quantity measured between the suffixes, e.g. $S_{1.2}$ = spheroidal distance between point 1 and point 2. Other indicators are defined when first introduced.

$$\left.\begin{array}{l} a = \text{Major semi-axis} = 20\,923\,713 \text{ feet of Bar } O_1 \\ b = \text{Minor semi-axis} = 20\,853\,810 \text{ feet of Bar } O_1 \end{array}\right\} \text{Airy's spheroid.}$$

e = eccentricity.
$e^2 = (a^2 - b^2)/a^2 = 0{\cdot}006\,670\,540\,000\ldots$
$n = (a-b)/(a+b) = 0{\cdot}001\,673\,220\,310\ldots$ (See § 2.229).
φ = Latitude, north (+), south (−).
λ = Longitude from Greenwich, east (+) or west (−).

$$\left.\begin{array}{l} E = \text{National Grid Eastings} \\ N = \text{National Grid Northings} \end{array}\right\} \text{metres.}$$

$y = E - 400{,}000$.
ν = Radius of curvature in the prime vertical = $a/(1 - e^2 \sin^2\varphi)^{1/2}$.
ρ = Radius of curvature in the meridian = $a(1 - e^2)/(1 - e^2 \sin^2\varphi)^{3/2}$.
$\eta^2 = \left(\frac{\nu}{\rho} - 1\right) = e^2 \cos^2\varphi/(1 - e^2)$.
F_0 = Transverse Mercator scale factor on the central meridian of the projection.
F = Transverse Mercator scale factor at a point, also called local scale factor.
S = Spheroidal distance between two points.
D = Plane National Grid distance between two points.
A = Azimuth, measured 0°–360° clockwise from true north.
α = Plane National Grid bearing measured 0°–360° clockwise from National Grid north.

t = The direction of a straight line joining two points on the transverse Mercator projection.
T = The direction on the transverse Mercator projection of the geodesic between two points.
C = Meridian convergence; it is used here more specifically to denote the angle at a point between true north and National Grid north.
ϵ = Spherical excess of a triangle.

2.20 The Spheroid and Unit of Length

Full details of the spheroid of reference and unit of length used in the calculation of the Retriangulation have been given by E. H. Thompson(9). The following description is based largely on Thompson's account.

The Figure of the Earth used by the Ordnance Survey for its work in Great Britain is that given by Sir George Airy in an article on the Figure of the Earth in the 'Encyclopaedia of Astronomy'. The latter forms part of the *Encyclopaedia Metropolitana* which was published in 1848. It is universally known as Airy's Figure of the Earth. The defining elements are the major and minor semi-axes:

$$a = 20\,923\,713 \text{ feet}$$
$$b = 20\,853\,810 \text{ feet}$$

The old Principal Triangulation was calculated on this figure using 'feet' defined by the Ordnance Survey standard 10 feet bar O_1. Doubts exist as to the permanence of the 'foot', which is defined as one-third of the Imperial Standard Yard. For example, in a Board of Trade report of 1930(10) it is stated on page 9 that:

> 'The new Copy No. VI (of the Imperial Standard Yard), made for the Board of Trade in pursuance of Section 5 of the Weights and Measures Act, 1878, has shown a progressive and regular shortening, relative to the older bars, amounting in all to two ten-thousandths of an inch. Since this bar was made as nearly as possible identical in material and construction with the original series, it appears not improbable that the earlier bars also, in the first years of their existence, all shortened by a similar amount, in which case the Imperial Standard Yard would now (1922) be about two ten-thousandths of an inch (1/180,000) shorter than when originally legalised.'

and on page 11:

> 'Although the results of the 1922–23 comparisons have established the values of both the Yard and the Pound on a firmer basis than for some decades past, it has become very apparent that neither series of standards is of a quality corresponding to that of more modern primary standards, or to the possibilities and requirements of present day scientific and industrial development.'

For this reason among others the planimetric results of all surveys in Great Britain based on the Retriangulation are expressed as rectangular co-ordinates in metres, International Metres being understood; and the geodetic calculations require that the elements of Airy's Figure should first be converted from feet to metres. For this purpose the following logarithm is added to log a and log b:

$$\text{Log (conversion ratio)} = \bar{1}\cdot484\,016\,03$$

from which the following natural value may be derived:

$$\text{Natural value of the ratio} = 0\cdot304\,800\,749\,1\ldots$$

Note that the defining figure is the logarithm to just eight significant figures, no more and no less. (See below.) Since the selection of the above, possibly unfamiliar, ratio may cause comment, an explanation of the choice is perhaps of some interest.

It is clear from Airy's article ('Encyclopaedia of Astronomy', p. 217) that the feet of his *a* and *b* are those of Sir George Shuckburgh's Five-Foot brass standard made by Edward Troughton in 1796. To arrive at the elements of the meridian ellipse, Airy examined and combined 14 arcs of meridian and four arcs of parallel of which the foreign arcs based on a metric standard were reduced to feet by assuming that one metre was the equivalent of 3·280 899 (Shuckburgh) feet. This ratio was obtained by Captain Henry Kater(11) by a comparison between the Shuckburgh standard and two platinum copies of the metre whose lengths had been accurately determined by Arago. The logarithm of the reciprocal of Kater's ratio is $\bar{1}$·484 007 14. It would at first sight seem logical, since Airy's synthesis included arcs based on a metric standard, to express his final result in metres by the employment of the ratio used in his own work. Kater's ratio referred to the original prototype metre, the *Mètre des Archives*, but there was no significant difference between this standard and the International Prototype Metre when the latter was adopted in 1889.

Unfortunately this straightforward procedure would not have been entirely satisfactory since the scale of the Retriangulation was obtained by fitting it as closely as possible to the old Principal Triangulation at 11 stations. (See § 2.27 below.) The elements of the new work are, therefore, effectively the same as those of the old which was computed on Airy's figure on the assumption that the *a* and *b* were feet of the Ordnance Survey 10-foot bar known as O_1, and in terms of which the lengths of sides of the old Principal Triangulation are also expressed. The bar O_1 was constructed for the Ordnance Survey in 1826/27 by Messrs. Troughton and Simms. It is still in existence, together with the intermediate bar OI_1, in the museum at Chessington. It follows that the conversion ratio to International Metres, for both old and Retriangulations, should be the O_1/International Metre ratio; and that this same ratio must, therefore, be applied to Airy's *a* and *b*.

There is no direct comparison between O_1 and the International Metre, but it is possible to establish a connection through the Ordnance Survey 10-foot intermediate bar OI_1. The length of the latter was determined at the Bureau International des Poids et Mesures at Breteuil in 1906 by M. J.-R. Benoit and Major W. J. Johnston, R.E. The result obtained(12) was

$$3{\cdot}047\,895\,34 \text{ International Metres at } 13^\circ\text{C } (=55^\circ{\cdot}4\text{F}).$$

The effect of temperature on this bar may be expressed,

$$\text{Length at } t^\circ\text{F} = \text{Length at } 50^\circ\text{F} + 20{\cdot}8229\,(t-50^\circ) + 0{\cdot}02135\,(t-50^\circ)^2$$

in units of 10^{-6} yards(12) (the kind of yard is not significant). Note that the reference gives the unit incorrectly as 10^{-6} feet.

The bar O_1 defined 10 feet at 62°F and the correction to OI_1 to give its length in metres at 62°F is

$$+0{\cdot}000\,127\,90 \text{ International Metres.}$$

(Again, the particular foot/metre ratio used to convert yards in the expansion formula to metres is not important for a difference in temperature of 6·6°F.)

At 62°F the difference in length between OI_1 and O_1 is(13)

$$OI_1 - O_1 = +0{\cdot}000\,015\,68 \text{ International Metres.}$$

Hence the length of O_1 at 62°F is

$$3{\cdot}048\,007\,56 \text{ International Metres.}$$

From which one foot of O_1 at 62°F is

$$0{\cdot}304\ 800\ 756 \text{ International Metres,}$$

the logarithm being $\bar{1}{\cdot}484\ 016\ 037$. It will be noted that the logarithm of this ratio to eight figures should have had 4 as the last digit, and not 3 as quoted above. It is supposed that this error took place in rounding off. The definitive 8-figure logarithm ending in a 3 and its appropriate anti-logarithm were used in all calculations of the Retriangulation, and should be used in any future calculations.

2.21 The Projection and the National Grid

Rectangular metric co-ordinates for the new work were calculated on a transverse Mercator projection with a modified scale. On the central meridian of the projection the scale factor, F_0, is defined exactly by the common logarithm:

$$\text{Log } F_0 = \bar{1}{\cdot}999\ 826\ 80$$

or approximately:

$$F_0 = 0{\cdot}999\ 601\ 271 \ldots$$

The two lines, or 'sub-meridians', on which the scale factor is unity are approximately 180 km either side of, and almost parallel with, the central meridian.

This scale modification ensures that over most of Great Britain the scale error of the projection does not exceed ± 1/2500. Exceptions are the Scilly Isles, where the scale error is about + 1/1280, and parts of the Western Isles of Scotland, notably the Outer Hebrides, where the scale error in the extreme west is about + 1/980.

The origin of the transverse Mercator rectangular co-ordinates is:

$$\varphi_0 = 49^\circ \text{ North } (+)$$
$$\lambda_0 = 2^\circ \text{ West } (-)$$

To convert transverse Mercator rectangular co-ordinates to National Grid co-ordinates, the former are referred to a false origin by applying:

+ 400,000 metres to the transverse Mercator eastings,
− 100,000 metres to the transverse Mercator northings.

The false origin of the National Grid thus lies 400,000 metres west and 100,000 metres north of the origin of the projection.

All points are defined by their National Grid co-ordinates, and not by their geographical co-ordinates. The latter are derived from the former.

2.22 Transverse Mercator Formulae

The formulae which are used for calculations on the projection are given in §§ 2.220 to 2.229 below. Full details of the derivation of the formulae are given by Jordan(14) and Hristow(15).

For convenience all the separate terms in the formulae were tabulated with latitude as argument, and were published in 1950 by Her Majesty's Stationery Office(16).

In all the formulae given below it is assumed that the linear quantities ρ, ν, and M are in International Metres and have been multiplied by F_0.

2.220 E AND N FROM φ AND λ

$$\text{I} = M - 100{,}000 \text{ (see para. 2.229 below)}$$

$$\text{II} = \frac{\nu}{2}\sin^2 1'' \sin\varphi \cos\varphi 10^8$$

$$\text{III} = \frac{\nu}{24}\sin^4 1'' \sin\varphi \cos^3\varphi(5-\tan^2\varphi+9\eta^2)10^{16}$$

$$\text{IIIA} = \frac{\nu}{720}\sin^6 1'' \sin\varphi \cos^5\varphi(61-58\tan^2\varphi+\tan^4\varphi)10^{24}$$

The term P^6(IIIA) is given in Graph A in the Projection Tables⁽¹⁶⁾.

$$P = (\lambda-\lambda_0)''10^{-4} \quad \text{All tabular quantities are for } \varphi.$$

Then

$$N = (\text{I})+P^2(\text{II})+P^4(\text{III})+P^6(\text{IIIA}) \text{ (See Graph A).}$$

$$\text{IV} = \nu \sin 1'' \cos\varphi 10^4$$

$$\text{V} = \frac{\nu}{6}\sin^3 1'' \cos^3\varphi\left(\frac{\nu}{\rho}-\tan^2\varphi\right)10^{12}$$

$$\text{VI} = \frac{\nu}{120}\sin^5 1'' \cos^5\varphi(5-18\tan^2\varphi+\tan^4\varphi+14\eta^2-58\tan^2\varphi\eta^2+2\tan^4\varphi\eta^2)10^{20}$$

Then

$$E = 400{,}000+P(\text{IV})+P^3(\text{V})+P^5(\text{VI})$$

2.221 φ AND λ FROM E AND N

$$\text{VII} = \frac{\tan\varphi 10^{12}}{2\rho\nu\sin 1''}$$

$$\text{VIII} = \frac{\tan\varphi}{24\rho\nu^3\sin 1''}(5+3\tan^2\varphi+\eta^2-9\tan^2\varphi\eta^2)10^{24}$$

$$\text{IX} = \frac{\tan\varphi}{720\rho\nu^5\sin 1''}(61+90\tan^2\varphi+45\tan^4\varphi)10^{36}$$

$$Q = (E-400{,}000)10^{-6}$$

All tabular quantities are for φ', φ' being obtained from Table I in the Projection Tables⁽¹⁶⁾ with N as argument.

Then

$$\varphi = \varphi' - Q^2(\mathrm{VII}) + Q^4(\mathrm{VIII}) - Q^6(\mathrm{IX})$$

$$\mathrm{X} = \frac{\sec\varphi 10^6}{\nu \sin 1''}$$

$$\mathrm{XI} = \frac{\sec\varphi}{6\nu^3 \sin 1''}\left(\frac{\nu}{\rho} + 2\tan^2\varphi\right)10^{18}$$

$$\mathrm{XII} = \frac{\sec\varphi}{120\nu^5 \sin 1''}(5 + 28\tan^2\varphi + 24\tan^4\varphi)10^{30}$$

$$\mathrm{XIIA} = \frac{\sec\varphi}{5040\nu^7 \sin 1''}(61 + 662\tan^2\varphi + 1320\tan^4\varphi + 720\tan^6\varphi)10^{42}$$

The term Q^7(XIIA) is given in Graph B in the Projection Tables⁽¹⁶⁾.

Then

$$\lambda = \lambda_0 + Q(\mathrm{X}) - Q^3(\mathrm{XI}) + Q^5(\mathrm{XII}) - Q^7(\mathrm{XIIA})\ \text{(See Graph B)}.$$

2.222 CONVERGENCE (C) FROM φ AND λ

$$\mathrm{XIII} = \sin\varphi 10^4$$

$$\mathrm{XIV} = \frac{\sin\varphi \cos^2\varphi \sin^2 1''}{3}(1 + 3\eta^2 + 2\eta^4)10^{12}$$

$$\mathrm{XV} = \frac{\sin\varphi \cos^4\varphi \sin^4 1''}{15}(2 - \tan^2\varphi)10^{20}$$

P as previously defined. All tabular quantities are for φ.

Then

$$C'' = P(\mathrm{XIII}) + P^3(\mathrm{XIV}) + P^5(\mathrm{XV})$$

2.223 CONVERGENCE (C) FROM E AND N

$$\mathrm{XVI} = \frac{\tan\varphi}{\nu \sin 1''}10^6$$

$$\mathrm{XVII} = \frac{\tan\varphi}{3\nu^3 \sin 1''}(1 + \tan^2\varphi - \eta^2 - 2\eta^4)10^{18}$$

$$\mathrm{XVIII} = \frac{\tan\varphi}{15\nu^5 \sin 1''}(2 + 5\tan^2\varphi + 3\tan^4\varphi)10^{30}$$

Q as previously defined. All tabular quantities are for φ'.

Then

$$C'' = Q(\mathrm{XVI}) - Q^3(\mathrm{XVII}) + Q^5(\mathrm{XVIII})$$

2.224 $(t-T)''$ FROM E AND N

$$\text{XXIII} = \frac{10^9}{6\rho\nu \sin 1''}$$

1 and 2 are the terminals of the line.

The tabular function is taken out for φ_m, being the interpolate from Table I in the Projection Tables(16) for

$$N_m = \frac{N_1+N_2}{2}$$

Then

$$(t-T)''_{1.2} = (2y_1+y_2)(N_1-N_2)(\text{XXIII})10^{-9}$$
$$(t-T)''_{2.1} = (2y_2+y_1)(N_2-N_1)(\text{XXIII})10^{-9}$$

See also § 2.241.

2.225 AZIMUTH (A) FROM PLANE NATIONAL GRID BEARING (α)

$$\alpha_{1.2} = \tan^{-1}(E_2-E_1)/(N_2-N_1)$$
$$A_{1.2} = \alpha_{1.2}-(t-T)''_{1.2}+C_1$$

2.226 SCALE FACTOR (F) FROM φ AND λ

$$\text{XIX} = \frac{\cos^2\varphi \sin^2 1''}{2}(1+\eta^2)10^8$$

$$\text{XX} = \frac{\cos^4\varphi \sin^4 1''}{24}(5-4\tan^2\varphi+14\eta^2-28\tan^2\varphi\eta^2)10^{16}$$

P as previously defined. All tabular quantities are for φ.

Then

$$F = F_0(1+P^2(\text{XIX})+P^4(\text{XX}))$$

2.227 SCALE FACTOR (F) FROM E AND N

$$\text{XXI} = \frac{10^{12}}{2\rho\nu}$$

$$\text{XXII} = \frac{(1+4\eta^2)}{24\rho^2\nu^2}10^{24}$$

Q as previously defined. All tabular quantities are for φ'.

Then

$$F = F_0(1+Q^2(\text{XXI})+Q^4(\text{XXII}))$$

2.228 SPHEROIDAL DISTANCE (S) FROM PLANE NATIONAL GRID DISTANCE (D)

$$D^2 = (E_2-E_1)^2+(N_2-N_1)^2$$

Let F_m = Local scale factor for point E_m, N_m, where

$$E_m = \tfrac{1}{2}(E_1+E_2); \quad N_m = \tfrac{1}{2}(N_1+N_2)$$

Let F_1, F_2 = Local scale factor at point 1 and point 2 respectively, and F' = Local scale factor over the whole line from 1 to 2.

Then

$$\frac{1}{F'} = \frac{1}{6}\left(\frac{1}{F_1}+\frac{4}{F_m}+\frac{1}{F_2}\right)$$

and

$$S = D/F'$$

2.229 ARC OF MERIDIAN

Arc of Meridian from φ_2 to φ_1 = $M\varphi_2 - M\varphi_1$

$$M\varphi_2-M\varphi_1 = b\left\{\left(1+n+\frac{5n^2}{4}+\frac{5n^3}{4}\right)(\varphi_2-\varphi_1)-\left(3n+3n^2+\frac{21n^3}{8}\right)\sin(\varphi_2-\varphi_1)\cos(\varphi_2+\varphi_1)+\right.$$
$$\left.+\left(\frac{15n^2}{8}+\frac{15n^3}{8}\right)\sin 2(\varphi_2-\varphi_1)\cos 2(\varphi_2+\varphi_1)-\frac{35n^3}{24}\sin 3(\varphi_2-\varphi_1)\cos 3(\varphi_2+\varphi_1)\right\}$$

N.B. M in § 2.220 is obtained by putting φ_1 in this formula equal to φ_0 = 49°.
The derivation of this formula has been given by Clarke(17).

2.23 Processing the Observations

'Processing' is a term which is used by the Ordnance Survey for the method of treating the observations at a station so as to get the most probable values for the mean directions according to the theory of least squares. The method was described originally by Clarke(1), but as this reference is out of print the derivation of the method is given here.

In the primary Retriangulation the observations at each station were processed, and each processed mean direction was given unit weight in the subsequent calculations.

Let:

$0_{r.s}$ = Mean of Face Left and Face Right pointings to station s on zero r. Pointings to the Referring Object (R.O.) are denoted by station suffix 0.

$p_1, p_2 \ldots p_n$ = Number of mean pointings in the 1, 2, . . . n zeros respectively. (Including R.O.)

$q_0, q_1, q_2 \ldots q_m$ = Number of mean pointings to the 0, 1, 2, . . . m stations respectively.

A group of observations for *m* stations on *n* zeros may be represented thus:

$$
\begin{array}{llll}
0_{1.0} & 0_{1.1} & 0_{1.2} \ldots & 0_{1.m} \\
0_{2.0} & 0_{2.1} & 0_{2.2} \ldots & 0_{2.m} \\
\cdot & \cdot & \cdot & \cdot \\
0_{n.0} & 0_{n.1} & 0_{n.2} \ldots & 0_{n.m}
\end{array}
$$

Here $0_{1.0}$, $0_{2.0} \ldots 0_{n.0}$ are the prescribed circle graduation settings for the respective zeros. (See Appendix 12.) Subtracting the R.O. reading in each zero from the readings in its particular zero gives:

$$
\begin{array}{llll}
00 & (0_{1.1}-0_{1.0}) & (0_{1.2}-0_{1.0}) \ldots & (0_{1.m}-0_{1.0}) \\
00 & (0_{2.1}-0_{2.0}) & (0_{2.2}-0_{2.0}) \ldots & (0_{2.m}-0_{2.0}) \\
\cdot & \cdot & \cdot & \cdot \\
00 & (0_{n.1}-0_{n.0}) & (0_{n.2}-0_{n.0}) \ldots & (0_{n.m}-0_{n.0})
\end{array}
$$

or, using the prime to indicate the new quantities:

$$
\begin{array}{llll}
00 & 0'_{1.1} & 0'_{1.2} \ldots & 0'_{1.m} \\
00 & 0'_{2.1} & 0'_{2.2} \ldots & 0'_{2.m} \\
\cdot & \cdot & \cdot & \cdot \\
00 & 0'_{n.1} & 0'_{n.2} \ldots & 0'_{n.m}
\end{array}
$$

The direction of the R.O. on each of the zeros 1, 2, ... *n* is now reduced to nought. Let $x_1, x_2, \ldots x_n$ be small corrections to these R.O. directions and let $B_1, B_2, \ldots B_m$ be the most probable values for the mean directions 0′ of stations 1, 2, ... *m*.

The observation equations are then:

$$
\begin{aligned}
-x_1 &= v_{1.0} \\
-x_2 &= v_{2.0} \\
&\cdots \\
-x_n &= v_{n.0}
\end{aligned}
$$

$$
\begin{aligned}
0'_{1.1}-B_1-x_1 &= v_{1.1} \\
0'_{2.1}-B_1-x_2 &= v_{2.1} \\
&\cdots \\
0'_{n.1}-B_1-x_n &= v_{n.1}
\end{aligned}
$$

$$
\begin{aligned}
0'_{1.2}-B_2-x_1 &= v_{1.2} \\
0'_{2.2}-B_2-x_2 &= v_{2.2} \\
&\cdots \\
0'_{n.2}-B_2-x_n &= v_{n.2}
\end{aligned}
$$

. . . .

$$\begin{aligned} 0'_{1.m}-B_m-x_1 &= v_{1.m} \\ 0'_{2.m}-B_m-x_2 &= v_{2.m} \\ &\;\;\ldots\ldots \\ 0'_{n.m}-B_m-x_n &= v_{n.m} \end{aligned}$$

where v = residual differences between observed and corrected directions.

The condition required by the theory of least squares is that Σv^2 should be a minimum. Taking partial derivatives of Σv^2 with respect to $x_1, x_2, \ldots x_n$, equating to zero and rearranging gives:

$$\left.\begin{aligned} p_1\,.\,x_1+B_1+B_2+\ldots+B_m &= 0'_{1.1}+0'_{1.2}+\ldots+0'_{1.m} \\ p_2\,.\,x_2+B_1+B_2+\ldots+B_m &= 0'_{2.1}+0'_{2.2}+\ldots+0'_{2.m} \\ &\ldots\ldots\ldots\ldots \\ p_n\,.\,x_n+B_1+B_2+\ldots+B_m &= 0'_{n.1}+0'_{n.2}+\ldots+0'_{n.m} \end{aligned}\right\} \tag{2.01}$$

Doing the same with respect to $B_1, B_2, \ldots B_m$ gives:

$$\left.\begin{aligned} q_1\,.\,B_1+x_1+x_2+\ldots+x_n &= 0'_{1.1}+0'_{2.1}+\ldots+0'_{n.1} \\ q_2\,.\,B_2+x_1+x_2+\ldots+x_n &= 0'_{1.2}+0'_{2.2}+\ldots+0'_{n.2} \\ &\ldots\ldots\ldots\ldots \\ q_m\,.\,B_m+x_1+x_2+\ldots+x_n &= 0'_{1.m}+0'_{2.m}+\ldots+0'_{n.m} \end{aligned}\right\} \tag{2.02}$$

A direct solution of (2.01) and (2.02) to find the values of $x_1 \ldots x_n$, and $B_1 \ldots B_m$, is usually impracticable, so a method of successive approximations is used.

First assume that $x_1 = x_2 = \ldots = x_n = 0$, then from (2.02):

$$\text{R.O.} = 00$$

$$\left.\begin{aligned} B'_1 &= (0'_{1.1}+0'_{2.1}+\ldots+0'_{n.1})/q_1 \\ B'_2 &= (0'_{1.2}+0'_{2.2}+\ldots+0'_{n.2})/q_2 \\ &\ldots\ldots\ldots\ldots \\ B'_m &= (0'_{1.m}+0'_{2.m}+\ldots+0'_{n.m})/q_m \end{aligned}\right\} \tag{2.03}$$

where B' indicates a first approximation.

Substituting in (2.01):

$$\left.\begin{aligned} x'_1 &= [(0'_{1.1}-B'_1)+(0'_{1.2}-B'_2)+\ldots+(0'_{1.m}-B'_m)]/p_1 \\ x'_2 &= [(0'_{2.1}-B'_1)+(0'_{2.2}-B'_2)+\ldots+(0'_{2.m}-B'_m)]/p_2 \\ &\ldots\ldots\ldots\ldots \\ x'_n &= [(0'_{n.1}-B'_1)+(0'_{n.2}-B'_2)+\ldots+(0'_{n.m}-B'_m)]/p_n \end{aligned}\right\} \tag{2.04}$$

where the x' are better approximations for the values of x than the first assumption that they were all zero.

From (2.02):

$$\left.\begin{aligned}
B''_1 &= (0'_{1.1}+0'_{2.1}+\ldots+0'_{n.1}-x'_1-x'_2-\ldots-x'_n)/q_1 \\
B''_2 &= (0'_{1.2}+0'_{2.2}+\ldots+0'_{n.2}-x'_1-x'_2-\ldots-x'_n)/q_2 \\
&\quad \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\
B''_m &= (0'_{1.m}+0'_{2.m}+\ldots+0'_{n.m}-x'_1-x'_2-\ldots-x'_n)/q_m
\end{aligned}\right\} \qquad (2.05)$$

or, substituting from (2.03):

$$\left.\begin{aligned}
B''_1 &= B'_1-(x'_1+x'_2+\ldots+x'_n)/q_1 \\
B''_2 &= B'_2-(x'_1+x'_2+\ldots+x'_n)/q_2 \\
&\quad \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\
B''_m &= B'_m-(x'_1+x'_2+\ldots+x'_n)/q_m
\end{aligned}\right\} \qquad (2.06)$$

and R.O. $= 00-(\Sigma x')/q_0 = -(\Sigma x')/n$.

If a mean pointing to a station is missing in any zero, say the mean pointing to station s in zero r, then x_r is omitted in the equations (2.02), (2.05), (2.06), for the Bs; in other words, when evaluating B for a particular station, corrections to the direction of the R.O. are only included for those zeros on which observations were made to that particular station.

New values of x are now calculated by substituting $B''_1 \ldots B''_m$ in equation (2.04), remembering that the direction of the R.O. is now $-(\Sigma x')/n$, so an extra term $(\Sigma x')/n$, must be included in each numerator in the equations for x. That is:

$$\begin{aligned}
x''_1 &= [(\Sigma x')/n+(0'_{1.1}-B''_1)+\ldots+(0'_{1.m}-B''_m)]/p_1 \\
x''_2 &= [(\Sigma x')/n+(0'_{2.1}-B''_1)+\ldots+(0'_{2.m}-B''_m)]/p_2 \\
&\quad \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\
x''_n &= [(\Sigma x')/n+(0'_{n.1}-B''_1)+\ldots+(0'_{n.m}-B''_m)]/p_n
\end{aligned}$$

By substituting $x''_1 \ldots x''_n$ in (2.06) new values for B, $B'''_1 \ldots B'''_m$, are found; these in turn give new values $x'''_1 \ldots x'''_n$. And so on.

In practice the values $x'_1 \ldots x'_n$ and $B''_1 \ldots B''_m$ are usually adequate, and this is checked by calculating $x''_1 \ldots x''_n$, which should not differ appreciably from $x'_1 \ldots x'_n$.

Accepted values are then:

$$\begin{aligned}
\text{R.O.} &= -(\Sigma x')/q_0 = -(\Sigma x')/n \\
\text{Station } 1 &= B''_1 \\
\text{Station } 2 &= B''_2 \\
&\cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\
\text{Station } m &= B''_m
\end{aligned}$$

and final abstract means are obtained by subtracting the direction of the R.O. from each B''.

The worked example which follows has been taken as far as $x_1'' \ldots x_n''$ and $B_1''' \ldots B_m'''$, with $x_1''' \ldots x_n'''$ as a check, to show the effect of the successive approximations on the values of x and B.

	Station Number							
Zero	(R.O.) 0 00° 00′	1 53° 16′	2 157° 49′	3 196° 40′	4 276° 01′	x'	x''	x'''
1	00″	17″83	″	″	63″68	+0″47	+0″49	+0″50
2	00	18·95				+0·35	+0·44	+0·46
3	00				59·38	−1·24	−1·27	−1·27
4	00				62·55	+0·35	+0·32	+0·32
5	00		50·65	37·40	60·10	−0·34	−0·38	−0·39
6	00		49·30	37·99	62·30	+0·02	−0·02	−0·03
7	00	21·00				+1·38	+1·46	+1·48
8	00	15·02				−1·62	−1·53	−1·51
9	00	18·76				+0·26	+0·34	+0·36
10	00	21·09				+1·42	+1·50	+1·52
11	00		49·35	34·57	62·04	−0·89	−0·93	−0·94
12	00		48·58	38·03	60·69	−0·55	−0·60	−0·60
13	00		50·28	37·72		+0·11	+0·10	+0·08
14	00	17·93				−0·16	−0·08	−0·06
15	00	16·28			62·42	−0·47	−0·45	−0·44
16	00	15·92	47·02	36·42	60·00	−1·68	−1·69	−1·69
17	00	19·33	52·30		62·87	+1·06	+1·07	+1·08
18	00	18·22	47·73		61·45	−0·72	−0·70	−0·70
19	00		49·85	36·63		−0·39	−0·41	−0·42
20	00		49·85	40·00		+0·73	+0·71	+0·70
21	00		51·42	38·83		+0·86	+0·85	+0·83
22	00	20·45	52·36		64·35	+1·72	+1·74	+1·74
23	00	17·05	48·20		60·37	−1·16	−1·15	−1·14
24	00	18·53	51·72		61·72	+0·42	+0·44	+0·44
25	00	16·60	50·17		62·17	−0·34	−0·32	−0·31
26	00	17·67	51·65		64·42	+0·86	+0·88	+0·89
27	00			36·30		−0·59	−0·62	−0·64
28	00			38.10		+0·31	+0·28	+0·26
29	00	19·23	50·62	35·48	59·83	−0·52	−0·53	−0·53
30	00			39·07		+0·80	+0·76	+0·75
31	00			37·51		+0·02	−0·02	−0·03
32	00			37·65		+0·08	+0·06	+0·04
33	00	18·67	52·27		62·97	+0·91	+0·92	+0·93
34	00			37·95		+0·24	+0·20	+0·19
B'	00	18·25	50·18	37·48	61·85			
$(\Sigma x')/q$	+ 0·05	+ 0·12	+ 0·01	− 0·11	− 0·12			
B''	−00·05	18·13	50·17	37·59	61·97			
$(\Sigma x'')/q$	+ 0·05	+ 0·16	0·00	− 0·14	− 0·12			
B'''	−00·05	18·09	50·18	37·62	61·97			
	00° 00′ 00″	53° 16′ 18″14	157° 49′ 50″23	196° 40′ 37″67	276° 02′ 02″02	Final abstract		

2.24 The Adjustment of the New Primary Network

2.240 INTRODUCTION

All primary work in the Retriangulation was adjusted by the method of least squares. To facilitate the computations, the network was divided into seven main portions, or figures.

Figures 1, 2, 3, 4 and initially Figure 5, were adjusted by the method of correlates with condition equations, and positions were computed spheroidally. Two small additions to the network —the Spurn Head Extension, and the reco-ordination of Liddington Castle (35)—were adjusted in the same way. In the re-adjustment of Figure 5, the adjustments of Figures 6 and 7, and in the adjustments of several subsequent small additions to the network, the method of variation of co-ordinates was used with plane rectangular co-ordinates.

A description of the method of variation of co-ordinates which is now used is given below. This method was adopted in 1949 for the primary work because it saved an appreciable amount of time.

2.241 THE METHOD OF ADJUSTMENT BY VARIATION OF CO-ORDINATES

Before the mean observations are used for this method of adjustment they are reduced to the projection; this enables the whole adjustment to be carried out in terms of plane trigonometry. On the transverse Mercator projection the correction, known as the $(t-T)$ correction, is easily computed from approximate rectangular co-ordinates for the stations.

It may be noted here that the $(t-T)$ correction reduces the direction of the geodesic to that on the plane, and therefore the mean observed directions should first be corrected from the normal section to the geodesic. This was not done, however, in the primary Retriangulation because this latter correction is very small—less than 0″03 anywhere. Two further corrections were also ignored. One was for the height of the observed station, and has a maximum in Great Britain of about 0″05. The other was for the deviation of the vertical; no data were available for the deviation corrections.

§ 2.224 above gives a formula for $(t-T)$, but on primary lines at some distance from the central meridian of the projection higher order terms become effective. The full formula is:

$$(t-T)''_{1.2} = \frac{(2y_1+y_2)(N_1-N_2)}{6\rho_m\nu_m \sin 1''} - \frac{\eta_m^2 \tan\varphi_m \,.\, y_1(N_1-N_2)^2}{3R_m^3 \sin 1''} + \frac{\eta_m^2 \tan\varphi_m (y_1-y_2)(3y_1^2+2y_1y_2+y_2^2)}{6R_m^3 \sin 1''}$$

$$R_m = \sqrt{(\rho_m\nu_m)}$$

The hypothetical example which follows shows the magnitude of the three terms in the $(t-T)$ correction on a line 180 km in length and remote from the central meridian.

Let:

$E_1 = 50{,}000$ $\quad$ $E_2 = 200{,}000$
$N_1 = 400{,}000$ $\quad$ $N_m = 450{,}000$ $\quad$ $N_2 = 500{,}000$
$y_1 = -350{,}000$ $\quad$ $\varphi_m = 53° \, 57'$ (approx.) $\quad$ $y_2 = -200{,}000$

$$(t-T)''_{1.2} = +75''974 + 0''003 - 0''035 = +75''942$$

The second term is always very small and is usually negligible. It was included where it significantly affected the third decimal place of the $(t-T)$ correction.

Approximate rectangular co-ordinates to the requisite accuracy of about a metre can usually be obtained from a preliminary calculation using the mean observations.

Derivation of the observation equation

The following analysis assumes that observations have been reduced to the projection as described above.

Consider two points whose rectangular co-ordinates are E_0, N_0; E_1, N_1 (Fig. 2.3).

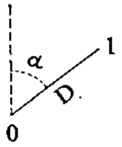

FIG. 2.3. Plane grid bearing

Then

$$\alpha_{0.1} = \tan^{-1}\left(\frac{E_1-E_0}{N_1-N_0}\right) = \cot^{-1}\left(\frac{N_1-N_0}{E_1-E_0}\right) \tag{2.07}$$

and

$$d\alpha'' = -P_{0.1}\,.\,dE_0 - Q_{0.1}\,.\,dN_0 + P_{0.1}\,.\,dE_1 + Q_{0.1}\,.\,dN_1 \tag{2.08}$$

where

$$P_{0.1} = (N_1-N_0)/D^2_{0.1}\sin 1''; \quad Q_{0.1} = -(E_1-E_0)/D^2_{0.1}\sin 1''$$

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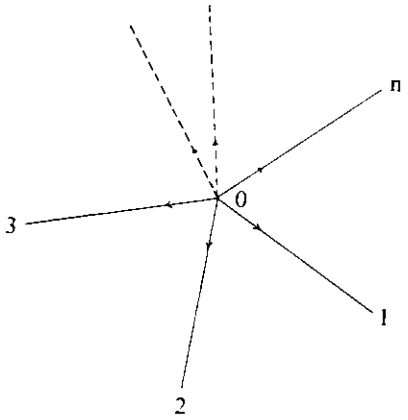

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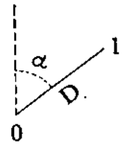

FIG. 2.3. Plane grid bearing

Then

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and

$$d\alpha'' = -P_{0.1}\,.\,dE_0 - Q_{0.1}\,.\,dN_0 + P_{0.1}\,.\,dE_1 + Q_{0.1}\,.\,dN_1 \tag{2.08}$$

where

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Consider now the case where a set of observations has been taken from point 0 to points 1, 2, 3, . . . n (Fig. 2.4). Assume that approximate rectangular co-ordinates E, N, are available for

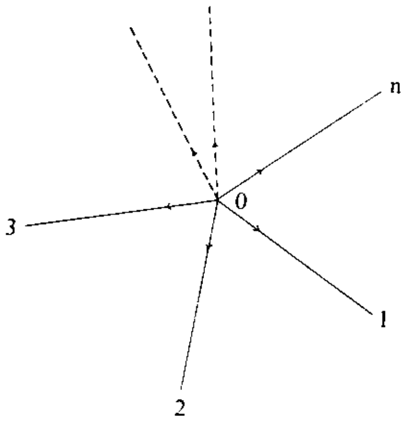


FIG. 2.4. Plane observations

the $(n+1)$ points, and that the plane National Grid bearings $0 \rightarrow 1$, $0 \rightarrow 2, \ldots 0 \rightarrow n$ have been computed from the co-ordinates using equation (2.07). Denote the computed bearings by α_c. The set of observations at 0, having been reduced to the projection by the $(t-T)$ correction, is now converted to a set of plane grid bearings by adding a constant, Z_0, to each observation at the station. The bearings derived in this way from the observations are denoted by α_o. The constant Z is usually found by assuming that α_c equals α_o on one particular line from the station—preferably the longest line; this gives the Z for that station.

In general, on any line α_o will not be equal to α_c, and the discrepancy will be due to:

(*a*) Random observational errors reflected in α_o.
(*b*) Error $(-\mathrm{d}Z_0)$ in the assumed orientation constant Z_0.
(*c*) Errors $(-\mathrm{d}E,\ -\mathrm{d}N)$ in the assumed rectangular co-ordinates of the points.

From (2.08):

$$\mathrm{d}\alpha'' = (\alpha_o-\alpha_c)'' = -P_{0.1}\,.\,\mathrm{d}E_0-Q_{0.1}\,.\,\mathrm{d}N_0+P_{0.1}\,.\,\mathrm{d}E_1+Q_{0.1}\,.\,\mathrm{d}N_1$$

But the correct value of α_o is $(\alpha_o+\mathrm{d}Z_o)$, therefore

$$\mathrm{d}\alpha'' = (\alpha_o+\mathrm{d}Z_0-\alpha_c)'' = -P_{0.1}\,.\,\mathrm{d}E_0-Q_{0.1}\,.\,\mathrm{d}N_0+P_{0.1}\,.\,\mathrm{d}E_1+Q_{0.1}\,.\,\mathrm{d}N_1$$

Or

$$-P_{0.1}\,.\,\mathrm{d}E_0-Q_{0.1}\,.\,\mathrm{d}N_0+P_{0.1}\,.\,\mathrm{d}E_1+Q_{0.1}\,.\,\mathrm{d}N_1-\mathrm{d}Z_0''+(\alpha_c-\alpha_o)'' = 0$$

This is the fundamental observation equation. In Fig. 2.4 there will be n such equations at point 0. Extending the principle to a network of triangulation containing m points, we have a set of such observation equations at each point. There are, then, $3m$ unknowns to find, namely $\mathrm{d}E$, $\mathrm{d}N$, and $\mathrm{d}Z$, at each of the m stations.

In practice a number of the co-ordinate values are definitive, being the results from previous adjustments, consequently the number of unknowns is invariably less than $3m$. If there are m stations of which p already have definitive co-ordinates, the number of unknowns will be $3m-2p$. This fact, coupled with the number of observations usually taken, means that the number of observation equations is always greater than $3m-2p$; the observation equations are therefore not equated to zero, but to the residual differences, v'', between the α_o bearings corrected for $\mathrm{d}Z$, and the bearings, α', computed from the final adjusted co-ordinates, that is:

$$v'' = \alpha'-(\alpha_o+\mathrm{d}Z)$$

At each point such as 0 in Fig. 2.4 we have then n observation equations, thus:

$$\left.\begin{array}{llll}
-P_{0.1}\,.\,\mathrm{d}E_0-Q_{0.1}\,.\,\mathrm{d}N_0+ & & & \\
\quad +P_{0.1}\,.\,\mathrm{d}E_1+Q_{0.1}\,.\,\mathrm{d}N_1 & & & -\mathrm{d}Z_0''+(\alpha_c-\alpha_o)_{0.1}'' = v_1'' \\
-P_{0.2}\,.\,\mathrm{d}E_0-Q_{0.2}\,.\,\mathrm{d}N_0 & +P_{0.2}\,.\,\mathrm{d}E_2+Q_{0.2}\,.\,\mathrm{d}N_2 & & -\mathrm{d}Z_0''+(\alpha_c-\alpha_o)_{0.2}'' = v_2'' \\
\ldots\ldots\ldots & \ldots\ldots\ldots & \ldots\ldots\ldots & \ldots\ldots\ldots \\
-P_{0.n}\,.\,\mathrm{d}E_0-Q_{0.n}\,.\,\mathrm{d}N_0 & & +P_{0.n}\,.\,\mathrm{d}E_n+Q_{0.n}\,.\,\mathrm{d}N_n & \\
 & & & -\mathrm{d}Z_0''+(\alpha_c-\alpha_o)_{0.n}'' = v_n''
\end{array}\right\} \quad (2.09)$$

The co-efficient of $\mathrm{d}Z_0''$ is -1, and:

$$P_{0.r} = (N_r-N_0)/D_{0.r}^2 \sin 1''; \quad Q_{0.r} = -(E_r-E_0)/D_{0.r}^2 \sin 1''$$

$$r = 1, 2, \ldots\ldots\ldots\ldots n.$$

Note that for practical purposes here:

$$D_{0.r}^2 = (E_r - E_0)^2 + (N_r - N_0)^2$$

Where a point (say q) has a fixed co-ordinate value, this value is kept unchanged by simply making $P_{0.q} \,.\, \mathrm{d}E_q$ and $Q_{0.q} \,.\, \mathrm{d}N_q$ zero in the observation equations. This does not affect dZ; an orientation correction is necessary at *all* stations. A corollary to this is that the observation equation at each end of a line between two points that are held fixed contains only one unknown, namely the dZ for the particular station. In these cases the observation equation at each end is of the form:

$$-\mathrm{d}Z'' + (\alpha_c - \alpha_o)'' = v''$$

A network thus gives rise to a system of Σn observation equations ($\Sigma n = n_1 + n_2 + \ldots + n_m$) containing $3m - 2p$ unknowns. The method of least squares gives the most likely values for the unknowns, the condition being that the sum of the squares of the residuals shall be a minimum. The least squares adjustment can be effected in the usual way by forming $3m - 2p$ normal equations from the Σn observation equations, and then solving the former for the values of the various differentials, dE, dN, and dZ''.

Schreiber's method of elimination

To reduce the number of normal equations it is possible to reduce the number of unknowns by eliminating the dZ'' terms from the solution, thus leaving $2(m-p)$ unknowns. The method is due to O. Schreiber, and the general theory has been given by Jordan(18).

Consider again the set of observation equations at (2.09) but in a more general form:

$$\left.\begin{aligned}
a_1 U_1 + b_1 U_2 + c_1 U_3 + \ldots - \mathrm{d}Z_0'' + k_1 &= v_1 \\
a_2 U_1 + b_2 U_2 + c_2 U_3 + \ldots - \mathrm{d}Z_0'' + k_2 &= v_2 \\
\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad & \\
a_n U_1 + b_n U_2 + c_n U_3 + \ldots - \mathrm{d}Z_0'' + k_n &= v_n
\end{aligned}\right\} \qquad (2.10)$$

where $U_1, \ldots U_n$, dZ'', are the unknowns.

The normal equations from (2.10) are:

$$\begin{aligned}
[aa]U_1 + [ab]U_2 + [ac]U_3 + \ldots - [a]\,\mathrm{d}Z_0'' + [ak] &= 0 \\
[ba]U_1 + [bb]U_2 + [bc]U_3 + \ldots - [b]\,\mathrm{d}Z_0'' + [bk] &= 0 \\
\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad & \\
-[a]U_1 - [b]U_2 - [c]U_3 - \ldots + n\,.\,\mathrm{d}Z_0'' - [k] &= 0
\end{aligned}$$

where

$$[aa] = a_1^2 + a_2^2 + \ldots + a_n^2, \quad [ab] = a_1b_1 + a_2b_2 + \ldots + a_nb_n,$$

and so on.

From the last normal equation:

$$\mathrm{d}Z_0'' = ([a]U_1 + [b]U_2 + [c]U_3 + \ldots + [k])/n \qquad (2.11)$$

Substituting from (2.11) for dZ_0'' in the other normal equations:

$$\left.\begin{aligned}\left([aa]-\frac{[a][a]}{n}\right)U_1+\left([ab]-\frac{[a][b]}{n}\right)U_2+\ldots+\left([ak]-\frac{[a][k]}{n}\right)&=0\\\left([ba]-\frac{[b][a]}{n}\right)U_1+\left([bb]-\frac{[b][b]}{n}\right)U_2+\ldots+\left([bk]-\frac{[b][k]}{n}\right)&=0\end{aligned}\right\}\qquad(2.12)$$

and so on.

Now re-write the equations at (2.10) thus:

$$\left.\begin{aligned}a_1U_1+b_1U_2+c_1U_3+\ldots+k_1&=v_1+dZ_0''\\a_2U_1+b_2U_2+c_2U_3+\ldots+k_2&=v_2+dZ_0''\\\ldots\ldots\ldots\ldots\ldots\ldots&\\a_nU_1+b_nU_2+c_nU_3+\ldots+k_n&=v_n+dZ_0''\end{aligned}\right\}\qquad(2.13)$$

Adding these equations, each of which has unit weight, a fictitious equation can be obtained with weight $-1/n$, and of the form:

$$i\frac{[a]}{\sqrt{n}}\,.\,U_1+i\frac{[b]}{\sqrt{n}}\,.\,U_2+i\frac{[c]}{\sqrt{n}}\,.\,U_3+\ldots+i\frac{[k]}{\sqrt{n}}=i\sqrt{n}\,.\,dZ_0''\qquad(2.14)$$

where $i^2=-1$, and $[v]=0$. The equality $[v]=0$ can be proved as follows.

Write the observation equations at (2.09) thus:

$$\begin{aligned}M_1-dZ_0''&=v_1\\M_2-dZ_0''&=v_2\\\ldots\ldots&\\M_n-dZ_0''&=v_n\end{aligned}$$

Where M includes all the left hand side of the observation equation except for dZ_0''.

Then

$$[M]-n\,dZ_0''=[v]_0$$

But from (2.11)

$$\begin{aligned}n\,dZ_0''&=[a]U_1+[b]U_2+\ldots+[k]\\&=[M]\end{aligned}$$

So

$$[M]-n\,dZ_0''=[v]_0=0$$

Adding (2.14) to (2.13) gives:

$$\left.\begin{aligned} a_1U_1+b_1U_2+c_1U_3+\ldots+k_1 &= v_1+\mathrm{d}Z_0'' \\ a_2U_1+b_2U_2+c_2U_3+\ldots+k_2 &= v_2+\mathrm{d}Z_0'' \\ \ldots\ldots\ldots\ldots\ldots\ldots & \ldots\ldots \\ a_nU_1+b_nU_2+c_nU_3+\ldots+k_n &= v_n+\mathrm{d}Z_0'' \\ i\frac{[a]}{\sqrt{n}}\,.\,U_1+i\frac{[b]}{\sqrt{n}}\,.\,U_2+i\frac{[c]}{\sqrt{n}}\,.\,U_3+\ldots+i\frac{[k]}{\sqrt{n}} &= i\sqrt{n}\,.\,\mathrm{d}Z_0'' \end{aligned}\right\} \tag{2.15}$$

The normal equations obtained from (2.15) are identical with those at (2.12).

Practical procedure

This modification to reduce the number of normal equations is used as standard practice by the Ordnance Survey. The observation equations as at (2.09) are stated for each point but the $\mathrm{d}Z''$ are omitted, and a fictitious observation equation as at (2.14) is formed additionally at each point. Formation of the normal equations is carried out in the usual way, except that when computing the squares or products of terms in the fictitious equations, the operator, i, effects a sign change since $i^2 = -1$. Solution of the normal equations gives the various values for $\mathrm{d}E$, $\mathrm{d}N$, and these are substituted in the observation equations (2.15) at each station to get $(v+\mathrm{d}Z'')$. Substituting the values for $\mathrm{d}E$, $\mathrm{d}N$, in the fictitious equation (2.14) at each station (ignoring the operator i) and multiplying by the appropriate $1/\sqrt{n}$ gives the values of $\mathrm{d}Z''$. (See equation (2.11)). Knowing the $\mathrm{d}Z''$, the values of v can now be found, then:

$$[v]_1 = [v]_2 = \ldots = [v]_m = 0$$

summation being at each of the m stations.

The final data required from an adjustment are:

Residuals $= v$

Plane Adjusted Bearings $= \alpha_o+v+\mathrm{d}Z'' = \alpha'$

Adjusted Co-ordinates $= E+\mathrm{d}E,\ N+\mathrm{d}N = E',\ N'$

where E, N, are the approximate values.

An arithmetic check on the internal consistency of the final data is as follows.
The adjusted co-ordinates are E', N', and the plane adjusted bearing is α'.
Then on any line:

$$\left.\begin{aligned} & E_1'+(N_2'-N_1')\tan\alpha_{1.2}'-E_2' = 0 \\ \text{Or}\qquad & \\ & N_1'+(E_2'-E_1')\cot\alpha_{1.2}'-N_2' = 0 \end{aligned}\right\} \tag{2.16}$$

This is done at each end of every line, taking the tan or cot, whichever is less than unity.

2.242 CONDITION EQUATIONS

Appendices 1 to 4 list in symbolic form the condition equations for Figures 1 to 4 respectively. In these lists the following notation is used.

Condition of angular closure

The condition is that the sum of the three angles of a triangle should equal $180^\circ + \epsilon$, and this is written by giving the three diagram letters of the stations forming the triangle. Occasionally the figure of closure is a polygon, and the letters are used to delineate the polygon in a similar way to the triangle. In this case the condition is that the sum of the n interior angles should equal $180^\circ . n - 360^\circ + \epsilon$.

Condition of side closure

In a closed polygon ABCD (see Fig. 2.5) the condition is:

$$\frac{\sin(ACB - \epsilon/3)}{\sin(ABC - \epsilon/3)} \cdot \frac{\sin(ADC - \epsilon/3)}{\sin(ACD - \epsilon/3)} \cdot \frac{\sin(ABD - \epsilon/3)}{\sin(ADB - \epsilon/3)} = 1$$

where ϵ is the spherical excess in the particular triangle indicated by the three letters of the angle.

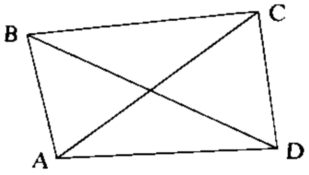


FIG. 2.5. Closed polygon

This condition is written thus:

A(BCD)

and A is called the pole.

The condition equation can be formed automatically using this notation by starting with the line AB radiating from the pole and working round in a cycle, thus:

$$\frac{\overset{\hat{C}}{A\quad B}}{\underset{\hat{B}}{A\quad C}} \cdot \frac{\overset{\hat{D}}{A\quad C}}{\underset{\hat{C}}{A\quad D}} \cdot \frac{\overset{\hat{B}}{A\quad D}}{\underset{\hat{D}}{A\quad B}}$$

Quite often it is preferable to put the pole at the intersection of two lines. See Fig. 2.6. In this case the pole is not a station, but the cyclic procedure described above gives the equation without ambiguity. When the intersection of two lines is used as the pole, the latter is indicated by a letter x (lower case), thus:

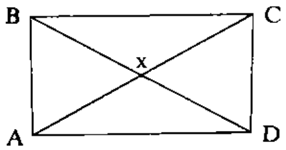


FIG. 2.6. Polygon with pole at intersection of diagonals

x(ABCD)

denoting:

$$\frac{\sin(xBA-\epsilon/3)}{\sin(xAB-\epsilon/3)}\cdot\frac{\sin(xCB-\epsilon/3)}{\sin(xBC-\epsilon/3)}\cdot\frac{\sin(xDC-\epsilon/3)}{\sin(xCD-\epsilon/3)}\cdot\frac{\sin(xAD-\epsilon/3)}{\sin(xDA-\epsilon/3)}=1$$

When two side lengths having a common terminal are to be held fixed from a previous adjustment, a condition of fixed side closure is required; this condition is stated as described above but is qualified, and has a slightly different interpretation. In these cases the pole is always the terminal common to the two adjacent fixed side lengths. See Fig. 2.7. The condition of fixed side closure is written thus:

A(BCD) - - - - - - Fixed Sides

where AB and AD are the two fixed lengths.

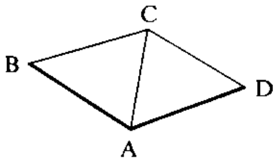


FIG. 2.7. Fixed sides

This equation then denotes:

$$\frac{\sin(ACB-\epsilon/3)}{\sin(ABC-\epsilon/3)}\cdot\frac{\sin(ADC-\epsilon/3)}{\sin(ACD-\epsilon/3)}\cdot\frac{AD}{AB}=1$$

Sometimes it is necessary to compute and use an artificial direction; that is, a direction which has not been observed but which is necessary for the purpose of stating a condition of side closure. In this case the requisite equation including the artificial direction is stated in the usual way, and a redundant equation including the artificial direction is also stated. When the two equations are evaluated, they are combined in such a way that the coefficients for the artificial direction are eliminated, and a single combined condition of side closure is obtained. Such pairs which are to be combined are qualified in the equation list, the redundant equation being called the 'eliminator for the artificial direction'.

2.243 OVERLAP

A feature of the adjustment of the primary figures was the overlap whereby the adjustment of a figure included part of a subsequent figure. Before the subsequent figure was adjusted the directions included in the overlap with the previous figure were altered slightly from the results of the previous adjustment. The subsequent adjustment was then done and definitive values obtained for the overlapping directions.

The method of dealing with an overlap is illustrated below in § 2.26. As all overlap corrections were computed in the same way, only one detailed example is given. In the overlap between Figures 1 and 2, the overlap corrections were applied only to directions actually included in the overlap; in subsequent work the overlap corrections were applied to *all* directions from the edge stations, except for those that were held fixed.

2.244 DIAGRAMS OF THE ADJUSTMENTS

Diagrams 5 to 13 show the figures as adjusted. These diagrams make clear at a glance the following distinctions.

(*a*) *Heavy lines*. These indicate definitive data from previous adjustments; that is, data which remained unchanged by the adjustment.

(*b*) *Medium lines*. These indicate directions which received definitive values in the adjustment.

(*c*) *Light lines*. These indicate overlapping directions which were included in the adjustment but which received definitive values in a subsequent adjustment.

2.25 New Primary Figure 1

This figure contained 28 stations, of which 4 were intersected, that is they were not occupied for observation. There was a total of 78 lines, of which 16 were observed in one direction only, thus giving 140 directions for which adjustment corrections were required. The figure was adjusted using condition equations, of which there were 39 for angular closure and 25 for side closure. Being the first figure, the adjustment gave shape only,—there were no fixed conditions. Approximate scale for spherical excess was obtained by accepting the old Principal Triangulation side length Dunnose (10) to Beacon Hill (15).

The data for Figure 1 are given at Appendix 1. These consist of the condition equations, the processed mean observed directions, the adjustment corrections and adjusted directions, the triangle misclosures and spherical excesses. See Diagram 5 for the diagram of the adjustment. For statistical details see Table 2.2. Standard errors were computed from the adjustment corrections.

The formulae used were:

$$\text{Standard error of an observed direction of unit weight} = \sqrt{(\Sigma v^2/n_c)} = \sigma_o$$

$$\text{Standard error of an adjusted direction of unit weight} = \sigma_o\sqrt{\{(n_o - n_c)/n_o\}}$$

where v is the adjustment correction, n_o is the number of observed directions and n_c is the number of conditions. Note that 'average' is used to denote a mean taken without regard to sign, whereas 'mean' has the usual algebraic connotation.

After the adjustment was completed, the adjusted angles were used to calculate the lengths of all the sides in Figure 1, starting with the old side length Dunnose (10) to Beacon Hill (15) taken from the Principal Triangulation. These two new stations were coincident with the old ones. The common logarithm of this initial length in feet was:

Log length Dunnose (10)—Beacon Hill (15) 5·378 64326

(See *Account of the Principal Triangulation*⁽¹⁾, p. 469).

These lengths will be referred to hereafter as provisional new side lengths.

2.26 New Primary Figure 2

There were 46 stations in this figure, of which 12 were intersected. There were 129 lines altogether, of which 2 were fixed from Figure 1, and 40 were observed only in one direction. This gave 214 directions for which adjustment corrections were required. The figure contained 97 condition equations, comprising 56 for angular closure, 40 for side closure, and 1 for fixed side closure. For statistical details see Table 2.2.

Fig. 2.8 shows the northern edge of the adjustment of primary Figure 1, with the primary Figure 2 overlap in dotted lines. The heavy lines show the sides which were held fixed in the adjustment

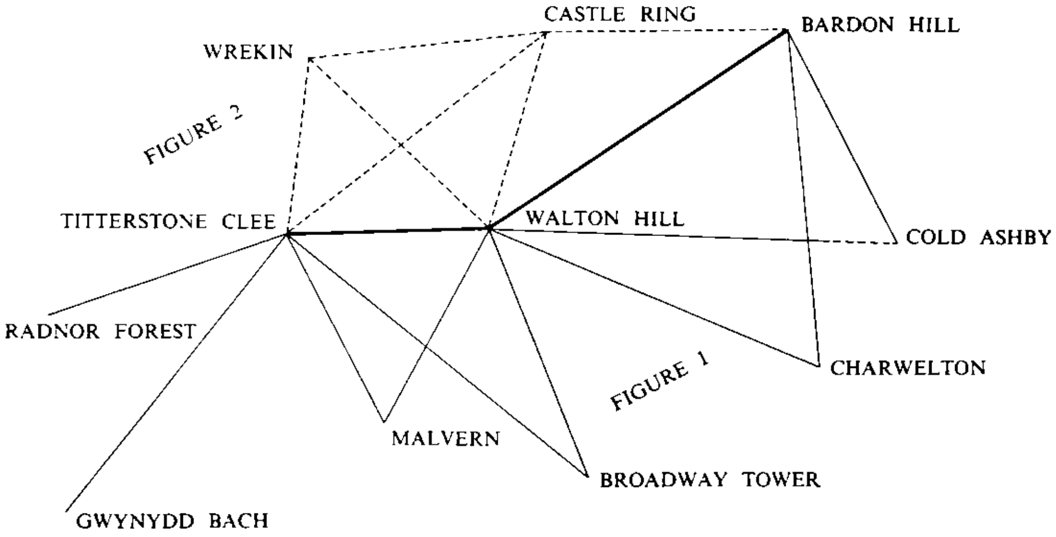


FIG. 2.8. Overlap between Figures 1 and 2

of Figure 2. Although the adjustment of Figure 1 gave corrections to all the directions shown in Fig. 2.8, the corrections to the directions shown in dotted lines were ignored, and the following procedure was adopted to give data for the adjustment of Figure 2. In the particular case of Figure 2, overlap corrections were only applied to directions which were included in the adjustment of Figure 1. Other directions from the edge stations into Figure 2 were unaffected.

At Wrekin (63) *and Castle Ring* (60)

Processed mean observed directions were used in the adjustment of Figure 2.

At Titterstone Clee (62)

The adjusted direction to Walton Hill (61) from Figure 1 was held fixed in the adjustment of Figure 2.

The mean of the adjustment corrections from Figure 1 to the directions to:

Walton Hill (61)
Broadway Tower (91)
Malvern (79)
Gwynydd Bach (72)
Radnor Forest (71)

was applied to the processed mean observed directions to Wrekin (63) and Castle Ring (60). These were then used as unadjusted directions in the adjustment of Figure 2.

At Walton Hill (61)

The adjusted directions to Titterstone Clee (62) and Bardon Hill (58) from Figure 1 were held fixed in the adjustment of Figure 2. The mean of the adjustment corrections from Figure 1 to the directions to:

Bardon Hill (58)
Cold Ashby (76)
Charwelton (78)
Broadway Tower (91)
Malvern (79)
Titterstone Clee (62)

was applied to the processed mean observed directions to Wrekin (63) and Castle Ring (60). These were then used as unadjusted directions in the adjustment of Figure 2.

At Bardon Hill (58)

The adjusted direction to Walton Hill (61) from Figure 1 was held fixed in the adjustment of Figure 2.

The mean of the adjustment corrections from Figure 1 to the directions to:

Cold Ashby (76)
Charwelton (78)
Walton Hill (61)

was applied to the processed mean observed direction to Castle Ring (60). This was then used as an unadjusted direction in the adjustment of Figure 2.

Directions corrected in this way for overlap are given in Appendix 2, together with the directions that were held fixed, processed mean observed directions, adjustment corrections, and adjusted directions. Appendix 2 also contains the condition equations, triangle misclosures and spherical excesses. See Diagram 6 for the diagram of Figure 2.

When Figure 2 had been adjusted, the adjusted angles were used to calculate provisional new side lengths, starting at the fixed edge from Figure 1. At this stage Figures 1 and 2 formed a self-consistent network with scale derived from the Dunnose (10)–Beacon Hill (15) side of the old Principal Triangulation.

2.27 Deriving Position, Orientation, and Scale, for the New Work

For the reasons stated in § 2.01 it was decided to position, orient, and scale the new work as closely as possible to the old, while keeping the shape of the new work undisturbed.

Eleven stations in the new Figures 1 and 2 had been sited precisely over stations of the old Principal Triangulation, and these were used to find the most probable fit of the new work on to the old. The 11 coincident stations were:

Bardon Hill (58)
Beacon Hill (15)
Butser (9)
Coringdon (11)
Dunnose (10)
Great Whernside (7)
Holme Moss (26)
Inkpen (33)
Malvern (79)
Rombalds Moor (70)
White Horse Hill (34)

Fig. 2.9 shows the distribution of these stations in the new Figures 1 and 2.

Provisional new geographical co-ordinates for these 11 stations were calculated by accepting:

(*a*) The geographical co-ordinates of Butser (9) in the old Principal Triangulation, namely:

$$\varphi\ 50^\circ\ 58'\ 38''.233, \quad \lambda\ -00^\circ\ 58'\ 43''.780$$

(*b*) The azimuth from Butser (9) to Beacon Hill (15) in the old Principal Triangulation, namely:

$$294^\circ\ 03'\ 09''.619$$

(*c*) The provisional new side lengths obtained from the adjustments of Figures 1 and 2, the scale having been taken from the old Principal Triangulation side length Dunnose (10) to Beacon Hill (15).

The following formulae were used:

$$\left.\begin{aligned}
\varphi_m &= \varphi_1 + \frac{S_{1.2}\cos A_{1.2}}{2\rho_1 \sin 1''} \\
\epsilon &= \frac{S_{1.2}^2 \sin A_{1.2}\cos A_{1.2}}{2\rho_m \nu_m \sin 1''} \\
\varphi_F &= \varphi_1 + \frac{S_{1.2}\cos(A_{1.2}-2\epsilon/3)}{\rho_m \sin 1''} \\
n &= \frac{S_{1.2}^2 \sin^2(A_{1.2}-\epsilon/3)\tan\varphi_F}{2\rho_F \nu_F \sin 1''} \\
\varphi_2 &= \varphi_F - n \\
\lambda_2-\lambda_1 &= \frac{S_{1.2}\sin(A_{1.2}-\epsilon/3)}{\cos(\varphi_F-2n/3)\nu_F \sin 1''} \\
A_{2.1} &= 180^\circ + A_{1.2} + (\lambda_2-\lambda_1)\sin(\varphi_F - n/3) - \epsilon
\end{aligned}\right\} \tag{2.17}$$

where φ_1, λ_1, $A_{1.2}$, $S_{1.2}$, are the known data, and φ_2, λ_2, $A_{2.1}$ are to be found. Eight-figure logarithms were used. The above formulae use the azimuth and length of the geodesic, not the normal section. Provisional new values and the comparable old Principal Triangulation values, were:

Station	*Provisional New Values*		*Old Principal Triangulation Values*	
	φ	λ	φ	λ
Bardon Hill (58)	52° 42′ 50″6764	−01° 19′ 08″7280	52° 42′ 50″754	−01° 19′ 08″751
Beacon Hill (15)	51 10 59·2320	−01 43 15·5042	51 10 59·233	−01 43 15·506
Butser (9)	As old value		50 58 38·233	−00 58 43·780
Coringdon (11)	50 37 47·2563	−01 59 17·5756	50 37 47·246	−01 59 17·568
Dunnose (10)	50 37 03·7491	−01 11 50·1290	50 37 03·748	−01 11 50·136
Great Whernside (7)	54 09 38·6255	−01 59 48·8144	54 09 38·809	−01 59 48·899
Holme Moss (26)	53 32 18·4700	−01 52 55·3051	53 32 18·628	−01 52 55·341
Inkpen (33)	51 21 07·0743	−01 27 49·1595	51 21 07·081	−01 27 49·157
Malvern (79)	52 06 15·7832	−02 20 15·1689	52 06 15·817	−02 20 15·210
Rombalds Moor (70)	53 54 10·0879	−01 49 31·5240	53 54 10·257	−01 49 31·591
White Horse Hill (34)	51 34 29·8417	−01 33 57·0240	51 34 29·8555	−01 33 57·0383

Although Rombalds Moor (70) was observed as a primary station in the old Principal Triangulation (and called Rumbles Moor), it was not included in the adjustment of the old work by A. R. Clarke. In 1929, Figure 21 of the old Principal Triangulation was re-adjusted(19), and the re-adjustment included five extra stations which had been omitted by Clarke in the original adjustment. Full details of the re-adjustment are not now available, but it appears certain that Rombalds Moor (70) was one of the five additional stations, and that the old Principal Triangulation geographical co-ordinates of this station given above came from the re-adjustment of the old Figure 21 done in 1929.

The accepted value shown above for White Horse Hill (34) from the old Principal Triangulation is given to four decimal places and differs very slightly from the original published value which was φ 51° 34′ 29″ 856 λ −01° 33′ 57″038. This trivial revision has resulted apparently from meaning several values subsequently computed from the original adjusted data.

The differences between the old and new values shown in the table above were used in a simple least squares adjustment to obtain the most likely values for small differential changes to the old Principal Triangulation position of Butser (9), the old Principal Triangulation azimuth from Butser (9) to Beacon Hill (15), and a small proportional scale change to all provisional new side lengths.

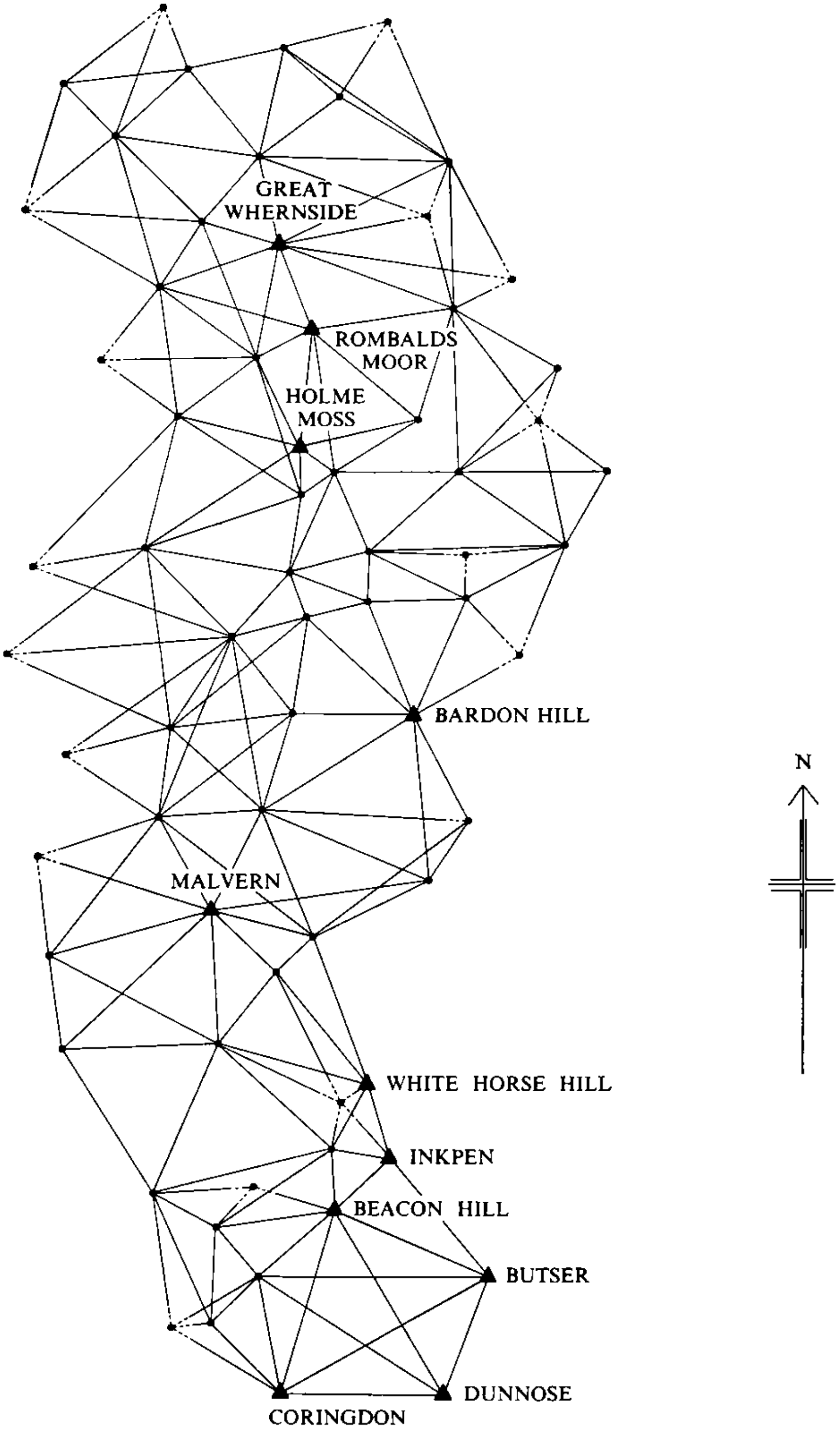


FIG. 2.9. Distribution in new Figures 1 and 2 of the eleven stations common to the old Principal Triangulation and the new primary Retriangulation

Fig. 2.10 shows a spherical triangle formed by the pole and two points with geographical co-ordinates $\varphi_1\lambda_1$, $\varphi_2\lambda_2$.

$\omega = \lambda_2 - \lambda_1$

A_1 = Azimuth at point 1 of the spherical direction 1 to 2

A_2 = Azimuth at point 2 of the spherical direction 1 to 2

By spherical trigonometry:

$$\sin\varphi_2 = \cos A_1 \,.\, \sin S \,.\, \cos\varphi_1 + \cos S \,.\, \sin\varphi_1$$

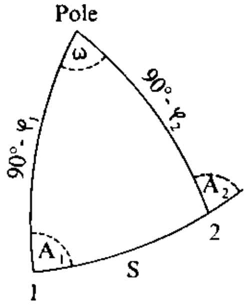


Fig. 2.10. Spherical triangle

Taking partial derivatives with respect to φ_1, A_1, and S, it can be shown that:

$$\frac{\partial\varphi_2}{\partial\varphi_1} = \cos\omega; \quad \frac{\partial\varphi_2}{\partial A_1} = -\sin A_2 \,.\, \sin S; \quad \frac{\partial\varphi_2}{\partial S} = \cos A_2$$

$$\therefore \quad d\varphi_2 = \cos\omega \,.\, d\varphi_1 - \sin A_2 \,.\, \sin S \,.\, dA_1 + \cos A_2 \,.\, dS$$

Let dS' be the proportional change in S, then:

$$dS = S \,.\, dS'$$

$$\therefore \quad d\varphi_2 = \cos\omega \,.\, d\varphi_1 - \sin A_2 \,.\, \sin S \,.\, dA_1 + S \,.\, \cos A_2 \,.\, dS' \qquad (2.18)$$

Also:

$$\cot\omega = (\cot S \,.\, \cos\varphi_1 - \sin\varphi_1 \,.\, \cos A_1)/\sin A_1$$

Taking partial derivatives with respect to φ_1, A_1, and S, it can be shown that:

$$\frac{\partial\omega}{\partial\varphi_1} = \sin\omega \,.\, \tan\varphi_2; \quad \frac{\partial\omega}{\partial A_1} = \sin\omega \,.\, \operatorname{cosec} A_1 \,.\, \cos A_2;$$

$$\frac{\partial\omega}{\partial S} = \sin A_2 \,.\, \sec\varphi_2$$

$$\therefore \quad d\omega = \sin\omega \,.\, \tan\varphi_2 \,.\, d\varphi_1 + \sin\omega \,.\, \operatorname{cosec} A_1 \,.\, \cos A_2 \,.\, dA_1 + S \,.\, \sin A_2 \,.\, \sec\varphi_2 \,.\, dS'$$

But

$$d\omega = d\lambda_2 - d\lambda_1$$

$$\therefore \quad d\lambda_2 = d\lambda_1 + \sin\omega \,.\, \tan\varphi_2 \,.\, d\varphi_1 + \sin\omega \,.\, \operatorname{cosec} A_1 \,.\, \cos A_2 \,.\, dA_1 + S \,.\, \sin A_2 \,.\, \sec\varphi_2 \,.\, dS' \qquad (2.19)$$

Equations (2.18) and (2.19) show the effects on φ_1, λ_1, A_1, and S, of small changes in φ_2 and λ_2, and are the observation equations used in the adjustment of the new triangulation to the old.

With Butser (9) and the pole constant, the remaining ten common stations tabulated above each gave a spherical triangle like Fig. 2.10 in which Butser (9) was point 1. Using the provisional new geographical co-ordinates each spherical triangle was solved to find A_1, A_2, and S, from φ_1, φ_2, and ω. This gave the necessary coefficients in the observation equations (2.18) and (2.19). Since the intention was to fit the new work to the old, the differences in latitude and longitude were taken as old minus new, that is:

$$d\varphi_2 = \varphi_2(\text{Old}) - \varphi_2(\text{New});\quad d\lambda_2 = \lambda_2(\text{Old}) - \lambda_2(\text{New})$$

The 10 common stations gave 20 observation equations of the form of (2.18) and (2.19). Four normal equations were formed and solved for $d\varphi_1$, $d\lambda_1$, dA_1, and dS'. The corrections $d\varphi_1$, $d\lambda_1$, were applied to the old value of Butser (9) given in (*a*) above, dA_1 to the old azimuth given in (*b*) above, and all provisional new side lengths were multiplied by $(1+dS')$. All the provisional new side lengths were actually in logarithmic form, so the scale correction was applied by adding $\log(1+dS')$ to all the log lengths. This gave final new log side lengths.

Numerical details of the adjustment are given below.

Observation Equations (2.18) *and* (2.19)

Station	$d\lambda_1$	$d\varphi_1$	dA_1	dS'	$-$(*Old* $-$ *New*)"	*Residual* v''
	$-$	$\cos\omega$	$-\sin A_2 \,.\, \sin S$	$S \,.\, \cos A_2$	$-d\varphi_2$	
	$+1$	$\sin\omega \,.\, \tan\varphi_2$	$\sin\omega \,.\, \operatorname{cosec} A_1 \,.\, \cos A_2$	$S \,.\, \sin A_2 \,.\, \sec\varphi_2$	$-d\lambda_2$	
Bardon Hill (58)	0	+0·999 982	+0·003 739	+0·625 039	−0·0776	+0·0176
	+1	−0·007 800	+0·050 019	−0·127 331	+0·0230	+0·0123
Beacon Hill (15)	0	+0·999 916	+0·008 156	+0·073 250	−0·0010	+0·0072
	+1	−0·016 102	+0·005 666	−0·268 353	+0·0018	−0·0080
Coringdon (11)	0	+0·999 845	+0·011 091	−0·126 663	+0·0103	−0·0137
	+1	−0·021 470	−0·009 681	−0·360 677	−0·0076	−0·0238
Dunnose (10)	0	+0·999 992	+0·002 401	−0·129 520	+0·0011	−0·0190
	+1	−0·004 642	−0·009 892	−0·078 032	+0·0070	+0·0343
Great Whernside (7)	0	+0·999 842	+0·011 187	+1·144 394	−0·1835	−0·0123
	+1	−0·024 600	+0·094 707	−0·394 307	+0·0846	+0·0101
Holme Moss (26)	0	+0·999 876	+0·009 925	+0·920 741	−0·1580	−0·0205
	+1	−0·021 336	+0·075 099	−0·344 596	+0·0359	−0·0209
Inkpen (33)	0	+0·999 964	+0·005 327	+0·134 532	−0·0067	+0·0123
	+1	−0·010 579	+0·010 441	−0·175 959	−0·0025	−0·0006
Malvern (79)	0	+0·999 719	+0·014 929	+0·402 889	−0·0338	+0·0216
	+1	−0·030 462	+0·031 795	−0·501 412	+0·0411	−0·0178
Rombalds Moor (70)	0	+0·999 891	+0·009 303	+1·052 058	−0·1691	−0·0111
	+1	−0·020 264	+0·086 533	−0·325 846	+0·0670	+0·0072
White Horse Hill (34)	0	+0·999 948	+0·006 450	+0·214 642	−0·0138	+0·0170
	+1	−0·012 913	+0·016 741	−0·214 100	+0·0143	+0·0072

The observation equations tabulated here have been equated to the residual, v.
S is in seconds of arc multiplied by 10^{-4}.

The normal equations formed from these observation equations were:

$$
\begin{aligned}
+10{\cdot}000\,000\,.\,d\lambda_1 - 0{\cdot}170\,168\,.\,d\varphi_1 + 0{\cdot}351\,428\,.\,dA_1 - 2{\cdot}790\,613\,.\,dS' + 0{\cdot}264\,600 &= 0\\
-\ 0{\cdot}170\,168\,.\,d\lambda_1 + 10{\cdot}001\,430\,.\,d\varphi_1 + 0{\cdot}075\,288\,.\,dA_1 + 4{\cdot}367\,802\,.\,dS' - 0{\cdot}637\,715 &= 0\\
+\ 0{\cdot}351\,428\,.\,d\lambda_1 + 0{\cdot}075\,288\,.\,d\varphi_1 + 0{\cdot}027\,035\,.\,dA_1 - 0{\cdot}075\,346\,.\,dS' + 0{\cdot}013\,186 &= 0\\
-\ 2{\cdot}790\,613\,.\,d\lambda_1 + 4{\cdot}367\,802\,.\,d\varphi_1 - 0{\cdot}075\,346\,.\,dA_1 + 4{\cdot}852\,593\,.\,dS' - 0{\cdot}692\,888 &= 0
\end{aligned}
$$

which gave:

$$
\begin{aligned}
d\lambda_1 &= +0{\cdot}034\,306; & dA_1 &= -0{\cdot}508\,401\\
d\varphi_1 &= +0{\cdot}001\,066; & dS'\,.\,10^4 &= +0{\cdot}153\,662\\
& & \therefore\ dS' &= +0{\cdot}000\,015\,3662\\
& & \mathrm{Log}(1+dS') &= +0{\cdot}000\,006\,67
\end{aligned}
$$

Applying these corrections:

Butser (9)	φ	50° 58′ 38″·2330	λ	− 00° 58′ 43″·7800
	$d\varphi_1$	+0·0011	$d\lambda_1$	+0·0343
Accepted new value		50° 58′ 38″·2341		− 00° 58′ 43″·7457

Azimuth Butser (9) to Beacon Hill (15)	294° 03′ 09″·6190
dA_1	−0·5084
Accepted new value	294° 03′ 09″·1106

and +0·000 006 67 was added to all the provisional new log side lengths.

Starting with the accepted new position of Butser (9), the accepted new azimuth Butser (9) to Beacon Hill (15), and the final new log side lengths, final new geographical co-ordinates were calculated for all stations in Figures 1 and 2 using formulae (2.17) above. These geographical co-ordinates were then converted to National Grid co-ordinates, this final stage being completed in April 1937.

2.28 New Primary Figure 3

There was a total of 80 stations in this figure of which 21 were intersected. The 80 stations were connected by 242 lines, of which 92 were observed in one direction only. Thirteen fully observed lines and 3 one-way lines were held fixed from Figure 2, leaving 363 directions for which adjustment corrections were required. The adjustment was done using condition equations, of which there were 89 for angular closure, 82 for side closure, and 8 for fixed side closure, making 179 altogether.

Before adjustment the overlap with Figure 2 was dealt with on the lines described in § 2.26, but *all* unfixed directions in Figure 3 at each of the edge stations received the appropriate overlap correction for the station.

For statistical details see Table 2.2.

Final side lengths were calculated using the adjusted angles and taking scale from the edge that was held fixed from the adjustment of Figure 2. Geographical co-ordinates were then computed, taking position and azimuth from the fixed edge, and using the formulae (2.17) in § 2.27. The conversion of geographical co-ordinates to National Grid co-ordinates completed the calculation of this figure in March 1938.

Full data for this figure are given in Appendix 3, together with triangle misclosures and spherical excesses. The diagram of the adjustment is shown in Diagram 7.

2.29 New Primary Figure 4

This figure contained 77 stations, of which 20 were intersected. There were 226 lines, which included 81 lines observed in one direction only, and 16 fully observed lines that were held fixed from previous adjustments. This gave 339 directions for which adjustment corrections were required. The adjustment was done using condition equations, there being 85 for angular closure, 72 for side closure, and 10 for fixed side closure, a total of 167.

The overlaps with Figures 1 and 2 were dealt with as described in § 2.26 except that *all* unfixed directions in Figure 4 at each of the edge stations received the appropriate overlap correction for the station.

For statistical details see Table 2.2.

Final side lengths in feet were calculated from the adjusted angles, taking initial scale from the edges held fixed from the adjustments of Figures 1 and 2.

This figure was the first one in which National Grid co-ordinates were calculated directly from adjusted data without the intermediate stage of geographical co-ordinates.

Using adjusted angles and final spheroidal side lengths in metres multiplied by F_0, National Grid co-ordinates were computed from the following formulae. Eight-figure logarithms were used.

$$E_2 = E_1 + S_{1.2}\sin(A_{1.2}-C_1) - \frac{S_{1.2}^2\cos^2(A_{1.2}-C_1)\,.\,y_1}{2\rho_m\nu_m} - $$

$$- \frac{S_{1.2}^2\cos^2(A_{1.2}-C_1)\,.\,S_{1.2}\sin(A_{1.2}-C_1)}{6\rho_m\nu_m} + \frac{(y_2^3-y_1^3)}{6\rho_m\nu_m}$$

$$N_2 = N_1 + S_{1.2}\cos(A_{1.2}-C_1) + \frac{S_{1.2}\cos(A_{1.2}-C_1)\,.\,y_2^2}{2\rho_m\nu_m} - \frac{S_{1.2}\cos(A_{1.2}-C_1)\,.\,S_{1.2}^2\sin^2(A_{1.2}-C_1)}{6\rho_m\nu_m}$$

$$(A_{2.1}-C_2) = (A_{1.2}-C_1) \pm 180^\circ - \left(\frac{S_{1.2}\cos(A_{1.2}-C_1)(y_1+y_2)}{2\rho_m\nu_m\sin 1''}\right)''$$

where E_1N_1, $(A_{1.2}-C_1)$, $S_{1.2}$, are known, and E_2N_2, $(A_{2.1}-C_2)$, are to be found.

$(A_{1.2}-C_1)$ = The bearing at station 1 of the geodesic from station 1 to station 2, measured 0°–360° clockwise from National Grid north. $(A_{2.1}-C_2)$ is the comparable bearing from station 2 to station 1 at station 2. (See also § 2.223 and § 2.225.)

$S_{1.2}$ = Spheroidal distance between stations 1 and 2 in international metres and multiplied by F_0.

$y = E - 400{,}000$ (metres).

Geodetic functions ρ_m and ν_m are for the mid-point (m) of the line.

Starting values of $(A-C)$ on the edge held fixed from previous adjustments were calculated from the National Grid co-ordinates of the fixed edge points using the following formulae:

$$\tan(A_{1.2}-C_1) = \frac{(y_2-y_1)-q}{(N_2-N_1)-p}$$

$$(A_{2.1}-C_2) = (A_{1.2}-C_1) \pm 180^\circ - \left(\frac{(N_2-N_1)(y_2+y_1)}{2\rho_m\nu_m \sin 1''}\right)''$$

where

$$q = -\frac{(N_2-N_1)^2 \,.\, y_1}{2\rho_m\nu_m} - \frac{(N_2-N_1)^2 \,.\, (y_2-y_1)}{6\rho_m\nu_m} + \frac{(y_2^3-y_1^3)}{6\rho_m\nu_m}$$

and

$$p = \frac{(N_2-N_1) \,.\, y_2^2}{2\rho_m\nu_m} - \frac{(N_2-N_1)(y_2-y_1)^2}{6\rho_m\nu_m}$$

The actual adjustment of Figure 4 was completed before the outbreak of the 1939/45 war, but the calculation of National Grid co-ordinates to complete the figure was done about 1941. There is no record of the actual date.

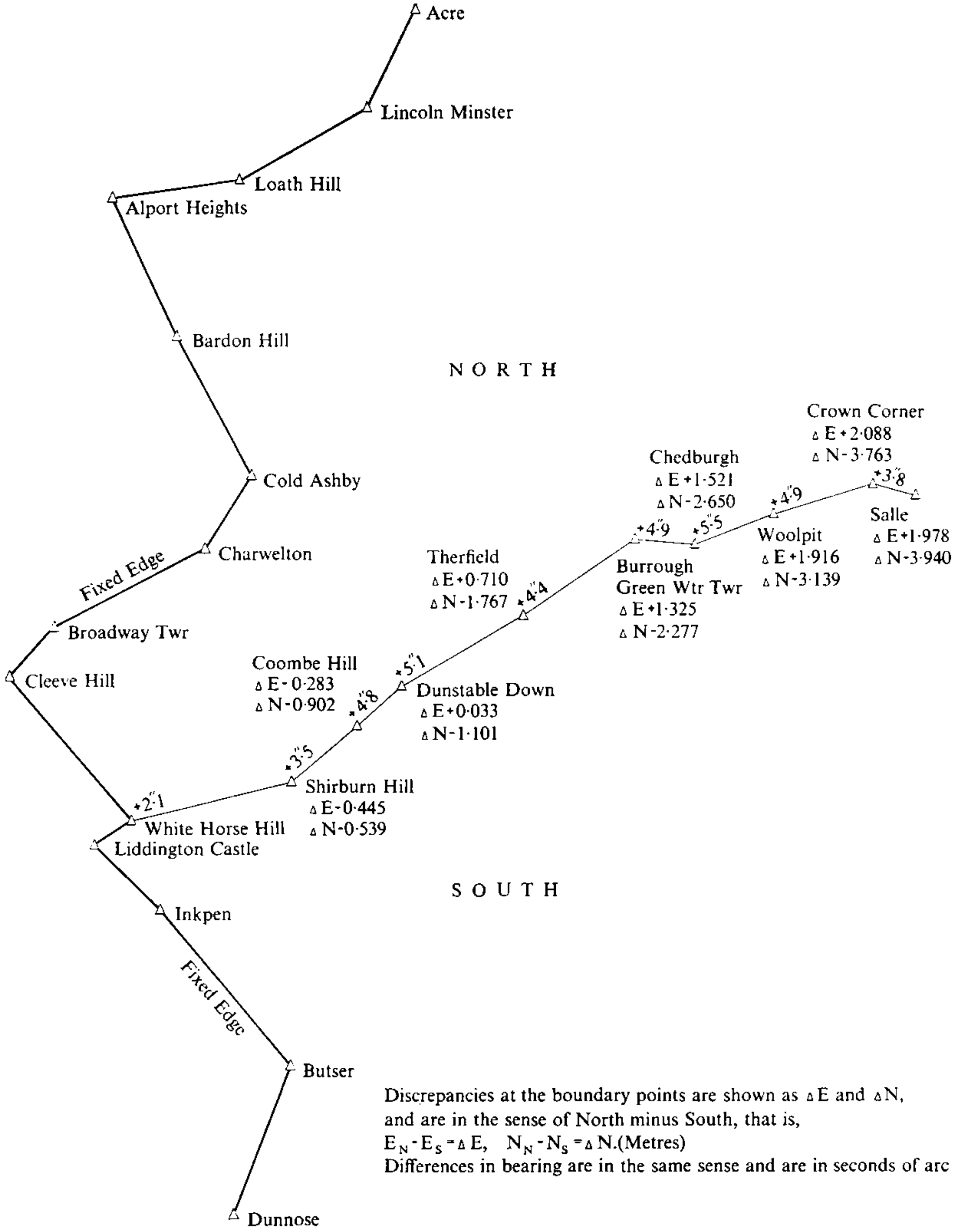
See Appendix 4 and Diagram 8 for all data relevant to Figure 4.

2.30 New Primary Figure 5

This figure contained 102 stations, of which 4 were intersected. There were 309 fully observed lines, of which 15, forming the whole western edge, were held fixed from previous adjustments; and there were 24 lines observed in one direction only. This gave 612 directions for which adjustment corrections were required. The adjustment was carried out using condition equations, of which there were 210 for angular closure, 130 for side closure, and 12 for fixed side closure, making a total of 352.

One station held fixed in this adjustment was Liddington Castle (35), which had been co-ordinated originally in Figure 1 as an intersected point. As it was to form one terminal of the Ridgeway Base, it was subsequently occupied and observations were taken to form a fully observed polygon, which was adjusted to give a new value for the station; all perimeter points of the polygon were held fixed. See § 2.33 below.

Work on the adjustment of Figure 5 started in 1939, and proceeded intermittently during the war years until the latter part of 1943. At that time plans were being considered for the re-survey of London, for which the secondary and tertiary trigonometrical control was an early requirement. The normal equations for the adjustment had been formed from the correlative equations starting from the southern edge of the figure, and by the end of 1943 some 180 of the 352 normal equations had been dealt with as far as the forward elimination of the unknowns. As the portion of the whole figure covered by these 180 equations included the London area, it was decided to terminate the southern half of the figure at a suitable boundary and complete it as a separate adjustment. The remaining equations were treated as a separate adjustment for the northern half of the figure.



Discrepancies at the boundary points are shown as ∆E and ∆N, and are in the sense of North minus South, that is, $E_N - E_S = \Delta E$, $N_N - N_S = \Delta N$. (Metres)
Differences in bearing are in the same sense and are in seconds of arc

FIG. 2.11. The discrepancies at the boundary stations between the northern and southern halves of Figure 5

The figure was not separated into two completely independent portions. Some modification of the condition equations was necessary to avoid lines crossing the selected boundary. Also, some duplication of condition equations across the boundary was accepted to avoid extensive alterations in the normal equations that had already been formed. One fully observed line had to be omitted, 7 of the original equations for side closure were cancelled and one new one was created; 10 equations for angular closure, 3 for side closure and 1 for fixed side closure were common to the adjustments of both halves of the figure.

Figure 5 North contained 116 conditions for angular closure, 60 for side closure, and 8 for fixed side closure, to find 337 adjustment corrections. Figure 5 South contained 103 conditions for angular closure, 66 for side closure, and 5 for fixed side closure, to find 317 adjustment corrections.

From the adjusted data final side lengths were calculated for both halves and National Grid co-ordinates were calculated directly using the formulae in § 2.29.

As expected, the two sets of co-ordinates for the common boundary stations disagreed. Fig. 2.11 shows the discrepancies in co-ordinates and bearings at common stations on the boundary between the northern and southern halves of the figure. A graphic method was used to distribute the discrepancies back through the two halves, and this is described below.

A weighted mean value was accepted for the co-ordinates of the common boundary stations, co-ordinates from the adjustment of the northern half being given twice the weight of co-ordinates from the adjustment of the southern half. So far as can be ascertained weights were based on the relative lengths of the fixed edges of the two halves. Accepted co-ordinates were, then:

$$E_M = (2E_N + E_S)/3; \quad N_M = (2N_N + N_S)/3$$

where suffixes M = Mean, N = North, S = South, and $E_N N_N$, $E_S N_S$, are the boundary station co-ordinates from the northern and southern adjustments respectively. Then at each boundary station there were two sets of discrepancies:

$$E_M - E_N = e; \quad N_M - N_N = n$$

to be distributed in the northern half, and

$$E_M - E_S = e'; \quad N_M - N_S = n'$$

to be distributed in the southern half.

Fig. 2.12 shows in diagrammatic form the junction of the fixed edge FPF', and the boundary P, A, B, etc., between the two halves of Figure 5. With centre at P, arcs were drawn through all the boundary stations cutting the northern and southern parts of the fixed edge in $A'A''$, $B'B''$, etc. Let Q be a point on an arc in the northern half, say on arc BB', and R a point in the southern half, say on arc AA''. Then the correction at Q was:

$$e_Q = e_B \,.\, a^2/(a^2+b^2)$$

$$n_Q = n_B \,.\, a^2/(a^2+b^2)$$

and the correction at R was:

$$e'_R = e'_A \,.\, a'^2/(a'^2+b'^2)$$

$$n'_R = n'_A \,.\, a'^2/(a'^2+b'^2)$$

a, b, a', b', being lengths of arcs as shown in Fig. 2.12.

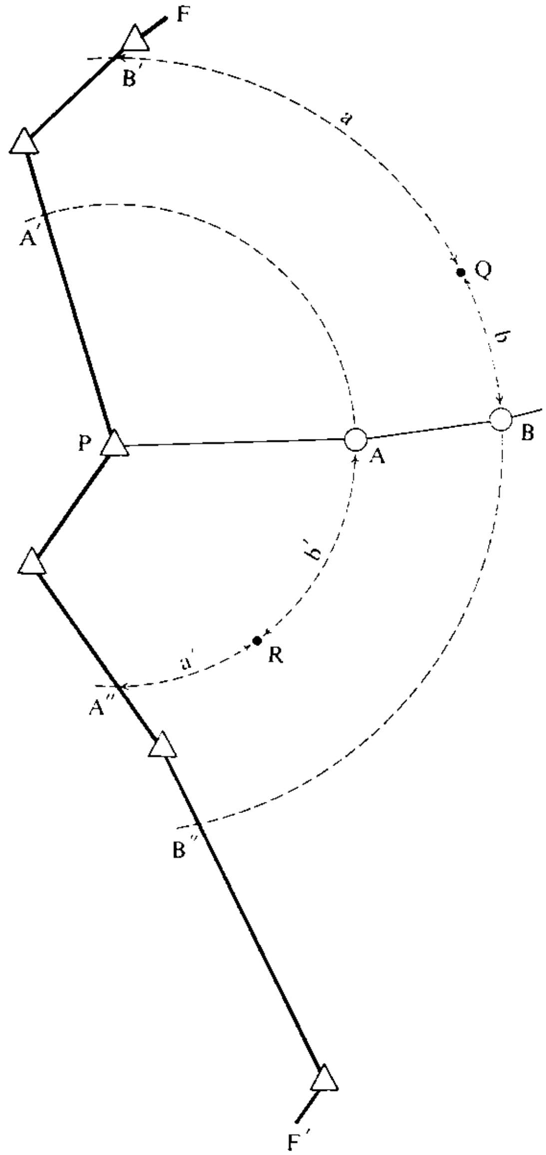


FIG. 2.12. Method of obtaining correction ratio

TABLE 2.1

NATIONAL GRID CO-ORDINATES FROM THE RE-ADJUSTMENT OF FIGURE 5 IN ONE BLOCK

N.B. These co-ordinates are not the accepted values for Figure 5; the latter values are listed in Appendix 10

Station	*Eastings (Metres)*	*Northings (Metres)*	*Difference: Readjusted value minus published value in metres*	
			Diff. E	*Diff. N*
Abberton Wtr Twr (230)	600 402·074	219 009·489	−0·699	−0·568
Beachy Head (194)	559 037·312	95 790·033	−1·065	+0·038
Belvoir Castle (81)	481 981·372	333 712·928	−0·070	−0·018
Benfleet Wtr Twr (219)	579 051·632	186 711·385	−0·680	−0·589
Bethersden Air Beacon (Int. 4)	593 123·269	140 583·440	−1·289	−0·347
Bignor Beacon (39)	496 596·931	113 116·218	−0·037	−0·058
Bolnhurst (433)	505 879·825	259 778·357	+0·209	−0·008
Boston Twr (264)	532 655·643	344 178·884	−0·141	−0·208
Brenchley Air Beacon (Int. 5)	567 964·664	142 235·861	−0·909	−0·342
Buckminster Wtr Twr (153)	488 170·119	322 950·860	−0·048	−0·048
Bunwell Ch Twr (255)	612 544·298	292 768·811	−0·620	−0·446
Burrough Green Wtr Twr (241)	563 214·520	256 399·880	+0·192	−0·235
Caister Wtr Twr (293)	651 408·445	313 177·032	−1·458	−0·239
Charnwood (57)	450 936·055	314 808·474	+0·002	0
Chedburgh (236)	578 690·701	255 856·774	+0·088	−0·359
Chipping Barnet Ch Twr (185)	524 538·568	196 462·799	+0·080	−0·389
Church Farm Wtr Twr (279)	654 026·690	294 349·481	−1·398	−0·178
Cold Harbour (266)	526 592·576	381 213·832	+0·038	−0·252
Collyweston (431)	500 078·956	303 198·991	−0·018	−0·054
Coombe Hill (204)	489 068·703	209 997·146	+0·506	+0·072
Crimplesham (424)	564 839·751	304 270·027	−0·214	−0·303
Crowborough (196)	551 168·491	130 760·932	−0·720	−0·252
Crown Corner (260)	625 513·507	270 169·751	−0·588	−0·468
Dexthorpe (265)	540 661·150	373 017·445	−0·105	−0·306
Ditchling (32)	533 162·237	113 062·943	−0·579	−0·094
Docking Ch Twr (284)	576 507·573	336 971·212	−0·437	−0·446
Dunmow (437)	564 886·594	222 349·814	−0·103	−0·592
Dunstable Down (94)	500 880·156	219 418·247	+0·433	+0·151
East Grinstead Ch Twr (170)	539 630·665	138 001·654	−0·514	−0·310
Ely Cathedral (430)	554 048·160	280 275·502	+0·021	−0·268
Epping Wtr Twr (188)	546 705·360	202 764·357	−0·081	−0·540
Fairlight Down (193)	584 338·936	111 923·175	−1·346	−0·164
Faxton (443)	480 589·647	275 413·422	+0·109	−0·020
Fayway (432)	506 679·181	278 492·651	+0·073	−0·052
Felixstowe Wtr Twr (233)	628 696·992	236 383·728	−0·977	−0·559
Firle Beacon (199)	548 556·329	105 922·217	−0·843	−0·038
Framingham (261)	626 237·276	302 646·067	−0·973	−0·348
Fransham (426)	592 507·193	310 417·631	−0·513	−0·417
Frog Hill (262)	587 199·680	291 089·757	−0·333	−0·326
Harrowby (429)	494 620·718	335 766·487	−0·070	−0·061
Helion Bumpstead (248)	562 492·920	241 622·534	+0·076	−0·408
Hindhead (31)	489 984·501	135 909·512	−0·079	−0·214

TABLE 2.1 *continued*

Station	*Eastings (Metres)*	*Northings (Metres)*	*Difference: Readjusted value minus published value in metres*	
			Diff. E	*Diff. N*
Hingham Ch Twr (287)	602 154·016	302 125·930	−0·570	−0·428
Hockley Wtr Twr (220)	582 440·788	192 207·901	−0·681	−0·597
Icomb Twr (67)	420 179·745	222 880·907	+0·055	−0·031
Ilketshall St. Andrews Ch Twr (290)	637 903·072	287 239·170	−1·042	−0·284
Kessingland Ch Twr (278)	652 764·488	286 264·713	−1·319	−0·173
Leith Hill Twr (50)	513 949·115	143 161·382	−0·166	−0·331
Lenham Wtr Twr (205)	592 573·490	152 842·290	−1·176	−0·461
Linch Ball (38)	484 804·576	117 371·720	−0·040	−0·014
Mablethorpe Wtr Twr (269)	550 554·251	384 163·677	−0·133	−0·439
Manningtree (245)	608 326·580	229 540·788	−0·722	−0·542
Maplestead (235)	583 017·110	234 470·370	−0·231	−0·527
Massingham (272)	579 482·504	320 138·656	−0·429	−0·399
Metfield (258)	631 245·127	280 008·869	−0·842	−0·375
Muswell Hill (100)	464 129·362	215 295·522	+0·279	−0·018
Nedging Tye (240)	601 971·558	249 713·375	−0·416	−0·424
North Walsham Wtr Twr (283)	627 844·956	329 200·114	−1·178	−0·391
Orford Castle (254)	641 943·393	249 878·018	−1·052	−0·438
Paddlesworth (190)	619 998·342	139 527·408	−1·669	−0·423
Peterborough Cathedral (447)	519 426·536	298 646·017	−0·010	−0·119
Piggs Grave (263)	602 652·585	332 998·083	−0·796	−0·476
Puttocks Hill (246)	589 820·093	269 582·853	−0·160	−0·311
Rollright (66)	427 878·047	229 860·003	+0·088	−0·050
Rumfields Wtr Twr (201)	637 752·413	167 766·571	−1·743	−0·620
Salle (259)	635 858·724	266 256·434	−0·810	−0·376
Selsey (47)	486 827·552	95 745·650	−0·081	−0·081
Severndroog Castle (189)	543 185·896	176 199·199	−0·247	−0·574
Shirburn Hill (207)	472 344·651	195 240·574	+0·288	−0·168
Shurland (191)	600 156·843	171 679·412	−1·127	−0·554
Sibleys Wtr Twr (434)	556 480·231	229 993·989	+0·038	−0·455
Skegness Wtr Twr (267)	555 782·563	364 407·737	−0·229	−0·388
South Lopham Ch Twr (237)	603 958·734	281 755·023	−0·442	−0·383
Southwold Ch Twr (280)	650 733·112	276 388·617	−1·196	−0·170
Stoke by Nayland Ch Twr (249)	598 596·154	236 273·391	−0·501	−0·508
Swaffham (425)	583 912·226	309 252·632	−0·418	−0·373
Swilland (244)	618 238·980	253 813·272	−0·635	−0·475
Therfield (441)	533 184·501	237 241·974	+0·326	−0·074
Tilton Pile (75)	476 739·953	305 904·118	−0·010	−0·006
Topcroft Ch Twr (296)	626 574·249	292 894·445	−0·878	−0·368
Uppingham (442)	485 119·987	298 887·173	+0·016	−0·020
Walpole St. Peters (427)	550 202·344	316 621·736	−0·175	−0·267
Walton on the Naze Twr (227)	626 485·452	223 538·283	−1·060	−0·583
Warley Wtr Twr (224)	559 102·593	191 526·523	−0·295	−0·636
Woolpit (247)	599 634·412	262 291·544	−0·240	−0·340
Wrotham (192)	559 322·166	160 004·495	−0·621	−0·493
Wyck Beacon (144)	420 190·129	220 792·701	+0·052	−0·051
Wyton Wtr Twr (444)	528 152·362	273 816·459	+0·095	−0·116

Proceeding in this way the corrections to eastings and northings were found at several points on each arc, thus distributing the discrepancies at the common boundary stations back along the appropriate arcs. Points on the arcs which had the same corrections were joined by smoothed curves, easting corrections and northing corrections being dealt with separately. This system of correction 'contours' was plotted on a diagram of Figure 5 drawn to scale, and corrections to the adjusted co-ordinates of each station in the two halves were read off the curves. The adjusted co-ordinates corrected from the 'contours' gave the accepted published National Grid co-ordinates for Figure 5, as given in Appendix 10. These co-ordinates, which were published in 1945, were the basis for the calculation of all lower order triangulation, and for all dependent re-surveys.

In September 1949 it was decided to adjust Figure 5 as a whole to see what the results would have been had the figure not been adjusted in two parts.

By 1949 all the secondary triangulation was being adjusted by observation equations using the method of variation of rectangular co-ordinates. (See § 2.241 for details.) The re-adjustment of Figure 5 was done this way, there being 647 observation equations to determine 278 unknowns, namely 88 values of dE, 88 of dN, and 102 of dZ. Actually there were 176 normal equations to solve as the 102 values of dZ were eliminated by Schreiber's method described in § 2.241.

The re-adjustment included two lines which had been excluded in the original adjustment. These lines were between fixed points, and were included to strengthen the determination of the orientation corrections, dZ. Work proceeded intermittently as a low priority task, and the re-adjustment was completed in July 1951. Table 2.1 shows the final co-ordinates obtained from the re-adjustment, and it must be emphasised that although these co-ordinates are the most probable values, they have not been used for controlling lower order work. All lower order triangulation which had been calculated from the accepted ('contoured') values had proved satisfactory, and there was therefore no need to change the accepted co-ordinates as a result of the re-calculation. In 1951 it was decided that the accepted values from the 'contoured' two-part adjustment would be used for all practical cartographic purposes. These values have also been used throughout this History for comparisons of azimuth, scale, and position, except where it is specifically stated otherwise.

For statistical details see Table 2.2.

Appendix 5 gives the processed mean observed directions, $(t-T)$ corrections, plane observed directions, adjustment corrections (from the re-adjustment), plane adjusted directions (from the re-adjustment), triangle misclosures, and spherical excesses. See Diagram 9 for the diagram of the adjustment.

Because the accepted co-ordinates were not derived directly from the least squares adjustment, no adjustment data for the two halves are shown in Appendix 5.

When computing standard errors for figures adjusted by the method of variation of rectangular co-ordinates the formulae in § 2.25 were used, but n_c was replaced by $n_o - n_u$, where n_u = the number of independent unknowns.

2.31 New Primary Figure 6

This figure was adjusted by the method of variation of rectangular co-ordinates. It contained 2 intersected stations and 73 observing stations; of the latter, 13 were held fixed from Figure 3. This gave 197 unknowns—62 values of dE, 62 of dN, and 73 of dZ—and there were 477 observation equations from which to find them. The 73 values of dZ were eliminated (see § 2.241) thus reducing the normal equations to 124. At most of the stations held fixed from previously adjusted figures observation equations for lines to other previously fixed stations outside the figure were included to strengthen orientation; this was standard practice when adjusting primary work by the variation of co-ordinates method. The adjustment of Figure 6 was completed in 1952.

For statistical details see Table 2.2.

Appendix 6 gives details of observations, $(t-T)$ corrections, adjustment corrections, triangle misclosures, etc. Diagram 10 shows the diagram of the adjustment.

2.32 New Primary Figure 7

Five of the seven stations to be co-ordinated in this figure were re-fixations of stations co-ordinated in earlier figures. Inshanks (361), Carleton Fell (362) and Rottington (1), were originally co-ordinated as intersected stations in Figure 3, and Rhiw (110) and Holyhead (117) as intersected stations in Figure 4. The angles at these five stations were now observed and the stations reco-ordinated, to strengthen the fixed edge subsequently used for the connection to Northern Ireland and Eire.

Adjustment was by the method of variation of rectangular co-ordinates. The figure contained 21 stations that were occupied, of which 14 were held fixed from previous adjustments. There were 135 observation equations to find 35 unknowns, namely, 7 values of dE, 7 of dN, and 21 of dZ; this gave 14 normal equations, the 21 values of dZ being eliminated as usual. The adjustment of Figure 7 was completed in 1952.

For statistical details see Table 2.2.

See Appendix 7 and Diagram 11 for all data relevant to Figure 7.

2.33 Additional Primary Work

Reco-ordination of Liddington Castle (35)

This station was first co-ordinated in Figure 1 as an intersected point, but as it was to be one of the terminals of the Ridgeway Base it was subsequently occupied and observations were taken at it. The resulting fully observed polygon was adjusted to give a new value for the station.

There were 6 stations altogether in the polygon, 5 being held fixed. Condition equations were used for the adjustment, there being 4 for angular closure and 3 for fixed side closure. The calculation was done in 1937.

For details see Appendix 8.1, and Diagram 12. Statistics are given in Table 2.2.

Spurn Head Extension

This extension to Figure 2 was undertaken in 1939. It contained 5 stations of which 2 were held fixed. There were 10 fully observed lines of which one was held fixed; this gave 18 directions for which adjustment corrections were required. The figure was adjusted by condition equations, there being 6 for angular closure, and 3 for side closure.

For details see Appendix 8.2, and Diagram 12. Statistics are given in Table 2.2.

Co-ordination of Frittenfield (480) *and Paddlesworth* (190)

As a preliminary to the connection with France, the south-eastern corner of Figure 5 was strengthened in 1951 by inserting a new station, Frittenfield (480). The fixation of Paddlesworth (190) in Figure 5 was not very strong, so when the adjustment of the Frittenfield (480) figure was put in hand Paddlesworth (190) was included as an unfixed station.

The figure contained 7 stations, of which 5 were fixed. It was adjusted by the method of variation of rectangular co-ordinates, and there were 31 observation equations to find 11 unknowns, namely 2 values of dE, 2 of dN, and 7 of dZ.

For details see Appendix 8.3, and Diagram 12. Statistics are given in Table 2.2.

Co-ordination of Hillhead Farm (478)

The side Spital Hill (398) to Warth Hill (399) in Figure 6 had been selected as a measured base—the Caithness Base. Part of the base is over good country, and part over bog. Hillhead Farm (478) is sited almost exactly on the line of the base, and marks the transition from good country to bog. It was an alternative terminal in case measurement over the boggy section became impracticable.

The adjustment figure to fix Hillhead Farm (478) contained 7 stations, of which 6 were held fixed. The adjustment was done using observation equations, and there were 38 equations to find 9 unknowns, namely, 1 value of dE, 1 of dN, and 7 of dZ. The calculation was done in 1952.

For details see Appendix 8.4, and Diagram 13. Statistics are given in Table 2.2.

Co-ordination of Herstmonceux (481)

This station was co-ordinated in 1953. It was sited to give a primary station which could be related to a fundamental position at the Royal Greenwich Observatory, Herstmonceux Castle, where the astronomical latitude, longitude and azimuth would be known. (See § 3.07.)

The figure contained 5 stations, 4 being held fixed. Observation equations were used, and there were 18 equations from which to find the 7 unknowns.

For details see Appendix 8.6, and Diagram 12. Statistics are given in Table 2.2.

Co-ordination of Greenwich Observatory (482)

This primary station was co-ordinated in 1954. It was sited so that the International Longitude Datum could be related to the primary triangulation, the station being placed on the zero meridian. See § 3.05 and § 3.06.

The figure contained 5 stations of which 4 were held fixed, and there were 20 observation equations to find the 7 unknowns. It was intended that Epping Wtr Twr (188) should be one of the fixed stations in this adjustment, but it was not intervisible with Greenwich Observatory (482). To overcome the difficulty an auxiliary station, called Epping (483), was co-ordinated by measurements from Epping Wtr Twr (188), and this auxiliary station was used as a fixed station in place of Epping Wtr Twr (188). Even then it was necessary to erect a steel tower over Epping (483).

For details see Appendix 8.7, and Diagram 12. Statistics are given in Table 2.2.

Co-ordination of North Tolsta (484)

This station was co-ordinated in 1955, and was sited for the Shoran connection to Iceland. See § 3.04.

The figure contained 6 stations of which 5 were held fixed, and there were 20 observation equations to find the 8 unknowns.

For details see Appendix 8.8, and Diagram 13. Statistics are given in Table 2.2.

Co-ordination of St. Kilda (486)

This station was co-ordinated in 1957. It was sited in connection with the establishment of a guided weapons range. The figure contained 6 stations, 5 being held fixed. There were 24 observation equations to find the 8 unknowns.

For details see Appendix 8.9, and Diagram 13. Statistics are given in Table 2.2.

TABLE 2.2

STATISTICAL DETAILS OF ANGULAR OBSERVATIONS IN THE PRIMARY RETRIANGULATION

Figure	*Triangle Misclosure*		*Average Direction Correction*	*Standard Error of Observation of Unit Weight*	
	Average	*Maximum*		*Observed*	*Adjusted*
1	1″09	3″48	0″31	0″56	0″42
2	1·12	4·46	0·33	0·65	0·49
3	1·07	2·65	0·31	0·58	0·41
4	1·10	3·58	0·37	0·68	0·47
5	1·37	4·96	0·52	0·90	0·59
6	1·09	3·86	0·35	0·60	0·39
7	1·07	2·65	0·39	0·61	0·31
Liddington Castle	1·74	4·00	0·59	0·88	0·48
Spurn Head	1·64	3·44	0·43	0·83	0·62
Frittenfield and Paddlesworth	1·01	2·48	0·86	1·28	0·76
Hillhead Farm	1·11	2·40	0·38	0·54	0·26
Herstmonceux	1·14	2·12	0·40	0·64	0·40
Greenwich	1·16	3·47	0·98	1·36	0·80
North Tolsta	1·28	1·88	0·34	0·58	0·37
St. Kilda	1·22	2·46	0·28	0·53	0·30

CHAPTER THREE

Supplementary Work connected with the Primary Retriangulation

CONNECTIONS WITH OTHER COUNTRIES

3.00 Introduction

The main primary Retriangulation was connected to France and Ireland by primary triangulation, and to Norway and Iceland by Shoran trilateration. The connections are described below in the order in which they were undertaken. The description of the French connection is based on the report produced jointly by the Directeur de l'Institut Géographique National and the Director General of the Ordnance Survey.

Connections between primary triangulation stations in the north of Great Britain to triangulation stations in Norway and Iceland were made by the United States Air Force as Phases I and II of Project 53 AFS-1—the North Atlantic Tie. This project was designed to establish a geodetic connection between North America and Europe by measuring a trilateration net by Shoran, in order to allow the positioning of European stations with reference to the North American 1927 datum. Shoran is a system for measuring lines up to a length of about 500 km. by means of microwave transmitters mounted in an aircraft operating in conjunction with transponder stations sited on the ground. By this means the distances between the aircraft and the ground stations are continuously measured as the aircraft crosses the line between them. The sum of the two measured distances becomes a minimum at the actual moment of crossing. Some confusion may exist between the terms Shoran and Hiran. Hiran, which was the system actually used for the work described in § 3·04, is, in fact, merely an improved version of Shoran; the latter term therefore is used throughout this book to denote all measurements by this technique. The description of the project given is taken from the following reports published by the United States Air Force.

Final Report of Results of Project 53 AFS-1 Scotland-Norway Tie 21st December 1953(20),

Progress Report of Project 53 AFS-1 North Atlantic Tie 1st February 1955(21).

(NOTE. The report dated 21st December 1953 is the 'Final' report of the Scotland-Norway Tie only and not of the whole project.)

3.01 Connection with Ireland

In 1951 in conjunction with the Survey Departments of Eire and Northern Ireland it was decided to extend the primary Retriangulation of Great Britain westwards to connect to Northern Ireland and Eire. The Ordnance Survey of Great Britain had also been asked by the Ordnance Survey of Northern Ireland to observe and compute the whole primary Retriangulation of Northern Ireland and it was decided that the two operations could conveniently be carried out in the same observing season.

At preliminary discussions between senior representatives of the Ordnance Survey of Great Britain, the Chief Survey Officer, Northern Ireland, and the Assistant Director, Ordnance Survey, Eire, held in Chessington, Belfast, and Dublin in 1951, it was agreed that the work would be carried out in 1952. All operations at stations in Eire would be undertaken by personnel from the Eire Survey Department and the remainder by the Ordnance Survey of Great Britain.

3.010 PLANNING

The triangulation scheme (Diagram 14) was based largely on the connection originally made in the Principal Triangulation (Diagram 1). All stations in Ireland were either coincident with, or very close to, the original stations of that connection. Existing stations of the Retriangulation were used in Great Britain, the majority of them being also coincident with the old stations of the Principal Triangulation. The main departure from the previous connection was in the omission of Snowdon which was replaced by Holyhead (117) and Rhiw (110), thereby avoiding some unduly long rays which were particularly subject to interference by cloud.

The Ordnance Survey of Great Britain supplied two observing parties, one in Northern Ireland and one on the mainland. Personnel from Eire were attached for training purposes to both parties during the Great Britain to Northern Ireland connection.

Observing procedure followed closely that laid down for the Retriangulation of Great Britain. It was however decided that all rays passing over the sea should be observed in the course of at least three nights' work, with a minimum of four, and a maximum of 16 zeros on any one night. The minimum number of zeros for any one ray was laid down as 24, with a desirable maximum of 48. In the event of a triangular misclosure exceeding three seconds, an immediate decision on the necessity for re-observation would be taken after reference to the Ordnance Surveys of Great Britain and of Eire.

All stations, both in Northern Ireland and in Eire, were marked with the standard concrete pillar used in the Retriangulation of Great Britain.

3.011 PROGRESS OF OBSERVATIONS

Observations were commenced on 19th April 1952, and, as so often elsewhere, were hampered at the outset by heavy rain and cloud. Indeed the ray between Trostan and Slieve Donard, scheduled to be observed at an early stage, was abandoned after repeated efforts, but was subsequently completed when Slieve Donard was later re-occupied to observe the Holyhead (117) ray. The Kippure to South Barrule (469) ray, 95 miles long and obscured by smoke from Dublin, had to be finally abandoned. By mid June the northern section of the connection had been completed.

Observations for the internal retriangulation of Northern Ireland were next put in hand, while the mainland party completed additional work designed to strengthen the western edge of the primary Retriangulation on the coast of Wales. On the 28th July, work was started on the southern half of the connection and reasonable progress was maintained. Slieve Donard however again proved a stumbling block and the observer at Holyhead (117) had perforce to wait for 25 nights, until the third night's observations could be completed. As the work progressed southwards the rays across the sea became progressively longer, but at the same time there was a welcome, and in the opinion of all concerned, a long overdue, improvement in the weather.

Prescelly (107), the last station allotted to the mainland observer, was occupied on 3rd September and the Eireann observer started work on Ballycreen. The statutory three nights sufficed for the completion of this, the longest ray (98 miles) in the connection, and indeed in the whole of the Retriangulation. The ray between Prescelly (107) and Kippure was not considered essential and, after partial observation, was abandoned.

The Eireann party then moved to Tara, and thence to Forth Mountain, but by that time the weather had again deteriorated and it was not until 8th October that the officer in charge of observations in Eire was able to inform the mainland observer that Forth Mountain, and with it the connection of the Retriangulation to Eire, had been completed.

After consultations between the Ordnance Survey Offices of Great Britain and Eire, the observers were informed that all triangular closures were acceptable. The average misclosure was 1·16 seconds, the same in fact as that of the Retriangulation. Apart from persistently bad weather, the operation had been uneventful, and its success was due to the hard work and excellent co-operation of all concerned.

3.012 COMPUTATIONS

The system was adjusted by the method of variation of rectangular co-ordinates (See § 2.241). The figure contained 18 stations of which eight were held fixed, and there were 123 observation equations from which to find 38 unknowns, namely, 10 dE, 10 dN, and 18 dZ (see Appendix 8.5).

Computations which were completed in November 1952, yielded the following statistical data:

Maximum triangle misclosure = 3″07
Average triangle misclosure = 1″16
Average adjustment correction = 0″71
Standard error of an observed direction of unit weight = ± 1″03
Standard error of an adjusted direction = ±0″57

This adjustment gave British National Grid co-ordinates for the Irish stations. These are given in Table 3.1 opposite together with derived geographical co-ordinates.

3.02 Connection with France

This work was carried out in accordance with a formal agreement drawn up between the Ordnance Survey of Great Britain and the Institut Géographique National of France.

3.020 PROCEDURE

The observations began on the evening of Wednesday, 2nd May 1951. At 1700 hours the French set up heliotropes, as leading lights, on the calculated directions and the British used these signals to align their lights. On those rays where the heliotropes were not seen lights were shone that evening on the calculated lines. As soon as each ray had been seen from each side, the light signal G.B. was shown at each end and the lights were extinguished for the night. By 2030 hours all the lamps were set up and properly aligned. No measurement of angles was carried out. As the erection of the Gravelines station on the water tower had not been completed by 2nd May, a lamp was installed that day on the reservoir there, firmly placed on one of the supporting pillars so that the British were able to check their alignment on 2nd May, and begin their observations next day. All alignments were completed on 2nd May and observations began on the night of 4th May.

Each country observed at the stations in its own territory. The British and French each had two teams of observers working simultaneously. All observations were made by night on lamps. The British used the geodetic Tavistock theodolite and the French the 'cercle azimuthal répétiteur I.G.N. Mle 40'.

The British measured their angles by zeros with three micrometer readings on each pointing in a round. A minimum of 16 zeros was to be acceptable if observations were stopped by bad visibility, but it was hoped that 24 zeros would be observed on each ray. In practice this was exceeded on all rays, the minimum number of zeros being 35 and the maximum 88.

The French measured their angles in 'series' of six repetitions. Each observation of a distant mark was the result of ten pointings with the moving-hair eyepiece with which their instrument is fitted; the result of a 'series' was thus equivalent to the mean of 60 single measures. Each angle was to be observed with a minimum of four 'series'. In practice, most angles were measured by more than 10 'series'.

Observations on each ray were to be spaced as widely as possible over six different nights. With the exception of one or two rays, all were observed on at least six nights and in some cases on as many as 10 nights. The quality of the results obtained is probably due to this spreading of the observations.

TABLE 3.1

Station	*British National Grid Co-ordinates*		*Geographical Co-ordinates Derived from British National Grid Co-ordinates*	
	E (*metres*)	*N* (*metres*)	φ	λ
Ballycreen	106 573·033	344 597·162	52° 55′ 05″·3228	−06° 21′ 56″·2620
Divis	140 627·153	531 435·167	54 36 40·2974	−06 01 02·7341
Slieve Donard (New)	144 244·571	483 148·483	54 10 48·2103	−05 55 11·9395
Forth Mountain	89 171·112	278 473·908	52 18 56·7860	−06 33 41·4251
Howth	129 422·309	393 992·473	53 22 23·1324	−06 04 06·0717
Kippure	110 638·506	373 331·736	53 10 40·5966	−06 19 52·0521
Knocklayd	129 338·656	593 560·930	55 09 43·5840	−06 15 00·2671
Slieve Snaght	60 699·205	602 117·976	55 11 47·1717	−07 20 00·4553
Tara	115 137·933	319 613·023	52 41 55·4830	−06 13 00·5317
Trostan	134 656·691	580 251·991	55 02 44·4934	−06 09 15·8460

NOTE: British National Grid co-ordinates were converted to geographical co-ordinates by means of the Projection Tables(16) mentioned in § 2.22.

On any given night observing began at nightfall and continued till between one and three o'clock in the morning, by which time the humidity of the air had reached a value which affected the quality of the lights and made it impossible to obtain good pointings.

Three of the British stations were established on Bilby towers, and the fourth was sited on a water tower. No measurement of torsion was made, satisfactory closure of each zero being taken as proof of the absence of torsion.

Three of the French stations were set up on double towers with the inner part of the scaffolding protected. The fourth was on a water tower.

Measurement of torsion was made, using a second telescope coaxial with and mounted vertically below the main instrument. This second telescope, which also has a moving-hair eyepiece, was aligned throughout on the referring object. Since torsion, if any, is small, it can be measured with the moving-hair eyepiece without movement of the telescope during observations with the main instrument. In fact, no observable torsion occurred at any of the stations.

In spite of poor atmospheric conditions the observations, begun on both sides of the Channel on 2nd May, were completed on 13th July by the French, and on 31st July by the British. The latter had trouble in sighting Mt. Lambert from Beachy Head (194).

3.021 RESULTS

By the 17th July the British had sent the French the mean values of all their observed angles with the exception of those on the Mt. Lambert—Beachy Head (194) ray which were sent in provisional form.

The French calculated the closures of the triangles which were found to be very satisfactory in 12 triangles out of 16.

Three of the closures which remained seemed to justify verification on the common side Paddlesworth (190) to Fairlight Down (193). It was finally agreed however to accept these observations since they were but little worse than others in the British primary network. The direction Paddlesworth (190)—Fairlight Down (193) was therefore retained.

At a final meeting on 29th January 1953 the results were agreed and signed by both parties.

The Ordnance Survey adjusted the connection by the method of variation of rectangular co-ordinates, and holding fixed the National Grid co-ordinates of the four English stations. For details see Appendix 8.10, and Diagram 12.

The following statistical data were obtained:

Maximum triangle misclosure = 2″85
Average triangle misclosure = 1″00
Average adjustment correction = 0″42
Standard error of an observed direction of unit weight = ±0″64
Standard error of an adjusted direction = ±0″43

3.03 The Shoran Connection to Norway

3.030 OUTLINE OF THE PROJECT

In July to September 1953 the United States Air Force carried out a connection from three geodetic stations in Norway to three in Scotland and the Shetland Islands by Shoran radar methods, as the first part of a great project (since completed) connecting Norway, Iceland, and Greenland, to Canada. Diagram 15(a) shows the lines measured.

The airborne equipment used was the AN/APN-3 (XA-5) Shoran set. The equipment and its method of use is fully described in standard U.S.A.F. manuals(50). Numerical results from which the following data have been abstracted are given in *The Final Report of the Results, Project 53AFS-1, Scotland-Norway Tie*, prepared by the 55th Strategic Reconnaissance Wing, 21st December 1953(20).

The net connecting Scotland to Norway, shown in Diagram 15(a) consists of 15 measured lines: three between the three Norwegian stations, whose lengths are given by the Norwegian triangulation: three between the British stations, whose lengths are given by the Ordnance Survey Retriangulation, and nine lines across the North Sea. The Shoran geodetic stations did not in general actually coincide with the geodetic triangulation stations, but were so close that no significant error can result in the transfer from one to the other. Numerical data, below, refer to the actual Shoran stations.

The Norwegian stations were

Number	*Name*
1	Skibmannshei Shoran
2	Eigeberg Shoran
3	Helliso Fyr Shoran

and the British were

Number	*Name*
4	Saxavord Shoran
5	Warth Hill Shoran
6	Mormond (338)

3.031 INTERNAL ACCURACY

Each of the 15 lines was measured by six line crossings at each of two altitude levels, 12 crossings in all, constituting a 'Mission'. Such a programme was accepted provided, (a) that at least four of the six crossings in each group did not deviate from the group mean by more than 0·003 miles (16 feet), (b) that the two group means agreed within 0·003 miles, and (c) that the condition of the flight appeared generally satisfactory.

Table 3.2 shows results of missions which were rejected for these reasons, accepted missions being marked A and rejected missions R. The Table shows that the worst of the rejected missions differed from the accepted measure by 0·0055 miles (29 feet), and that the average difference between a rejected measure and the mean of the accepted measures is 0·0013 miles (6 feet). The least satisfactory line is perhaps 1–5 in which four separate missions, all accepted, range through 0·0048 miles (25 feet), but this is only one of the nine independent lines crossing the sea.

3.032 COMPARISON WITH TRIANGULATION MEASURES

Table 3.3 compares the distance 1–2, 1–3 and 2–3 as given by the Norwegian triangulation with (*a*) the adopted observed Shoran distance and (*b*) that given by the 'free' adjustment of the Shoran net (see § 3.033).

TABLE 3.2

Line	Date Flown	Shoran Distance (miles)
1–2	24th July 1953	78·3933 (R)
	29th July 1953	78·3917 (A)
1–3	24th July 1953	206·3818 (A)
	19th Aug. 1953	206·3840 (A)
1–4	12th Aug. 1953	339·2592 (R)
	15th Aug. 1953	339·2588 (A)
1–5	15th Aug. 1953	371·1808 (A)
	29th Aug. 1953	371·1803 (A)
	4th Sept. 1953	371·1760 (A)
	7th Sept. 1953	371·1786 (A)
1–6	29th Aug. 1953	336·6987 (A)
	4th Sept. 1953	336·6972 (A)
	7th Sept. 1953	336·6975 (R)
2–3	24th July 1953	135·1176 (R)
	17th Aug. 1953	135·1178 (A)
2–4	12th Aug. 1953	262·2596 (R)
	15th Aug. 1953	262·2585 (A)
2–6	23rd Aug. 1953	291·0998 (A)
	7th Sept. 1953	291·0992 (A)
3–4	29th July 1953	187·8928 (R)
	31st July 1953	187·8874 (R)
	12th Aug. 1953	187·8873 (A)
3–5	31st July 1953	310·2000 (R)
	17th Aug. 1953	310·1996 (A)

TABLE 3.3

(IN MILES)

Line	Triangulated Distance	Adopted Shoran Distance	From Shoran free Adjustment	Triangulated minus Adjusted Shoran	Triangulated minus Adjusted in PPM
1–2	78·3924	78·3921	78·3925	−0·0001	− 1
1–3	206·3742	206·3829	206·3823	−0·0081	−39
2–3	135·1113	135·1178	135·1187	−0·0074	−55

It is noticeable that the lines 1–3 and 2–3 both differ from their triangulated values by about 0·008 miles (40 feet). This is discussed in § 3.034 and § 3.036.

Table 3.4 gives similar details for the British stations. The first column of triangulated distances is in terms of the Retriangulation (adjusted to the Principal Triangulation scale as explained in Chapter 2), while the second is in terms of scale given by the Caithness Base and the Saxavord (463)—Fetlar (459) side measured by Geodimeter.

TABLE 3.4

(IN MILES)

Line	*Distance from the Retriangulation*	*Triangulated Distance with Corrected Scale*	*Adopted Shoran Distance*	*From Shoran free Adjustment*	*Triangulated (corrected scale) minus Adjusted Shoran*	*Triangulated (corrected scale) minus Adjusted Shoran PPM*
4–5	172·2493	172·2476	172·2493	172·2480	−0·0004	− 2
4–6	227·2128	227·2101	227·2078	227·2095	+0·0006	+ 3
5–6	79·7685	79·7672	79·7700	79·7682	−0·0010	−12

Here the agreement is good.

In Tables 3.3 and 3.4 and elsewhere, the Shoran distances have been computed using 186,282·42 miles/sec for the velocity *in vacuo*, with meteorological correction given by

$$10^6(\mu-1)=\frac{77{\cdot}54(p-e)}{T}+67{\cdot}88\,\frac{e}{T}+\frac{37{\cdot}84\times10^4 e}{T^2}$$

where

μ = refractive index
p = total atmospheric pressure in millibars
T = temperature in degrees Kelvin
e = water vapour pressure in millibars.

These figures are in accord with the general experience of the U.S. Air Force, and substantially agree with those which were accepted in 1962.

3.033 ADJUSTMENT OF THE SHORAN NET

The net was adjusted by least squares on three different systems as follows:

(*a*) A 'free' adjustment was made ignoring the Norwegian and British triangulated azimuths and distances. Such an adjustment shows the consistency of the observations, but cannot reveal the existence of any systematic error proportional to distance.

This adjustment gives the probable error of a single (unadjusted) adopted observed distance as ±0·00126 miles (7 feet). The average discrepancy between an adopted observed value and that given by the adjustment was 0·0010 miles (5 feet), and the maximum was 0·0022 (12 feet). This consistency (confirming what might be expected from § 3·031) renders more remarkable the relatively great discrepancies between Shoran and triangulation in lines 1–3 and 2–3 (Table 3.3). Differences between observed and adjusted values are given in Tables 3.3 and 3.4 for the six triangulated lines and in Table 3.5 for the remaining nine lines.

TABLE 3.5

(IN MILES)

Line	*Adopted Shoran*	*Free Adjustment*
1–4	339·2590	339·2594
1–5	371·1799	371·1803
1–6	336·6979	336·6973
2–4	262·2585	262·2569
2–5	312·2620	312·2642
2–6	291·0996	291·0987
3–4	187·8872	187·8880
3–5	310·1996	310·1983
3–6	323·6542	323·6547

(*b*) The 'Final' adjustment accepted the positions of the three Norwegian stations, the distances between the three British stations as given by the Retriangulation (Table 3.4), and the Retriangulation azimuth 4–6 which was treated as an observed astronomical azimuth.

In this adjustment the probable error of an observed distance increased to ±0·00185 miles (9 feet) with a maximum of 0·0059 miles (31 feet) in 3–4, these figures excluding changes in 1–2, 1–3 and 2–3 whose observed values were not introduced into the adjustment. They would, of course, have increased the probable and maximum errors.

As a result of this adjustment the change of position from British Datum (Airy spheroid based on the Retriangulation) to European Datum is given as:

TABLE 3.6

ADD TO BRITISH DATUM: EAST LONGITUDE POSITIVE

Station	*Co-ordinate*	*Change in Seconds*
4	Latitude	−0·184
	Longitude	−1·022
5	Latitude	+0·890
	Longitude	+0·179
6	Latitude	+1·377
	Longitude	−0·337

(*c*) A third adjustment was made differing from the 'Final' adjustment only in ignoring the position of station 3. The probable error of an observed distance was then given as ±0·00137 miles (7 feet) which is very little greater than that given by the free adjustment.

3.034 STATION 3, HELLISO FYR

The residuals in these adjustments suggest trouble at Helliso Fyr. The following are the more possible causes.

(*a*) The Shoran beacon was placed about 10 m. south of a large iron lighthouse, and this may have reflected the signal transmitted from the aircraft or the signal transponded from the ground station. This latter is more likely, but any reflection from the lighthouse will have the apparent effect of increasing the length of the measured line and this would go far towards explaining the discrepancy.

(*b*) A delay error in the Shoran beacon at Helliso Fyr could have made the measurements of lines 1–3 and 2–3 too long; this would also help to explain the discrepancy. There is a built-in delay measuring circuit in the Shoran set and in normal operation the delay is set to a known figure. It is conceivable that an error in the setting, or subsequent drift in the instrument delay, might have occurred.

(*c*) Error in the geodetic position of Helliso Fyr. But an error of 30 feet in a single geodetic station, all others being correct, is inconceivable. In any case since this project was completed the Norwegians have made Tellurometer measurements of the six triangulation lines between Eigeberg and Helliso Fyr, and the Tellurometer length agrees with the triangulation value to about 1/130,000. This alternative cause of the trouble can therefore be discounted.

3.035 CIRCUIT CLOSING ERROR

As indicated in § 3.033(*b*) Table 3.6 shows the differences at the three British stations between British Retriangulation co-ordinates and European Datum co-ordinates. To find the closing error on European Datum of the circuit Norway-Germany-France-Great Britain-Shoran-Norway it is necessary to eliminate from the differences in Table 3.6,

(*a*) the amount due to converting the British Retriangulation from its own origin and spheroid to the European Datum,

(*b*) the amount due to correcting the British Retriangulation for its known scale and azimuth errors. (As explained in § 2.01 and § 2.27 of Chapter 2 the British Retriangulation was adjusted without using measured bases and Laplace stations).

In 1955 Brigadier G. Bomford of Oxford University made an assessment of (*a*) and (*b*) from data available at that time, European Datum being carried across the Straits of Dover from France. (See § 6.03 of Chapter 6). This assessment has been used to compile Table 3.7 below. In this Table the second column gives corrections to the British Retriangulation to convert to European Datum, the third column gives the correction to the British Retriangulation for scale and azimuth errors, the fifth column repeats Table 3.6, and the sixth gives the closing errors of the circuit at the British stations.

Relative to the British triangulation (adjusted for scale and azimuth errors and corrected to European Datum) the Shoran positions are thus 47 feet south and 53 feet west, all three stations agreeing with this figure within 3 feet. The length of the circuit is about 2,000 miles, so the closing error is 1/140,000 of the length of the circuit.

A reasonable distribution of the error would be to put one quarter into the Shoran, one quarter into Great Britain and one half into the European section, which is about twice as long as the British.

TABLE 3.7

(1)	(2)		(3)		(4)		(5)		(6)	
Point	*Conversion to European Datum*		*Correction for Scale and Azimuth Errors*		*Total*		*From Table 3.6*		*European Datum minus Shoran*	
	Lat.	Long.	Lat.	Long.	Lat.	Long.	Lat.	Long.	Lat.	Long.
4	+0″60	−0″57	−0″34	+0″63	+0″26	+0″06	−0″18	−1″02	+0″44 44 ft.	+1″08 52 ft.
5	+1″65	+0″50	−0″27	+0″72	+1″38	+1″22	+0″89	+0″18	+0″49 49 ft.	+1″04 54 ft.
6	+2″10	0	−0″25	+0″61	+1″85	+0″61	+1″38	−0″34	+0″47 47 ft.	+0″95 50 ft.

3.036 GRAPHIC REPRESENTATION

The reliability of the Shoran and the significance of the suggested changes is most easily seen graphically as in Diagram 15(c). In this diagram the three left-hand squares 4, 5 and 6, show (in firm lines) the positions of the loci given by the nine observed Shoran distances from the Norwegian stations relative to the accepted British positions (on European Datum, and corrected for scale and azimuth errors) as marked by small triangles.

The broken lines 55, 44, etc. represent Shoran distances between the three British stations, but these are not fixed loci since 44 (at 5) may be moved by any amount provided 55 (at 4) is moved by an equal amount. Small circles show the positions given by the 'Final' Shoran adjustment, their separation from the small triangles being as in Table 3.7.

The loci make it quite clear that the British stations need to be moved about 50 feet to the west. This is about 1/40,000 of the sea crossing and could not possibly be due to error in the accepted basic velocity of light. It could result from a systematic error of 5 mbs. of water-vapour pressure, but it is thought that the methods used to record it make such an error impossible.

The suggested changes in latitude are less conclusively established. A change of 20 feet would add nothing significant to the residuals.

The right-hand diagrams of Diagram 15(c) similarly show the changes in the positions of the Norwegian stations which are suggested if the adjusted European Datum positions of the British stations are accepted. Small triangles 3, 2, 1 show the positions given by the Norwegian triangulation, and small circles 3′, 2′, 1′ are positions deduced from the plotted loci.

The shifts from 1 to 1′ and from 2 to 2′ are substantially the same and amount to about 60 feet to the east and 40 feet to the north. The shift from 3 to 3′ amounts to about 40 feet to the east and 70 feet to the north. This again suggests that there is trouble of some sort at Helliso Fyr.

3.04 The Shoran Connection to Iceland

The Iceland-Scotland connection forms Phase II of the main project and fieldwork was completed in 1954 using Shoran equipment (see Diagram 15(b)). The positions so far computed are, however, only of a preliminary nature. Final results and evaluation await a redetermination of ground survey positions in Iceland and The Faeroes.

3.040 THE TRILATERATION NET

The lack of intermediate ground station sites between Iceland and Great Britain, other than The Faeroes, complicated the construction of the net and necessitated the inclusion of very long lines. Four of the lines measured were over 500 miles in length and were longer than any lines hitherto measured by Shoran.

3.041 CONTROL

The ground stations used as control were:

	Station No.	*Station Name*	*Remarks*
Norway	3	Helliso Fyr Shoran	Used in Phase I (Scotland-Norway Tie).
Great Britain	4	Saxavord	This station coincided with the Ordnance Survey primary station No. 463. The Shoran station used in Phase I could not be used again, due to construction work at the site.
	5	Warth Hill Shoran	Used in Phase I. Adjacent to Ordnance Survey primary station No. 399.
	7	Fair Isle	Coincides with Ordnance Survey primary station No. 458.
	8	North Tolsta	Coincides with Ordnance Survey primary station No. 484.
The Faeroes	9	Milk	—
	10	Nigvan	—
Iceland	11	Rey	—
	12	Fago	—
	13	Hofn	—
	14	Paul	—

3.042 CONNECTIONS OF GROUND STATIONS TO LOCAL TRIANGULATIONS

Great Britain

Station 8 was not originally a station of the primary Retriangulation and was provisionally connected to the primary Retriangulation by personnel from the School of Military Survey in 1954. The provisional values were used in the Shoran computations. Subsequently the station was connected by first order methods to the primary Retriangulation by the Ordnance Survey in 1955. The difference between the provisional and the final values was 0·3 m. in Eastings and 0·5 m. in Northings. European Datum values were obtained for stations 4, 5 and 6 through the Shoran connection from Norway in Phase I. European Datum values for stations 7 and 8 were determined using the positions of these stations relative to stations 4, 5, and 6, as defined by the Retriangulation.

The Faeroes

Station 9 was connected to the local triangulation by Danish Survey personnel.

Iceland

Stations 11, 12, 13 and 14 were connected at the time of observations to the existing triangulation but the connections were not considered adequate for geodetic work. Precise connections are expected to be accomplished in due course.

3.043 ADJUSTMENT

The triangulation information available for Iceland and The Faeroes in 1955, when the progress report on Phase II of the project was issued, was not considered to be of geodetic standard. Consequently only provisional results were published. Several adjustments were made to attempt to determine the precision of the Shoran net, but again these can only be provisional due to the inadequacy of the fixed data. The availability of possible ground station locations necessitated a design of net which was greatly dependent on the inclusion of triangulation data for accurate results.

Some indications of the consistency of the Shoran net were obtained from an adjustment holding only the positions of Stations 3, 4, 5, 7, and 8 fixed. The probable error of a single observation from this adjustment was ±0·0020 miles which indicated that the consistency of the field measurements was comparable to that of previous projects.

The results of Phase II have not been included in this publication due to their provisional nature and to the fact that they can, by themselves, contribute little to the study of the Retriangulation of Great Britain.

THE CONNECTION TO THE GREENWICH MERIDIAN

3.05 Introduction

Prior to 1851 the zero meridian of astronomic longitude was defined as that passing through the centre of the instrument known as the Pond Transit Instrument then located at the Royal Observatory at Greenwich. Clarke accepted the astronomical co-ordinates of this point as the origin of geodetic co-ordinates for the Principal Triangulation.

From 1851 onwards the zero meridian has been defined by the centre of the instrument known as the Airy Transit Circle which was used in place of the earlier Pond Transit Instrument. It was therefore the centre of this Airy Transit Circle that was co-ordinated when a connection was made with the Retriangulation in 1949. Since the Retriangulation had been fitted as closely as possible to the Principal Triangulation it was expected that there would be close agreement between the two triangulations at the zero meridian, because at that time there was no reason for supposing that the two transits were not on the same meridian. But in fact it transpired that the Retriangulation gave a longitude value for the Airy Transit Circle of 00° 00′ 00″418 east of Greenwich, revealing a discrepancy of 0·418 seconds of arc, or 8·06 m. The discovery of this discrepancy and the investigation into it are described below.

3.06 Investigation into the apparent Longitude Discrepancy at Greenwich

3.060 THE CONNECTION OF THE PRINCIPAL TRIANGULATION TO THE ROYAL OBSERVATORY, GREENWICH

The Principal Triangulation was connected to the Pond Transit Instrument by observations from the primary stations Epping Cupola, Chingford, and Severndroog Castle. From the latter station the Observatory Dome was taken, but the plan of the building afforded means of calculating the angle subtended at Severndroog Castle by the Dome and Transit; this observation was thus reduced to the Transit. The scheme, which is shown in Fig. 3.1(a), was rigorously adjusted prior to calculating the three side lengths to the Transit.

The previously adjusted triangle Chingford-Wrotham-Leith Hill gave the side length Chingford to Leith Hill. See Fig. 3.1(b). With the side lengths Chingford to Leith Hill and Chingford to Transit, and the included angle at Chingford, the triangle Chingford-Leith Hill-Transit was solved to find the angle at the Transit between Chingford and Leith Hill, and the side length Transit to Leith Hill.

The primary station at Chingford was found by measurement to be 0·454 feet west of the meridian plane passing through the centre of the Greenwich North Meridian Mark at Chingford. This distance subtended 1″62 at the Transit. The 'Greenwich Observations' for 1842 gave the azimuth at the Transit of the centre of the Meridian Mark as 0″02 west of north, so the azimuth of Chingford primary station was 00° 00′ 01″64 west of north. Applying the calculated angle between Chingford and Leith Hill gave the azimuth from the Transit to Leith Hill. With this azimuth, the calculated side length Leith Hill to Transit, and the astronomic latitude and longitude (zero) of the Transit, Leith Hill was co-ordinated, and from this the remainder of the co-ordinates of the Principal Triangulation stations were successively obtained.

On page 672 of the account of the Principal Triangulation(1) it is explicitly stated that 'for the latitude at Greenwich the quantity 51° 28′ 38″30 has been used in all calculations'. Furthermore paragraph III on pages 674 to 676 together with the first entry in the table on page 677 make it certain that the origin of all longitudes was that transit instrument which was in position in 1848 or earlier, that is, the Pond Transit Instrument. In other words, the longitude of the Pond Transit Instrument was accepted as 00° 00′ 00″00, and all other geodetic longitudes in the Principal Triangulation were derived from that longitude. The above statements regarding latitude and longitude are borne out by the values given in the table of latitudes and longitudes of the primary stations, etc., calculated on Airy's figure, given on page 23 at the end of Major Wolff's pamphlet *The Mathematical Basis of the Ordnance Maps of the United Kingdom* (dated 1919)(22).

3.061 THE CONNECTION OF THE RETRIANGULATION TO THE ROYAL OBSERVATORY, GREENWICH

In 1949 when the first connection with the Retriangulation was made it was not possible to triangulate directly into the Airy Transit Circle. A mark was therefore established on the roof of the Astronomer Royal's house near the Time Ball lobby, and was fixed by four rays in from and four rays out to the following stations of the Retriangulation:

Severndroog Castle (189) (Primary Station)
St. Aubyn's Church Tower (Secondary Station)
McDougall's Silo (Tertiary Station)
C.W.S. Silo (Tertiary Station)

The two tertiary stations were accurately co-ordinated from adjacent secondary stations.

From the roof station a traverse was run to the centre of the Airy Transit Circle.

The roof station was co-ordinated semi-graphically, and from the plotted graph it would seem unlikely that the value for this station on the Astronomer Royal's house could be in error by as much as 0·1 m. relative to the triangulation control. A re-observation of the traverse gave co-ordinates differing by less than 0·04 m. from the first value.

The resulting geographical co-ordinates of the Airy Transit Circle were as follows:

$$\varphi\ \ 51^\circ\ 28'\ 38''\!.261\ \mathrm{N}$$
$$\lambda + 00\ \ 00\ \ 00{\cdot}417\ \mathrm{E}$$

These differ from Clarke's value for the Pond Transit Instrument by:

In latitude: 0·039 seconds of arc (= 3·95 feet = 1·21 m.)
In longitude: 0·417 seconds of arc (= 26·39 feet = 8·04 m.)

There seemed to be reasonable agreement in latitude but the longitude difference appeared to be too large to be attributable to errors in either triangulation.

3.062 INVESTIGATION OF THE DISCREPANCY

In the course of investigations into this discrepancy it was noticed that the difference between the old triangulation value of St. Paul's Cathedral Cross and its new value on the Retriangulation was 2·3 m. in eastings. Similarly, the Retriangulation easting co-ordinate of the Observatory Time Ball differed from the old value by 2·4 m. in the same sense.

It is clear that there is a systematic difference of about 2·4 m. in eastings between the old triangulation and the new in this area. But even allowing for this there was still a discrepancy of 8·04 − 2·4 = 5·6 m. to be explained.

3.063 THE ESTABLISHMENT IN 1850 OF THE NEW AIRY TRANSIT CIRCLE IN A NEW TRANSIT ROOM ADJOINING AND EAST OF THE OLD TRANSIT ROOM

At this stage Dr. R. d'E. Atkinson, Chief Assistant at the Royal Observatory, was consulted, and was able to clarify the matter. It transpired that in 1850 the Astronomer Royal of that time, Sir George Airy, erected a new transit instrument, called the Airy Transit Circle, in a new room

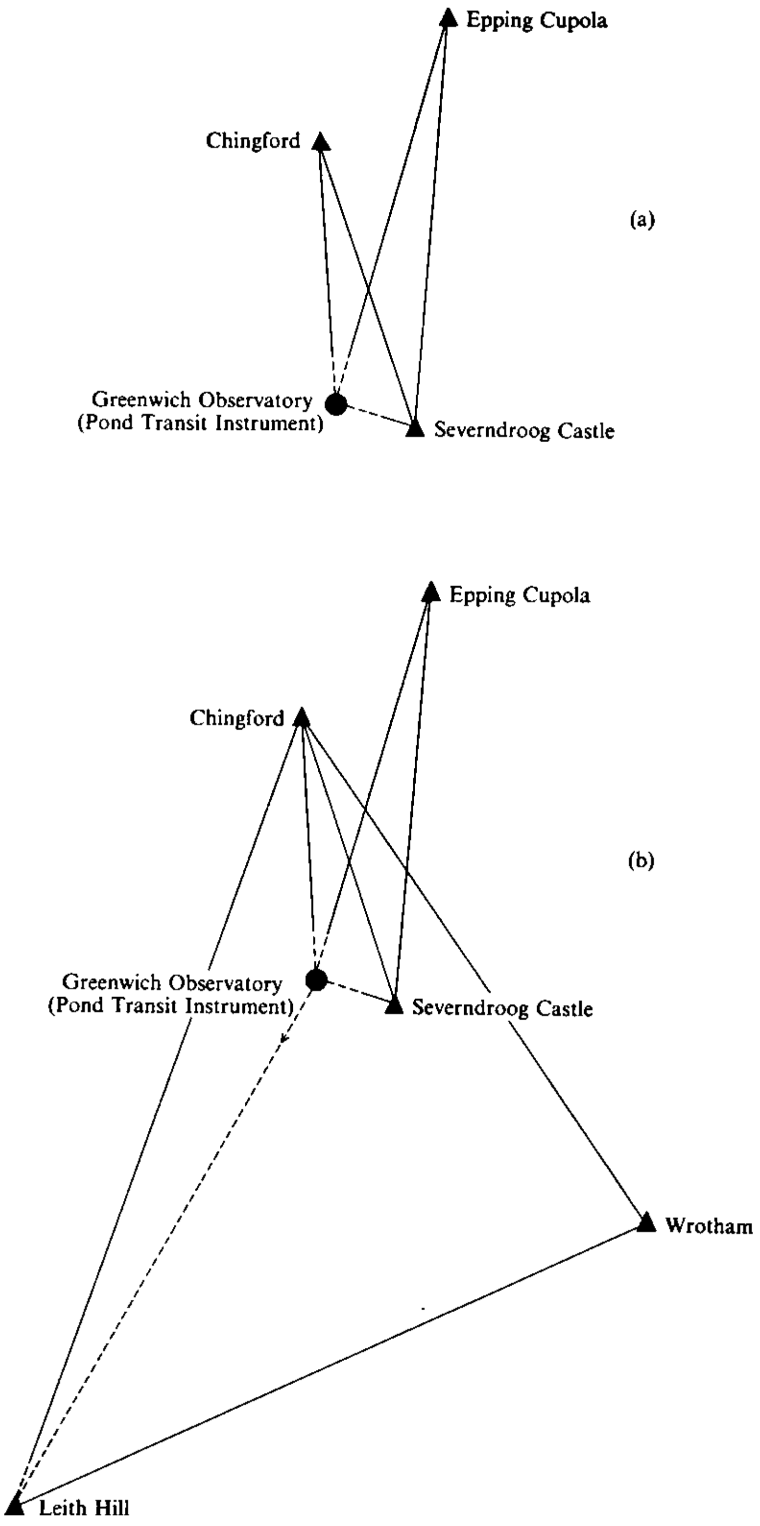


FIG. 3.1. Connection of the Principal Triangulation to the Pond Transit Instrument

alongside the old transit room. This new instrument was first used in 1851, after which the old Pond Transit was taken away. The Pond Transit had been erected on the site of Bradley's original instrument in 1816, and up to 1851 it had defined the Greenwich zero meridian. There can be no doubt that the Greenwich transit instrument referred to by Clarke was the Pond Transit Instrument. But since 1850 the Greenwich zero meridian has been the meridian through the centre of the Airy Transit Circle. This was agreed at a conference held in Washington in 1884 at which this meridian was accepted internationally as the zero meridian of longitude.

3.064 DOCUMENTARY EVIDENCE

The matter therefore hinges on the relative positions of the Pond and Airy Transits regarding which evidence is contained in various documents cited below.

(*a*) Extracts from a letter written by Sir George Airy in March 1849 to Captain Yolland, R.E., of the Ordnance Map Office:

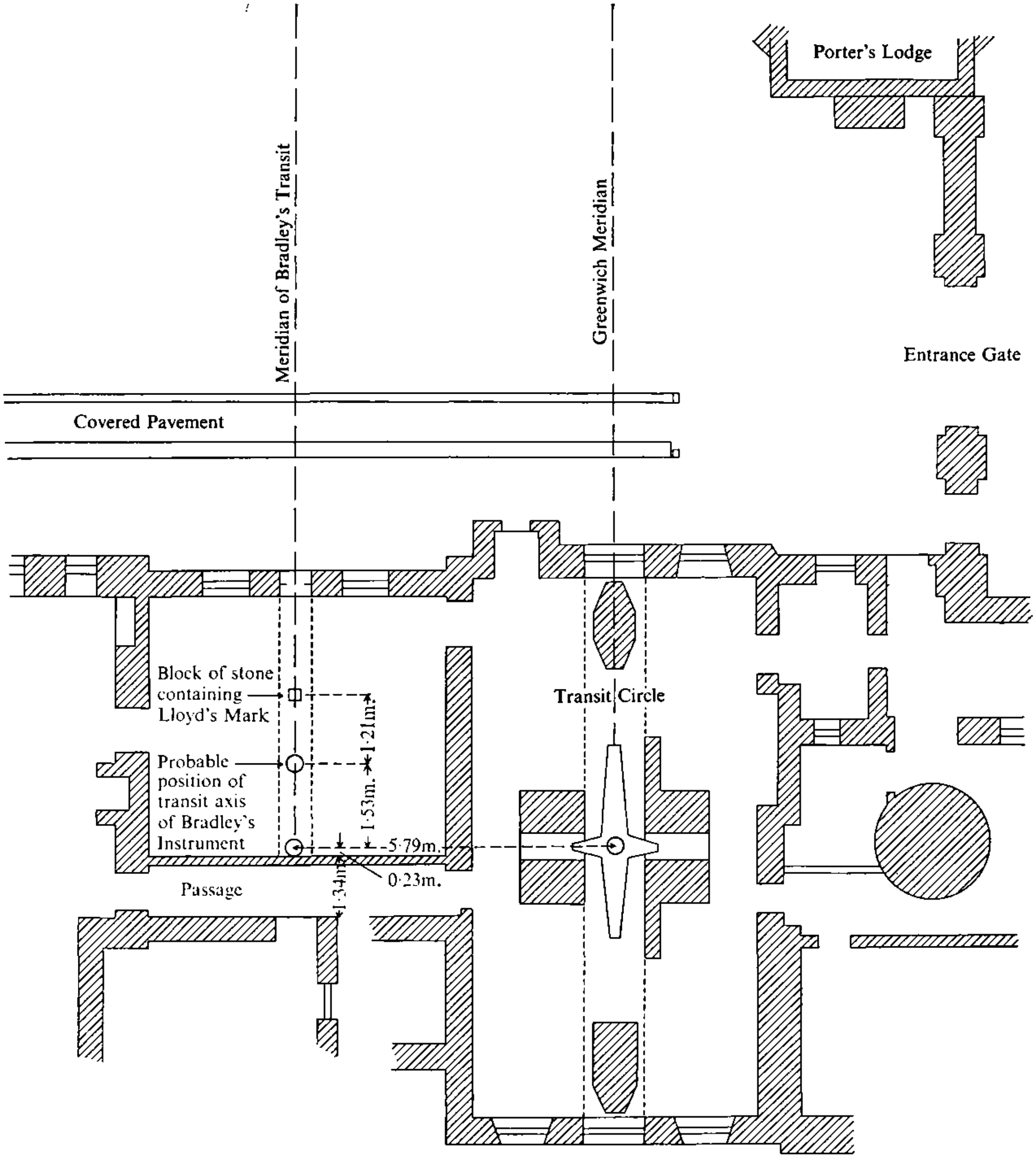
> R.O.
> March 1849
>
> 'The brass standard in the transit room of the Royal Observatory to which the surveyor under the Ordnance Map Department has lately levelled, is the same to which Mr. Lloyd refers in his paper "An account of operations carried on for ascertaining the difference in level between the River Thames at London Bridge and the sea etc." (*Philosophical Transactions* 1831, page 184) in the following words:
>
> > "I levelled up to a small brass standard already placed for me by the direction of the Astronomer Royal in the block of stone immediately under the eye end of the transit instrument pointing southwards . . ."
>
> In levelling to this mark it may be proper to observe that I contemplate applying the transit room to another purpose, and it may be desirable to qualify the description of the place of the brass standard by adding to it "the transit instrument in the position it occupied in 1830 and 1848".
>
> (Signed) George Airy'

From the letter it is clear that:

(i) Airy was intending in 1849 to move the transit instrument to a new position.
(ii) That the brass standard referred to by Mr. Lloyd was in the old transit room.

(*b*) The diagram at Fig. 3.2 is reproduced from *Determinations of Longitude 1888–1902* published by the Board of Admiralty 1906(23). It shows the old transit room alongside the present transit room with the 'Meridian of Bradley's Transit' running through the centre of it. There is a stone in the floor of this room with a small brass rivet in it known as Lloyd's mark, which must clearly be identical with the 'small brass standard' referred to in (*a*) above. There can be no doubt that this room was the site of the transit instrument until 1849 or 1850, and that it was to the Pond Transit Instrument located there that Clarke made his trigonometrical observations. There is also no doubt that the Pond Transit Instrument stood on the 'Meridian of Bradley's Transit' shown in the diagram.

(*c*) The Introduction to the annual volume of the *Greenwich Observations* contained in Airy's day and later a great deal of matter repeated verbatim from one year to the next. The volume for 1851 states 'The centre of the instrument (The Airy T.C.) is about 7½ feet south and 19 feet east of the old transit instrument.' This statement is repeated year by year until 1861 when '7½ feet' is changed to '5½ feet'. The statement establishes the change of the zero meridian when the Airy Transit Circle was installed. It also proves that the new latitude differed from the old. The later change from '7½ feet' to '5½ feet' was concluded by Dr. Atkinson to be the result of an error detected in 1860 or thereabouts which is further discussed in § 3.065.



Details in red do not appear on the original plan, but have been added to illustrate measurements described in paragraph 3·064

FIG. 3.2. Copied from an old plan of the Royal Observatory, Greenwich

(*d*) Further evidence is contained in the following letters exchanged between Sir George Airy and Lieutenant A. R. Clarke in 1855.

Extract from Lieutenant A. R. Clarke's letter to Sir George Airy:

> 'In order to reconcile a slight discrepancy in azimuth I beg leave to enquire whether the object observed and recorded in the *Greenwich Observations* as "the Meridian Mark" (at Chingford) is the *centre* of the stone pillar or a vertical line drawn on it either East or West of the centre, and if the latter be the case, is there any record of its place on the stone.'

Extract from Sir George Airy's letter to Lieutenant A. R. Clarke:

> 'The object observed as the Chingford Meridian Mark was the centre of the pillar as estimated.
>
> There is no mark on the stone; nor, if there had been any, could it have been seen. The pillar was never well defined.
>
> The observations were made with the Transit Instrument in the *old* Transit Room.
>
> I believe that any observations recorded in your office will refer to the same point: but I state this in caution if any late observations on the survey shall have been referred to the *new* Transit Room.'

It is thus quite clear that Clarke knew of the move to the new transit room, although no mention of it was made in his account of the Principal Triangulation.

Measurement on the plan at Fig. 3.2 indicates that the distance between the two meridians is about 19 feet. This was checked on site and a result of 18·9 feet was obtained despite the fact that there was some doubt as to the two exact positions to be measured to.

3.065 THE RELATIVE POSITIONS OF THE POND AND AIRY TRANSITS

The Bradley Transit Instrument was replaced in 1816 by the Pond Transit Instrument which stood on the identical site of the Bradley. The position of the centre of the axis of the Pond Transit Instrument is not now marked, but old plans all place it in the centre of the transit room. The room now has a passage which was constructed sometime subsequent to the original building along its south wall. (See Fig. 3.2.) The original roof of the building is still in place and is symmetrical. It would be reasonable to expect the centre of the room to be vertically below the roof ridge, and measurements from the old walls ignoring the passage confirm this. The measurements place the centre of the roof as the most probable position of the old transit instrument at a point about 47 inches south of Lloyd's mark. The subsequently constructed passage is 3·3 feet in width and its dividing wall 1·1 feet thick. Thus the centre of the room was shifted by (4·4)/2 feet = 2·2 feet when the passage was constructed. It seems very likely that failure to allow for this shift was the cause of the latitude error evidently occurring in the *Greenwich Observations* volumes before 1861.

With the above assumptions modified as a result of actual measurements made on the site in 1949 the following is the best estimate of the relative positions of the Pond and Airy Transits.

The Airy Transit Circle is 19 feet 0 inches east of the Pond Transit Instrument (or 5·79 m. = 0·300 seconds).

The Airy Transit Circle is 5 feet 0 inches south of the Pond Transit Instrument (or 1·52 m. = 0·049 seconds).

3.066 THE RESIDUAL DISCREPANCY IN LONGITUDE BETWEEN THE TWO TRIANGULATIONS

The existence of the discrepancy between the geodetic longitudes for the Greenwich zero meridian in the two triangulations having been established beyond doubt, it was decided in 1954 to co-ordinate the centre of the Airy Transit Circle directly from the primary Retriangulation, in order to confirm the position obtained from the lower order triangulation in 1949. A 40-foot steel tower was erected in the grounds of the observatory on the meridian immediately outside the room containing the Airy Transit Circle (see Fig. 5.11 in Chapter 5). From the steel tower the centre of the Airy Transit Circle was fixed by bearing and distance. Reciprocal observations were made between the steel tower and the following primary stations:

Epping (483)
Warley Water Tower (224)
Severndroog Castle (189)
Chipping Barnet Church Tower (185).

In addition, observations were made to the Pole Hill Obelisk, which was the azimuth mark used in azimuth determinations by the Royal Observatory. For details of observations see Appendix 8.7.

The Retriangulation value of the Airy Transit Circle obtained from this primary connection was:

E 538 882·88 m. N 177 321·61 m.
or φ 51° 28′ 38″265 N λ+00° 00′ 00″418 E

The following Tables, 3.8 and 3.9, give the geodetic positions of the Airy Transit Circle in both triangulations, and the discrepancies between them. It will be noted that two Retriangulation values, differing by 0·016 seconds in latitude and longitude, are given. This is due to the different methods of adjustment of Figure 5; the published National Grid values are based on the adjustment of Figure 5 in two parts, whereas the second value is based on the more correct adjustment of the complete Figure 5 as a single unit. See Chapter 2, § 2.30.

Thus, the residual discrepancy between the deduced value for the Airy Transit Circle in the Principal Triangulation and its value from published Retriangulation co-ordinates is 0·014 seconds in latitude (0·43 m.) and 0·118 seconds in longitude (2·29 m.) The resulting vector is 2·33 m. Vectors of similar direction and magnitude occur between the two triangulations in this area, for example the comparable vector at St. Paul's Cathedral is 2·28 m. See Chapter 6. There is little doubt therefore that the residual discrepancy is to be attributed to the errors in both triangulations. Using the values obtained from the adjustment of Figure 5 as one figure, the discrepancy is reduced to 0·002 seconds in latitude and 0·102 seconds in longitude, or a vector of 1·95 m.

3.067 AZIMUTH CONNECTION

When Greenwich Observatory (482) primary station was co-ordinated in 1954 the observations at this station included pointings to the old Greenwich North Meridian Mark at Chingford, now called Pole Hill Obelisk. (See Appendix 8.7 for details of the observations.)

TABLE 3.8

Item No.	*Item*	*Latitude*	*Longitude*
	Geodetic position of the centre of the Pond Transit Instrument, accepted by Clarke for the Principal Triangulation.	51° 28′ 38″300 N	00° 00′ 00″000
	Distances measured between the assumed centre of the Pond Transit Instrument and the actual centre of the Airy Transit Circle.	−0″049	+0″300
1	Deduced geodetic position of the Airy Transit Circle in the Principal Triangulation.	51° 28′ 38″251 N	+00° 00′ 00″300 E
2	Geodetic position of the Airy Transit Circle from the Retriangulation (derived from published National Grid values).	51° 28′ 38″265 N	+00° 00′ 00″418 E
3	Geodetic position of the Airy Transit Circle from the Retriangulation (from the adjustment of Figure 5 as one figure).	51° 28′ 38″249 N	+00° 00′ 00″402 E

TABLE 3.9

Difference	*Discrepancy*				
	Latitude		*Longitude*		*Vector Metres*
	Seconds	*Metres*	*Seconds*	*Metres*	
Item No. 2 minus Item No. 1	+0·014	+0·43	+0·118	+2·29	2·33
Item No. 3 minus Item No. 1	−0·002	−0·06	+0·102	+1·95	1·95

During the periods 8th June–7th August and 11th September–9th October of 1953 the staff of the Royal Observatory made some azimuth observations with the Airy Transit Circle to an Ordnance Survey beacon lamp set on the top of Pole Hill Obelisk; the position of the lamp was 0·122 m. east of the vane in the centre of the Obelisk. This distance subtends 1″43 at Greenwich Observatory (482).

The mean geodetic azimuth from Greenwich Observatory (482) to the Obelisk lamp position (called Reference Mark in Appendix 8.7) was

359° 58′ 55″25

Reducing to the Obelisk centre by applying $-1''.43$ gave

$$359^\circ\ 58'\ 53''.82$$

as the geodetic azimuth to the Obelisk centre. The Airy Transit Circle is 7·390 m. from the primary station, and the correction to reduce the azimuth from the primary station to the Transit Circle is $-0''.10$. The geodetic azimuth of the Pole Hill Obelisk centre at the Airy Transit Circle was therefore

$$359^\circ\ 58'\ 53''.72$$

The geodetic latitude and longitude of the Airy Transit Circle were

$$\varphi_G = 51^\circ\ 28'\ 38''.265 \text{ N} \qquad \lambda_G = +00^\circ\ 00'\ 00''.418 \text{ E} \quad (\text{See § 3.066})$$

so the Laplace correction to astronomic azimuth to get geodetic azimuth was

$$(\lambda_G - \lambda_A) \sin\varphi = +0''.418 \times \sin\varphi = +0''.33$$

λ_A being zero. See § 3.09.

The results of the 1953 azimuth observations by the Royal Observatory gave an astronomic azimuth from the Airy Transit Circle to Pole Hill Obelisk centre of

$$359^\circ\ 58'\ 52''.62$$

with a probable error of $\pm 0''.05$.
Applying the Laplace correction, the Laplace azimuth was

$$359^\circ\ 58'\ 52''.95$$

which differed by $0''.77$ from the geodetic azimuth.

The Airy Transit Circle is not reversible, consequently any uncertainty in the determination of the collimation error will have entered systematically into the astronomic azimuth of the Pole Hill Obelisk.

THE CONNECTION TO THE ROYAL GREENWICH OBSERVATORY, HERSTMONCEUX

3.07 Introduction

In 1949 the Royal Observatory started to move from Greenwich to Herstmonceux in Sussex because atmospheric conditions at Greenwich were no longer suitable for precise astronomic observation. It was decided to establish a connection between the Retriangulation and the new observatory. The ideal arrangement would have been to co-ordinate the point over which the main meridian transit of the observatory, called the Cooke Transit Circle, was to be centred. Had this been possible, and had observations to the azimuth marks which were to be used in conjunction with the Cooke Transit Circle been included, it would have been possible to make a complete comparison between the astronomic latitude, longitude, and azimuth, and the geodetic values.

Unfortunately this could not be done, because the exact location of the Transit Circle could not be determined before the instrument was installed. Furthermore, intervening trees prevented a connection from the site of the Transit Circle to the surrounding primary stations being made from ground level. To clear these local obstructions a 103-foot steel tower would have been necessary and the foundations for such a tower would have interfered with the foundations for the Transit Circle which were already being prepared.

Consequently it was decided to do the work in two parts, first to co-ordinate the position of the Transit Circle, and later, after the installation of the instrument, to make the azimuth connection. As a preliminary to co-ordinating the Transit Circle a standard triangulation pillar was erected and co-ordinated. The site chosen for this pillar was about 3,000 feet to the south-east of the Transit Circle and observations were made to and from the primary stations:

Fairlight Down (193); Beachy Head (194); Firle Beacon (199); Ditchling (32).
(See Diagram 12)

This work was carried out in August 1953, and the pillar was known as Herstmonceux (481).

3.08 Co-ordination of the Cooke Transit Circle

As soon as the base plate of the Transit Circle was in position in June 1953, a temporary mark on it was connected to Herstmonceux pillar by the scheme shown in Fig. 3.3. At the time it was understood that the Transit Circle would be accurately centred over this temporary mark, but subsequently in 1956 from discussions with the members of the staff of the Observatory responsible for the erection of the Transit Circle it transpired that the centring of the Transit Circle over that mark could not be guaranteed, but they believed it to be within half an inch. Re-observations taken at Herstmonceux pillar and Solar have established that the temporary mark is indeed located under the Transit Circle, but it is not possible to determine its exact location in plan relative to the actual centre of the Transit Circle, that is, to the point of intersection of the axis of collimation with the trunnion axis.

The results of the 1953 observations to connect the temporary mark were:

From	*To*	*Mean Observed Direction*
Herstmonceux (481)	Fairlight Down (193) Firle Beacon (199) Solar Transit Circle (Temporary Mark)	00° 00′ 00″ 171 49 48·7 198 29 29·1 238 03 28·2
Solar	Transit Circle (Temporary Mark) Herstmonceux (481)	00° 00′ 00″ 94 05 44
Transit Circle (Temporary Mark)	Solar Herstmonceux (481)	00° 00′ 00″ 313 39 37

Horizontal Distance: Solar to Transit Circle (Temporary Mark) 567·641 m.

From these results it was calculated that the Retriangulation value for the Cooke Transit Circle (Temporary Mark) was:

	E 564 531·38 m.	N 110 704·21 m.
or	φ 50° 52′ 18″597 N	λ +00° 20′ 19″273 E

These results can be accepted as applicable to the centre of the Transit Circle as the small uncertainty in position discussed above is not significant in relation to the accuracy of the triangulation.

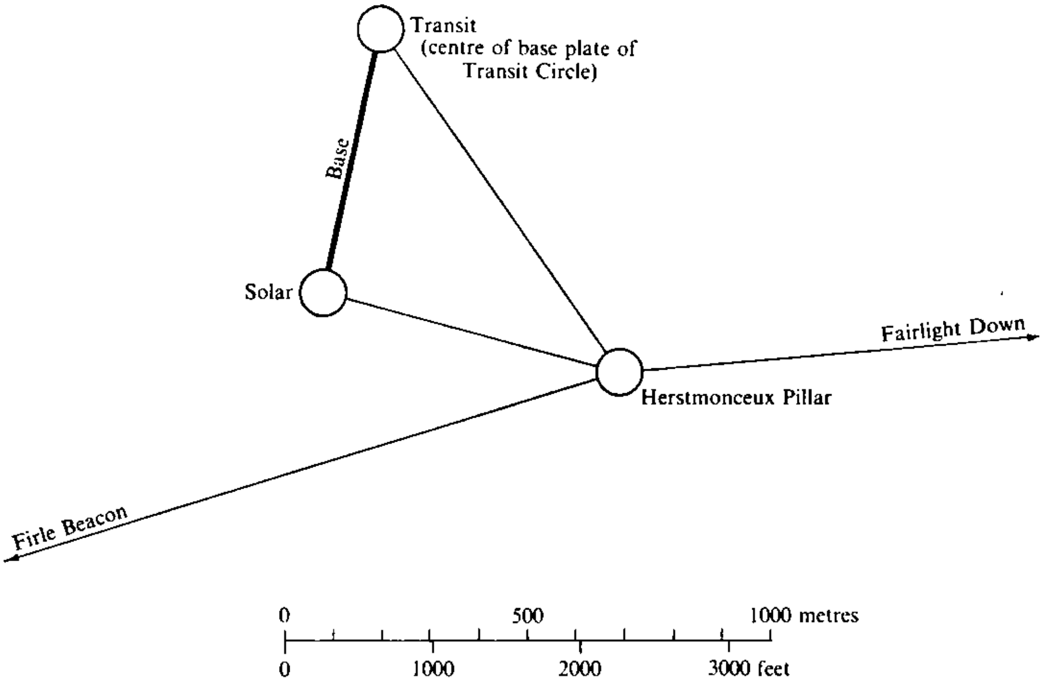


FIG. 3.3. Triangulation scheme to co-ordinate the Cooke Transit Circle at the Royal Greenwich Observatory, Herstmonceux

3.09 Longitude Difference Greenwich-Herstmonceux

From the above it may be seen that the geodetic longitude difference between the Airy Transit Circle at Greenwich and the Cooke Transit Circle at Herstmonceux is:

Geodetic longitude of Cooke Transit Circle	+00° 20′ 19″273 E
Geodetic longitude of Airy Transit Circle	+00° 00′ 00″418 E
Geodetic longitude difference	00° 20′ 18″855

From this the following geodetic longitude difference between the Photo Zenith Tube at Herstmonceux and the Airy Transit Circle at Greenwich is deduced from ground measurements:

$$00^\circ\ 20'\ 19''.755.$$

When the Observatory at Herstmonceux opened it was necessary for the Meridian Department to adopt a value for the astronomic longitude difference between Greenwich and Herstmonceux. For this purpose they used calculations of the deviations of the vertical at Greenwich and Herstmonceux made by A. H. Cook from available gravity observations(51). This gave the following astronomic longitude difference between the Airy Transit Circle at Greenwich and the Photo Zenith Tube at Herstmonceux.

Geodetic longitude difference	$00^\circ\ 20'\ 19''.755$
Deviation at Greenwich	$-2''.135$
Minus deviation at Herstmonceux	$-1''.394$
Astronomic longitude difference	$00^\circ\ 20'\ 16''.226$

Or, expressed in time, $00^{\mathrm{h}}\ 01^{\mathrm{m}}\ 21^{\mathrm{s}}.082$.

In fact owing to a misunderstanding of the correct value of the geodetic longitude of the Airy Transit Circle site at Greenwich, which arose from the slight discrepancy between the Principal Triangulation and the Retriangulation at this place (see § 3.066), the Meridian Department adopted a provisional value $00^{\mathrm{s}}.009$ different from the above, i.e.,

$$00^{\mathrm{h}}\ 01^{\mathrm{m}}\ 21^{\mathrm{s}}.091$$

This provisional value agreed well with the provisional values of astronomic longitude observations of the Ordnance Survey at Greenwich and Herstmonceux which then gave the difference (later revised) as:

$$00^{\mathrm{h}}\ 01^{\mathrm{m}}\ 21^{\mathrm{s}}.092$$

A thorough study of the observations made with an instrument known as the Small Transit before and after it was moved from Greenwich to Herstmonceux in April 1957, and comparisons against the Photo Zenith Tube at Herstmonceux, showed that the provisional value was satisfactory within one or two milliseconds, and the provisional value was therefore adopted.

Later, in 1962, however, a definitive value was allotted to the Photo Zenith Tube at Herstmonceux by the Bureau International de l'Heure of

$$00^{\mathrm{h}}\ 01^{\mathrm{m}}\ 21^{\mathrm{s}}.102$$

It is this value that has been used in calculating all azimuth results. (See § 3.103, and § 5.08 in Chapter 5.) It should be noted however that if this value is adopted for Herstmonceux, and if Cook's deviations derived from gravity observations are accepted, an adjustment of about $00^{\mathrm{s}}.020$ should be made to the adopted longitude of the Airy Transit Circle at Greenwich. However, since the deviations based on the gravity survey are of somewhat uncertain accuracy, and since in a sense the Airy Transit Circle site continues to define the zero meridian of astronomic longitudes, a zero value for astronomic longitude at Greenwich has been used for calculating the azimuth there in § 3.067.

3.10 Azimuth Connection

As part of the installation of the Meridian Group of the Observatory it was proposed to erect azimuth marks or referring objects, for which the meridian observations would in due course yield azimuth values of very high accuracy.

For these marks obviously a high degree of east-west stability is desirable, although slight changes in azimuth occurring steadily over a period of time would not seriously affect the observations since the effect of such changes would become apparent from the results and could be eliminated. From the geodetic point of view the existence of a line of which the astronomic azimuth is established to a small fraction of a second is of obvious value, particularly when as in this case its azimuth is continuously checked by astronomic observations of the highest accuracy. Clearly therefore a geodetic connection to one of these lines was desirable.

The ideal way to make the connection would have been to take observations from the actual centre of the Cooke Transit Circle itself or from a point vertically above it, but for the reasons given in § 3.07 this could not be done. Various schemes for getting over this difficulty were considered but rejected because they all involved a degree of uncertainty in centring which would have invalidated observations on the rather short rays concerned. It was therefore necessary to make the connection by observation back from one of the azimuth marks into the Transit Circle telescope. With this procedure if the comparison between astronomic and geodetic azimuth is made at the station of geodetic observation, that is at the azimuth mark, an unknown error is incurred since the astronomic azimuth of the line is known only at its other end, unknown variation in the deviation of the vertical rendering its exact value at the azimuth mark uncertain. On the other hand if a comparison is made at the Transit Circle end a deduced value of the geodetic azimuth must be used which is necessarily weaker than an azimuth derived from direct observation from the point of comparison. In this case the unknown deviation of the vertical again exerts an effect but only insofar as it makes horizontal angle measurements slightly erroneous, a factor which has been ignored throughout the Retriangulation.

Probably a more serious source of error arises from the grazing nature of all the rays used in the scheme. Observations must certainly as a result have suffered from lateral refraction for which there is no effective check, although to reduce this error observations were spread over a period of four nights.

3.100 THE PEVENSEY AZIMUTH MARK

The Astronomer Royal agreed to design and site one of the proposed azimuth marks so that the necessary observations for the connection could be made there. The mark is about 3 miles south of the Cooke Transit Circle, near Pevensey.

Details of the design were proposed by the Royal Greenwich Observatory in consultation with the Ordnance Survey. The mark itself consists of a concrete pier rising about 8 feet above ground level set on foundations resting on deep piles. A normal triangulation spider is inserted on the top of the pier. The pier is protected from direct sunlight and other causes of temperature variation by a wooden shelter with shaded walls. This shelter is arranged so that theodolite observations can be made from the spider on top of the pier. The actual azimuth mark is a small hole (about $\frac{1}{4}$ in. diameter) in an inclined metal plate illuminated by a light placed behind it. The spider was plumbed vertically over the small hole during construction of the pier, and is so designed that the relationship can be checked subsequently.

3.101 FIXATION OF THE AZIMUTH MARK

Observations to determine the geodetic co-ordinates of the azimuth mark were made after removing the roof of the wooden shelter, and were taken to one primary and three tertiary stations from a temporary point 0·2 m. away from the azimuth mark centre. The latter was co-ordinated by bearing and distance from the temporary point.

This was not a first order fixation. It has already been stated that it was decided to make the azimuth connection by observing the geodetic azimuth of the line from the Pevensey Azimuth Mark to the Transit Circle. This could have been done by fixing the azimuth mark to geodetic accuracy and incorporating into the fixation horizontal observations from the azimuth mark to the Transit Circle. But this was unnecessary. Herstmonceux (481) was already co-ordinated and a geodetic azimuth of the line from the azimuth mark to the Transit Circle could be determined by observing from Herstmonceux (481) to the azimuth mark and then observing the included angle at the azimuth mark between Herstmonceux (481) and the Transit Circle. For this determination a lower order fix of the azimuth mark would suffice, and this is what was done.

From this fixation the National Grid co-ordinates of the Pevensey Azimuth Mark (centre) are:

E 564 692·94 m.   N 105 602·38 m.

3.102 OBSERVATIONS FOR MAKING THE AZIMUTH CONNECTION

The results of the 1961 first-order observations to effect the azimuth connection were:

From	*To*	*Mean Observed Direction*
Pevensey Azimuth Mark (centre)	Transit Circle Eyepiece	00° 00′ 00″
	Herstmonceux (481)	06 46 07·61
Herstmonceux (481)	Firle Beacon (199)	00 00 00
	Fairlight Down (193)	188 10 10·78
	Pevensey Azimuth Mark (centre)	288 49 26·37
	Beachy Head (194)	306 52 52·18

To obtain a suitable target at the Transit Circle and to eliminate the possibility of centring errors, which would be critical on this 3-mile ray, the observations from the azimuth mark centre were made to the micrometer eyepiece of the Transit Circle. The Transit Circle was positioned so that the eyepiece end of the telescope pointed south, and the eyepiece was adjusted by the Observatory staff so that it was exactly centred on the line between the azimuth mark and the centre of the instrument. The eyepiece was then removed and the aperture illuminated by a light placed behind the objective of the telescope, the light being positioned so that a suitable observing target was obtained.

3.103 THE AZIMUTH CONNECTION AND COMPARISON

Fig. 3.4 shows the connection. From Herstmonceux (481) plane grid bearings were computed to Firle Beacon (199), Fairlight Down (193), and Beachy Head (194), and a mean plane grid

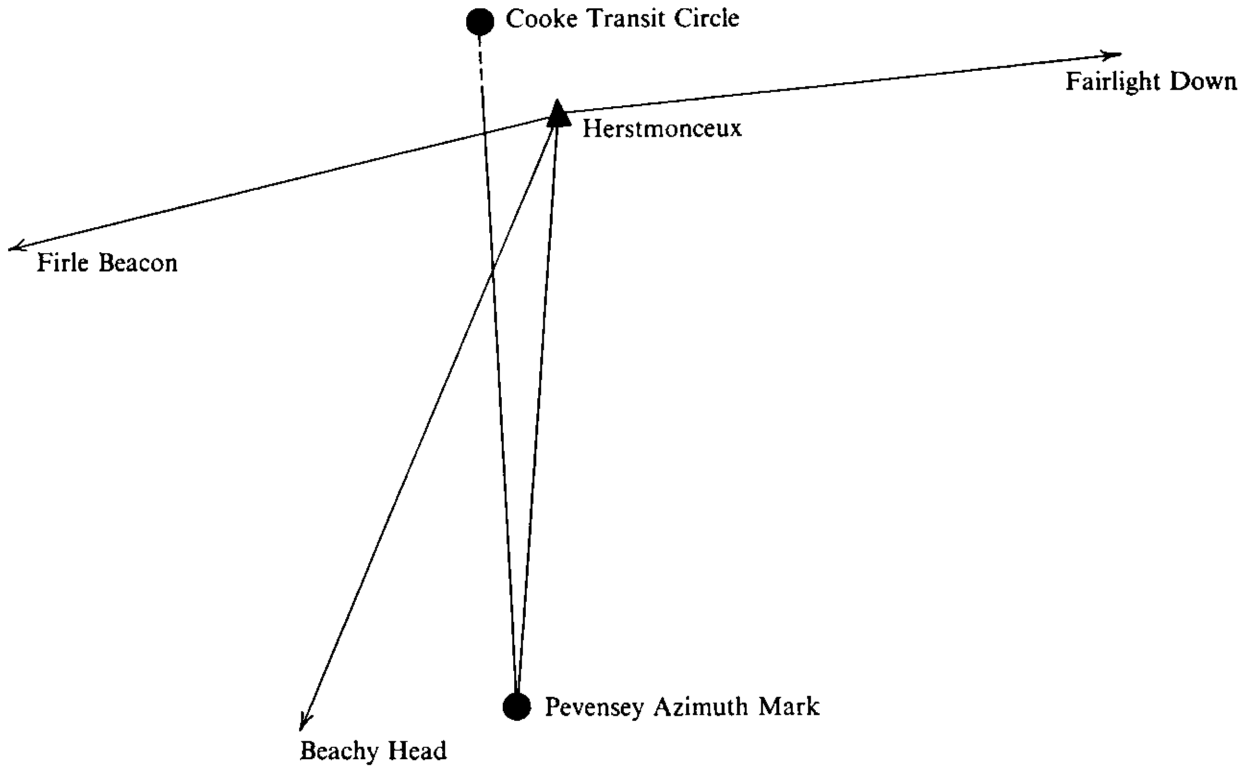


FIG. 3.4. Azimuth connection at the Royal Greenwich Observatory, Herstmonceux

bearing obtained for the line Herstmonceux (481) to Pevensey Azimuth Mark (centre) using the observations in § 3.102. These observations were first reduced to the plane by the $(t-T)$ correction. (See Chapter 2, § 2.241). Reversing the mean plane grid bearing Herstmonceux (481) to Azimuth Mark (centre) by 180° gave the plane bearing Azimuth Mark (centre) to Herstmonceux (481). This was transferred to the Transit Circle by applying the plane angle at the Azimuth Mark (centre); reversing this plane bearing by 180° gave the plane grid bearing from Transit Circle to Azimuth Mark (centre). Applying grid convergence and $(t-T)$ correction to this plane grid bearing gave the transferred geodetic azimuth of the line from Transit Circle to Azimuth Mark (centre) as

$$180^\circ\ 00'\ 00''.63$$

The geodetic latitude and longitude of the Transit Circle is given in § 3.08, and is

$$\varphi_G = 50^\circ\ 52'\ 18''.597\ \mathrm{N} \qquad \lambda_G = +00^\circ\ 20'\ 19''.273\ \mathrm{E}$$

The astronomic latitude and longitude of the Transit Circle were given by the Astronomer Royal in 1962 as

$$\varphi_A = 50^\circ\ 52'\ 17''.95\ \mathrm{N} \qquad \lambda_A = +00^\circ\ 20'\ 15''.630\ \mathrm{E}$$

The value of λ_A was derived from the definitive value allotted to the longitude of the Photo Zenith Tube at Herstmonceux by the Bureau International de l'Heure in 1962. (See § 3.09.) So the Laplace correction to astronomic azimuth to get geodetic azimuth was:

$$(\lambda_G - \lambda_A) \sin \varphi = +3''.643 \times \sin \varphi = +2''.83$$

The astronomic azimuth from the Transit Circle to the Pevensey Azimuth Mark (centre) was observed by the staff of the Observatory during the period April–June 1960, the result being:

$$179^\circ\ 59'\ 56''.46$$

with a probable error of $\pm 0''.04$.

Applying the Laplace correction gave the Laplace azimuth from the Transit Circle to Pevensey Azimuth Mark (centre) as

$$179^\circ\ 59'\ 59''.29$$

which differed by $1''.34$ from the transferred geodetic azimuth. The difference of $1''.34$, and that of $0''.77$ at Greenwich (see § 3.067), are typical of the small errors in the geodetic azimuths of the Retriangulation found at the other Laplace stations. See Table 5.6 in Chapter 5.

CHAPTER FOUR

Measurement of Bases

CATENARY MEASUREMENTS

4.00 Introduction

As explained in Chapter 2, the scale of the Retriangulation was determined by fitting it to the old Principal Triangulation; the more normal method of using measured bases was excluded. Hence the Retriangulation is computed on Airy's Figure of the Earth in terms of feet of the Ordnance Survey Standard Bar O_1 (see § 2.20). Nevertheless, when the Retriangulation was planned in 1935, provision was made for the measurement of bases in order to determine the scale of the Retriangulation in terms of the International Metre, the accepted universal standard of length. Two bases were projected, one in Southern England and one in Northern Scotland. It was not possible to select and measure these bases before using the co-ordinate values of the new primary triangulation in England and Wales, which were urgently required to control lower order triangulation. It was therefore impossible to introduce a length equation between bases in the adjustment of the main chain, but in view of the geometrical strength of the network and the greater probability of local error in the base extensions, it may be doubted whether in any case it would have been sound to introduce such a length equation into which only a very few of the intermediate observations could enter.

It was fully realised that the procedure of fitting the new work to the old would mean that any scale error existing in the Principal Triangulation would inevitably be introduced into the new work, but, so far as could be foreseen at that time (1935), there was no practical application of the Retriangulation which would have required a more accurate knowledge of absolute length, and such has proved to be the case until recently. But scientific and military developments have now created a demand for a knowledge of accurate lengths for such purposes as determining artificial satellite orbits. Fortuitously this need has arisen at a time when electronic methods of distance measurement have been developed, and it is possible to determine these lengths to the required accuracy by using the Retriangulation in conjunction with Geodimeter and Tellurometer measurements.

The main reason for the decision to fit the Retriangulation as closely as possible to the old Principal Triangulation was to minimise the differences between the new and old large scale plans, and thus reduce the inconvenience to the users. Events have justified this decision.

Two bases were initially selected—one the Ridgeway Base, on the Ridge Way on the Berkshire Downs, and the other the Lossiemouth Base near Lossiemouth in Morayshire, Scotland. The latter had previously been established as a base in 1909 to test the accuracy of the old Principal Triangulation(12). The Ridgeway Base was measured in November–December 1937 and the Lossiemouth Base in July–August 1938 under the direction of Major Hotine. For reasons given later, the Lossiemouth Base with its extension was considered to be unsatisfactory and a further base was reconnoitred in the extreme north of Scotland where a complete side of the triangulation could be measured. This base, near Thurso in Caithness, and known as the Caithness Base, was measured in April–June 1952 under the direction of Major M. H. Cobb. The Caithness Base crosses a peat bog for nearly half its total length of 25 km. and was known to be fraught with difficulties for catenary base measurement. Consequently, the Ridgeway Base was remeasured in 1951 to train the measuring party and to test certain modifications made to the Macca base measurement equipment.

Before deciding on the initial measurement of the Ridgeway Base, due consideration was given to the remeasurement of the Salisbury Plain Base of the old Principal Triangulation. Originally measured with Ramsden's steel chains in 1794 and securely marked with a pair of buried guns, the Salisbury Plain Base was remeasured in 1849 with Colby's compensation bars; and there is little doubt that this latter measure would compare favourably, as regards accuracy if not as regards speed, with modern bases. Consequently a third measure in 1937 might have thrown further light on the relation between the modern International Metre and the 1849–1866 values of the Ordnance Survey 10-foot bar standards, which had been included with most other contemporary geodetic standards in Clarke's 1866 comparisons. Although a definitive scale for the old Principal Triangulation of the British Isles in terms of modern standards is no longer a burning question, such a remeasurement of the Salisbury Plain Base might well have been helpful in other ways such as the unification of the primary surveys of the African Continent. Unfortunately, for some reason which is no longer obvious, the terminals were not sited in 1794 to be intervisible. A 32-foot scaffolding was erected over the south terminal in 1849, and it may be concluded from the fact that the misclosures of the main extension triangles exceeded 4 seconds, that the air line was even then far too close to intervening ground for accurate base extension. Apart from the fact that there are now many obstructions on the base line, a remeasurement would not therefore have served to fix the scale of the new triangulation to the required accuracy.

The subsequent invention of electronic instruments for base measurement, such as the Geodimeter and Tellurometer, have enabled further measurements to be made of the Ridgeway and Caithness Bases. These are discussed later in this Chapter.

THE RIDGEWAY BASE 1937

4.01 Description of Site

The Ridgeway Base runs along the ancient Ridge Way at a general level of about 700 feet, mainly across open downland between two primary stations, White Horse Hill (34) (856 feet) and Liddington Castle (35) (910 feet) and is about 11 km. (7 miles) in length. The base is shown in elevation in Fig. 4.1. The site provides a well-conditioned connection into the main primary network and the base terminals themselves are sharp clean features. Several sections had, however, to be measured over steeper slopes than had hitherto been considered permissible in first class bases; even so it was felt that with good levelling any resulting linear inaccuracies would not be comparable with the less obvious loss of accuracy inherent in a weak extension. Nothing is achieved in measuring a completely level base to an accuracy of one part in 1,000,000, if this accuracy is at

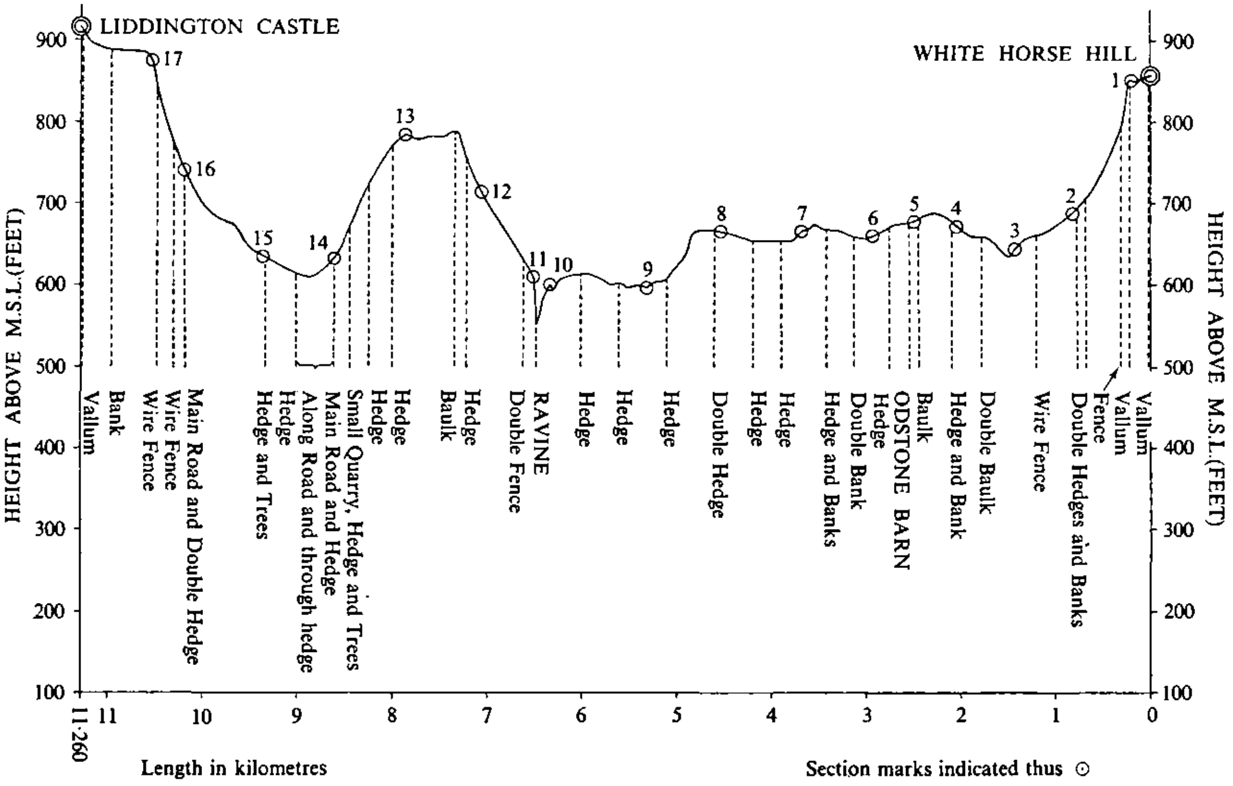


FIG. 4.1. Ridgeway Base: Elevation showing section marks and obstructions

once reduced to 1 in 100,000 or less by the introduction of angular errors of no more than 2 seconds in the first triangles of the base extension. These figures in no way exaggerate the effect of accidental or systematic atmospheric errors, which may occur even in a temperate country, in badly conditioned extension triangles, which are apt to be associated with a level base.

The White Horse Hill (34) terminal was already an observing station in the primary triangulation, whereas the Liddington Castle (35) terminal was initially intersected from five primary stations. After the decision to adopt this base site, Liddington Castle (35) was occupied for the outward observations to the five primary stations (see § 2.33). The inclusion of these later observations in a re-adjustment altered the position of Liddington Castle (35) only by 0·059 m., or one part in 190,000 of the base length, which gives some indication of the reliability of the extension.

4.02 Equipment and Procedure

The standard Macca base measurement equipment manufactured by Messrs. Cooke, Troughton & Simms was used in the 1937–38 measurements. This equipment was fully described in articles in the *Empire Survey Review*(24), in connection with the measurement of the Kate Base of the East African Arc 1931–33. The measuring procedure described in those articles was followed closely in the 1937–38 measurements on the Ridgeway and Lossiemouth Bases. Six new (but artificially 'aged') tapes, each 24 m. × ⅛ in. × 1/50 in., were obtained; three as working tapes and three for field standard tapes to control the working tapes.

The only modifications to the original procedure were the introduction of a levelling party and the inclusion of a booker. These had been excluded from the Kate measurement due to scarcity of trained surveyors. The inclusion of a levelling party eased the load of work on the surveyor in charge of the aligning party, who previously had measured the slope between measuring heads by vertical angles on the aligning theodolite. Since slopes of over 25° existed on the Ridgeway Base, it was thought that vertical angles would not have been sufficiently accurate and therefore precise levels and invar staves were used. A special short invar staff, for setting on the measuring heads, was ordered but was not available until the forward measurement had started. Consequently a normal levelling staff with a paper face was used on the flatter sections of the base. The paper-faced staff gave adequate results after calibration, but those section measures in which it was used were not incorporated in the finally accepted length.

4.03 Measurement

The measurement could not be started until late in 1937, as the personnel concerned were engaged on primary observations in Figure 3 in Scotland until October. An advance party left Southampton on the 1st November to prepare the base by clearing obstacles on the line which included hedges, traffic signs, and probably the most formidable of all, a large stack of very ripe manure! Measurement began on 10th November and since the party had had no previous training, there were inevitable delays through the tripods being knocked, tapes misread and other human factors. Initially about 40 bays per day were completed but as the team gained experience this improved to over 60 bays. The maximum rate was 91 bays, which was a considerable achievement for a short December day.

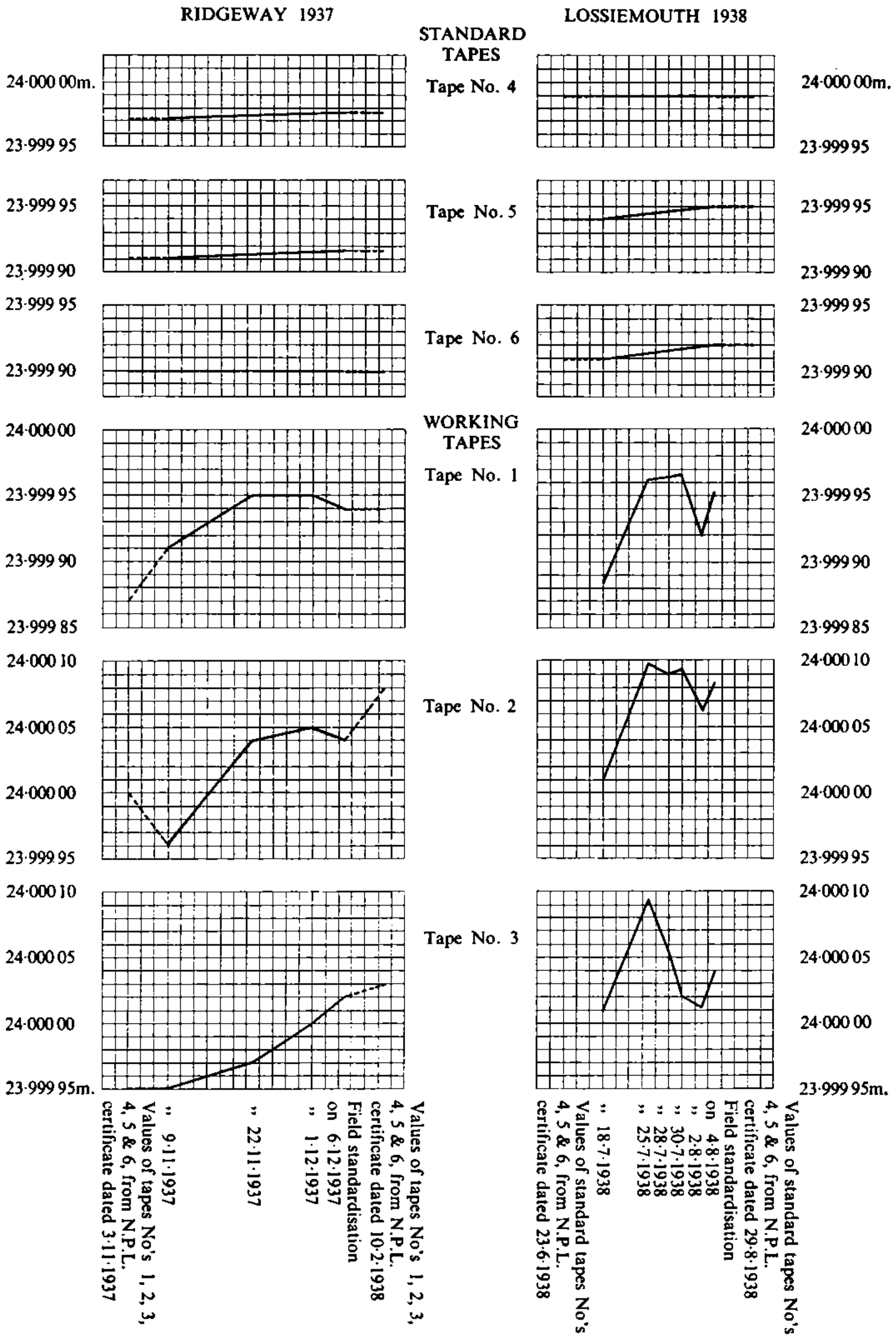


FIG. 4.2. Standardisations for 1937–1938 measures

4.04 Obstacles

The main obstacles were a deep ravine some 50 feet deep and 300 feet wide in Section 8, and the deep ditches and ramparts of the Iron Age hill forts on which both terminals were located. Several attempts to span the ravine by a 300-foot tape between prepared approaches and emplacements were defeated by wind. The gap was then triangulated, initially by a single triangle, and finally by a braced quadrilateral with offset bases on both sides. Zeiss traversing equipment was used and satisfactory results obtained despite the fact that the design of this equipment left much to be desired from the point of view of precise interchangeability of theodolite, target, and measuring head. The horizontal observations and measurements for the quadrilateral were repeated on different days and the two results agreed to 1·4 mm. in 191 m. or 1 in 136,000. Finally, the gap was negotiated with the 24-metre tapes to gain experience on very steep slopes, amounting in some cases to almost 40°. This result disagreed with the mean of the two triangulation sets by only 1·0 mm. This may well have been fortuitous.

Direct taping	191·3904		191·3904
Triangulation	191·3921	mean	191·3914
	191·3907		
		Accepted mean	191·3909 m.

The ditches and ramparts near the terminals could be spanned by the 24-metre tapes, but severe fluttering of the tapes resulted from the wind which seemed to be canalised by the ditches, where the tape could not be adequately screened from ground level. Eventually, satisfactory measurements were obtained by flying the whole screen up to the required level on long guys in the manner of a kite, an extremely perilous procedure requiring reliable men on the guys.

The line also ran straight through a barn which was negotiated by threading tapes through holes cut in the walls in preference to offsetting or triangulating.

4.05 Standardisation

All six tapes were calibrated by the National Physical Laboratory to Class 'A' accuracy (1 in 1,000,000) before and after the complete Ridgeway measurement. Field standardisation was carried out at sheltered parts on the line by comparing the working tapes against the field standard tapes. No field standardisation took place at the beginning of the measurement as it followed shortly after the calibration at the National Physical Laboratory.

The working tapes were standardised at the end of the measurement and on two occasions in the course of it. Intervals between standardisation were 13 days, 9 days, and 4 days. Figure 4.2 shows the results of these standardisations. The working tapes stretched during measurement by about 1 in 300,000 and there was a tendency for the greatest change to occur in the first period of 13 days. The standard tapes showed little or no appreciable change between the NPL calibrations.

4.06 Results

The measurements of individual sections are given at Table 4.1. It will be noted that for Sections 8 (the ravine) and 9, the conventional forward and back measurements were not taken. In the case of the ravine, measurements were made three times by two independent methods; for Section 9 two measurements were taken on separate days but in the same direction. The discrepancy between the two measures of Sections 7, 8, 9 and 11 exceeded the limit of 1 in 200,000 laid down. A possible explanation is that at section marks the tape was read against the image of the ground mark in the optical system of the centring head and not against the fiduciary mark in the plane of the tape. However, as the tripods were left overnight in position over section marks, and were thought not to have moved, it was considered that this had no effect on the overall length of the base. Subsequently at Lossiemouth this faulty procedure was corrected by transferring from the measuring head down to section marks at ground level by theodolite whenever there was a pause in the measurement.

TABLE 4.1

RIDGEWAY BASE: 1937

Section No.	*No. of 24-m. Bays*	*Date*	*Forward Measure (F) (Metres)*	*Return Measure (R) (Metres)*	*Accepted Length (Metres)*	*Standard Error of accepted length in mm.* (σ_m)	*Difference = Δ*		*Remarks*
							R−F (mm.)	*In p.p.m.*	
1	13	4.12.37	301·4507						Rejected because of strong wind.
		4.12.37		301·4532					
1A	9	6.12.37	212·0387						
		6.12.37	(301·4475)	212·0385					
1B	4	6.12.37	89·4088	(301·4476)	301·4476	0	+0·1	0·3	
		6.12.37		89·4091					
1B		Computed	89·4161						Computed from quadrilateral. Rejected. Angular observations incomplete.
			Mean	89·4140					
		Computed	89·4120						
2	40	10.11.37	958·8039						Rejected. Paper staff, also connecting tripod moved.
		3.12.37	958·8047						
		5.12.37		958·8080	958·8092	±1·2	−2·4	3	
		7.12.37	958·8104						
3	51	11.11.37	1223·2843						Rejected. Paper staff, also connecting tripod moved.
		5.12.37		1223·2743	1223·2754	±1·1	−2·2	2	
		7.12.37	1223·2765						
4	3	12.11.37	71·8968						
		12.11.37		71·8968	71·8968	0	0	0	
5	64	13.11.37	1534·9295						
		1.12.37		1534·9291	1534·9293	±0·2	−0·4	0·3	
6	68	15.11.37	1630·0007						
		30.11.37		1629·9955	1629·9981	±2·6	−5·2	3	

TABLE 4.1 *continued*

RIDGEWAY BASE: 1937 *continued*

Section No.	*No. of 24-m. Bays*	*Date*	*Forward Measure (F) (Metres)*	*Return Measure (R) (Metres)*	*Accepted Length (Metres)*	*Standard Error of accepted length in mm.* (σ_m)	*Difference = Δ*		*Remarks*
							R − F (mm.)	*In p.p.m.*	
7	26	16.11.37	623·2262						
		30.11.37		623·2320	623·2291	±2·9	+5·8	9	
		3.12.37	623·2323						Rejected. Measurement not continuous, i.e. terminal tripods may not have been centred.
8	7	6.12.37	191·3904						
		Computed	191·4002						Rejected. Probable errors in centring of transferring heads.
		Computed	191·3962						
					191·3909	±0·5	+1·0	5	Insufficient angular measures.
		Computed	191·3921						Computed from quadrilateral.
			Mean	191·3914					
		Computed	191·3907						
9	25	16.11.37	598·2591						Rejected. Paper staff on steep slopes. Alignment error.
		25.11.37	598·2574						
					598·2556	±1·8	3·5	6	Difference is between two forwards.
		29.11.37	598·2539						
10	26	17.11.37	623·1936						Rejected. Paper staff. Connecting tripod to 9 probably not centred.
		24.11.37		623·1965	623·1968	±0·2	−0·5	1	
		25.11.37	623·1970						
11	26	18.11.37	621·8397						Rejected. Paper staff.
		24.11.37		621·8441					
		26.11.37	621·8425		621·8418	±1·5	+3·4	5	
		29.11.37	621·8389						
12	8	18.11.37	190·6400						Rejected. Paper staff.
		24.11.37		190·6421	190·6418	±0·3	+0·6	3	
		26.11.37	190·6415						
13	10	18.11.37	239·4314						Rejected. Paper staff, and measurement not continuous.
		24.11.37		239·4267	239·4270	±0·2	−0·5	2	
		26.11.37	239·4272						

TABLE 4.1 *continued*

RIDGEWAY BASE: 1937 *continued*

Section No.	No. of 24-m. Bays	Date	Forward Measure (F) (Metres)	Return Measure (R) (Metres)	Accepted Length (Metres)	Standard Error of accepted length in mm. (σ_m)	Difference = Δ R − F (mm.)	Difference = Δ In p.p.m.	Remarks
14	27	19.11.37	646·7105						
		19.11.37		646·7113					
			Mean of F & R	646·7109	646·7094	±1·4	2·9	4	
		26.11.37	646·7080						
15	66	20.11.37	1577·9949						Rejected. Paper staff.
		23.11.37		1577·9916	1577·9929	±1·3	−2·6	2	
		27.11.37	1577·9942						
16	9	22.11.37	227·5222						
		22.11.37		227·5219	227·5225	±0·4	−0·9	4	
		27.11.37	227·5233						

The measured length of the base, corrected for temperature, inclination of the scales, inclined catenary, and slope = 11,260·56406 m.

Reduction to sea-level (mean height of base = 690 feet) = −0·37081 m.

Correction for gravity = −0·00017 m.

(g at National Physical Laboratory, Teddington = 981·18 cm./s^2
g at Ridgeway = 981·14 cm./s^2)

Final accepted length = 11,260·19308 m.

Standard Error of the Base

The sources of error affecting the accuracy of the measured length are detailed in § 4.23, thus:

$\sigma_a = 0{\cdot}6 \times 10^{-6} \times 11{,}260$ m. $= \pm 0{\cdot}0068$ m.

$\sigma_b = 0{\cdot}015 \times 10^{-6} \times \frac{22}{\sqrt{3}} \times 11{,}260$ m. $= \pm 0{\cdot}0021$ m.

$\sigma_c = \sqrt{\left(\frac{\Sigma\Delta^2}{4}\right)}$ $= \pm 0{\cdot}0053$ m.

$\sigma_d = 37 \times 10^{-8} \times 11{,}260$ m. $= \pm\ 0{\cdot}0042$ m.

$\sigma_{\text{Base}} = \pm 0{\cdot}0098$ m. or 1 in 1,149,000

LOSSIEMOUTH BASE 1938

4.07 Introduction

This base was originally established in 1909⁽¹²⁾ to test the accuracy of the old Principal Triangulation by measuring a base remote from the two main bases at Salisbury Plain and Lough Foyle in Northern Ireland. Thus by observing a conventional base extension a comparison was possible between the lengths of the primary side Corryhabbie to Knock obtained from the base measurement and from the triangulation.

The base was remeasured in 1938, partly to provide a check on the accuracy of the new triangulation carried through Figures 1, 2, and 3, and partly in the hope that some light might be thrown on the stability of invar tapes, which were relatively new at that time.

4.08 Description

The base has a mean height of about 24 feet, and is sited on the south shore of the Moray Firth east of the town of Lossiemouth. The terminals are not sharp, and the intervening ground is somewhat rough.

4.09 Measurement

The procedure and equipment of the Ridgeway Base were also used at Lossiemouth, except that the measuring heads at section marks were centred by two theodolites at right angles. This ensured that the measuring head was accurately positioned over the mark.

The measurement party included the majority of those employed on the Ridgeway Base. An advance party arrived on 13th July 1938 to undertake the preparatory work. The two main obstacles were the River Lossie and a canal. The water in each was about 12 in. to 18 in. deep, but the river was liable to rise suddenly if it came into spate, as did in fact happen in the 1909 measurement. The beds of both river and canal were firm.

Measurements began on the 19th July and good progress was made, as indeed was to be expected of such a trained team. Forty-seven bays were completed in the first day and 61 on the second, including the crossing of the River Lossie. The first complete forward measurement was carried out in five working days and the return measurement in four days, on the last of which no less than 98 bays were completed.

It was found, however, that the results of the standardisation at the beginning and at the end of the first forward measurement revealed disturbing changes in lengths of the working tapes. After 6 months storage on the drums the initial standardisation indicated that the working tapes had shortened from the lengths obtained from the calibration at the National Physical Laboratory 6 months previously. Standardisation at the end of the forward measurement revealed that they had stretched again, but owing to the absence of any intermediate standardisation it could not be established whether this fairly large extension had been uniform, or whether it had occurred in the first section of the forward measurement (see Fig. 4.2). The whole of the forward measurement was accordingly rejected and repeated. During the remaining measurements more frequent standardisation (at the risk of fatiguing the field standard tapes) indicated that the working tapes had settled down to a sufficient degree of accuracy after the rejected measurement, although their behaviour was not altogether uniform. The two final measurements, as will be seen, agreed closely but it was felt that these tapes should not be used again for further precise measurements. The whole question of fatigue in tapes, and in particular, the possible fatigue due to storage on small diameter drums, was considered by Hotine, whose investigation was published in an article in the *Empire Survey Review*(25).

4.10 Results

The results of duplicate measurements of the sections of the base are given at Table 4.2. It will be seen that the discrepancy between section lengths was nowhere greater than 1 in 590,000 and the total forward measurement agreed with the reverse to 0·2 mm. Furthermore, a satisfactory agreement was obtained with the 1909 results. The discrepancy between the 1909 and 1938 measures was 0·0115 m. or about 1 part in 620,000. In view of the fact that the two measures were carried out in totally different circumstances, with different procedure and apparatus, and on the basis of different fundamental length standards, this must be considered a very satisfactory agreement. (The 1909 measurement was made in feet, and the 1938 measurement in metres.)

4.11 Accuracy of the Lossiemouth Base Extension

Following the measurement of the Lossiemouth Base and the observation of the horizontal angles in the base extension (see Diagram 7), it was possible to compare the results with those obtained in 1910–11 during the investigation into the accuracy of the Principal Triangulation(8). Both extensions were observed to first order standards. Although the 1937 extension included a few additional rays, the lay-out was for the most part identical; but whereas the 1937 observations were carried through in about a month of more or less uniform weather, the 1910–11 observations occupied a considerably longer period and are for that reason likely to be the more reliable, if in fact, there is present any systematic error due to lateral refraction. Differences in the logarithms of the primary side Corryhabbie (342)–Knock (339) and of the base are as follows:

1910–11 measures	0·709 51658
1937	0·709 51037
Difference	621

or about 1 part in 70,000

It is rare that the two well-observed, but entirely independent, measures of an identical base extension can be compared. The present comparison emphasises the inaccuracies which are inevitable in a badly designed and sited system. The only available extension stations were emplaced on flat-topped hillocks, where systematic angular inaccuracies might well be expected. Certain of the extension lines which were found on examination to introduce unusually large misclosures in both angle and side equations, were for this reason rejected from the adjustment. The extension moreover, lies entirely to one side—the landward side—of the base. The length of the base itself, being about 4 miles, is insufficient for rapid extension into a 30-mile primary side.

TABLE 4.2

LOSSIEMOUTH BASE: 1938

Section No.	*No. of 24-m. Bays*	*Date*	*Forward Measure (F) (Metres)*	*Return Measure (R) (Metres)*	*Accepted Length (Metres)*	*Standard Error of accepted length in mm.* (σ_m)	*Difference* = Δ		*Remarks*
							R−F (mm.)	*In p.p.m.*	
1	47	19.7.38	1125·8303						* Rejected
		29.7.38		1125·8296	1125·8306	±1·0	−1·9	2	
		1.8.38	1125·8315						
2	51	20.7.38	1221·8857						* Rejected
		29.7.38		1221·8842	1221·8846	±0·4	−0·8	1	
		1.8.38	1221·8850						
3	5	20.7.38	119·8246						* Rejected
		20.7.38		119·8246					Rejected
		1.8.38	119·8242						
		28.7.38		119·8241	119·8242	0	−0·1	1	
4	65	21.7.38	1575·8242						* Rejected
		28.7.38		1575·8270	1575·8258	±1·2	+2·4	2	
		2.8.38	1575·8246						
5	77	22.7.38	1862·7258						* Rejected
	77	27.7.38		1862·7268	1862·7267	±0·1	+0·2	0·1	
	78	3.8.38	1862·7266						
6	52	25.7.38	1264·6336						* Rejected
		26.7.38		1264·6381	1264·6379	±0·2	+0·4	0·3	
		4.8.38	1264·6377						

* All first forward measurements rejected due to uncertainty as to the length of the working tapes (see § 4.09).

The measured length of the base, corrected for temperature, inclination of the scales, inclined catenary and slope	= 7,170·72982 m.
Reduction to sea-level (mean height of base 23·5 feet)	= −0·00803 m.
Correction for gravity (g = 981·76 cm./s^2)	= +0·00161 m.
Final accepted length	= 7,170·72340 m.

Standard Error of the Base

The sources of error affecting the accuracy of the measured length are detailed in § 4.23, thus:

$\sigma_a = 0{\cdot}6 \times 10^{-6} \times 7{,}171$ m. $= \pm 0{\cdot}0043$ m.

$\sigma_b = 0{\cdot}015 \times 10^{-6} \times \frac{2}{\sqrt{3}} \times 7{,}171$ m. $= \pm 0{\cdot}0001$ m.

$\sigma_c = \sqrt{\left(\frac{\Sigma\Delta^2}{4}\right)}$ $= \pm 0{\cdot}0016$ m.

$\sigma_d = 37 \times 10^{-8} \times 7{,}171$ m. $= \pm 0{\cdot}0027$ m.

$\sigma_{\text{Base}} = \pm 0{\cdot}0053$ m. or 1 in 1,353,000.

THE REMEASUREMENT
OF THE RIDGEWAY BASE 1951

4.12 Introduction

The following account of the remeasurement of the Ridgeway Base in 1951 is based on *Professional Paper No. 18* of the Ordnance Survey(26), to which readers are referred for greater detail.

Due to the urgent post-war requirements for second and third order triangulation, work on the primary Retriangulation did not recommence until 1949, but by 1951 personnel became available for base measurement. By this time a suitable site for a base in the extreme north of Scotland had been selected and marked in Caithness. As only one senior member of the 1937–38 base measuring parties was then available, and as it was thought that catenary measurement across the bog in the Caithness Base would be an extremely difficult undertaking, it was decided to remeasure the Ridgeway Base as a training exercise for the party. Furthermore the Macca base measurement equipment had been extensively modified as a result of experience gained in the 1937–38 measurement and it was desirable to test it under field conditions.

In articles published in the *Empire Survey Review*(25,27), Brigadier Hotine expressed certain doubts about the original measurement of the Ridgeway Base in 1937 which are summarised below.

(*a*) The measurement was made in temperatures around freezing point; this made it difficult for the men to produce their best results. Furthermore the National Physical Laboratory had not calibrated the tapes for so low a temperature range.

(*b*) Woven cord was used instead of piano wire to attach the tapes to the weights; there were thus differing end tensions which were not measurable.

(*c*) The mark in the measuring head was transferred down to the section mark at the end of a day's work by means of the optical plummet in the transferring head. This was not sufficiently accurate due to parallax, and transferring should be done by two theodolites at right angles to each other.

(*d*) During the triangulation across the ravine, angles were measured with Zeiss traversing equipment which was not designed to fit the tripods of the Macca equipment. Consequently it is possible that small errors exist owing to the centre of the theodolite not being exactly over the mark to which a measurement had been taken.

(*e*) Before the short invar staves arrived, paper-faced substitutes were used for levelling. The section measures in which these were used were not incorporated in the final answer which as a result contains the means of many sections with a differing number of forward and return measures.

(*f*) It was theoretically possible that the tapes were strained beyond their elastic limit by being wound on a small drum. It was thought that the use of a drum of larger diameter would reduce this possibility.

It was decided therefore to remeasure the Ridgeway Base in the autumn of 1951 to find out whether the 1937 value could be improved upon, and to eliminate all teething troubles in men and equipment before sending the team to the bog of Caithness in the summer of 1952.

4.13 Modifications to the Original Macca Base Measurement Equipment

The equipment used for the measurement of the bases was fundamentally the same as that described in the *Empire Survey Review* of 1935(24) but certain modifications to this equipment were made before the 1951–52 measurements.

- (*a*) Three working tapes were used, instead of two, all being graduated for use with 15 lb. weights.
- (*b*) The ring at each tape end was 6 in. from the nearest graduation, instead of 18 in. as previously.
- (*c*) The tape used for short measurements was 6 m., graduated throughout its length, instead of 20 feet.
- (*d*) Piano wire replaced woven cord to connect the weights to the tapes.
- (*e*) The design of the V notch and its surround on the measuring head was altered.
- (*f*) A clamping device was introduced into the eye-piece focusing of the measuring head.
- (*g*) The pulley wheels of the straining trestles were considerably modified by the addition of a brake and clamping device.
- (*h*) The alignment telescope was fitted with a slow motion horizontal screw.
- (*i*) The measuring tapes were provided with winding drums 4 feet 6 inches in diameter.
- (*j*) A length of electric fence was added to the equipment to discourage cattle from the section marks.

4.14 Changes in Accepted Procedure

4.140 POSITION OF THE TAPE-RACK

On the Ridgeway Base, the tape-rack was placed on the opposite side of the measuring head to that occupied by the observer. Local factors, particularly the instability of the ground at Caithness suggested that it would be preferable to maintain the tape-rack on the same side as the observer, in spite of some minor physical inconvenience caused thereby.

In the course of the measurement of the Ridgeway Base, some movement was detected at tripods which had been left in position over station marks for any length of time. All such cases were thereafter treated as suspect and the two previous bays were re-measured. As a precautionary measure a ground block was always inserted when any appreciable delay was anticipated. At night tripods were left in position over the blocks, but were invariably checked and re-centred the following morning.

4.141 HANDLING THE TAPES

The old-time embargo on touching the tapes with the bare hands was lifted, for practical reasons. Naturally contact with the hands was limited to the absolute minimum. All tapes were greased each night and carefully cleaned each morning. Where heavy dew, or even a light rain was present, the tapes were specially wiped before measurement, though of course measurement in such conditions is not to be recommended.

4.142 TEMPERATURE MEASUREMENT

On the Ridgeway Base four thermometers were used. One was hung on each tape-rack and two were carried near the booker. The mean of the four readings was accepted. Readings were taken at the opening and closing of each bay. As a result of experiments carried out by the National

Physical Laboratory, however, this method was not used on the Caithness Base. In its place, two thermometers enclosed in a brass sheath, and two with an exposed bulb were carried on a gallows by the booker's assistant and the mean of all four readings was accepted. Experiments in casing the bulb with invar were shown to produce unreliable results at the lower temperatures.

4.143 SECTION MARKS

A section mark, normally inserted at the end of each day's measuring, was additionally used at the commencement of the mid-day break. Each mark consisted of a pre-cast concrete block, 1 foot cube, with a zinc insert. It was convenient to foresee the need for emplacement about ten bays in advance. The transference was carried out by a pair of theodolites, as some doubts existed as to the accuracy of the Macca Base transferring head.

4.144 BOOKING

During the 1937–38 measurements of the bases in Great Britain the value of a booker was established. At each bay he recorded three pairs of readings from the observers, adding up each pair, checking that they fell within the required agreement, and then extracted the mean. The process was repeated for the other two tapes and the mean of all three readings was accepted as the final bay length. Temperatures were recorded in a similar manner. The booker also kept a running total of the distance from the previous section mark in order to ensure during the return measurement that the tripods were being put in at approximately the correct spacing to close on the section mark already emplaced. Sections were numbered consecutively, as were the bays in each section, and also the measuring heads. The relative positions of the observers were also noted for each bay. A daily diary was maintained, and on the return journey, the difference between the outward and return value was recorded, with reasons for any rejection.

4.145 NUMBER OF TRIPODS

In the 1937 measurements, eight measuring heads were used compared to six in previous measurements. In the 1951–52 measures two further tripods, making ten, were used; this ensured that the aligning party kept well clear of the measuring party, which was essential over unstable ground.

4.146 MEASUREMENT OF INCLINATION OF THE TAPES

In 1951–52, differences in height between measuring heads were determined by levelling instead of by vertical angles on the aligning theodolite. The original Macca equipment was designed for vertical angles to be read from the aligning theodolite to the nearest 20 seconds, but although reciprocal angles were observed at each bay consisting of two readings on both faces, giving a mean angle to about 2½ seconds, the resulting accuracy remained at two or three times that figure. Brigadier Hotine had shown(27) that on hilly sections levelling was essential, since the angular errors of the aligning theodolite gave rise to length errors greater than the errors of reading the tapes; this is discussed in detail later. It was therefore decided to use geodetic levelling equipment and invar staves and, as a check on a single leveller's work, two men were used working independently of each other, but reading to the same staves. The standard of mutual agreement was laid down and the measurement was repeated if this was exceeded.

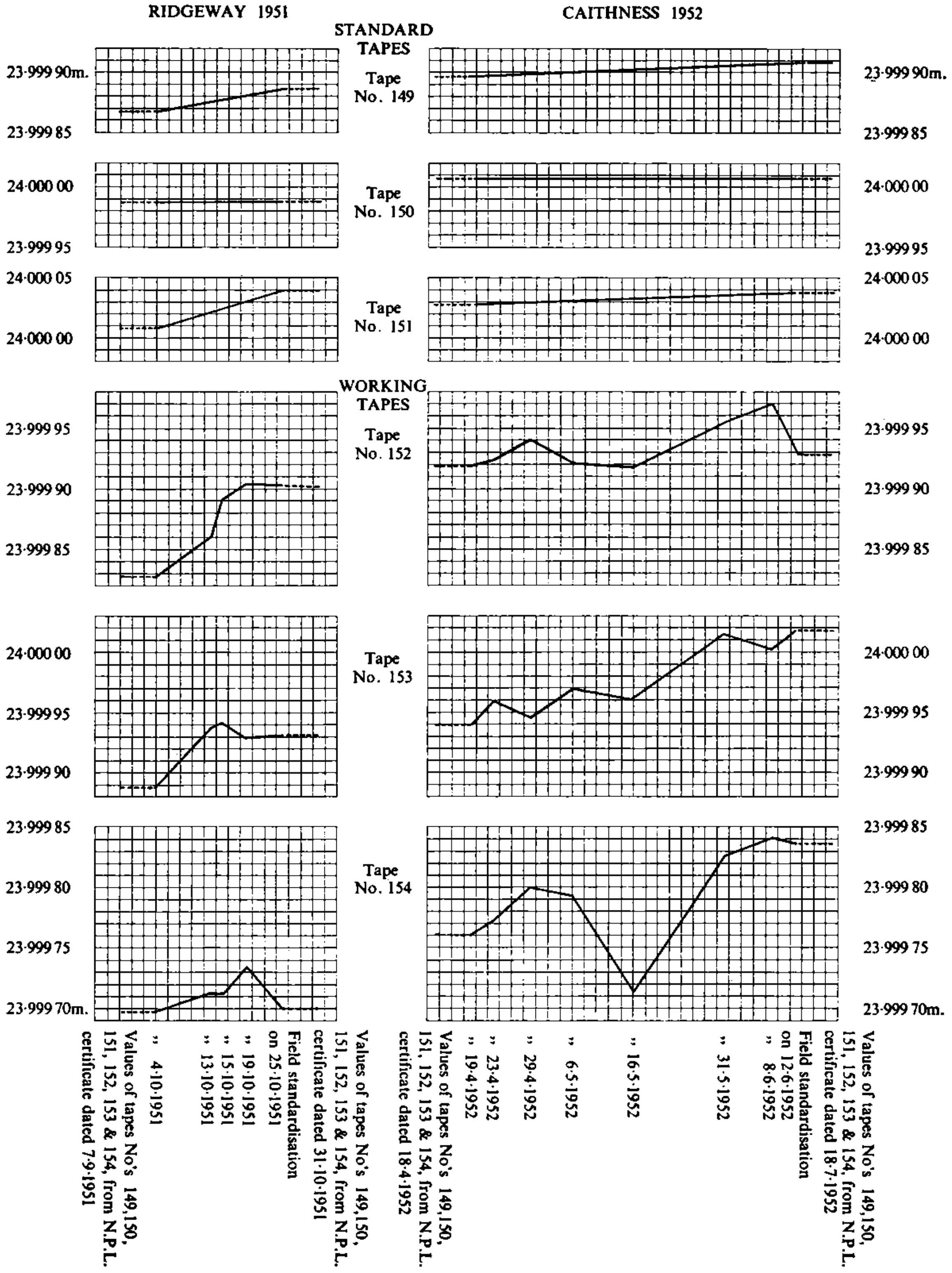


FIG. 4.3. Standardisations for 1951–1952 measures

4.15 Standards of Accuracy

4.150 FIELD STANDARDISATION

Field standardisation of the working tapes against the standard tapes was carried out at the beginning and end of any complete measurement, after any real or fancied mishap, and otherwise after an interval of about a week. Some authorities advocate the use of two working tapes with daily standardisation. The disadvantage of such repeated standardisations, in addition to the time consumed, is that extra fatigue may be caused in the standard tapes with consequent loss of accuracy. The use of three tapes, however, ensures that should any departure from the normal difference in length become apparent, the offender can be readily detected, and immediate re-standardisation can be put in hand. In the 1951–52 measurements only one serious alteration in the tape length occurred, on the Caithness Base when one working tape (No. 154) decreased between 16th and 31st May by as much as 0·08 mm., or 1/300,000 from its normal path of elongation, to which it returned at the next standardisation (see Fig. 4.3). As a result all readings with that tape over that period were omitted. Except when minor accidents occurred, the only extra standardisations were before and after measurement across the ravine on the Ridgeway Base, when it was quite possible for the tapes to have been damaged. From previous experience, it was desirable when standardising to secure agreement between the different sets of readings for each tape to 0·0001 m. (1/240,000). Eight sets had been normally taken in the 1937–38 measures, but to save time four sets within this tolerance were accepted initially in 1951–52. Later, six or seven sets were taken in order to make additionally sure of the required accuracy. Normal drill of alternating working tapes with standard tapes was followed during standardisation.

4.151 AGREEMENT BETWEEN MEASURES OF EACH BAY

During the measurement each tape was read three times to 0·1 mm, and the three readings were required to agree to 0·2 mm. If there was a range of 0·3 or 0·4 mm., a fourth reading was taken and the mean of all four accepted. If one reading differed by more than 0·4 mm. from the remainder it was assumed to be a gross error; a fourth reading was taken, the gross error was discarded and the three readings were meaned. On a number of occasions an extra reading became necessary because one reading differed by 0·3 mm. It was very rare for a departure of 0·4 mm. to occur and it was only in the early days at the Ridgeway Base that there were many gross errors.

From previous experience, an arbitrary figure had been derived of 1/200,000 for the maximum permissible discrepancy between the two measurements of the section lengths. In the event this was not strictly adhered to.

4.152 ALIGNMENT OF MEASURING HEADS

A misalignment of measuring heads of 1 cm. will produce an error of $1/10^7$ in a bay length of 24 metres. This accuracy of alignment was achieved without difficulty.

4.153 SLOPES

To obtain the differences in height between measuring heads, geodetic levels and invar staves were used. On the Ridgeway Base the following maximum discrepancies between the two readings of the height difference along one bay were laid down:

Slopes less than 6°	0·003 feet
Slopes from 6° to 10°	0·002 feet
Slopes greater than 10°	0·001 feet

The tolerances represented an error in one tape length of about 1/200,000.

On the Caithness Base, where slopes greater than 6° seldom occurred, an overall tolerance of 0·01 feet was allowed. In fact, despite the instability of the ground the discrepancy seldom exceeded 0·003 feet. Probably these tolerances were too large and it is now felt that the use of geodetic levels cannot be justified unless all possibility of errors due to slopes is to be eliminated.

The following tolerances are therefore suggested:

Slopes less than 3½°	0·003 feet
Slopes from 3½° to 8° (where the tape runs)	0·002 feet
Slopes greater than 8°	0·001 feet

4.16 The Measurement of the Ridgeway Base

The Ridgeway Base has been described in § 4.01. It was essential to complete its measurement during 1951 as it was planned to measure the Caithness Base in 1952. The equipment was not ready in the spring and the base could not be measured during the summer because of damage to crops. One of the main criticisms of the 1937 measurement was that it had been carried out when the weather was too cold. It was therefore most important to measure the base as soon as the crops were cleared. The base is very exposed to winds and to avoid the time of the September gales, a start was made on the 1st October. For the next four weeks the weather was near perfection, and the whole measurement was completed before the end of the month.

4.160 ADVANCE PARTY

The week before the measurement started, an advance party established aligning pegs between the pillars so placed that either pillars or pegs could be seen from any point on the base. This party also cleared any moveable obstacles known to be on line, such as haystacks and hedges. The barn on the line was dealt with as previously. This party also prepared the ravine with section marks on either side, pegged out the sites of measuring heads on the ground measurement, and located the centres of the Bilby towers which were to be used for the ravine crossing.

4.161 PRELIMINARY TRAINING

The personnel had no previous experience of this type of work. Preliminary training lasted one week, during which the equipment was demonstrated and the measurement procedure illustrated by a model. This model was also used to demonstrate the proposed methods to those with previous experience whose comments and criticisms were likely to be of value.

Two days were spent in practice measurement with old tapes on the actual base.

4.162 PROGRESS

On the first day progress averaged four bays per hour. By the end of the second day this figure had increased to nine. On the third day ten bays an hour were achieved. By the end of ten days a rate of 13 bays an hour was maintained comfortably (4½ minutes a bay). The base was measured between 2nd and 25th October 1951, as follows:

Training	2 days
Visitors' days	2 days

Ravine measurement, 39 bays	2½ days
Measurement of 932 bays	17 days
Lost by weather	½ day

The average daily progress was 55 bays a day.

4.163 OBSTACLES

Each end of the Ridgeway Base is located on an Iron Age hill fort whose 12-foot deep ditches were responsible for considerable delays. White Horse Hill (34) was completed in calm weather but at Liddington Castle (35) a strong cross wind made it impossible to provide effective shelter at the bottom of the ditch. Once again, as in 1937, it was necessary to fly the wind screen like a kite and even then the tape flutter due to the 15 m.p.h. wind rendered it advisable to take more than the prescribed number of readings before good agreement between forward and back measures could be obtained. Ultimately the difference was only 0·4 mm., a result which may perhaps reflect the fact that the wind and weather remained reasonably constant throughout the period of the measurement.

The ravine which provided the major obstacle to the measurement of the base is about 50 feet deep and about 300 feet wide. To avoid an offset measurement round it, it was decided to measure down across the ground and also measure across the top of previously erected Bilby towers. Three towers 30 feet, 50 feet, and 40 feet in height were emplaced at 24-metre intervals. Each tower was equipped with special platforms to accommodate the straining trestles. All equipment, including the tapes, was hauled up to the tops of the towers by rope. To each tower was attached a tape-rack and the tapes attached in the usual way. To move the tapes from the first to the second tower, string was attached to both ends and the tape transported across, one man pulling from the second tower, one man on the ground paying out the back end of the tape by the piece of string which acted as a counter-weight. This of course was not a speedy operation, and the measurement of the six bays took up half a day. The first day's work was abortive, owing to discrepancies between the tape readings. During the next two days, however, forward and back measurements, both over the ground and from the towers, were completed as below.

RAVINE MEASUREMENTS

Towers			*Ground*		
Forward	143·754285	Agreement 1/339,000	Forward	143·753541	Agreement 1/210,000
Return	143·753861		Return	143·752856	
Mean	143·754073		Mean	143·753199	

Agreement between the means 1/164,000

The agreement shown for Section 11 in Table 4.3 was produced by taking the mean of the two forward and the two return measurements.

4.164 TEMPERATURES OF STANDARDISATION AND FIELD MEASUREMENTS

The National Physical Laboratory standardised the tapes at a temperature of 68°F. The temperature on the Ridgeway Base was generally between 38°F and 55°F with an average of about 45°F. Consequently the temperature correction curve had to be extrapolated. At the end of the base measurement the tapes were re-calibrated at 37·4°F in order to verify the extrapolation. The two curves agreed very closely.

TABLE 4.3

RIDGEWAY BASE: 1951

Section No.	*No. of 24-m. Bays*	*Date*	*Forward Measure (F) (Metres)*	*Return Measure (R) (Metres)*	*Accepted Length (Metres)*	*Standard Error of accepted length in mm.* (σ_m)	*Difference = Δ*		*Remarks*
							R−F (mm.)	*In p.p.m.*	
1	8	4.10.51	189·4759						
		15.10.51		189·4757	189·4758	±0·1	−0·2	1	
2	26	5.10.51	620·8876						
		15.10.51		620·8914	620·8895	±1·9	+3·8	6	
3	25	6.10.51	599·7425						
		25.10.51		599·7429	599·7427	±0·2	+0·4	1	
4	25	6.10.51	599·7631						
		25.10.51		599·7617	599·7624	±0·7	−1·4	2	
5	19	7.10.51	455·8516						
		25.10.51		455·8470	455·8493	±2·3	−4·6	10	
6	18	7.10.51	431·9635						
		24.10.51		431·9616	431·9626	±1·0	−1·9	4	
7	31	8.10.51	744·0325						
		24.10.51		744·0334	744·0330	±0·4	+0·9	1	
8	37	8.10.51	888·0863						
		24.10.51		888·0900	888·0882	±1·8	+3·7	4	
9	32	9.10.51	767·3020						
		23.10.51		767·3026	767·3023	±0·3	+0·6	1	
10	44	9.10.51	1058·3843						
		23.10.51		1058·3887	1058·3865	±2·2	+4·4	4	
11	6	12.10.51	143·7535						Towers. Rejected.
		12.10.51		143·7519					Ground. Rejected.
		13.10.51	143·7535						Ground. Accepted.
		13.10.51		143·7539					Towers. Accepted.
		14.10.51	143·7543		143·7536	±0·3	−0·5	3	Towers. Accepted.
		14.10.51		143·7529					Ground. Accepted.
12	23	10.10.51	550·9427						
		22.10.51		550·9391	550·9409	±1·8	−3·6	7	
13	33	10.10.51	791·1848						
		22.10.51		791·1820	791·1834	±1·4	−2·8	4	
14	31	18.10.51	741·8022						
		20.10.51		741·8016	741·8019	±0·3	−0·6	1	
15	32	18.10.51	767·9111						
		20.10.51		767·9102	767·9106	±0·4	−0·9	1	
16	34	19.10.51	815·1601						
		21.10.51		815·1560	815·1580	±2·0	−4·1	5	
17	15	19.10.51	351·0775						
		21.10.51		351·0780	351·0778	±0·2	+0·5	1	
18	31	17.10.51	743·2399						
		17.10.51		743·2403	743·2401	±0·2	+0·4	1	

NOTE TO TABLE 4.3

The measured length of the base, corrected for temperature, inclination of the scales, inclined catenary and slope = 11,260·55867 m.
Reduction to sea-level (mean height of base = 692 feet) = −0·37205 m.
Correction for gravity ($g = 981{\cdot}14$ cm./s^2) = −0·00012 m.
Final accepted length = 11,260·18650 m.

Standard Error of the Base

The sources of error affecting the accuracy of the measured length are detailed in § 4.23, thus:

$\sigma_a = 0{\cdot}6 \times 10^{-6} \times 11{,}260$ m. $= \pm 0{\cdot}0068$ m.

$\sigma_b = 0{\cdot}015 \times 10^{-6} \times \frac{11}{\sqrt{3}} \times 11{,}260$ m. $= \pm 0{\cdot}0011$ m.

$\sigma_c = \sqrt{\left(\frac{\Sigma\Delta^2}{4}\right)}$ $= \pm 0{\cdot}0054$ m.

$\sigma_d = 47 \times 10^{-8} \times 11{,}260$ m. $= \pm 0{\cdot}0053$ m.

$\sigma_{\text{Base}} = \pm 0{\cdot}0102$ m. or 1 in 1,104,000.

4.165 ACCIDENTS

The measurement was particularly free from accidents, though some damage to the pulley spindles was caused by the braking system on the pulley wheels. Two minor mishaps occurred. A gust of wind blew over one tape-rack during a lunch break but no damage appeared to be done. A standardisation had just been completed. The tape was re-standardised at once and gave no evidence of any damage. Later, one of the straining trestles collapsed on a road with the tape on it, but no undue strain came on the tape as the weight hit the ground immediately and the observer then took the tape off.

One possible source of error, which did not become apparent until after the measurement, was caused on steep slopes by the weight man having to dig a hole to allow the weight to descend low enough. This digging may perhaps have disturbed the tripod.

4.166 BEHAVIOUR OF TAPES

Throughout the Ridgeway measurement the tapes behaved uniformly as can be seen in Fig. 4.3. The standard tapes on re-calibration by the National Physical Laboratory had increased as follows:

Standard Tape No. 149 +1/1,300,000
150 +1/24,000,000
151 +1/800,000

The working tapes had increased, though not quite so uniformly, viz.:

Working Tape No. 152 +1/320,000
153 +1/24,000,000
154 +1/8,000,000

The field working tapes were standardised with the standard tapes five times during the course of the measurement; at the beginning, before and after the ravine measurement, at the end of the forward measurement, and on completion. The intervals in working days between each were 8 days, 3 days, and 6 days.

4.167 SECTION AGREEMENTS

At Table 4.3 are listed the final agreements between the two measures of each section using the re-standardisation figures from the National Physical Laboratory. Sections 2 and 12, which agreed in the field to 1/170,000 and 1/150,000 respectively, are on the border line of those which should be measured again. Both were on steep slopes and it is possible that personnel went too near the tripods and probably moved them. Section 5 with a disagreement of 1/99,000 should certainly have been measured again. Unfortunately it was the last section to be computed and by the time the figures were available, all the gear had been greased and put away. The difference between the back and forward measures of the base was 5·9 mm. or 1/1,910,000, and as the remeasurement was being made solely as a check on the 1937 value, these doubtful sections were not remeasured.

THE MEASUREMENT OF THE CAITHNESS BASE 1952

4.17 Preparatory Work

4.170 DESCRIPTION OF THE SITE

The Caithness Base is about 15½ miles long and lies between the two stations of the primary Retriangulation which had been specifically sited as base terminals. The general level of the base is between 150 and 200 feet above sea level and the two terminals, Warth Hill (399) (406 feet) and Spital Hill (398) (577 feet) stand up well above the surrounding country. As can be seen from the sectional plan in Fig. 4.4 nearly half the base consisted of peat bog, and it was not known if precise catenary taping was possible over such unstable ground. Therefore, in 1951, a pillar was erected halfway along the base at Hillhead Farm (478) so that if measurement over the bog was impossible the base could be reduced to the south-western half (Hillhead Farm (478) to Spital Hill (398)), which at that time was thought to contain no bog. This would have been an unsatisfactory alternative as the station at Hillhead Farm (478) would not have provided a good terminal since two of the rays in the extension were grazing.

4.171 RECONNAISSANCE

After consultation with the Meteorological Office it was decided that May and June were the most suitable months in the year in Caithness for the base measurement, since local records showed that during this period rainfall was at its lowest and the average wind velocity nearly a minimum. It was thought that wind was likely to prove a major hindrance on the base since there were no features or vegetation to provide any shelter.

Preliminary reconnaissances were carried out in 1951 and in March 1952. Soundings were taken over the whole length of the bog section and the depth of peat was found to average between 10 and 15 feet with a maximum of 21 feet. Section marks had been planned at intervals of about 30 bays (720 m.), but in practice the actual intervals were dictated by the fact that the only stable ground suitable for section marks lay near the beds of the numerous small streams which crossed the base.

4.172 SPECIAL ARRANGEMENTS FOR MEASUREMENT ACROSS THE BOG

A stable platform for the measuring heads was evolved after much experiment. It consisted of three wooden pickets (of standard Army pattern 5 feet in length) which supported a wooden frame made of 9 in. × 1½ in. boards in the shape of an 'A'. The pickets fitted into holes in the 'A' frame.

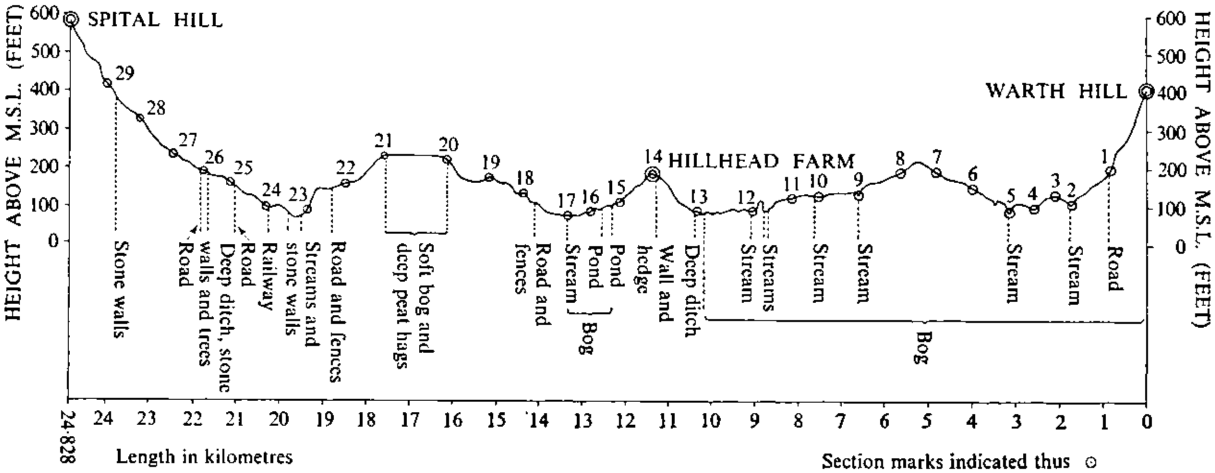


FIG. 4.4. Caithness Base: Elevation showing section marks and obstructions

The tripod legs of the measuring head rested on the heads of the three pickets. The observer stood on a duckboard 6 feet × 1½ feet which rested on four smaller pickets 2½ feet in length. It was decided that special supports were not required for the straining trestle, but with the latter in its normal position (1 to 1·5 m. from the measuring head) the measuring head would be disturbed by personnel emplacing and removing the straining trestles. The length of piano wire between the swivel hook and the weight was successively increased from 2 m. to 3 m. and finally to 4½ m. At 4½ m. tests showed that no disturbance was caused, provided the men erecting the trestle moved with caution. These tests were carried out in one of the worst parts of the bog with a theodolite mounted on a concrete platform supported on metal piles driven down through the peat into solid ground. As a result of these tests it was found that in different categories of bog, the area varied within which a person walking would disturb the measuring head. The bog was accordingly divided into three categories called One, Two and Three star, similar to the Automobile Association system for grading hotels, but with the order reversed, a Three Star bog being worse than a Two Star. In a One Star area any movement 10 feet or more away from the measuring head was safe, and in a Two Star area the safe radius was 15 feet. In a Three Star area the safe radius was 30 feet but personnel could approach to within 10 feet if great care was exercised.

Using the 4½-metre wire between the tape and the weight, a slight change was caused in the catenary taken up by the tape. The error from this cause was calculated as 1 in 7,000,000 of the tape length compared to 1 in 10,000,000 with the standard equipment. This increase in the error was considered to be negligible.

4.18 Measurement

4.180 SPECIAL PROCEDURE IN BOG SECTIONS

Measurement started on 20th April 1952, with two days' trial on the bog with old tapes, but using the modified procedure with all the new equipment. On the One Star ground, the normal length piano wire, i.e. 2 m., and short pickets only were used for the measuring heads, with no 'A' frame or duckboard. On the Two Star ground, the measuring heads were put on 5-foot pickets braced with 'A' frames, the observer was on a duckboard which was laid on the ground, and the straining trestle head was 2½ m. away with a 3-metre length of piano wire. On the Three Star ground the procedure was the same, except that the observer's duckboard was on short pickets and the straining trestle had a 4½-metre length of wire, with only a single weight man. The weight man had to carry the weight and the straining tripod away from the measuring head at the end of each bay to his assistant, who then took over his load. Later no distinction was made between the Two Star and Three Star ground and the latter procedure was used for both. The tape-rack was always on the same side as the observer. On the really bad Three Star ground, of which there were about two sections, even the tape-rack man could not approach near enough to the observer. At each end of the bay a man was therefore stationed in between the observer and the tape-rack, with a banderole to the end of which was attached a hook. The tape-rack man took the tape off the tape-rack, hooked it on to the banderole and the banderole man swung it over to the observer who took it off the hook and continued with the observations in the normal way.

Early on over the bog, a theodolite was set up near a measuring head at right-angles to the line of the base and observations were made for movement during the complete process of measurement. Each time this was done, no movement could be detected.

An actual standardisation was carried out using all the different lengths of piano wire, 2 m., 3 m. and 4½ m.; the results were identical within the limits of observation.

The essence of success over the bog was to keep the progress going. A measuring head should never be left long under such conditions with a measurement into it but not one out of it, in case of movement. The party therefore started in the morning and did not stop until they reached the section mark, until it rained, or until the wind was so great they were forced to stop. All the normal stores had to be carried, and because of the bog no wheeled vehicle could approach the middle of the base; consequently one hour of walking time often preceded and followed the day's measurement. This reduced the normal day's progress but even so it was discovered that at intervals of 35 bays the section blocks were too close. These marks however had all been sited on firm ground, and since two sections could not be completed in a day, it was not possible to make any improvement. The whole method worked well. It was decided, owing to the instability of the ground, that the tape-rack should remain on the same side of the tape as the observer, a procedure which was maintained throughout the whole length of the base. For the same reason the observers leapfrogged over the whole of the bog section because it was clearly essential that the observer should stay on his duckboard by his own measuring head and not attempt to move while the measurement was held in that head.

4.181 TRANSPORT

Normal wheeled transport was useless on the bog, therefore a pre-war ex-army tracked vehicle, a Carden–Loyd carrier, was used for transporting forward pickets, 'A' frames, and duckboards. A separate party was responsible for the pickets, of which there were enough to cover two sections. Consequently in the morning the carrier picked up the pickets on the section that had been measured, took them forward to the section to be measured on the following day, and spaced them out during the afternoon for the aligning party to knock in when they came to them later.

4.182 OTHER OBSTACLES

During the measurement of the south-western section, two patches of hitherto undiscovered bog were encountered and were given Three Star treatment.

4.183 PROGRESS

During the first day's measurement over the bog progress averaged between 3½ bays and 5½ bays an hour. By the second day this rate had speeded up to 8 bays an hour and by the fourth day a rate of 10 bays an hour was obtained which was the maximum progress throughout the bog. Progress on normal ground averaged 12 bays an hour with a maximum of 14. The maximum rate was achieved on the penultimate day when 94 bays were measured.

Measurement started on 20th April 1952 and finished on 12th June, two days under eight weeks.

The summary of the work was as follows:

Measurement of 2,193 bays	43½ days
Move from one end of the base to the other	1 day
Loss owing to rain and wind	9½ days
TOTAL =	54 days

The average daily progress was therefore 50½ bays a day.

4.184 EFFECT OF TEMPERATURE AND WIND

As on the Ridgeway Base, the temperatures encountered were considerably lower than that of the original National Physical Laboratory standardisation. Only twice did the temperature reach 68°F. Wind, most of which blew across the line of the base, interfered with the measurements; at the outset the first section was measured downhill with a wind along the line beating on the tapes. This section had to be remeasured. Later on, two more sections had to be remeasured for the same reason. The screen, which was used continuously, was not strong enough, and the eyelets by which it was attached to the carrying poles continually tore away. The screen was 5 feet 3 inches high and no trouble was encountered due to eddies of wind coming over the top. On the whole the weather was very fine over the period, only 9½ days being lost in 54. March had been the driest month in Caithness for some years which was fortunate, for the bog was exceptionally dry.

4.185 PIANO WIRE

The piano wire tended to curl and was apt to become kinked, especially in the longer lengths. It was later discovered that nylon line with a breaking strain of 80 lb., though somewhat liable to stretch, was in all other respects satisfactory.

4.186 BEHAVIOUR OF THE TAPES

The changes in the lengths of the tapes, illustrated at Fig. 4.3 were as follows:

Standard Tape No.	149	+1/2,400,000
	150	Nil
	151	+1/2,400,000
Working Tape No.	152	+1/2,700,000
	153	+1/310,000
	154	+1/310,000

Combining these with the Ridgeway Base results (§ 4.166), it will be seen that over the whole measurement the standard tapes had increased by a negligible amount and the three working tapes had all increased about the same amount. The field standardisations showed nothing irregular except in the case of Tape No. 154 which on one occasion reduced its length by 1/300,000 for one standardisation only (see § 4.150). The intervals between standardisation in working days were:

3 days, 13 days, 9 days, 12 days, 5 days, and 4 days.

Some of these long intervals were due to there being no suitable places for standardisation in some sections of the bog. There is no doubt that standardisation should have been carried out more often. Fig. 4.3 shows the movement of the six tapes over the whole periods of the Ridgeway and Caithness measurements. In November the working tapes were re-standardised and reductions in length were found in all three as follows:

Tape No. 152	Reduction 1/2½ million
Tape No. 153	Reduction 1/6 million
Tape No. 154	Reduction 1/600,000

4.187 SECTION AGREEMENTS

The agreements between the two measures of each section are shown at Table 4.4. Except for three sections which had to be remeasured, due to wind along the base, it will be seen that the average agreement was about 1/700,000. This good result reflects credit on the work of the team, especially as nearly all the poor measurements were over the bog, where some loss of accuracy was perhaps inevitable. Sections 3 and 4 were combined as one section; although the forward and backward measurements of both section 3 and section 4 differed appreciably, the signs of the differences were opposite and the total summation of these sections was just above the permissible limit. This was in the worst section of the bog, over which remeasurements were to be avoided if possible. Section mark 3 was the concrete raft used for test which was itself unlikely to have moved, though it is possible that the whole mass of peat changed position. In both forward and back directions, measurements were made straight through from section mark 2 to section mark 4, with very little delay on the concrete raft. Owing to the bog, the position of the measuring head over the mark on the zinc plate could only be checked by a theodolite set up on the same pickets which had been used by the reconnaissance party for testing movement. Unfortunately these pickets were at an angle of 65° to the Base, consequently the centring of the measuring head may have been suspect. It will be seen that the final figure for the unstable half of the base is 1/820,000, which would not be considered a high standard over normal ground, but acceptable in the extremely difficult conditions encountered. The stable half of the base, however, has a figure of 1/2,320,000, while it is interesting to note that most of the section agreements are 1/500,000 or better, representing an improvement on the Ridgeway Base results.

TABLE 4.4

CAITHNESS BASE: 1952

Section No.	*No. of 24-m. Bays*	*Date*	*Forward Measure (F) (Metres)*	*Return Measure (R) (Metres)*	*Accepted Length (Metres)*	*Standard error of accepted length in mm.* (σ_m)	*Difference = Δ*		*Remarks*
							R − F (mm.)	*In p.p.m.*	
BOG SECTION									
1	34	20.4.52	812·2615						Rejected. Wind along base.
		24.4.52		812·2544	812·2547	±0·3	−0·6	1	
		25.4.52	812·2550						
2	37	21.4.52	892·4808						
		23.4.52		892·4788	892·4798	±1·0	−2·0	2	
3	15	26.4.52	359·6482						
		16.5.52	(863·3818)	359·6508					
4	21	26.4.52	503·7336	(863·3780)	863·3799	±1·9	−3·8	4	Sections 3 and 4 combined.
		16.5.52		503·7272					
5	25	27.4.52	602·1740						
		15.5.52		602·1721	602·1730	±1·0	−1·9	3	
6	34	28.4.52	819·4757						
		15.5.52		819·4771	819·4764	±0·7	+1·4	2	
7	36	29.4.52	863·7479						
		14.5.52		863·7451	863·7465	±1·4	−2·8	3	

TABLE 4.4 *continued*

CAITHNESS BASE: 1952 *continued*

Section No.	No. of 24-m. Bays	Date	Forward Measure (F) (Metres)	Return Measure (R) (Metres)	Accepted Length (Metres)	Standard error of accepted length in mm. (σ_m)	Difference = Δ R − F (mm.)	Difference = Δ In p.p.m.	Remarks
BOG SECTION *continued*									
8	35	30.4.52 13.5.52	839·6608	839·6614	839·6611	±0·3	+0·6	1	
9	39	1.5.52 12.5.52	931·2066	931·2076	931·2071	±0·5	+1·0	1	
10	38	2.5.52 11.5.52	915·7146	915·7150	915·7148	±0·2	+0·4	0·4	
11	26	3.5.52 10.5.52	629·8618	629·8628	629·8623	±0·5	+1·0	2	
12	37	4.5.52 10.5.52	892·2398	892·2357	892·2378	±2·0	−4·1	5	
13	52	5.5.52 9.5.52	1253·3719	1253·3660	1253·3690	±3·0	−5·9	5	
14	44	6.5.52 8.5.52	1056·1580	1056·1608	1056·1594	±1·4	+2·8	3	
EASY SECTION									
15	32	18.5.52 18.5.52	767·4505	767·4519	767·4512	±0·7	+1·4	2	
16	28	19.5.52 11.6.52	665·7244	665·7241	665·7242	±0·2	−0·3	0·5	
17	23	20.5.52 11.6.52	557·8084	557·8063	557·8074	±1·0	−2·1	4	
18	42	20.5.52 11.6.52	1007·6423	1007·6426	1007·6424	±0·2	+0·3	0·3	
19	33	21.5.52 9.6.52	791·7911	791·7918	791·7914	±0·4	+0·7	1	
20	40	21.5.52 9.6.52	959·8410	959·8387	959·8398	±1·2	−2·3	2	
21	58	22.5.52 8.6.52	1397·7206	1397·7193	1397·7200	±0·6	−1·3	1	
22	38	23.5.52 7.6.52	911·5767	911·5744	911·5756	±1·2	−2·3	3	
23	37	23.5.52 6.6.52	887·5971	887·5963	887·5967	±0·4	−0·8	1	
24	38	24.5.52 6.6.52	921·6556	921·6587	921·6572	±1·6	+3·1	3	

TABLE 4.4 *continued*

CAITHNESS BASE: 1952 *continued*

Section No.	No. of 24-m. Bays	Date	Forward Measure (*F*) (*Metres*)	Return Measure (*R*) (*Metres*)	Accepted Length (*Metres*)	Standard error of accepted length in mm. (σ_m)	Difference = Δ		Remarks
							R − F (*mm.*)	*In p.p.m.*	
EASY SECTION		*continued*							
25	34	24.5.52	815·8375						
		5.6.52		815·8365	815·8370	±0·5	−1·0	1	
26	26	26.5.52	617·9949						
		5.6.52		618·0019					Rejected. Wind along base.
		12.6.52		617·9930	617·9940	±1·0	−1·9	3	
27	30	27.5.52	719·5207						
		4.6.52		719·5206	719·5206	0	−0·1	0·1	
28	31	29.5.52	743·5506						
		4.6.52		743·5522	743·5514	±0·8	+1·6	2	
29	31	30.5.52	743·2631						
		3.6.52		743·2651					Rejected. Wind along base.
		12.6.52		743·2632	743·2632	0	+0·1	0·1	
30	39	31.5.52	947·5030						
		3.6.52		947·5021	947·5026	±0·4	−0·9	1	

The measured length of the base, corrected for temperature, inclination of the scales, inclined catenary and slope = 24,828·19639 m.
Reduction to sea-level (mean height of base = 170 feet) = −0·20135 m.
Correction for gravity (g = 981·80 cm./s^2) = +0·00450 m.
Final accepted length = 24,827·99954 m.

Standard Error of the Base

The sources of error affecting the accuracy of the measured length are detailed in § 4.23, thus:

	Bog Section	*Easy Section*	*Whole Base*
σ_a	$0{\cdot}6\times10^{-6}\times11372$ m. = ±0·0068 m.	$0{\cdot}6\times10^{-6}\times13456$ m. = ±0·0081 m.	$0{\cdot}6\times10^{-6}\times24828$ m. = ±0·0149 m.
σ_b	$0{\cdot}015\times10^{-6}\times\frac{15}{\sqrt{3}}\times11372$ m. = ±0·0015 m.	$0{\cdot}015\times10^{-6}\times\frac{15}{\sqrt{3}}\times13456$ m. = ±0·0017 m.	$0{\cdot}015\times10^{-6}\times\frac{15}{\sqrt{3}}\times24828$m. = ±0·0032 m.
σ_c	$\sqrt{\left(\frac{\Sigma\Delta^2}{4}\right)}$ = ±0·0049 m.	$\sqrt{\left(\frac{\Sigma\Delta^2}{4}\right)}$ = ±0·0031 m.	$\sqrt{\left(\frac{\Sigma\Delta^2}{4}\right)}$ = ±0·0057 m.
σ_d	$47\times10^{-8}\times11372$ m. = ±0·0053 m.	$47\times10^{-8}\times13456$ m. = ±0·0063 m.	$47\times10^{-8}\times24828$ m. = ±0·0117 m.
	σ_{Bog} = ±0·0100 m. or 1 in 1,137,000	σ_{Easy} = ±0·0109 m. or 1 in 1,235,000	σ_{Base} = ±0·0200 m. or 1 in 1,241,000

CONCLUSIONS FROM THE 1951–52 MEASUREMENTS

4.19 Effect of Wind

By the use of screens measurements could be made in cross winds of 15 and possibly 20 m.p.h. No screening was, however, possible against winds blowing down the line of the base. Measurements made in these circumstances with winds of greater velocity than about 10 m.p.h. were invariably unsatisfactory.

4.20 Errors due to Tripod Movement

The possibilities of errors being caused by movement of the measuring head tripods was not fully appreciated at the outset of the 1951–52 measurement. Not until halfway through the Ridgeway measurement was movement of a tripod over a section mark detected. Thereafter elaborate precautions were taken to guard tripods to prevent personnel moving round them unnecessarily. At Caithness the bog conditions made such precautions imperative, and the improved results in the earlier section of the base reflect this.

4.21 Size of Party

The size of the party depended on the number of men needed on the screen. On the Ridgeway Base the party was composed of 40 men as follows:

(*a*) Forward party; four men, who cut down hedges, aligned pegs, erected banderoles and helped on the screen when they were available. One of the senior men in this party organised the marking and emplacing of all section marks.

(*b*) Aligning party; four men, under the charge of a senior surveyor, who used the aligning theodolite. One man assisted him at the rear end of the setting out wire, booking the approximate vertical angles. Two men worked forward, one on the end of the setting out wire and the other erecting and centring the tripod.

(*c*) Measuring party; eight men, consisting of two observers (Grade II surveyors), two weight men, two straining-trestle men, one booker, and one additional man to carry the thermometer rack, read the thermometers and steady the tapes when they were being moved.

(*d*) Tape-rack party; two men.

(*e*) Screen party; normally 10 men.

(*f*) Tripod party; three men to carry forward the tripods.

(*g*) Levelling party; six men, consisting of two levellers, two bookers and two staff holders.

(*h*) Computing party; two men.

(*j*) Surveyor in charge.

On the Caithness Base the tripod party was increased to nine as additional men were required for carrying ‘A’ frames, duckboards and pickets.

FINAL RESULTS AND ACCURACY

4.22 Comparison between the several Measurements

The final differences between the forward and reverse measurements were satisfactory. These were:

Lossiemouth Base (1938), difference 0·2 mm. (1/3,590,000)
Ridgeway Base (1951), difference 5·9 mm. (1/1,910,000)
Caithness Base (1952), Bog area, difference 13·9 mm. (1/820,000)
Caithness Base (1952), Stable area, difference 5·8 mm. (1/2,320,000)
Caithness Base (1952), (Overall), difference 19·7 mm. (1/1,260,000)
No complete reverse measurement of the Ridgeway Base was made in 1937.

See Tables 4.1, 4.2, 4.3, and 4.4.

The 1951 measurement of the Ridgeway Base was 6·6 mm. (1/1,710,000) shorter than that of the 1937 measurement. This result is very satisfactory in view of the fact that, apart from a time interval of 14 years between measurements, the conditions of the two measurements were different in respect of tapes, weights, and wires, method of bridging the ravine, teams, and weather.

4.23 Statistical Assessment of Accuracy

In Tables 4.1, 4.2, 4.3, and 4.4, an assessment of standard error (σ) is given. The following factors were taken into consideration in calculating the total standard error.

(*a*) Standard error of the National Physical Laboratory standardisation of the field standard tapes; this is given by the Laboratory as $\pm 0{\cdot}6 \times 10^{-6}$.

(*b*) Standard error of the coefficients of expansion of the field standards. The error arises from the difference of temperature between the laboratory and field standardisations, and the standard error of the coefficient of expansion of a tape has been estimated as $\pm 0{\cdot}015 \times 10^{-6}$ per 1°F. The following mean differences of temperature have been calculated for the various measurements.

Ridgeway Base (1937)	22°F
Lossiemouth Base (1938)	2°F
Ridgeway Base (1951)	11°F
Caithness Base (1952)	15°F

In all cases three field standards were used.

(*c*) Accidental errors of tape reading, temperature recording, and levelling. These have been determined from the discrepancies of the section measures.

(*d*) Error due to lack of knowledge of the true temperature of the tapes in the field. The uncertainty in temperature has been estimated to be ±1°F.

Calling the errors from (*a*), (*b*), (*c*), and (*d*), σ_a, σ_b, σ_c, and σ_d, respectively, then the total standard error of the base has been calculated from:

$$\sigma_{\text{Base}} = \sqrt{(\sigma_a^2 + \sigma_b^2 + \sigma_c^2 + \sigma_d^2)}$$

4.24 Accepted Lengths of the Various Bases

For convenience of reference the accepted lengths are given below together with their standard errors.

To complete the comparisons, the result of the Lossiemouth Base measurement of 1909 is included. In this case the result and the probable error given in *Professional Paper* (*New Series*) *No. 1*⁽¹²⁾ have been converted from feet to metres and the probable error expressed as standard error. The 1909 tapes were standardised against the International Metre at Sévres through an Ordnance Survey Intermediate 10-foot Bar, the length of the latter being converted to feet by the ratio:

$$1 \text{ m.} = 39{\cdot}370113 \text{ inches.}$$

This ratio was used to obtain the 1909 metric result given here. A very small correction of −0·4 mm. for difference of terminals has also been applied.

ACCEPTED LENGTHS

Lossiemouth Base (1909)	7,170·7119 m. ±0·0114 m.
Lossiemouth Base (1938)	7,170·7234 m. ±0·0053 m.
Ridgeway Base (1937)	11,260·1931 m. ±0·0098 m.
Ridgeway Base (1951)	11,260·1865 m. ±0·0102 m.
Caithness Base (1952)	24,827·9995 m. ±0·0200 m.

For the determination of the velocity of light and for the Geodimeter and Tellurometer length comparisons described below, a weighted mean value of 11,260·1887 m. was accepted for the Ridgeway Base. Although the assessed standard errors of the 1937 and 1951 measures of the Ridgeway Base were about the same, the 1951 measure was considered to be in fact superior because of the additional precautions taken. It was therefore given twice the weight of the 1937 measure.

GEODIMETER MEASUREMENTS

4.25 Introduction

4.250 THE INSTRUMENT

The AGA Geodimeter has been fully described in articles by the inventor Dr. E. Bergstrand of the Swedish Geographical Survey and others⁽²⁹⁾. Briefly the instrument uses a pulsed light beam to measure the distance to a distant reflector. The pulse (or modulation) frequency is accurately controlled by a crystal in conjunction with a Kerr cell, the distance to the reflector being deduced by measuring the distance by which returning pulses are out of phase with those emitted. In the Geodimeter Model NASM 1, which was used by the Ordnance Survey, two modulating frequencies, respectively 1% less than and greater than 10 Mc/s, were used in order to resolve ambiguity and to provide a check.

4.251 THE PROGRAMME AND PROCEDURE

The Ordnance Survey work was carried out in co-operation with the United States Army Map Service who in 1953 sent the instrument to Great Britain for trials accompanied by two technicians, Mr. John S. McCall, a surveyor, and Mr. Donald Mears, an electronics engineer. These technicians trained certain Ordnance Survey personnel who later carried out trials on the Ridgeway and Caithness Bases and used the instrument to measure a primary triangulation side at the northern extremity of the Shetland Islands. All this work has been described in an Ordnance Survey Professional Paper(30) by Major I. C. C. Mackenzie who was the officer responsible. The measuring procedures of the Army Map Service were followed in general although it was found that their practice of taking four sets of readings did not suit the unpredictable British climate since a complete observation thus took about 2½ hours—too long a period over which to expect continuously perfect visibility. Instead smaller numbers of observations were tried and in the end only two sets on each frequency were taken.

4.252 THE ACCEPTED VELOCITY OF LIGHT

At the time the work was undertaken the most widely accepted value for the velocity of light *in vacuo* (C_0), upon which of course measured distances directly depend, was that previously determined by Dr. Bergstrand(31), i.e.

$$C_0 = 299{,}793{\cdot}1 \text{ km/s.}$$

This value was used by the Ordnance Survey for all published results. Later, however, a lower value was recommended by the International Association of Geodesy in a resolution adopted at the General Assembly of the International Union of Geodesy and Geophysics at Toronto in 1957; viz.

$$C_0 = 299{,}792{\cdot}5 \text{ km/s} \pm 0{\cdot}4 \text{ km/s.}$$

This value has been used for all results given in this volume, previously published values for distances being revised accordingly. This matter is dealt with further in § 4.31.

4.26 Field Measurements

4.260 RIDGEWAY BASE

After initial attempts to measure the Caithness Base in June 1953 had failed due to the lack of complete darkness in summer in these northern latitudes, the Geodimeter was brought to the Ridgeway Base. Between the 5th and 29th July 1953, 17 measurements of the base were made, despite unfavourable weather. The results are given in Tables 4.5 and 4.6, and are discussed in § 4.27. The instrument was then calibrated at the National Physical Laboratory prior to moving north again to the Caithness Base.

4.261 CAITHNESS BASE

A second and successful attempt to measure the Caithness Base was made between the 13th and 26th August. During this period observations were only possible on seven nights; considerable difficulty being caused by ground and sea mists which are prevalent in this area. Subsequently, more measurements were made between 20th September and 5th October. The results are given in Tables 4.5 and 4.6, and are discussed in § 4.27 below.

4.262 THE RETRIANGULATION SIDE SAXAVORD (463) TO FETLAR (459)

This line was selected as it was at the extreme northern limit of the triangulation. Despite strong gales, 12 complete measures, each consisting of two sets of readings on both frequencies were obtained between the 4th and 13th September. See Tables 4.5 and 4.6.

At the conclusion of the fieldwork the Geodimeter was again calibrated at the National Physical Laboratory before being returned to the United States.

TABLE 4.5

GEODIMETER MEASUREMENTS

White Horse Hill (34) to Liddington Castle (35) (Ridgeway Base)

Date 1953	*Observer*	*Spheroidal Distances (Metres)*			*No. of Sets*	*Remarks*
		From f_1	*From f_2*	*Mean*		
5th July	Smith	11260·244	11260·184	11260·214	4 2	Not accepted. Very poor mirror readings on f_2.
5th July	Mears	11260·229	11260·134	11260·182	4 4	
6th July	Smith	11260·217	11260·210	11260·214	3 4	
7th July	Mears	11260·187	11260·173	11260·180	4 4	
7th July	Smith	11260·205	11260·187	11260·196	4 4	
8th July	Mears	11260·195	11260·189	11260·192	5 4	Not accepted. Rain.
8th July	Smith	11260·214			4	Not accepted, f_1 only.
9th July	Smith	11260·199	11260·191	11260·195	4 4	
9th July	Mears	11260·191	11260·187	11260·189	4 4	

TABLE 4.5 *continued*

GEODIMETER MEASUREMENTS

White Horse Hill (34) to Liddington Castle (35) (Ridgeway Base) *continued*

Date 1953	*Observer*	*Spheroidal Distances (Metres)*			*No. of Sets*	*Remarks*
		From f_1	*From f_2*	*Mean*		
10th July	Smith	11260·200	11260·178	11260·189	4 3	4 sets taken, 3 used.
10th July	Mears	11260·202	11260·181	11260·192	4 2	4 sets taken, 2 used.
12th July	Mears	11260·152	11260·166	11260·159	4 2	Not accepted. Very poor weather; f_2 particularly bad.
13th July	Smith	11260·224	11260·205	11260·214	4 4	
14th July	Mears	11260·187	11260·191	11260·189	4 3	4 sets taken, 3 used.
14th July	Smith	11260·193	11260·187	11260·190	4 4	
15th July	Mears	11260·189			3	Not accepted, f_1 only.
15th July	Smith	11260·212	11260·172	11260·192	4 4	
28th July	Smith	11260·197	11260·191	11260·194	3 2	4 sets taken, 3 used. 4 sets taken, 2 used.
29th July	Smith	11260·200	11260·177	11260·188	4 4	
28th July	Bickers	11260·157	11260·097	11260·127	4 4	Not accepted. Observer learning in bad weather.
29th July	Bickers		11260·118		3	
30th July	Bickers	11260·177			4	

TABLE 4.5 *continued*

GEODIMETER MEASUREMENTS

Saxavord (463) to Fetlar (459)

Date 1953	*Observer*	*Spheroidal Distances (Metres)*			*No. of Sets*	*Remarks*
		From f_1	*From f_2*	*Mean*		
4th Sept.	Smith	23126·982 23126·976	23126·953	23126·970	2 2 2	Two observations taken on f_1.
4th Sept.	Bickers	23127·008	23126·960	23126·984	2 2	
5th Sept.	Smith	23127·008	23126·959	23126·984	3 2	
5th Sept.	Bickers	23127·008			2	Not accepted, f_1 only.
6th Sept.	Smith	23126·985			2	Not accepted, f_1 only.
9th Sept.	Smith	23127·010	23126·986	23126·998	2 2	
9th Sept.	Bickers	23127·010	23126·993	23127·002	2 2	
9th Sept.	Smith	23126·992	23126·990	23126·991	2 2	
10th Sept.	Bickers	23126·987	23126·980	23126·984	2 2	
10th Sept.	Smith	23126·986	23126·984	23126·985	2 2	
13th Sept.	Smith	23127·003	23126·967	23126·985	2 2	

TABLE 4.5 *continued*

GEODIMETER MEASUREMENTS

Saxavord (463) to Fetlar (459) ***continued***

Date 1953	*Observer*	*Spheroidal Distances (Metres)*			*No. of Sets*	*Remarks*
		From f_1	*From f_2*	*Mean*		
13th Sept.	Smith	23127·009	23126·964	23126·986	2 2	
12th Sept.	Bickers	23127·027	23127·075	23127·051	2 2	Not accepted. Very still air with mist at Geodimeter.
12th Sept.	Smith	23127·051	23127·020	23127·036	2 2	
Warth Hill (399) to Spital Hill (398) (Caithness Base)						
13th Aug.	Smith	24828·111	24828·070	24828·090	4 4	Not accepted. Rain.
14th Aug.	Smith	24828·123	24827·997	24828·060	4 2	Range of 1°C in temperature at Geodimeter. Not accepted. Rain.
15th Aug.	Smith	24828·060	24827·954	24828·007	3 4	4 sets taken, 3 used. Range of 1·8°C in temperature at Mirror.
16th Aug.	Smith	24828·011	24827·989	24828·000	4 4	
16th Aug.	Bickers		24827·996		4	Not accepted, f_2 only. Range of 1°C in temperature at Mirror.
17th Aug.	Smith	24828·009	24827·983	24827·996	3 4	4 sets taken, 3 used. 1 hour delay during sets.
17th Aug.	Bickers	24828·011			4	
18th Aug.	Smith	24828·070	24828·045	24828·058	4 4	Not accepted. Rain. ½ hour delay during observations.

TABLE 4.5 *continued*

GEODIMETER MEASUREMENTS

Warth Hill (399) to Spital Hill (398) (Caithness Base) *continued*

Date 1953	*Observer*	*Spheroidal Distances (Metres)*			*No. of Sets*	*Remarks*
		From f_1	*From f_2*	*Mean*		
19th Aug.	Smith	24828·040			2	Not accepted, f_1 only. 3 sets actually taken.
24th Aug.	Bickers	24827·998			2	Not accepted, f_1 only.
20th Sept.	Smith	24828·026			2	Not accepted, f_1 only.
26th Sept.	Bickers	24827·999		24827·966	2	
			24827·933		2	
27th Sept.	Bickers	24828·067		24828·086	2	Not accepted. Rain.
			24828·105		3	
27th Sept.	Smith	24828·063		24828·050	2	
			24828·036		2	
28th Sept.	Bickers	24828·006		24827·994	2	
			24827·982		2	
28th Sept.	Smith	24827·990		24827·987	2	Temperature at Mirror estimated; thermometer broken.
			24827·984		2	
28th Sept.	Smith	24828·044		24828·040	2	
			24828·037		2	
29th Sept.	Bickers	24828·076		24828·058	2	Range of 1°C in temperature at Mirror.
			24828·039		2	
29th Sept.	Smith	24828·073		24828·047	2	
			24828·021		2	Range of 1·3°C in temperature at Geodimeter.

TABLE 4.5 *continued*

GEODIMETER MEASUREMENTS

Warth Hill (399) to Spital Hill (398) (Caithness Base) *continued*

Date 1953	*Observer*	*Spheroidal Distances (Metres)*			*No. of Sets*	*Remarks*
		From f_1	*From f_2*	*Mean*		
30th Sept.	Smith	24828·080		24828·074	2	
			24828·067		2	
30th Sept.	Bickers	24828·040		24828·052	2	
			24828·065		2	
1st Oct.	Smith	24828·147		24828·068	2	Ranges in temperature of 2·2°C at Geodimeter, 1·7°C at Mirror. Not accepted. Rain.
			24827·989		2	
1st Oct.	Bickers	24828·181		24828·086	2	Not accepted. Rain. Range of 1·1°C in temperature at Geodimeter.
			24827·991		2	
4th Oct.	Smith	24828·029		24828·013	3	
			24827·997		2	
4th Oct.	Bickers	24828·044		24828·020	2	
			24827·997		2	
3rd Oct.	Smith		24828·010		2	Not accepted, f_2 only.
5th Oct.	Smith	24827·998		24827·999	2	Range of 1·4°C in temperature at Mirror.
			24828·000		2	Range of 1·8°C in temperature at Geodimeter.
5th Oct.	Smith	24828·072		24828·056	2	Not accepted. Rain.
			24828·039		2	
5th Oct.	Bickers	24828·055		24828·028	2	
			24828·002		2	

TABLE 4.6

SUMMARY OF RESULTS

Date 1953	*Line: Geodimeter Station first*	*Spheroidal Distance (S)*		*Observer*	*No. of Obs.*	*Standard Error of a Single Observation*	*Standard Error of the Mean*	*Diff. G−C in p.p.m.*
		Geodimeter (G)	*Catenary (C)*					
(1) 5–29th July	White Horse Hill (34) Liddington Castle (35)	11260·186 m.		D. S. Mears	5	±0·004 m.	±0·003 m.	0·3
		0·197 m.	11260·189 m.	H. J. W. Smith	9	±0·010 m.	±0·003 m.	0·7
		Mean 0·193 m.		Mears & Smith	14	±0·010 m.	±0·003 m.	0·4
13–24th Aug. 20th Sept. to 5th Oct.(2)	Warth Hill (399) Spital Hill (398)	24828·001 m.		H. J. W. Smith	3	±0·006 m.	±0·003 m.	0
		0·030 m.		H. J. W. Smith	7	±0·031 m.	±0·012 m.	1·2
		0·020 m.	24828·000 m.	A. E. Bickers	6	±0·036 m.	±0·015 m.	0·8
		Mean 0·021 m.		Smith & Bickers	16	±0·030 m.	±0·007 m.	0·8
(3) 4–13th Sept.	Saxavord (463) Fetlar (459)	23126·986 m.	*S* from the triangulation is: 23127·130 m. Not measured in catenary.	H. J. W. Smith	7	±0·009 m.	±0·003 m.	
		0·990 m.		A. E. Bickers	3	±0·010 m.	±0·006 m.	
		Mean 0·987 m.		Smith & Bickers	10	±0·009 m.	±0·003 m.	

(1) Three observations were rejected. Some results by Bickers were not used as he was learning.

(2) Of the 23 observations 7 were rejected, 3 from the first visit and the remainder from the second.

(3) Two observations, one by each observer, on 12th September were rejected owing to the conditions. Although no catenary comparison is possible here, the results are very good giving an excellent pattern.

4.27 Results

4.270 INTRODUCTION

The main purpose of the trials of the Geodimeter was to test its accuracy by comparison with the tape measurements of the Ridgeway and Caithness Bases, respectively 11 and 25 km. in length. There was no time for more comprehensive testing of its operating characteristics or accuracy over other distances, although it was also used to check the length of the primary side Saxavord (463)-Fetlar (459) in the Shetlands (23 km.). Measurement was carried out whenever possible regardless of weather. Observations on a single frequency were discarded and the remaining results were plotted as graphs (Figs. 4.5, 4.6, 4.7). Of these results some were excluded from the accepted mean for the reasons given in Table 4.5. These reasons often arose directly or indirectly from the weather (see § 4.274).

4.271 EFFECTS OF ERRORS IN CALCULATING THE REFRACTIVE INDEX

Distances deduced from observations depend upon the calculated refractive index of the atmosphere along the light path, since changes in the velocity of propagation are inversely proportional to changes in the refractive index. These errors are considered below.

Temperature

Fig. 4.8 shows the correction for temperature which is almost linear—in fact it increases slightly as the temperature lowers. A positive error of 1°C produces a positive error of about 1/1,000,000 in distance.

Barometric pressure

The effects of errors in readings of barometric pressure vary slightly according to the temperature, and Fig. 4.9 shows that for a given error the effect is increased as the temperature is lowered. A positive 10 mm. error in pressure gives a negative error in distance of about 1/250,000.

Humidity

Fig. 4.10 shows that the corrections for humidity are very small although not linear, the effect of an error increasing as the temperature rises. At the maximum temperature on the graph, 30°C, a change from 60% to 100% makes a positive change in apparent distance of approximately 1/1,700,000.

Colour

See Fig. 4.11. There should be negligible error from this source. The group wavelength of the filter is ascertained accurately by the makers, and there is no reason to suppose that this is liable to appreciable change.

4.272 EFFECTS OF ERRORS IN OTHER DATA

Height errors

These affect the reduction of air distance (D) to spheroidal distance (S), or *vice versa*.

Considering the reverse case, it can be shown that for errors dh_1, dh_2, in heights h_1, h_2:

$$dD = \frac{1}{2D}\left\{\frac{K^2}{R}(dh_1 + dh_2) + 2(dh_1 - dh_2)(h_1 - h_2)\right\}$$

where K is the chord distance corresponding to S, and R is the radius of curvature along S.

The first term in this differential varies directly as the length of the line—assuming $K = D$ and dh_1, dh_2 constant—and is usually small, depending of course on dh_1, dh_2. Assume that $dh_1 = dh_2 = 5$ feet $= 1{\cdot}5$ m. Then clearly the second term is zero, and when D is about 30 km. the first term $= dD = 0{\cdot}008$ m. If dh_1 and dh_2 are opposite in sign, and there is a considerable difference in height between h_1 and h_2, the second term predominates, and is increasingly effective as the line gets shorter since it varies inversely as D, assuming dh_1, dh_2, $(h_1 - h_2)$, constant. Let $dh_1 = -dh_2 = 5$ feet $= 1{\cdot}5$ m. and $(h_1 - h_2) = 100$ m., then the first term is zero and the second term gives dD:

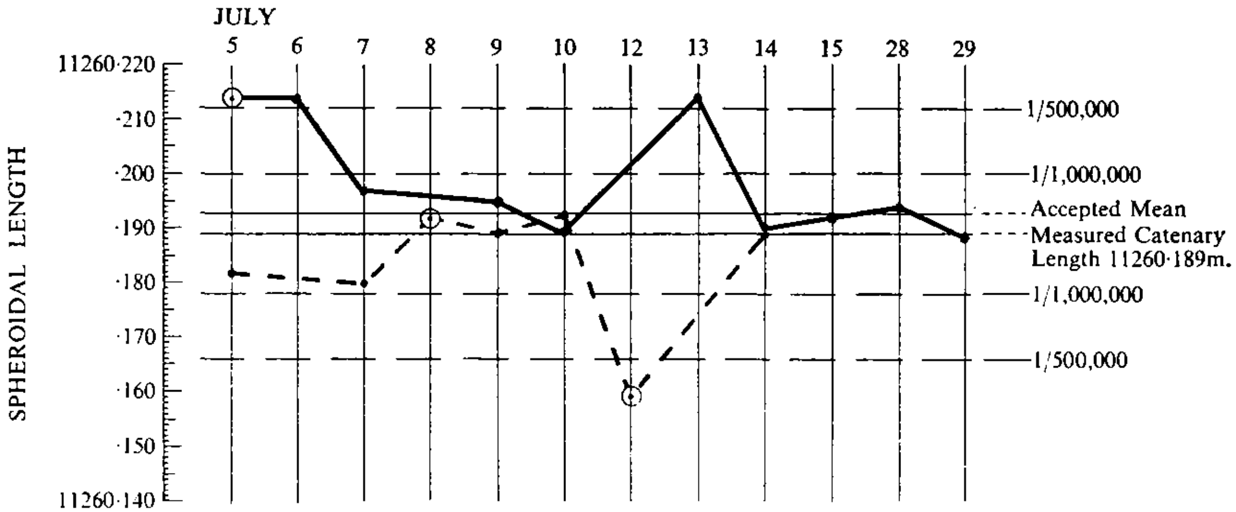
$$dD = 300/D \text{ m.}$$

which is 0·010 m. at 30 km., and 0·100 m. at 3 km.

Crystal frequencies

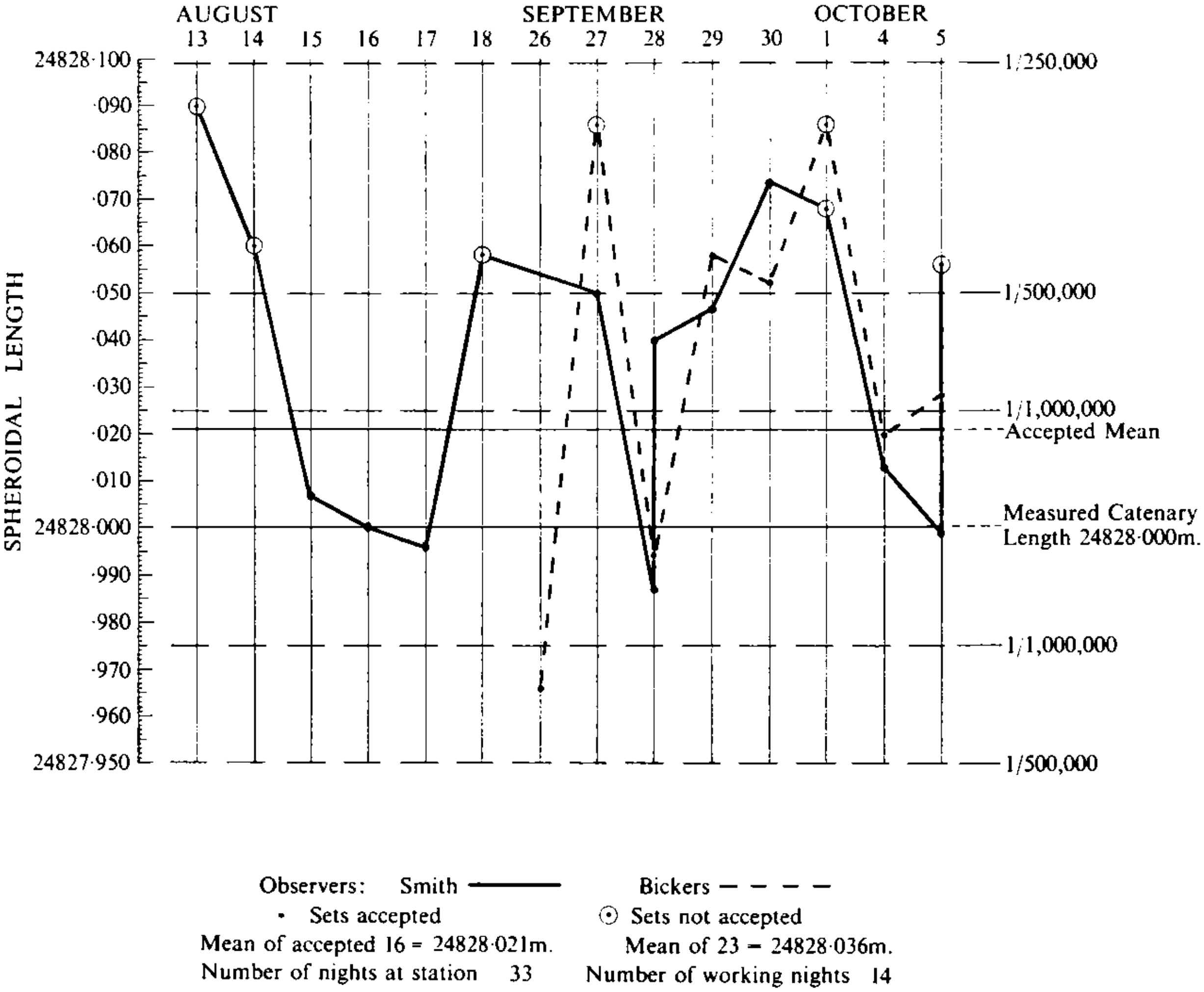
A positive error in the modulation frequency (approximately 10 Mc/s) gives rise to a directly proportionate negative error in the apparent distance. Thus an error of 1 cycle per second (c.p.s.) produces a distance error of 1 in 10^7.

The crystal frequencies were calibrated in the U.S.A. before the instrument arrived and later at the National Physical Laboratory on 7th August and 12th October, 1953, respectively before and



Observers : Smith ———	Mears — — — —
• Sets accepted	⊙ Sets not accepted
Mean of accepted 14 = 11260·193m.	Mean of 17 = 11260·192m.
Number of nights at station 25	Number of working nights 12

FIG. 4.5. Geodimeter measurements of the Ridgeway Base



Observers: Smith ——— Bickers — — — —

• Sets accepted ⊙ Sets not accepted

Mean of accepted 16 = 24828·021m. Mean of 23 = 24828·036m.

Number of nights at station 33 Number of working nights 14

FIG. 4.6. Geodimeter measurements of the Caithness Base

after the Scottish measurements. The results (after a warming up period of an hour) were:

f_1 9 999 944 c.p.s. U.S.A. figure, June 1953.
9 999 932 c.p.s. on 7.8.53.
9 999 945 c.p.s. on 12.10.53.
f_2 10 099 854 c.p.s. U.S.A. figure, June 1953.
10 099 855 c.p.s. on 7.8.53.
10 099 853 c.p.s. on 12.10.53.

A variation of ± 7 c.p.s. (1/1,400,000) about the mean was noted during calibration. f_1 was also found to decrease by about 1/400,000 in 20 minutes during warming up.

In general measurements in the field on f_1 gave slightly but consistently higher values for the distance than those on f_2 (see Table 4.5) amounting to an average of 0·02 m. for the Ridgeway Base or 1/550,000, and 0·05 m. for the Caithness Base or 1/500,000. The consistency indicated by these two figures points strongly to a differential error in the crystal frequencies. Although calibrations at the National Physical Laboratory do not indicate a liability to serious drift, in the rather different conditions of the field a slight drift possibly due to imperfect temperature control may have occurred.

Instrumental constants

Two constants, namely the length of one unit of the light conductor, and the distance from the Geodimeter reference point to the beginning of the light conductor, were checked by the National Physical Laboratory using their October determinations of f_1, f_2. They reached the conclusion that there were no significant errors in the accepted values.

4.273 ANALYSIS

The summary of results is given in Table 4.6 with remarks. This summary gives the standard errors of a single observation and of the mean of each series of observations; these provide a measure of the consistency of the observations but of course take no account of systematic errors due to errors in the calculated refractive index, the frequencies and heights. These systematic errors must contribute, albeit slightly, to the uncertainty of the results.

4.274 WEATHER AND CHOICE OF SITE

The weather is apt to be regarded more as a deterrent to observation than as a source of observational inaccuracy. From § 4.271 it can be seen that at normal temperatures a change of 1°C will cause a change in the distance of about one part per million; the same change in distance will be caused by a change in pressure of about 3 mm. Changes in humidity have little effect. Ideally measurements should be made when meteorological conditions are stable and uniform or varying uniformly along the line. Unfortunately owing to the variation of temperature with height above ground, it is difficult to obtain the temperature of the actual light path. In Great Britain, at 300 feet above ground level on calm, clear, summer nights over land, a 5°C variation of temperature can be expected, while 8°C is possible though exceptional. On the other hand on overcast windy nights the variation is as little as 0–2°C. Over the sea for all nights it is even less provided the wind is off the sea(32). To obtain a higher degree of accuracy, therefore, it seems better to measure over the sea, and this is borne out by the pattern of results in the Shetlands. For observations over the land, temperature readings taken along the light path by means of a moored kite balloon might help. Otherwise observations should be confined to nights with ten-tenths low cloud and a stiff breeze blowing. Observations should not be undertaken in rain or mist, and it should be remembered that the visibility required is virtually twice that of the distance to be measured.

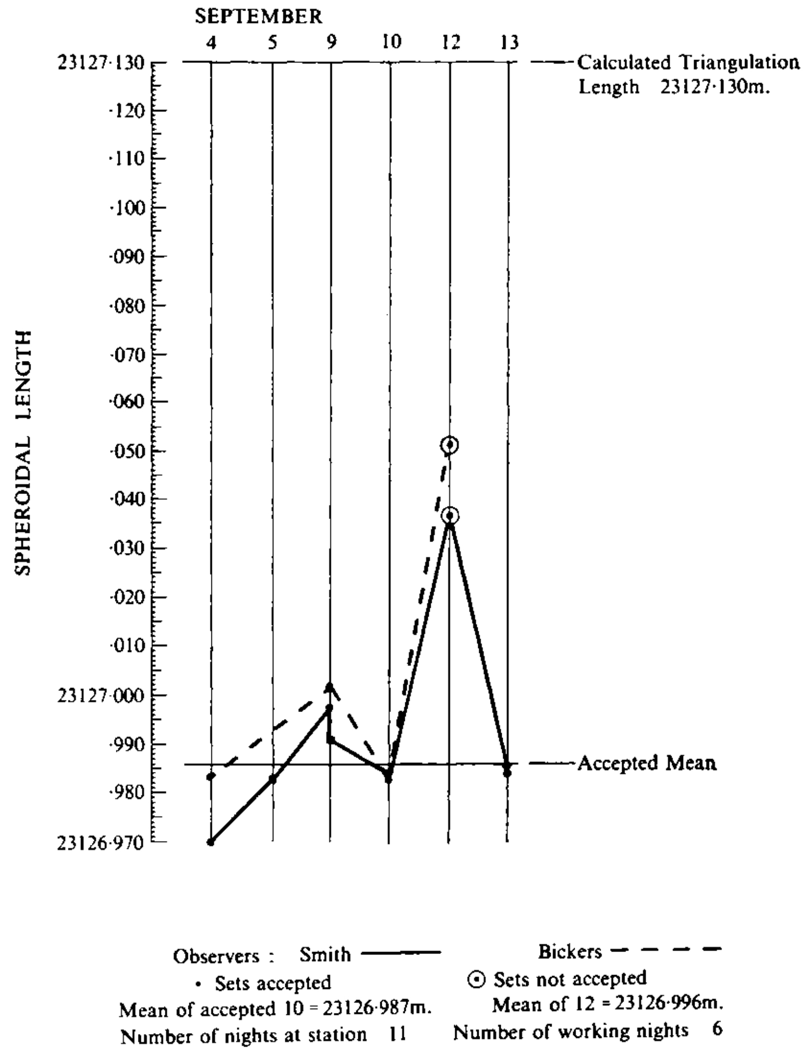


FIG. 4.7. Geodimeter measurements of the primary side Saxavord–Fetlar

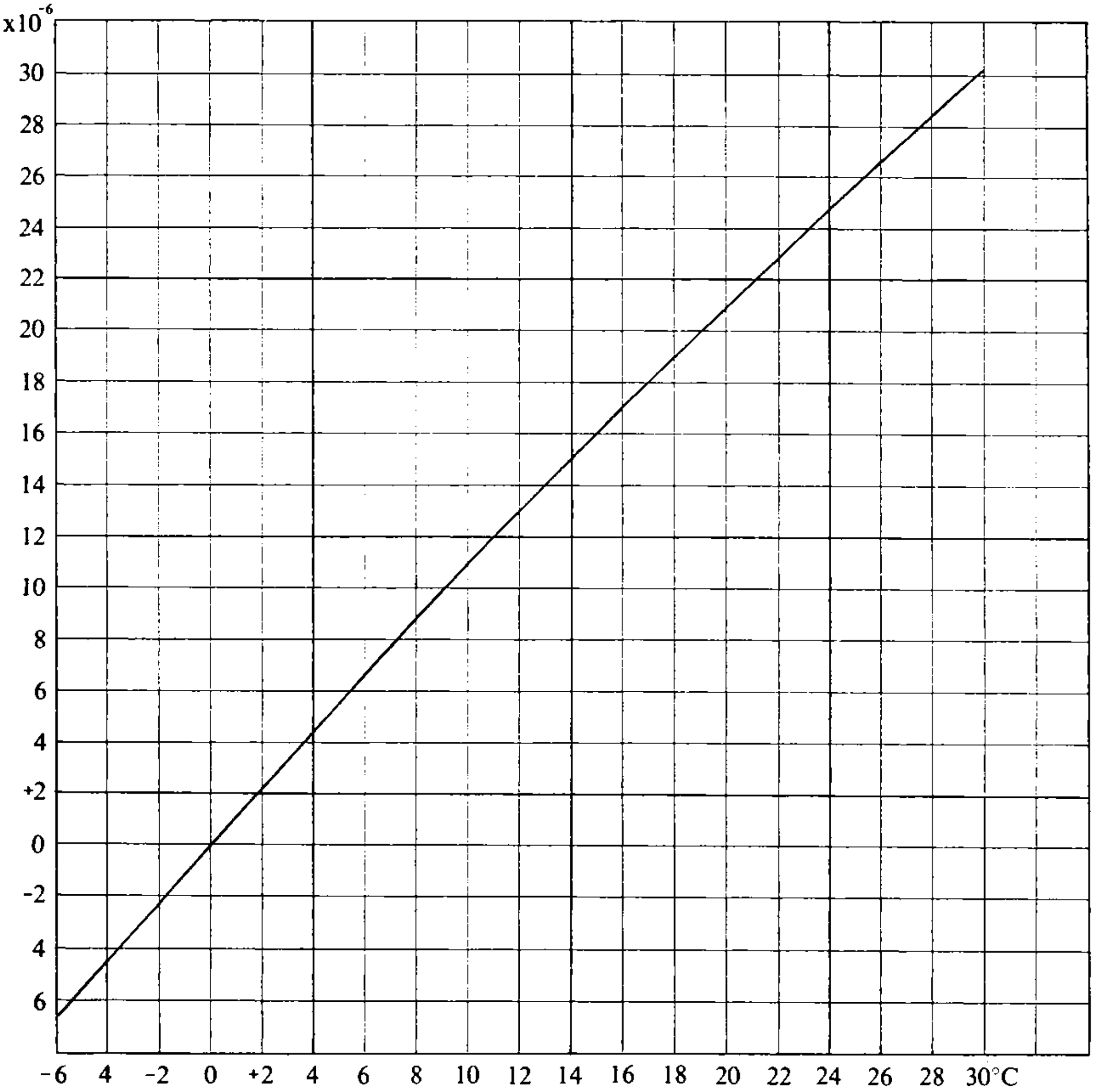


FIG. 4.8. Correction for temperature

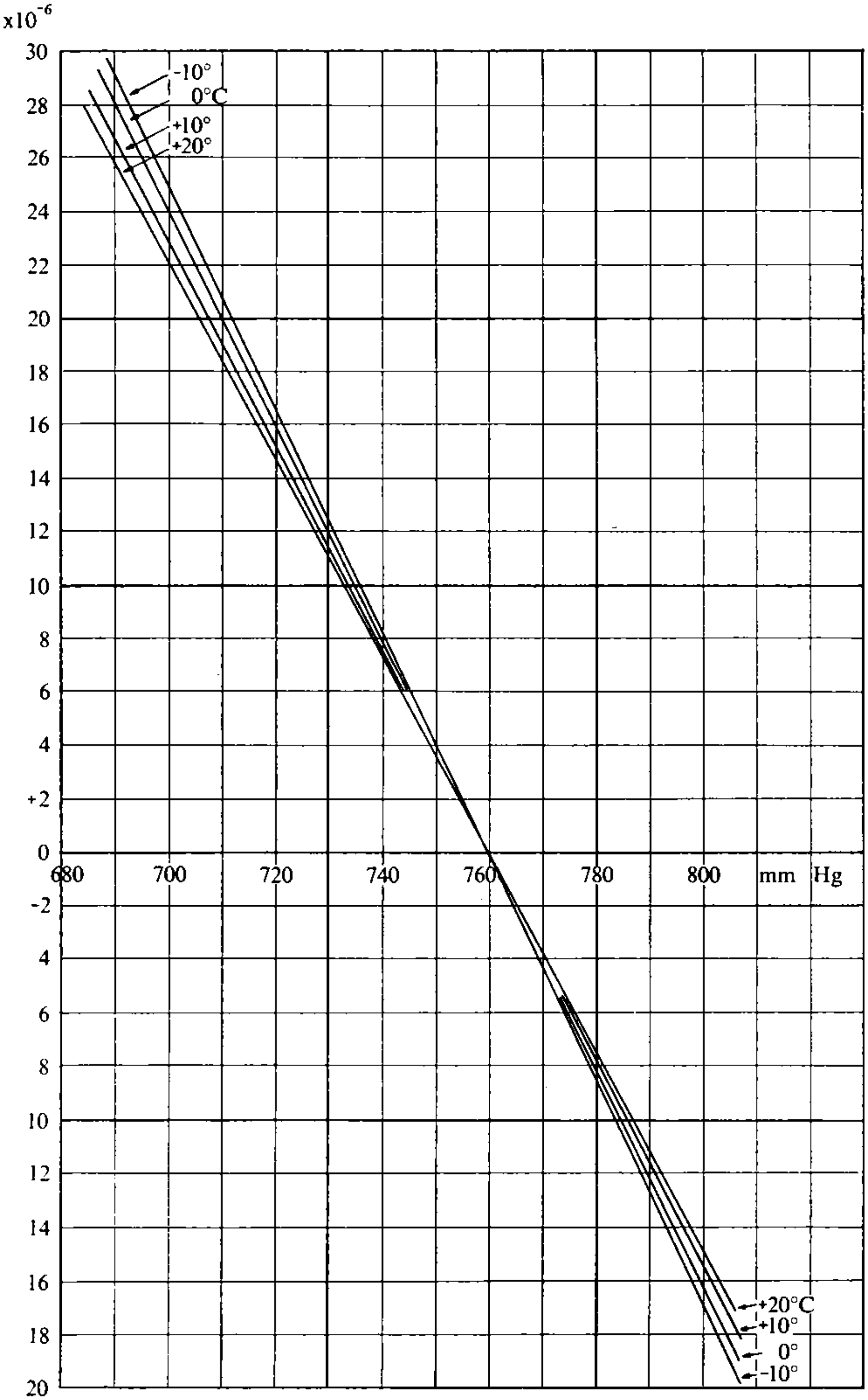


FIG. 4.9. Correction for pressure

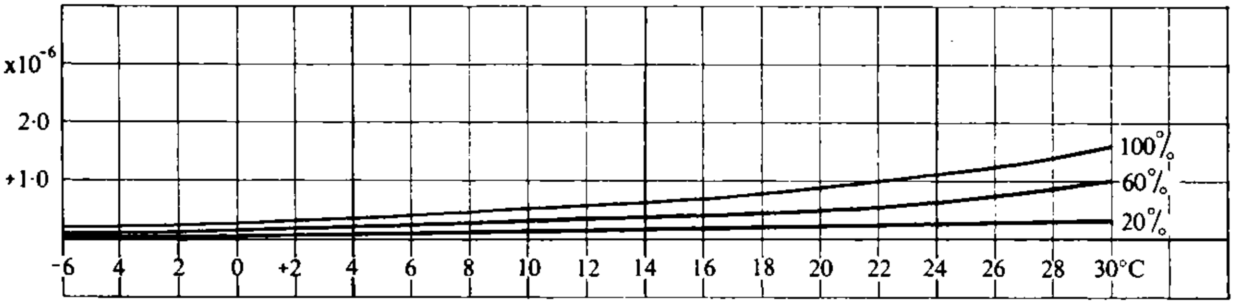


FIG. 4.10. Correction for humidity

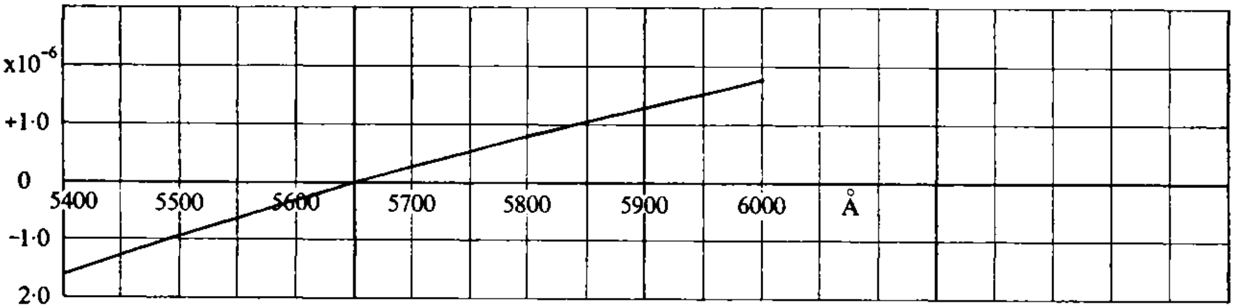
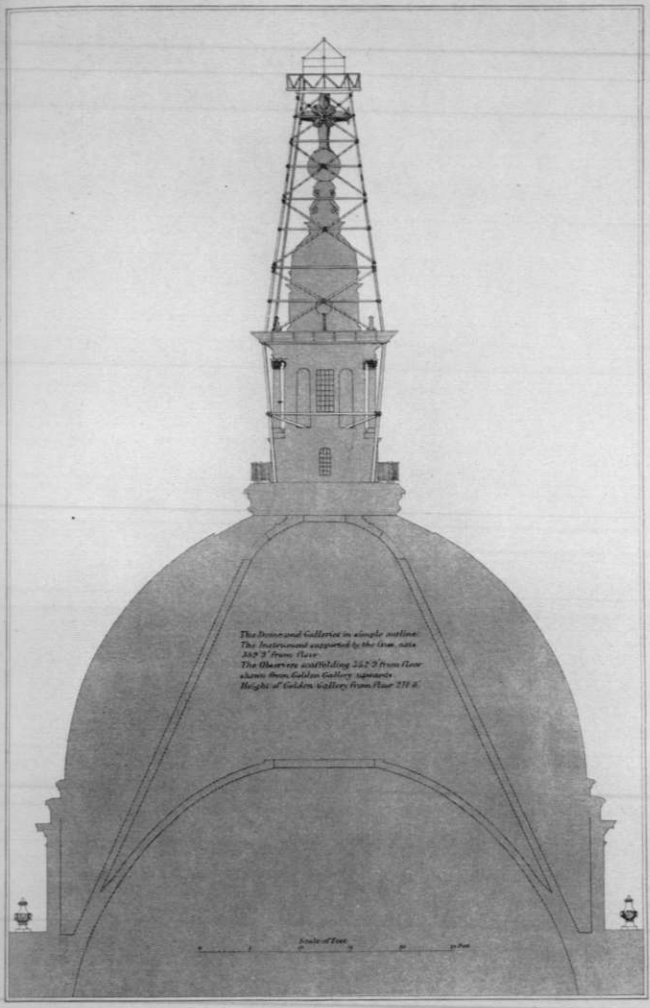
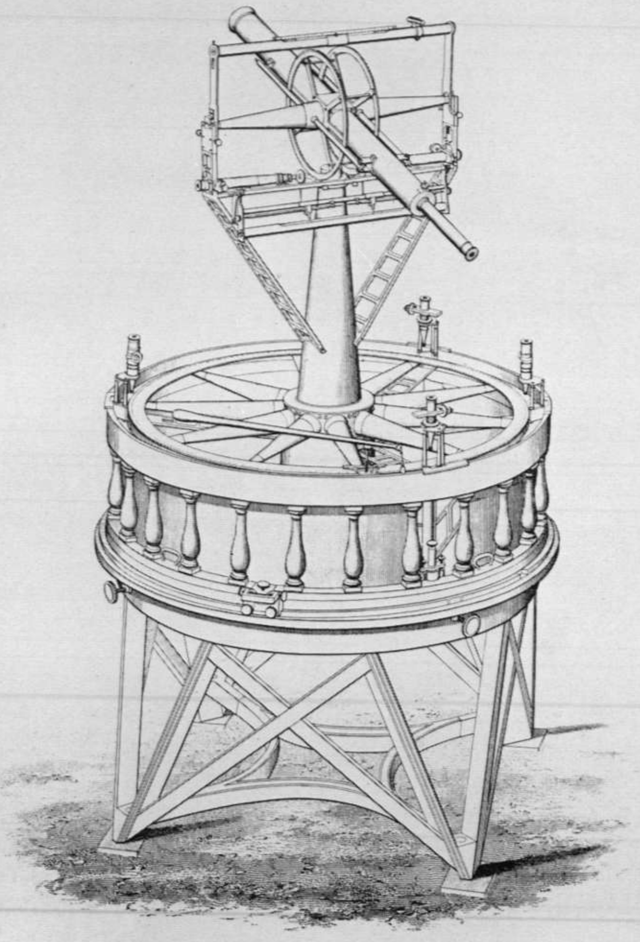


FIG. 4.11. Correction for colour



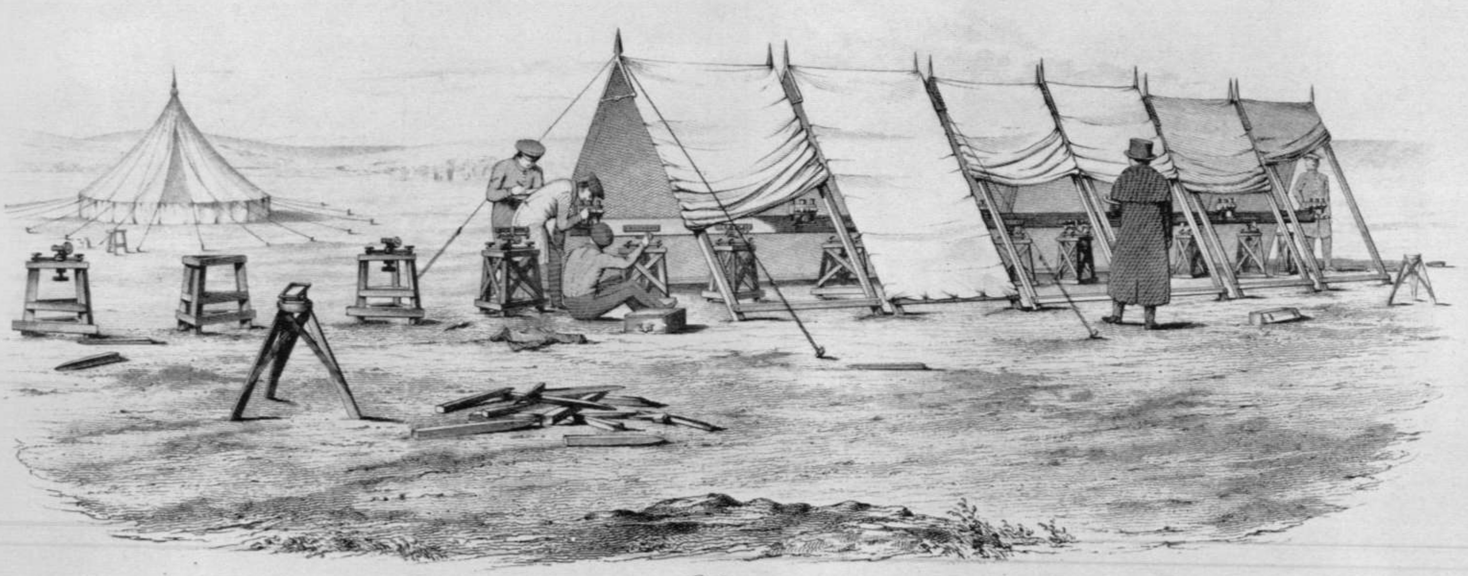
S^t^ PAULS CATHEDRAL DOME DEC^r^ 1848

RAMSDENS GREAT THEODOLITE
(Ordnance)



above, left An example of one of the many special scaffoldings erected (St. Paul's Cathedral Dome, 1848)
below The measurement of the Lough Foyle Base, 1827–8 *above, right* Ramsden's Great Theodolite

1. THE PRINCIPAL TRIANGULATION (see § 1.02)



Sketch
shewing the mode of proceeding
in measuring
THE LOUGH FOYLE BASE
(The original by Sir J.F.W. Herschel Bar^t^)
Engraved by J.C. Poole at the Ordnance Map Office Southampton, March 1847.

2. STATION MARKS
(see § 2.060, 2.061)

above Principal Triangulation mark (the ruler is one foot long)

below The Retriangulation pillar erected at Miltonhead, near Lanark, on the site of the birthplace of Major-General Roy

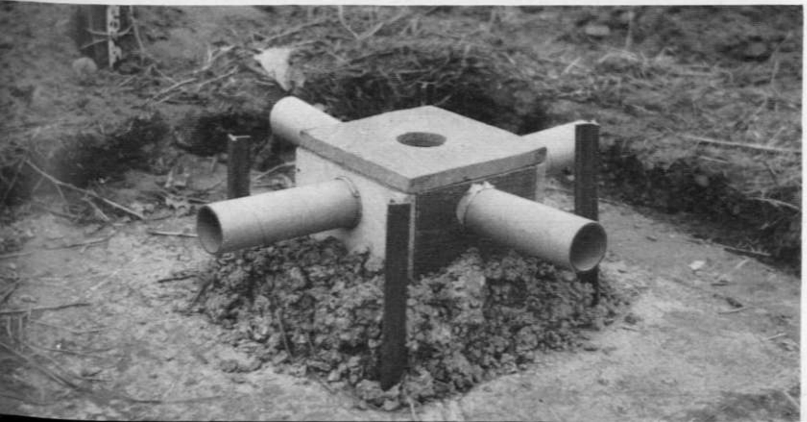
3. PILLAR CONSTRUCTION (see § 2.062)


Lower bolt and block in position


Plumbing device


Completing base


Top bolt, box, and angle-irons in position


Completed box with sighting tubes

Filling the shuttering


Facing

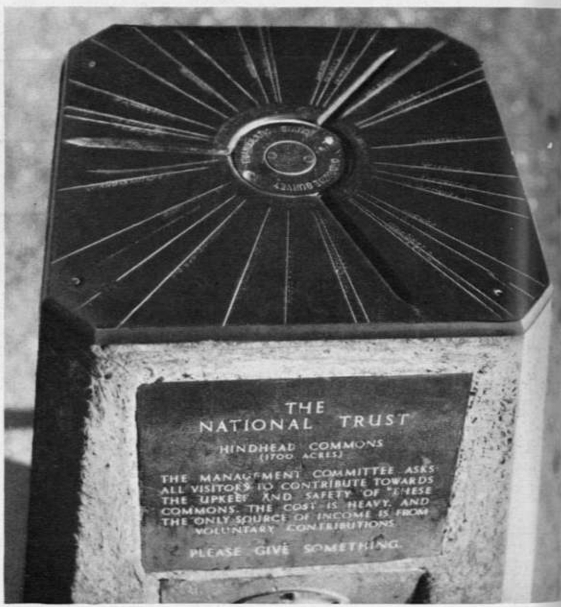

The completed pillar

4a. PRIMARY ROOF STATIONS 1936 (see § 2.064)

Observing on Lincoln Minster

. . . and beacon lamps set up on York Minster

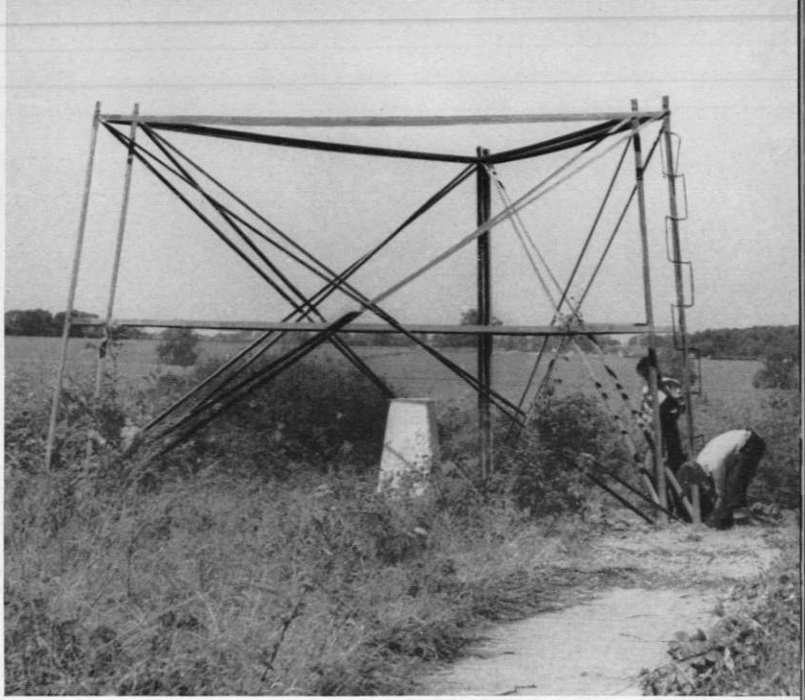
4b. HINDHEAD (see § 2.063) *below, left* A pillar on National Trust property, embodying the emblem, collecting-box, and (*right*) topograph



4c. PILLARS OF LOCAL STONE (see § 2.063)

below, left Foel Ispri, a secondary station near Dolgelley
below, right Loughrigg Fell, a tertiary station in Westmorland

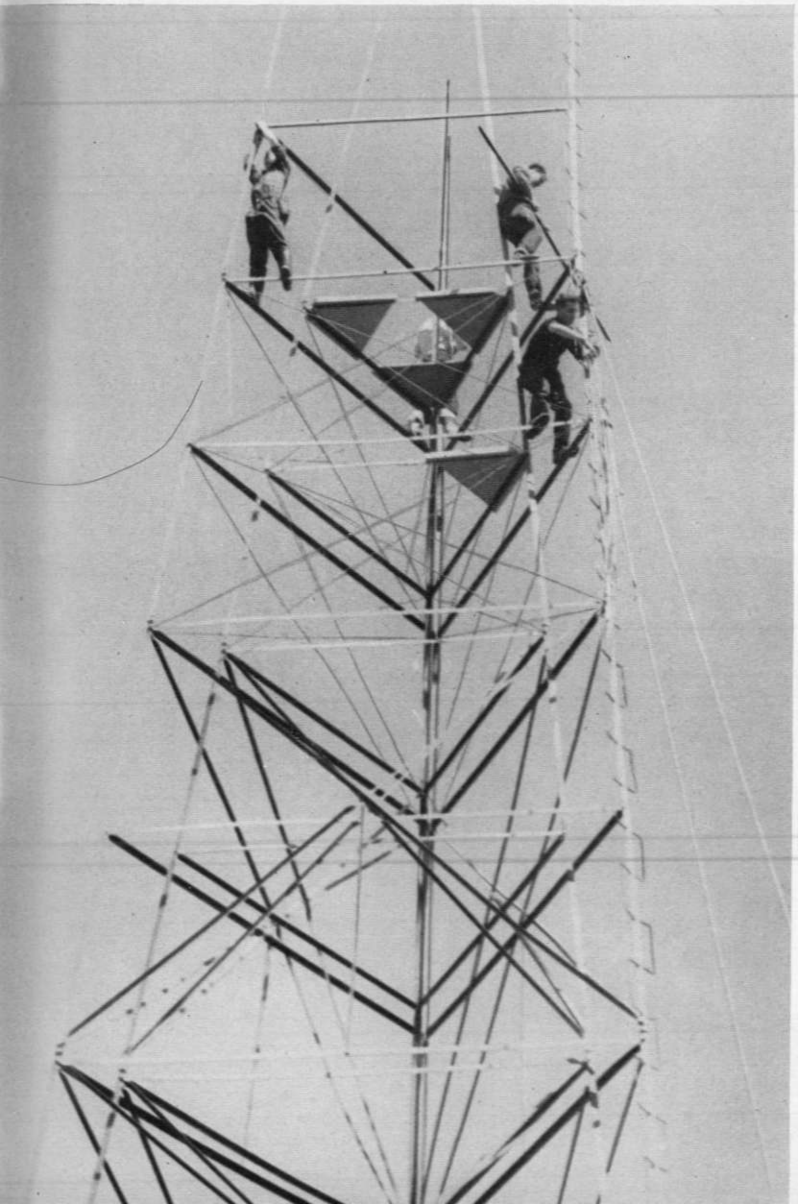
Excavating for footings



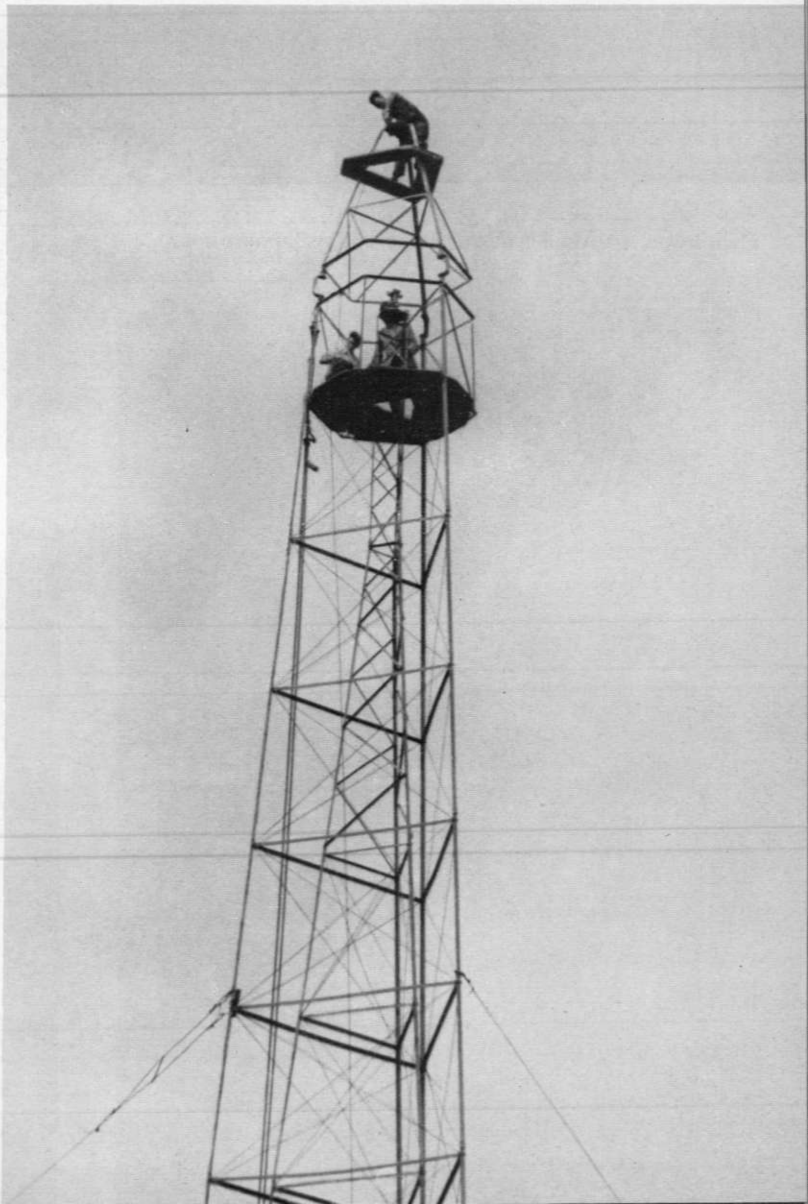
First sections of inner and outer towers erected

5. STEEL TOWER ERECTION (see § 2.07)

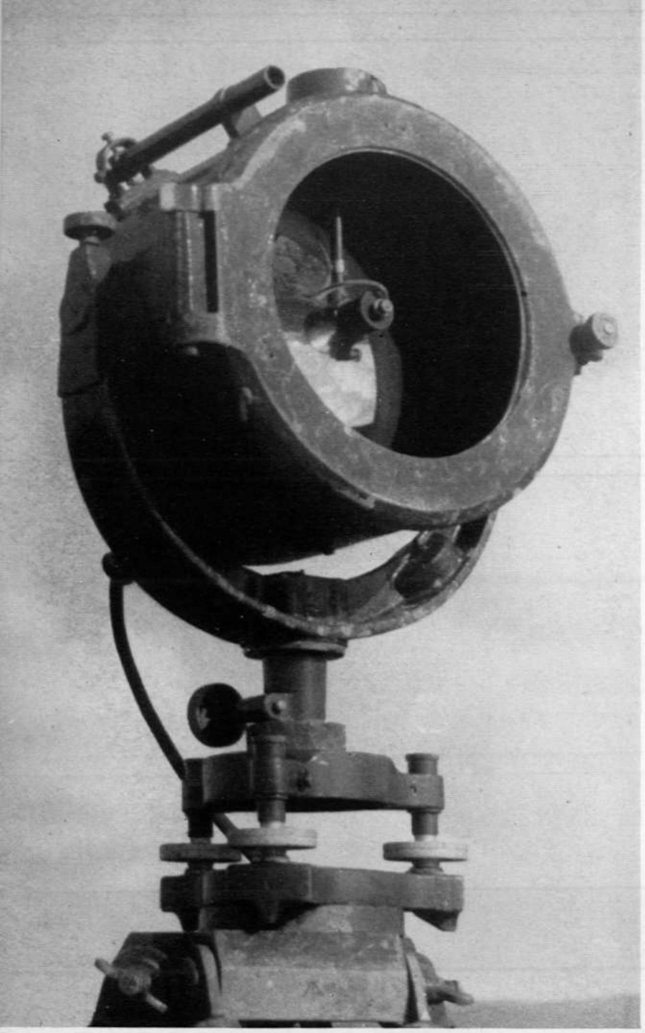
Nearing completion



Final plumbing of tribrachs over pillar

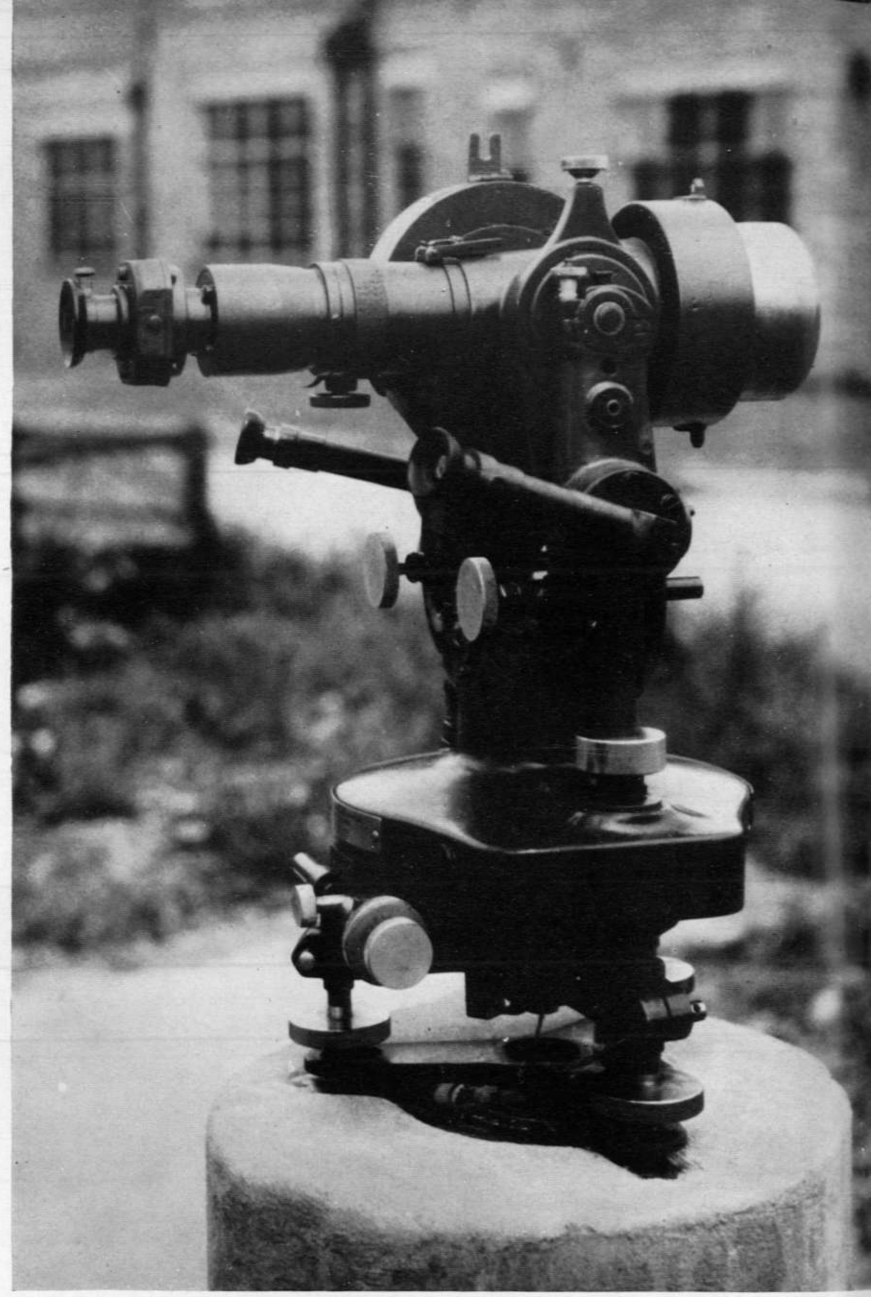


6. GEODETIC TAVISTOCK THEODOLITES; AND BEACONS (see § 2.080, 2.081)



Beacon lamp

Heliotrope, with duplex mirror in use, and semaphore board



Original model

Later model

G. F. Mullinger (observer) A. C. Wilde (observer) AT TURIFF, 1937 Major M. Hotine, RE (officer-in-charge) A. R. Martin (observer)

7. PRIMARY RETRIANGULATION 1936-37 (see §2.11, 2.12)

A. R. Martin
at Gwynydd
Bach 1936

The original
(1847)
O.S. hut on
Ben Macdhui
1936

Lightkeeping
at Wuddy Law
1937

Transport for
pillar stores at
Radnor Forest
1936

Transport lined up at the start of the season, at the Ordnance Survey Office, Southampton

8. PRIMARY RETRIANGULATION 1938 (see § 2.13)

Boston Stump, on which was erected two sections of a Bilby tower to clear the pinnacles

Practising tower erection, Southampton

R. J. Stone, lightkeeping at Dunstable Down

F. L. Harris observing at South Barrule, 1951

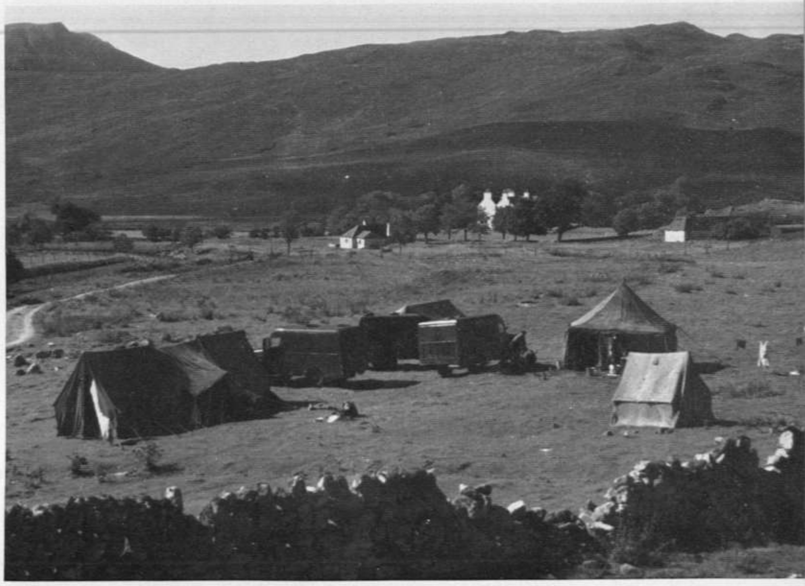
B. Willis (observer) planning his programme at Glencoe*

*Photograph with acknowledgements to A. D. S. McPherson

9. PRIMARY RETRIANGULATION 1949-51 (see § 2.15, 2.16, 2.17)

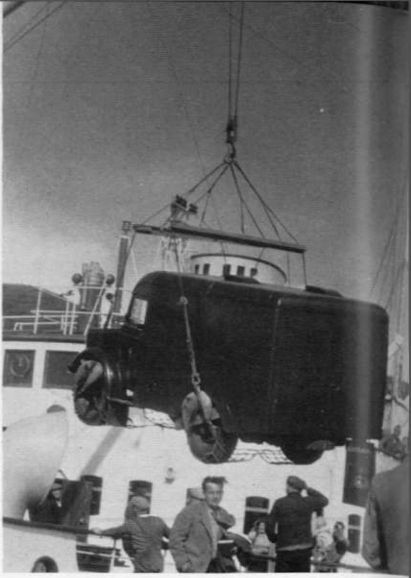
below, top Embarking at Walls, Shetland, for Foula, 1949
below, bottom Approaching Foula, 1949

below, top Camp-site at Inchnadamph, near Conival, 1949
below, bottom Yell, Shetland, 1950



Kinlochquoich, the debussing point and camp-site, five miles from Sgurr na Ciche

On South Barrule summit



Embarking for the Outer Hebrides

10a. PRIMARY RETRIANGULATION 1951 (see § 2.17)

Mheall Mhor; debussing point and camp-site for Sliabh Gaoil

Approaching Garnedd Ugain, Snowdonia, first day of the season

10b. ST KILDA PRIMARY CONNECTION 1957 (see § 2.33)

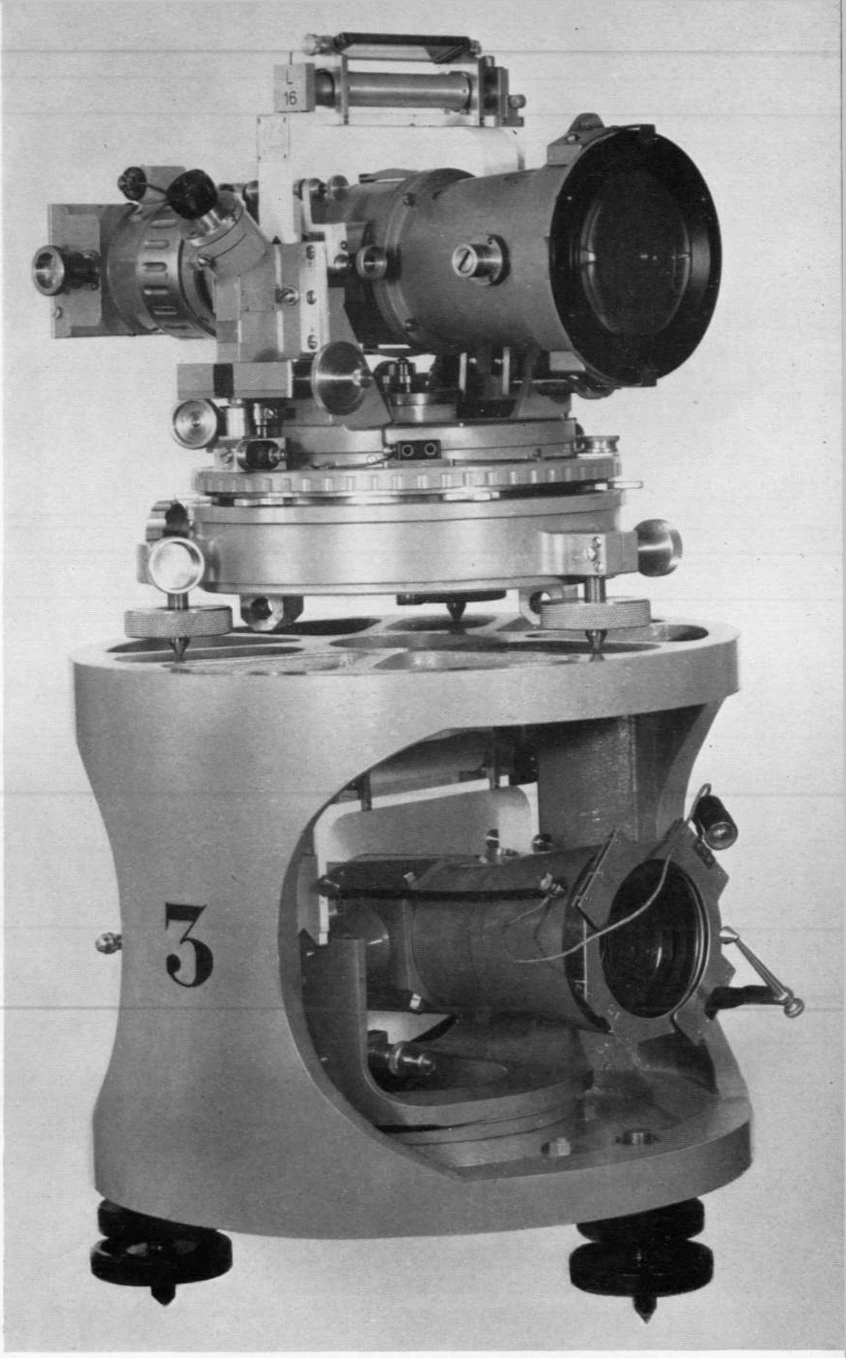
Climbing Conachair, St Kilda*

H. J. W. Smith observing*

**Photographs with acknowledgements to Tom Weir*

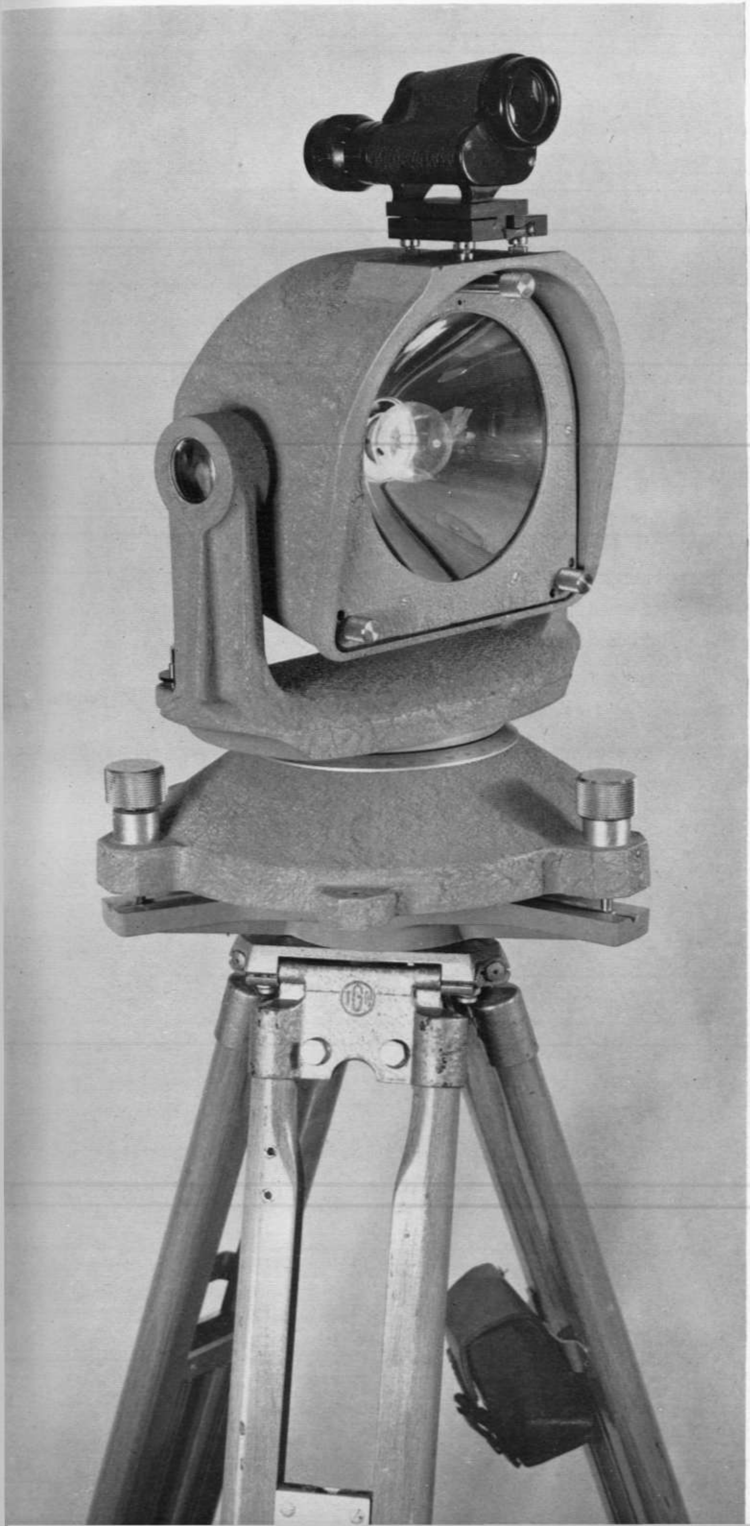
11. THE CONNECTION WITH FRANCE

(see § 3.02)



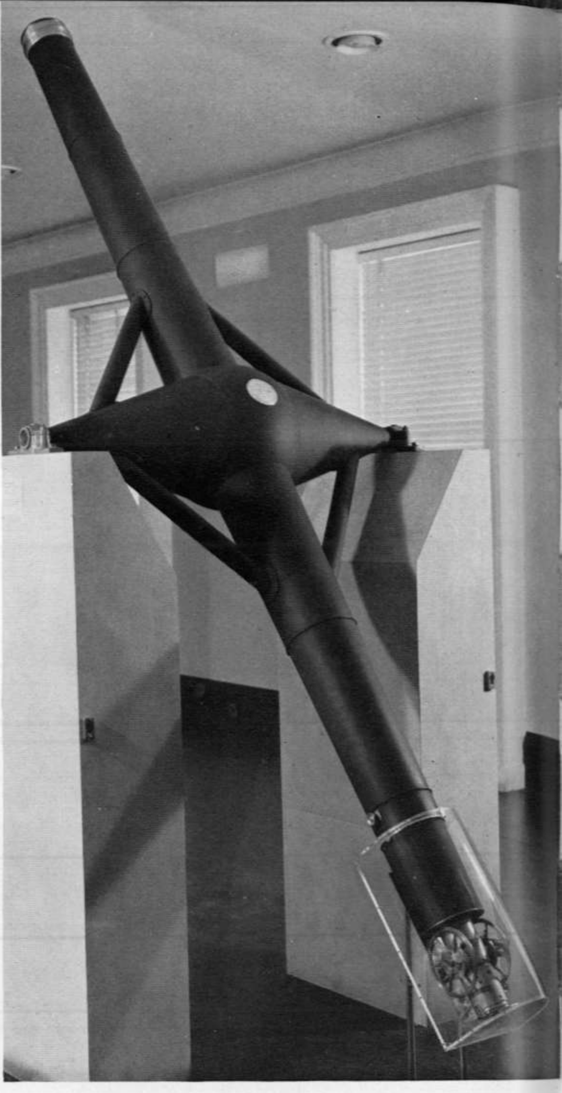
The 'Cercle Azimuthal Répétiteur' – the French geodetic theodolite. The lower telescope is for the measurement of torsion, and is aligned on the referring object at each measurement

A French beacon lamp



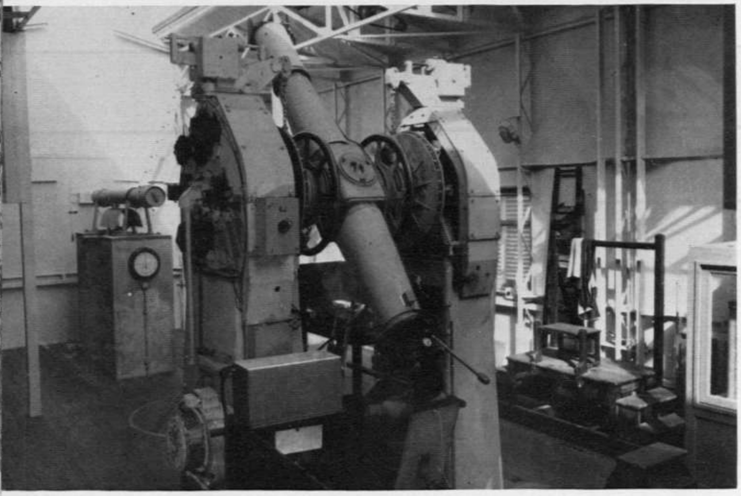
The station at Beachy Head

The Airy Transit Circle is housed in the left-hand building, behind the white panelled woodwork. The Pond Transit Instrument (now in the National Maritime Museum) was in the right-hand building beneath the glassed-in portion



The Pond Transit Instrument*

**Photograph with acknowledgements to National Maritime Museum*

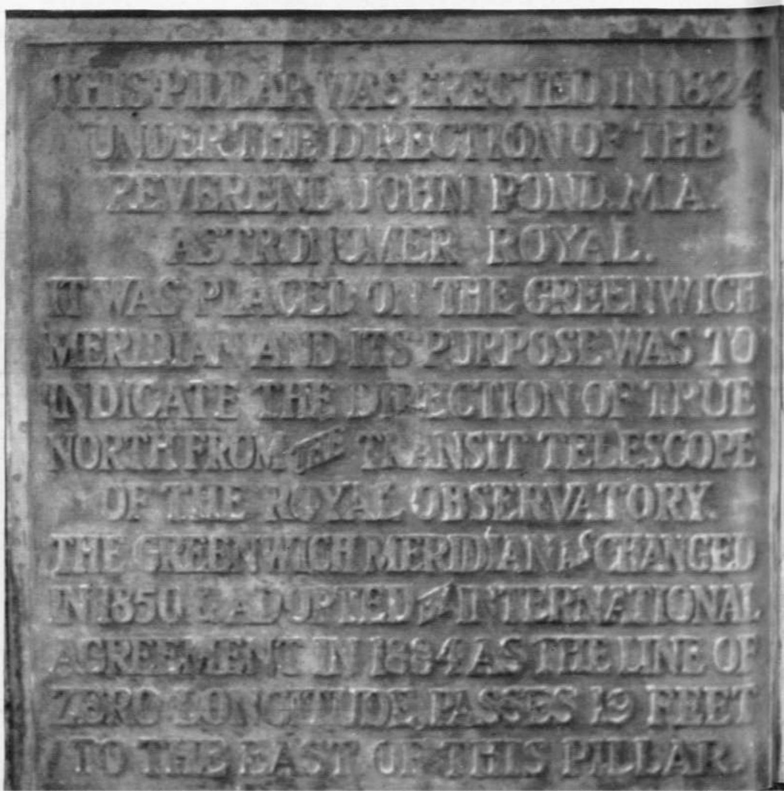


The Airy Transit Circle

12. THE ROYAL OBSERVATORY, GREENWICH
(see § 3.060 to 3.066)

below, left The obelisk and pillar at Pole Hill, Chingford

below, right The inscription on the obelisk plaque



THIS PILLAR WAS ERECTED IN 1824
UNDER THE DIRECTION OF THE
REVEREND JOHN POND M.A.
ASTRONOMER ROYAL.
IT WAS PLACED ON THE GREENWICH
MERIDIAN AND ITS PURPOSE WAS TO
INDICATE THE DIRECTION OF TRUE
NORTH FROM THE TRANSIT TELESCOPE
OF THE ROYAL OBSERVATORY.
THE GREENWICH MERIDIAN AS CHANGED
IN 1850 & ADOPTED BY INTERNATIONAL
AGREEMENT IN 1884 AS THE LINE OF
ZERO LONGITUDE, PASSES 19 FEET
TO THE EAST OF THIS PILLAR.

Exterior view of protecting hut, from south

13. PEVENSEY AZIMUTH MARK (see § 3.100)

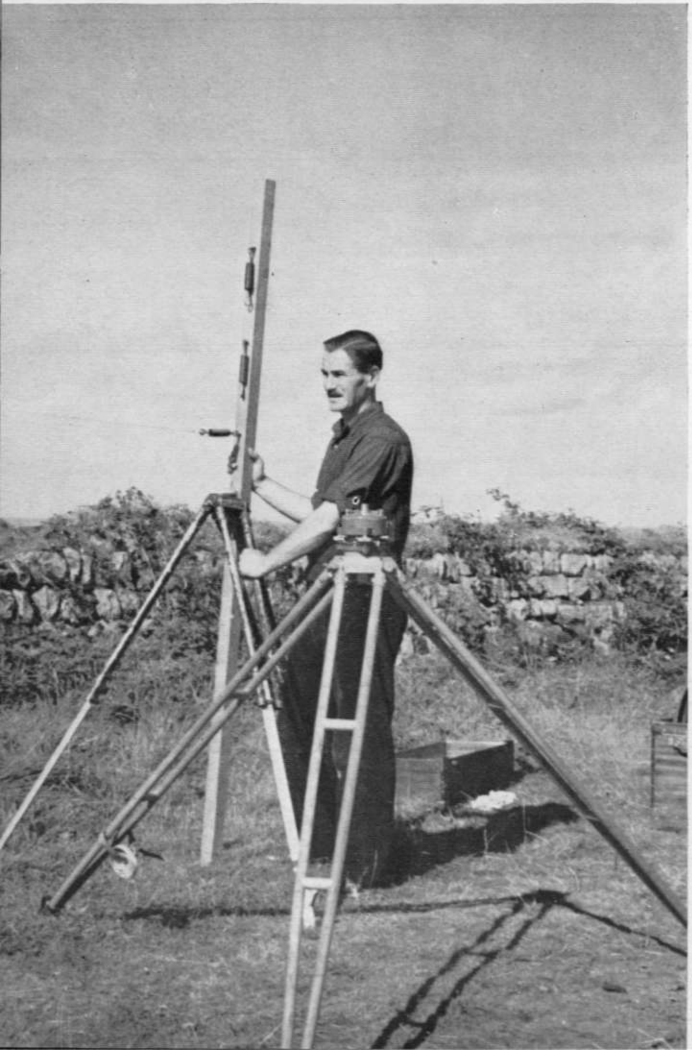
Artist's impression showing interior (9ft. square) from north. The concrete pedestal is 8ft.6in. high. It contains a flush-bracket, and a tribrach is set exactly over the illuminated azimuth mark

A. R. Martin reading

Dropping a
section mark

14. RIDGEWAY BASE 1937

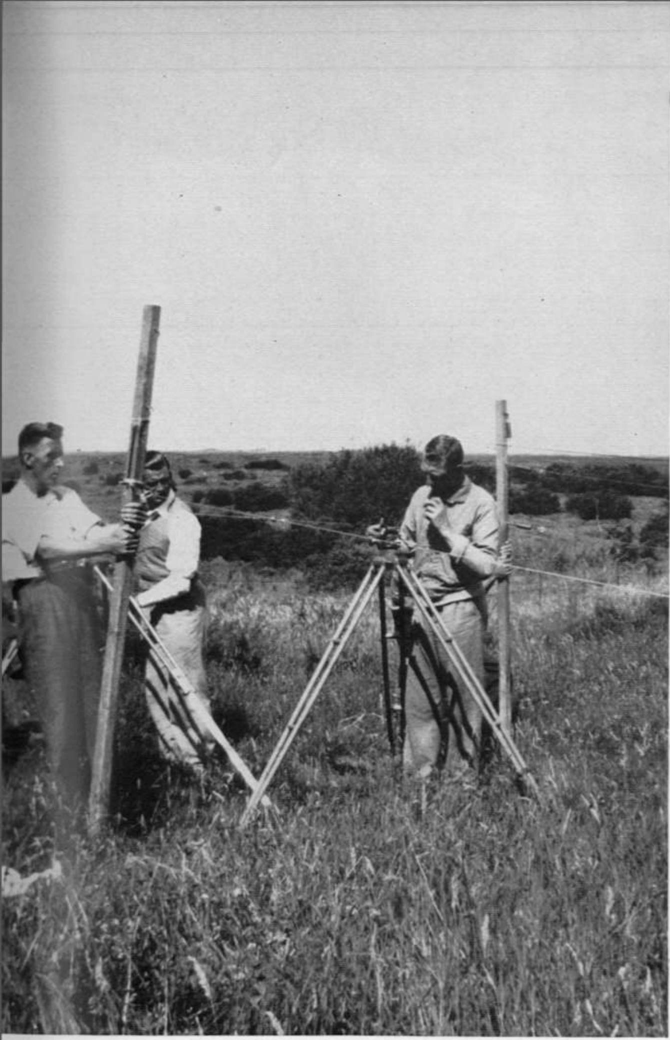
(see § 4.01 to 4.06)



T. B. Edwards,
booker on
this base and
Lossiemouth

Negotiating the slope on the side of the ravine

Aligning party; R. Pauling at the instrument, assisted by N. T. Forster



Standardising

15. LOSSIEMOUTH BASE 1938

(see § 4.07 to 4.11)

A difficult set-up for
the levellers at the
western terminal –
a pillar erected for the
1910 measurement

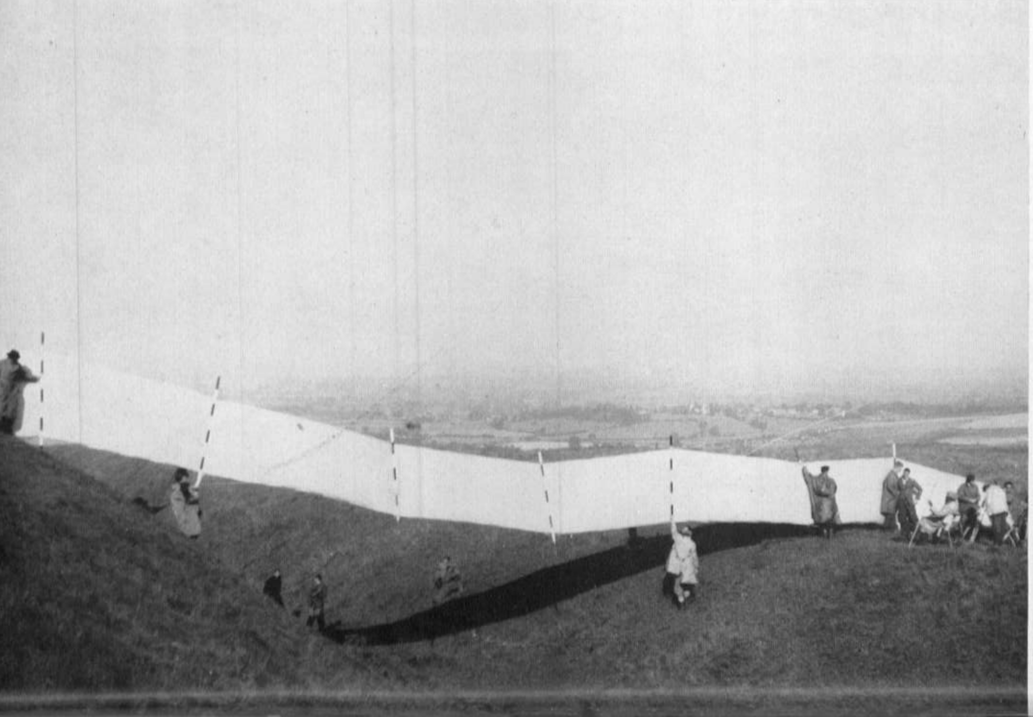
Crossing the River Lossie. A. C. Wilde is holding the tape. W. Stuart, then superintendent (who also took part in the 1910 measurement) is on the right

Near Liddington Castle, looking east

F. G. Bellamy reading at Liddington Castle

16. RIDGEWAY BASE 1951 (see § 4.12 to 4.16)

Crossing the ditch at Liddington Castle



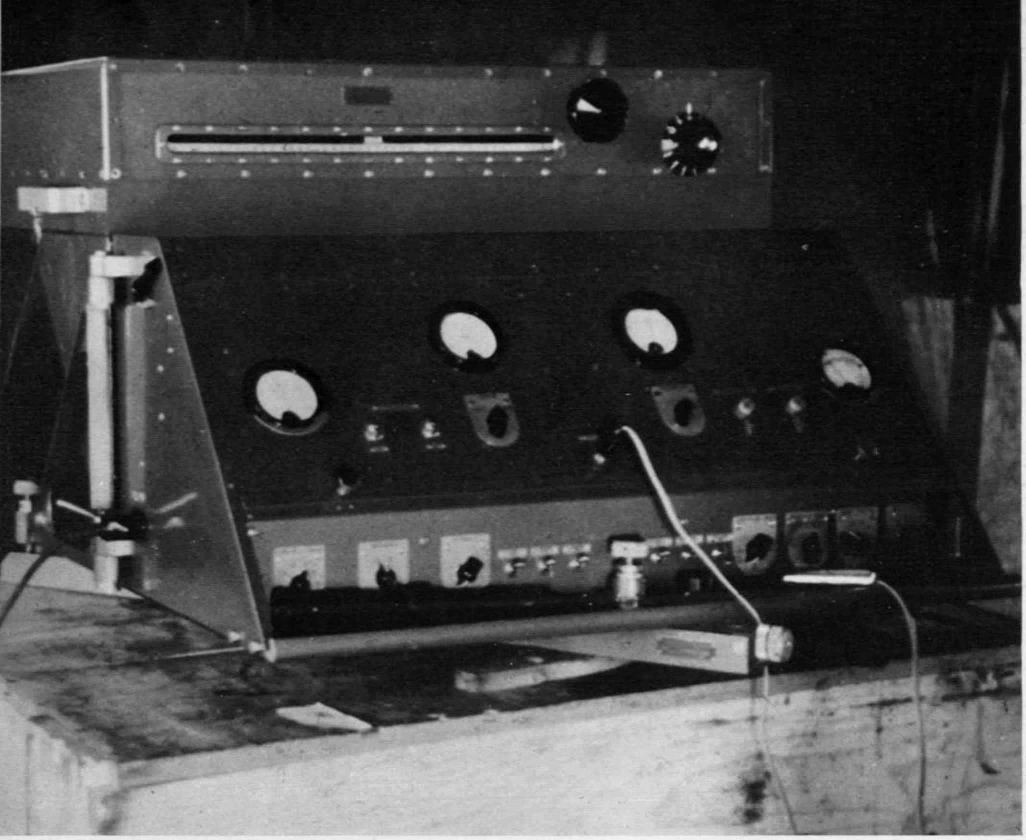
The ravine

Long and short levelling staves in use

17. CAITHNESS BASE 1952 (see § 4.17, 4.18)

Screen party in difficulties while crossing burn

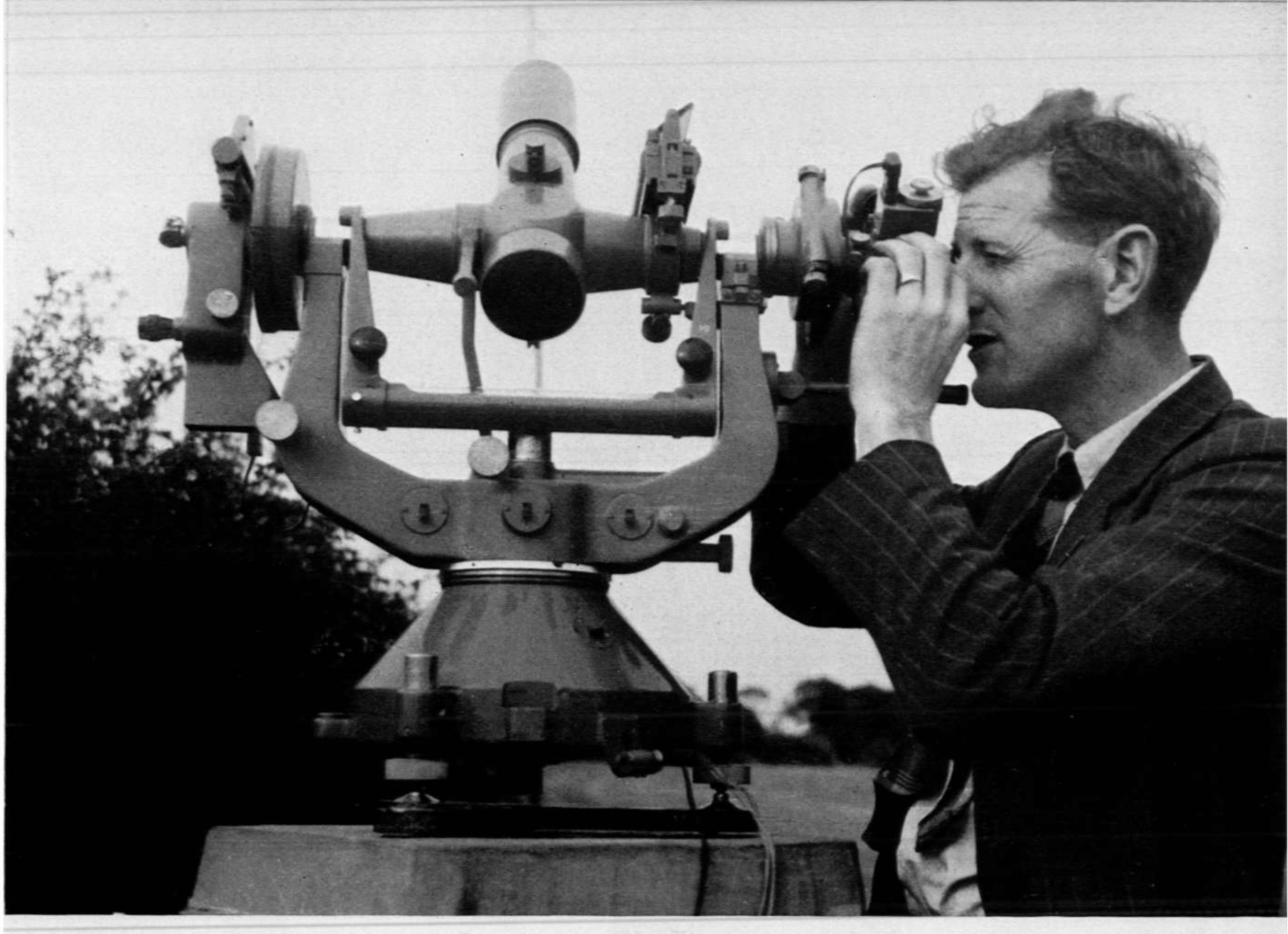
Crossing the Burn of Lyth just north of Hillhead Farm. 'A' frame in use



The Geodimeter and (*right*) prism reflector

18. THE GEODIMETER AND THE TELLUROMETER (see § 4.25 to 4.31)

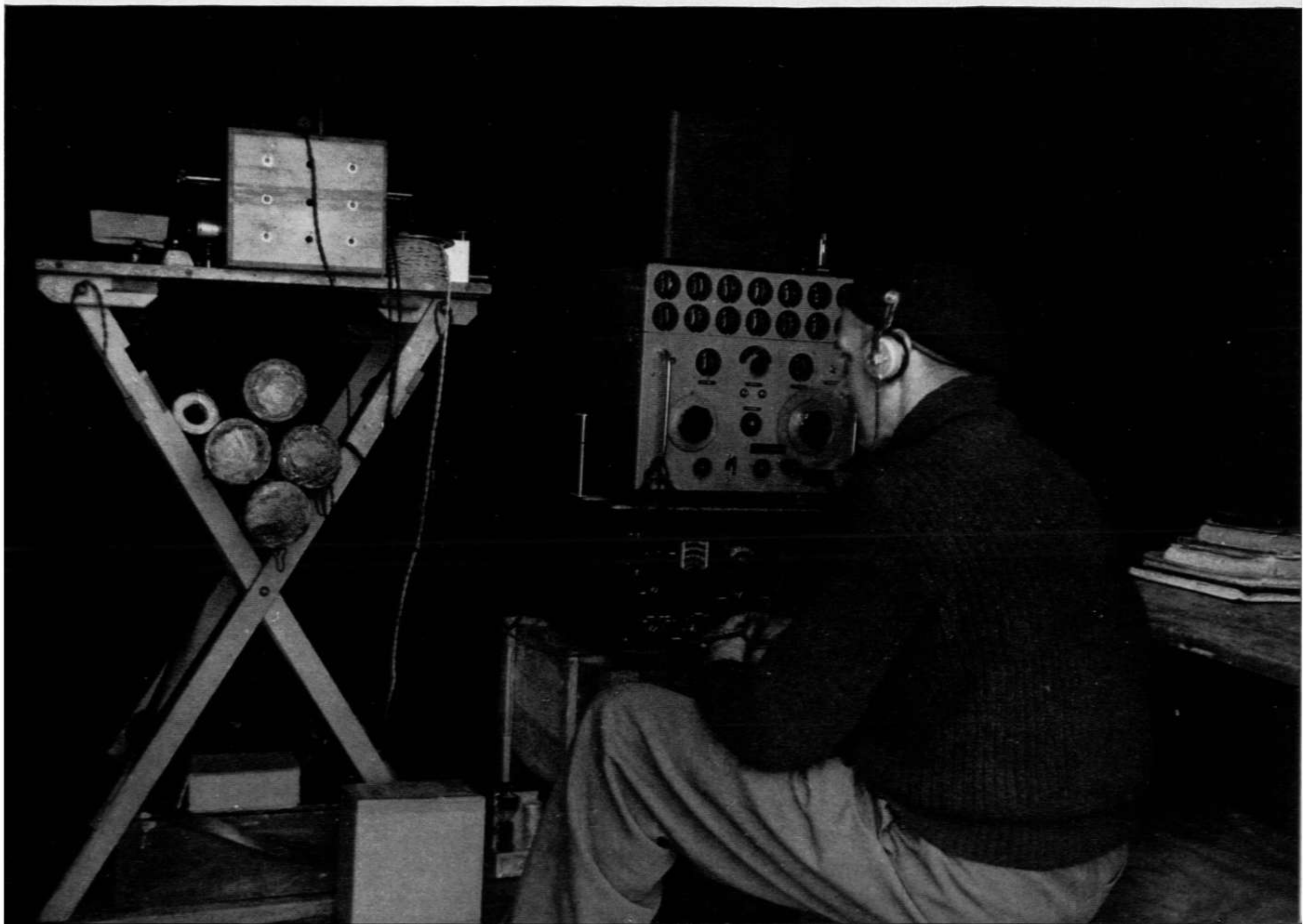
The Tellurometer in use in the Highlands

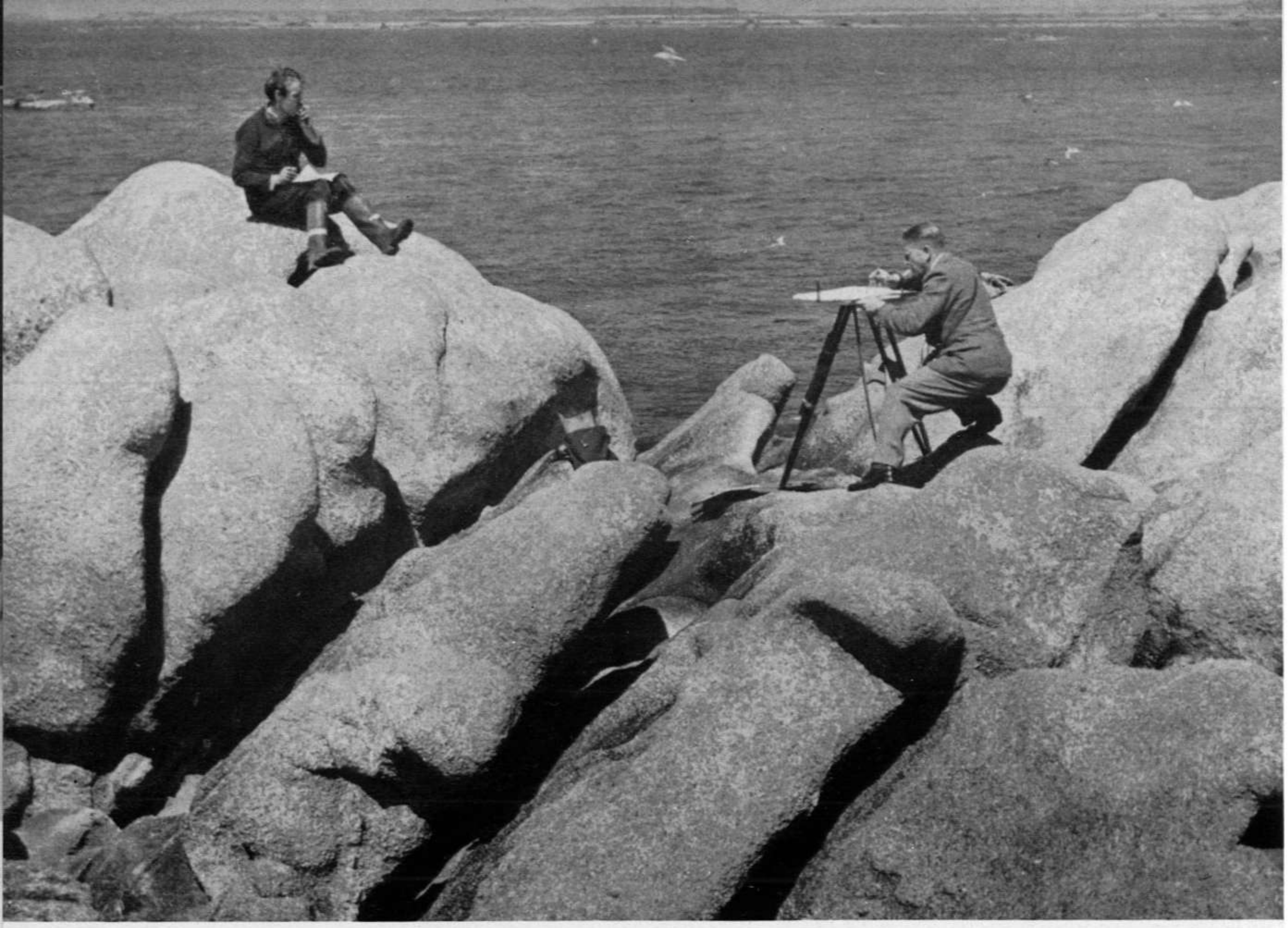


C. P. Clark (who took all the astronomic observations) with the Wild T4 theodolite

19. ASTRONOMIC EQUIPMENT (see § 5.030 to 5.033)

Receiving time-signals





E. Curry verifying 'rays open' in the Isles of Scilly

20. SECONDARY AND TERTIARY RECONNAISSANCE (see § 7.02)

Marking sites for witness marks (Leeds Town Control)

Caterpillar tractor and sledge

Man-haulage

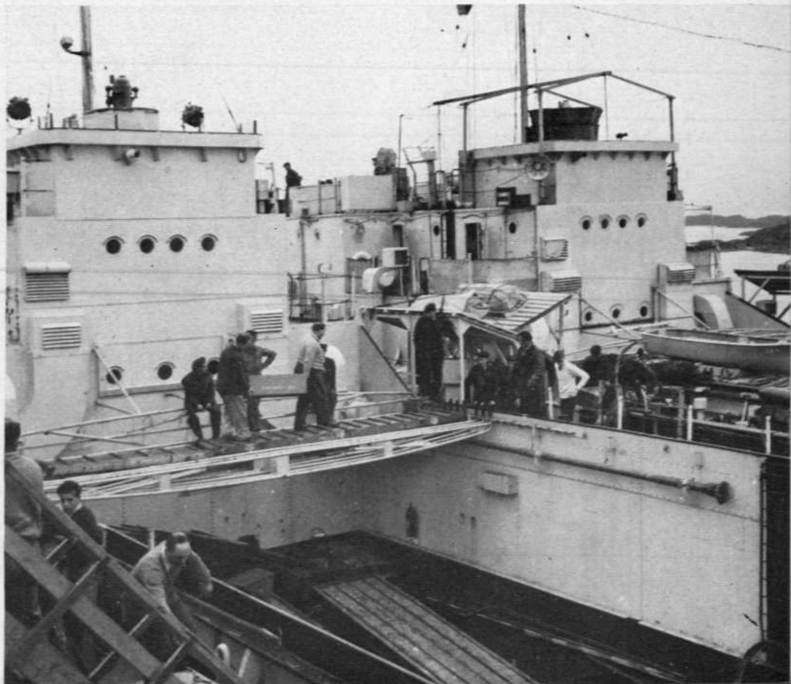
Carden-Loyd
carrier

**21.
SOME
OF THE
VARIOUS
FORMS OF
TRANSPORT
USED**
(see § 7.030)

Weasel
crossing
bog

Naval launch at Lochmaddy

Landing craft (tank) at Lochboisdale ; embarking for St Kilda



Major
J. Kelsey, RE
and a
completed
circular
pillar

22a.
CIRCULAR PILLARS
(see § 7.030, 7.031)

Shuttering,
centre-tube,
boxes and
some tools;
a one-man
load

22b. HELICOPTERS (see § 7.030)

Coming in with a load; pillar construction in progress (Wester Ross)

Hovering for attachment of load (Isle of Harris)

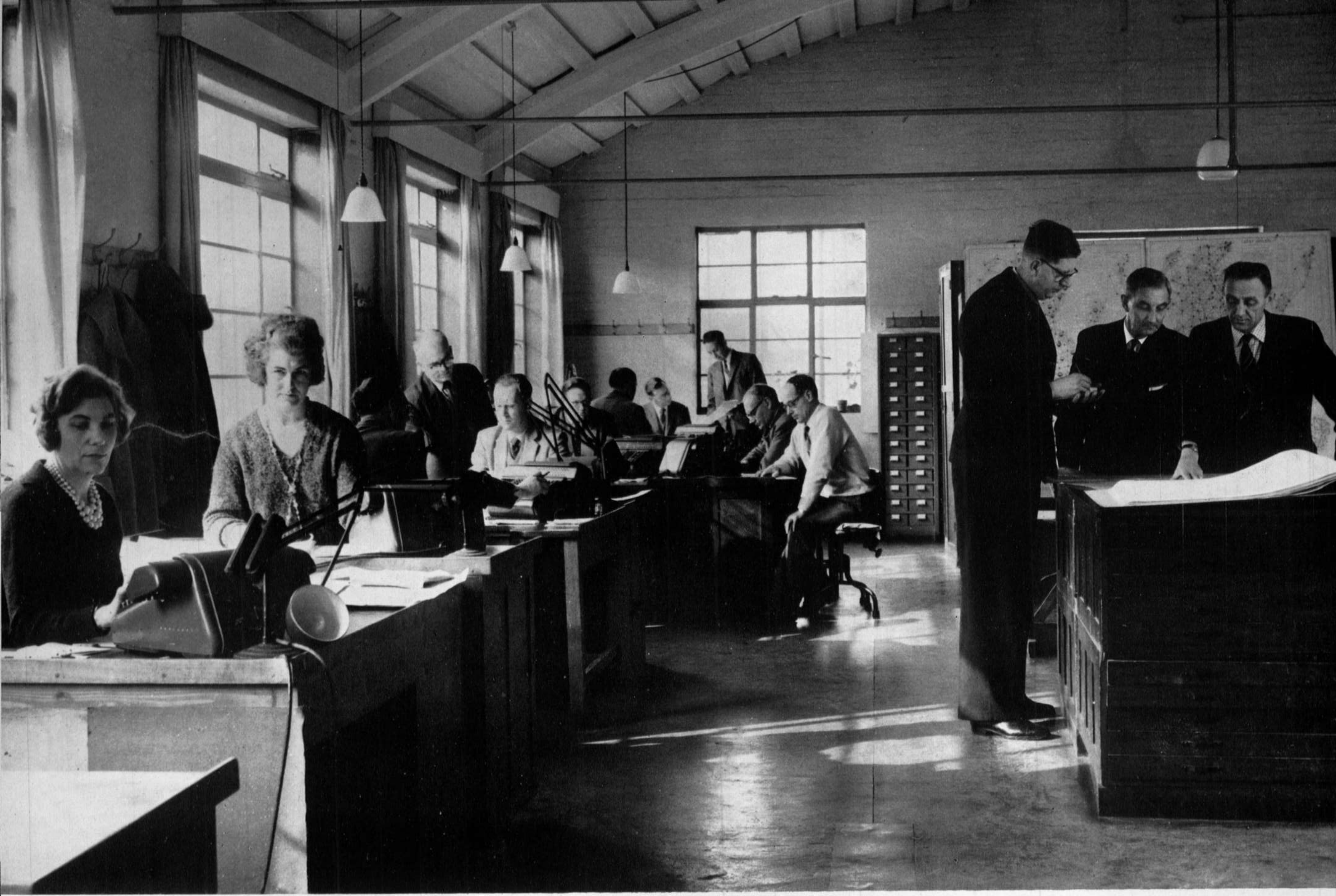
23. DAMAGED PILLARS (see § 7.04)

Maes Knoll, a secondary station
near Bristol,
wantonly damaged twice

Ben Cruachan (primary);
frost damage

Wilford Brickworks (tertiary) near
Nottingham; an unsuccessful attempt
to remove the tribrach

Bwrdd Arthur (secondary)
Anglesey; an attempt at
destruction with explosives



24. COMPUTATIONS (see § 7.07)

Some of the computers. A photograph taken early in 1962, showing the few remaining staff at the end of the Retriangulation. R. G. Curtis, who assisted, and then succeeded E. T. Bateman as superintendent in 1951, is in the centre of the group of three on the extreme right. J. K. Holt, the senior computer, is standing on the left, by the window

TELLUROMETER MEASUREMENTS

4.28 Introduction

The Tellurometer is an instrument for determining the distance between points by measuring the transit time, outward and return, of radio micro-waves emitted from a master station situated at one point and retransmitted from a remote station at the other. It has been fully described elsewhere, notably in a series of articles by the inventor, T. L. Wadley, in the *Empire Survey Review*(33) and in the *Transactions of the South African Institute of Electrical Engineers*(34). It has also been fully investigated by Special Study Group No. 19 of the International Association of Geodesy which published its conclusions regarding the accuracy of the instrument and recommendations as to its use for geodetic purposes in a report to the Association at the General Assembly of the International Union of Geodesy and Geophysics at Helsinki in 1960(43).

The instrument was first demonstrated in the United Kingdom in April 1957 during trials on the Ridgeway Base conducted jointly by the South African Council for Scientific and Industrial Research and the Ordnance Survey. During the course of these trials the Ridgeway Base itself was remeasured as well as the base extension figure. These measurements established the accuracy of the instrument and incidentally enabled a revised value for the velocity of radio waves *in vacuo* to be determined (see § 4.31). The results were reported in a paper presented to the International Association of Geodesy at the 1957 General Assembly of the International Union of Geodesy and Geophysics at Toronto(35).

4.29 Primary Scale Checks

During the remainder of 1957 and 1958 the Tellurometer was used on various trials in connection with second and lower order work, but whenever opportunity occurred it was employed to check the lengths of sides of the primary triangulation. These checks normally perforce consisted of single measurements, and were mostly made before the procedure for geodetic measurements now recommended by the International Association of Geodesy(43) had been evolved. Normally a measurement consisted of 30–36 sets of fine readings taken right across the scale together with the normal coarse readings. Each set of fine readings consisted of the usual four 'A' pattern readings. If the total range of the fine readings, or 'ground swing', exceeded 10 mμsec further readings were taken, usually from a different instrument position. Meteorological measurements to determine the refractive index were taken at the beginning and end of observations at each terminal, but no special precautions were taken to ensure that these were truly representative of the mean conditions for the line as a whole (see § 4.302 below). Between April 1957 and December 1961 68 primary sides were measured the results being recorded in Table 4.7.

TABLE 4.7

SIDES OF PRIMARY RETRIANGULATION MEASURED BY TELLUROMETER: 1957–61

All measurements in metres: Velocity 'in vacuo' 299 792·5 Kms/sec.

	Comparison of Tellurometer Lengths with Primary Retriangulation Lengths									
Serial No.	*From*	*To*	*No. of Measures*	*No. of Observing Days*	*Spheroidal Length*		*Triangulation minus Tellurometer*	*Standard Error of a Single Tellurometer Measurement*	*Standard Error of the Mean Tellurometer Measurement*	*Parts per Million*
					Triangulation	*Tellurometer*				
1*	White Horse Hill (34)	Liddington Castle (35)	*	3	11 260·266	11 260·178	+0·088			8
2*	White Horse Hill (34)	Inkpen (33)	*	1	25 803·785	25 803·496	+0·289			11
3*	White Horse Hill (34)	Martinsell (68)	*	1	25 633·888	25 633·739	+0·149			6
4*	White Horse Hill (34)	Cleeve Hill (69)	*	1	48 846·649	48 846·311	+0·338			7
5*	Liddington Castle (35)	Inkpen (33)	*	1	24 431·019	24 430·813	+0·206			8
6*	Liddington Castle (35)	Martinsell (68)	*	1	16 200·209	16 200·151	+0·058			4
7*	Liddington Castle (35)	Cleeve Hill (69)	*	1	49 656·183	49 655·842	+0·341			7
8*	Inkpen (33)	Martinsell (68)	*	1	19 645·353	19 645·268	+0·085			4
9*	Inkpen (33)	Cleeve Hill (69)	*	1	73 395·253	73 395·086	+0·167			2
10*	Martinsell (68)	Cleeve Hill (69)	*	1	63 402·702	63 402·474	+0·228			4
11*	Toney Wood	Cleeve Hill (69)	*	1	32 072·340	32 072·371	−0·031			1
12*	Toney Wood	Martinsell (68)	*	1	52 838·286	52 837·970	+0·316			6
13*	Toney Wood	Liddington Castle (35)	*	1	46 381·724	46 381·616	+0·108			2
14*	Toney Wood	White Horse Hill (34)	*	1	52 644·504	52 644·175	+0·329			6
15*	Malvern (79)	Cleeve Hill (69)	*	1	30 765·815	30 765·278	+0·537			17
16*	Malvern (79)	Peglers Tump (88)	*	1	45 254·134	45 253·913	+0·221			5
17	Malvern (79)	Broadway Tower (91)	6	4	35 655·547	35 655·243	+0·304	±0·113	±0·046	9
18	Leith Hill Twr (50)	Hindhead (31)	5	3	25 044·709	25 044·513	+0·196	±0·078	±0·035	8
19	Leith Hill Twr (50)	Bignor Beacon (39)	3	2	34 705·372	34 705·184	+0·188	±0·026	±0·015	5
20	Caister Wtr Twr (293)	N. Walsham Wtr Twr (283)	1	1	28 486·789	28 486·521	+0·268			9
21	Caister Wtr Twr (293)	Framingham (261)	1	1	27 277·487	27 276·999	+0·488			18
22	N. Walsham Wtr Twr (283)	Framingham (261)	1	1	26 596·497	26 596·346	+0·151			6
23	N. Walsham Wtr Twr (283)	Piggs Grave (263)	1	1	25 473·092	25 472·905	+0·187			7
24	Framingham (261)	Piggs Grave (263)	1	1	38 431·848	38 431·403	+0·445			12
25	Rombalds Moor (70)	Great Whernside (7)	1	1	30 826·733	30 825·904	+0·829			27
26	Rombalds Moor (70)	Boulsworth (16)	1	1	20 842·486	20 842·174	+0·312			15
27	Rottington (1)	Black Combe (2)	6	2	33 379·021	33 378·276	+0·745	±0·043	±0·018	22
28	Whitelyne Common (93)	Wisp Hill (317)	1	1	28 320·589	28 319·487	+1·102			39
29	Whitelyne Common (93)	Tosson Hill (95)	1	1	43 922·070	43 920·781	+1·289			29
30	Cairnsmore of Fleet (343)	Criffell (96)	1	1	45 874·056	45 872·453	+1·603			35
31	Cairnsmore of Fleet (343)	Cairn Pat (360)	1	1	46 971·091	46 968·982	+2·109			45
32	Cairnsmore of Fleet (343)	Carleton Fell (362)	2	2	30 822·489	30 820·926	+1·563	±0·111	±0·078	51
33	Cairnsmore of Fleet (343)	Merrick (301)	1	1	19 903·082	19 902·284	+0·798			40
34	Inshanks (361)	Carleton Fell (362)	3	1	28 929·950	28 929·008	+0·942	±0·125	±0·072	33
35	Black Mount (352)	Tinto (318)	4	2	17 213·074	17 212·659	+0·415	±0·049	±0·025	24
36	Black Mount (352)	Hart Fell (320)	2	1	32 579·261	32 578·171	+1·090	±0·016	±0·011	33
37	Tinto (318)	Hart Fell (320)	2	1	26 299·818	26 299·170	+0·648	±0·019	±0·014	25
38	Creach Bheinn (372)	Ben Nevis (323)	1	1	32 600·162	32 599·194	+0·968			30
39	Creach Bheinn (372)	Sgurr na Ciche (371)	1	1	39 155·929	39 154·716	+1·213			31
40	Beinn Bhreac Mhor (356)	Carn an Fhreiceadain (331)	1	1	13 589·609	13 589·278	+0·331			24
41	Ben Macdhui (302)	Carn Gower (332)	3	2	25 821·027	25 820·240	+0·787	±0·069	±0·040	30
42	Ben Macdhui (302)	Beinn Bhreac Mhor (356)	2	2	37 482·806	37 481·970	+0·836	±0·012	±0·009	22
43(a)*	Warth Hill (399)	Spital Hill (398)	3	3	24 828·423	24 828·399	+0·024	±0·233	±0·135	1
43(b)	Warth Hill (399)	Spital Hill (398)	6	5	24 828·423	24 828·045	+0·378	±0·060	±0·024	15

TABLE 4.7 *continued*

SIDES OF PRIMARY RETRIANGULATION MEASURED BY TELLUROMETER: 1957–61

All measurements in metres: Velocity 'in vacuo' 299 792·5 Kms/sec.

	Comparison of Tellurometer Lengths with Primary Retriangulation Lengths									
Serial No.	*From*	*To*	*No. of Measures*	*No. of Observing Days*	*Spheroidal Length*		*Triangulation minus Tellurometer*	*Standard Error of a Single Tellurometer Measurement*	*Standard Error of the Mean Tellurometer Measurement*	*Parts per Million*
					Triangulation	*Tellurometer*				
44(a)*	Warth Hill (399)	Hillhead Farm (478)	3	2	11 371·805	11 371·644	+0·161	±0·032	±0·019	14
44(b)	Warth Hill (399)	Hillhead Farm (478)	4	4	11 371·805	11 371·671	+0·134	±0·030	±0·015	12
45(a)*	Spital Hill (398)	Hillhead Farm (478)	3	2	13 456·618	13 456·419	+0·199	±0·041	±0·023	15
45(b)	Spital Hill (398)	Hillhead Farm (478)	4	4	13 456·618	13 456·465	+0·153	±0·042	±0·021	11
46	Dunnet Head (388)	Spital Hill (398)	2	2	21 210·035	21 209·693	+0·342	±0·027	±0·019	16
47	Dunnet Head (388)	Hill of Yarrows (391)	3	2	34 929·070	34 928·393	+0·677	±0·060	±0·035	19
48	Warth Hill (399)	Hill of Yarrows (391)	2	2	28 097·650	28 097·232	+0·418	±0·052	±0·037	15
49	Hill of Yarrows (391)	Spital Hill (398)	4	2	18 165·605	18 165·304	+0·301	±0·038	±0·019	17
50	Hill of Yarrows (391)	Hillhead Farm (478)	1	1	20 639·998	20 639·597	+0·401			19
51	Dunnet Head (388)	Hillhead Farm (478)	1	1	15 043·066	15 042·855	+0·211			14
52	Warth Hill (399)	Dunnet Head (388)	2	2	17 880·686	17 880·482	+0·204	±0·042	±0·030	11
53	Hockley Wtr Twr (220)	Abberton Wtr Twr (230)	8	2	32 261·814	32 261·571	+0·243	±0·034	±0·012	8
54	Warley Wtr Twr (224)	Chipping Barnet Ch Twr (185)	4	2	34 920·326	34 919·776	+0·550	±0·030	±0·015	16
55	St. Agnes Beacon (175)	Hensbarrow (174)	4	1	29 585·026	29 584·644	+0·382	±0·025	±0·012	13
56	Tregonning Hill (181)	St. Agnes Beacon (175)	2	1	23 014·631	23 014·422	+0·209	±0·065	±0·046	9
57	Tregonning Hill (181)	Trendrine Hill (178)	4	2	14 865·927	14 865·922	+0·005	±0·038	±0·019	0
58	Carnmenellis (177)	Trendrine Hill (178)	4	2	21 798·629	21 798·426	+0·203	±0·108	±0·054	9
59	Carnmenellis (177)	Bartinney (180)	2	1	30 928·001	30 927·718	+0·283	±0·011	±0·008	9
60	Carnmenellis (177)	Tregonning Hill (181)	4	2	11 562·501	11 562·401	+0·100	±0·039	±0·019	9
61	Bartinney (180)	Trendrine Hill (178)	4	2	12 642·310	12 642·226	+0·084	±0·022	±0·011	7
62	Bartinney (180)	Tregonning Hill (181)	7	3	20 480·862	20 480·841	+0·021	±0·075	±0·028	1
63	Pendine (149)	Prescelly (107)	4	3	25 542·652	25 542·389	+0·263	±0·052	±0·026	10
64	Prescelly (107)	Capel Cynon (114)	4	2	33 328·029	33 327·850	+0·179	±0·107	±0·054	5
65	Ronas Hill (462)	Yell (467)	4	2	19 631·476	19 631·270	+0·206	±0·030	±0·015	10
66	Ronas Hill (462)	Saxavord (463)	6	3	46 495·844	46 495·450	+0·394	±0·084	±0·034	8
67	Yell (467)	Saxavord (463)	5	2	34 126·954	34 126·835	+0·119	±0·048	±0·022	3
68	Fetlar (459)	Saxavord (463)	8	2	23 127·129	23 127·010	+0·119	±0·076	±0·027	5

* Measurements made by Mr. T. L. Wadley of the National Telecommunications Research Laboratory, South African Council for Scientific and Industrial Research in original trials. Details of individual measurements not known.

Serial Nos. 11, 12, 13, 14 Toney Wood is an auxiliary station to Peglers Tump.

Serial No. 34 Very poor line for measuring.

Serial No. 43(a) Doubtful measure.

Area (see Diagram 16)	*Mean Scale Error of Area*	*Area (see Diagram* 16)	*Mean Scale Error of Area*
Serial Nos. 1–17	6 p.p.m.	Serial Nos. 35–37	27 p.p.m.
Serial Nos. 18–19	6 p.p.m.	Serial Nos. 38–39	30 p.p.m.
Serial Nos. 20–24	10 p.p.m.	Serial Nos. 40–42	25 p.p.m.
Serial Nos. 25–26	21 p.p.m.	Serial Nos. 43–52	14 p.p.m.
Serial Nos. 28–29	34 p.p.m.	Serial Nos. 55–62	7 p.p.m.
Serial Nos. 30–34	41 p.p.m.	Serial Nos. 63–64	8 p.p.m.
		Serial Nos. 65–68	7 p.p.m.

In each area the Retriangulation is too large by the stated p.p.m.

4.30 Investigations into Tellurometer Accuracy

4.300 CAITHNESS TRIALS

The results of Table 4.7 revealed apparent variations in the scale of the triangulation which were unexpectedly large, especially in south-west Scotland and the Border Country (serials 28 to 37 of Table 4.7) suggesting the possibility that the Tellurometer might not be maintaining the accuracy it had achieved in the area of the Ridgeway Base (serials 1 to 17). To verify this therefore, and in particular to see if there was appreciable systematic error, a triangulation figure centred on the Caithness Base was measured (see Fig. 4.12). This area was chosen as it did not provide particularly good observation conditions for the Tellurometer, ground reflectivity being high, and might therefore be expected to show up any tendency to error; and also because the scale of the figure used was closely controlled by the accurately measured Caithness Base.

Before comparison with the Tellurometer measurement of its sides, the triangulation figure was readjusted by least squares to the mean taped length of the Base. The comparisons are shown in Table 4.8. They indicate that no significant systematic error was present in the Tellurometer observations, the slight systematic difference between unadjusted Tellurometer measurements and triangulated lengths (about 2 p.p.m.) being insignificant having regard to the standard errors of the triangulation. These measurements are also included in Table 4.7 (serials 43(b), 44(b), 45(b), and 46 to 52) which gives an average standard error of about 2½ p.p.m. for a single Tellurometer measurement in this series. There is therefore nothing in these results to lead one to suppose that scale variations of the order of 10 to 20 p.p.m. or greater are ascribable to Tellurometer errors.

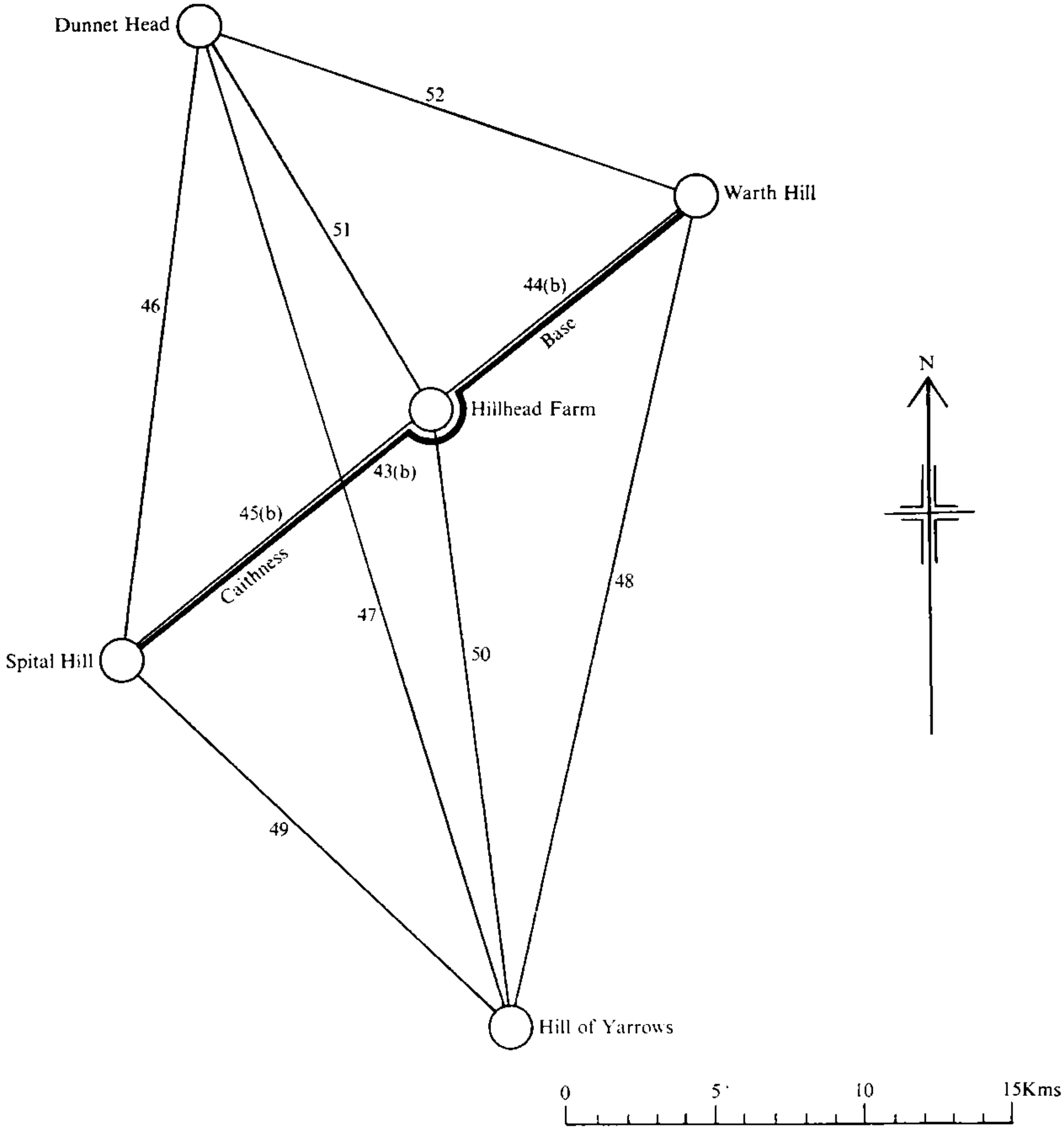
4.301 MEASUREMENTS IN KIRKCUDBRIGHTSHIRE

In the course of experimental work a second order block of control was provided by means of Tellurometer traverses in Kirkcudbrightshire where the greatest scale variations had been found. These traverses were themselves controlled by primary triangulation points within the block and by primary and secondary triangulation points at the edges. When the traverses were adjusted by least squares to fit this control the overall scale had to be expanded by 38 p.p.m. This provides confirmation of the results of the Tellurometer checks of primary sides in this area (serials 30 to 34 of Table 4.7) which give a mean scale error for the triangulation of +41 p.p.m.

4.302 PROBABLE ERRORS OF TELLUROMETER OBSERVATIONS

Single Tellurometer measurements, or multiple measurements taken within a short interval of each other on a single day, such as some of those in Table 4.7, are particularly liable to systematic errors due to inaccurate determination of refractive index. This is because any non-linear variation of the meteorological quantities (pressure, temperature, humidity) between the two terminals at which they are measured is liable to have persisted throughout the period of measurement. If the mean of the terminal measurements is not the mean for the line as a whole the resulting errors (for normal conditions) are as follows:

Error of Mean Met. Measurement		*Resulting Error of Length Measurement*
Pressure	1 mm. Hg	⅖ p.p.m.
Temperature	1°C	1⅓ p.p.m.
Humidity (Temperature dry minus wet bulb)	1°C	7 p.p.m.



Serial numbers refer to Table 4.7

FIG. 4.12. **Triangulation sides measured by Tellurometer in October 1958 in the Caithness Base area**

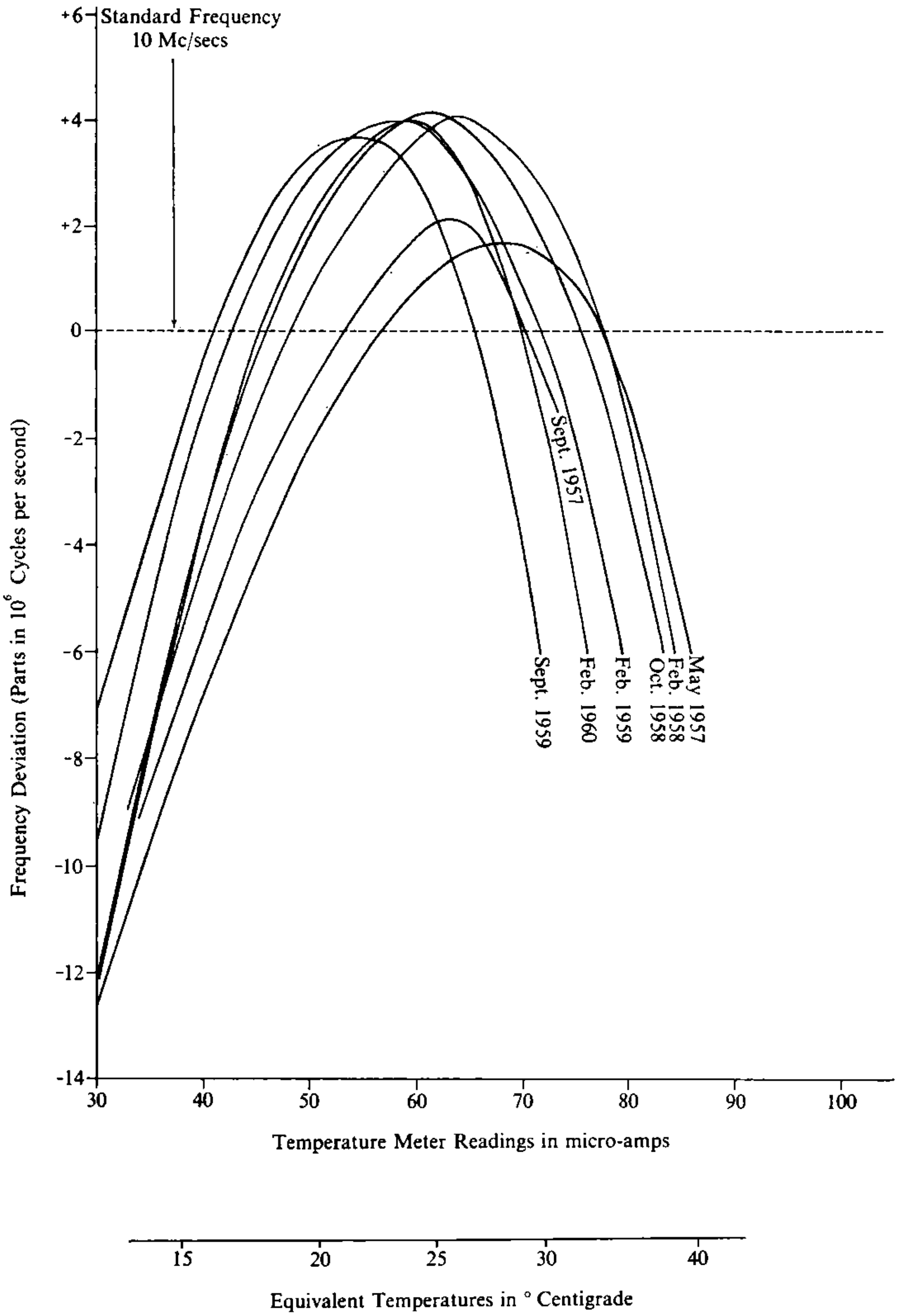


FIG. 4.13. Frequency–Temperature curves for the A crystal in Master No. MA 22

Special Study Group No. 19 of the International Association of Geodesy in its report⁽⁴³⁾ has estimated the total probable error of a single Tellurometer measurement as ±3·3 cm. ±4·1 p.p.m. Virtually the whole of the second quantity is due to refractive index errors. It is clear therefore that many of the individual length measurements of Table 4.7 are liable to errors of perhaps 5 to 10 p.p.m. However when the measurements are grouped as shown at the foot of the table and in Diagram 16, the scale derived from the group mean should be very much more accurate—probably within 5 p.p.m.

4.303 STABILITY OF MASTER CRYSTAL

The accuracy of Tellurometer measurements depends on the stability of the crystal which controls the operating frequency. This in turn depends on the accuracy of the temperature control of the crystal. Provided this is carried out correctly and provided the crystal frequency is calibrated at reasonably close intervals before and after measurement there is no reason why significant error should arise from this cause. For the measurements of Table 4.7 the master crystal (MA 22) was calibrated against standard frequencies at six-monthly intervals. The resulting frequency–temperature curves are shown at Fig. 4.13. The somewhat marked change in the curve between September 1957 and February 1958 was probably due to a change in the position of the thermistor which was moved to eliminate a slight lag in crystal temperature measurement when the heater was switched on. Apart from this a gradual drift of the frequency (about 1 p.p.m. every 6 months) may be noted. This was allowed for in all measurements and no significant error should therefore have resulted.

4.304 CONCLUSION

It may be concluded therefore that the errors of the Tellurometer scale checks are small compared with the apparent scale changes revealed, and that there is no evidence for any tendency to systematic error for the series as a whole. The scale changes are further discussed in Chapter 6.

THE FUNDAMENTAL CONSTANT

4.31 The Velocity of Electromagnetic Waves in vacuo

The value of C_0, the velocity of light and other electromagnetic waves *in vacuo*, is of fundamental importance when using instruments such as the Geodimeter and Tellurometer. The Ordnance Survey has been interested in measuring the lengths of its bases and triangle sides rather than in determining the value of this constant, but in the course of the fieldwork described in this Chapter observations have been made which have proved of considerable significance in the latter

TABLE 4.8

CAITHNESS BASE FIGURE: COMPARISON OF TELLUROMETER AND TRIANGULATION LENGTHS

All lengths in metres

Serial No.	*Line*		*Triangulation Spheroidal Lengths Scaled to Taped Base*	*Tellurometer Spheroidal Lengths*		*Differences Triangulation minus Adjusted Tellurometer*	*Diff. in p.p.m.*
	From	*To*		*Unadjusted*	*Adjusted*		
46	Dunnet Head (388)	Spital Hill (398)	21 209·649	21 209·693	21 209·656	−0·007	0
47	Dunnet Head (388)	Hill of Yarrows (391)	34 928·479	34 928·393	34 928·438	+0·041	1
51	Dunnet Head (388)	Hillhead Farm (478)	15 042·815	15 042·855	15 042·855	−0·040	3
48	Warth Hill (399)	Hill of Yarrows (391)	28 097·175	28 097·232	28 097·206	−0·031	1
44(b)	Warth Hill (399)	Hillhead Farm (478)	11 371·634	11 371·671	11 371·651	−0·017	1
45(b)	Hillhead Farm (478)	Spital Hill (398)	13 456·365	13 456·465	13 456·445	−0·080	6
50	Hillhead Farm (478)	Hill of Yarrows (391)	20 639·643	20 639·597	20 639·597	+0·046	2
43(b)	Warth Hill (399)	Spital Hill (398)	24 828·000	24 828·045	24 828·096	−0·096	4
	(Caithness	Base)					
49	Spital Hill (398)	Hill of Yarrows (391)	18 165·310	18 165·304	18 165·277	+0·033	2
52	Dunnet Head (388)	Warth Hill (399)	17 880·412	17 880·482	17 880·462	−0·050	3

Root mean square of the adjustment corrections = ±0·029 Average difference = 0·044

connection, and have had a material influence on the value adopted by the International Scientific Unions concerned. As a by-product of other work the following values for C_0 have been arrived at:

Serial No.	*Date*	*Location*	*Instrument Used*	*Value km/s*	*Standard Error*
1	July 1953	Ridgeway Base	Geodimeter	299,792·4	±0·5
2	Sept./Oct. 1953	Caithness Base	Geodimeter	299,792·2	±0·4
3	April 1957	Ridgeway Base	Tellurometer	299,792·6*	
4	April 1957	Ridgeway Base extension figure	Tellurometer	299,792·4*	
5	April 1957	Mean of 3 and 4 above	Tellurometer	299,792·5*	±0.3

* Revised values using the Essen-Froome formula for refractive index[47,48,49].

Of the above determinations serials 1 and 5 are probably the strongest. Serial 2 (Caithness Base) is less strong than Serial 1 (Ridgeway Base) in spite of its lower standard error because the accuracy of the taped length of the Ridgeway Base is probably superior to that of the Caithness

Base. It is of interest to note that both these determinations conform closely to the value now recommended by the International Union of Scientific Radio and the International Union of Geodesy and Geophysics in September 1957, viz.:

$$C_0 = 299{,}792{\cdot}5 \text{ km./s} \pm 0{\cdot}4.$$

CHAPTER FIVE

Geodetic Astronomy

5.00 Introduction

In order to check the orientation of the new primary net, observations for Laplace geodetic azimuths were made in 1953, the azimuths being observed as twins, that is, forward and back azimuths on each line. By observing the geodetic azimuths as twins it was hoped to minimize certain indeterminate errors such as those due to lateral refraction. Ultimately the back azimuth was transferred by the computed geodetic difference of azimuth, thus giving two values for the forward Laplace geodetic azimuth. It was envisaged that the Laplace azimuths would also be incorporated in any future re-adjustment of the primary net which might be made for scientific purposes.

Observations were also made at certain of the stations for astronomic latitude and longitude. It was anticipated that these results would be of use for any geoidal section work. Some of the position determinations were also needed in the Laplace azimuth programme for finding the observer's personal equation (see § 5.042(*g*) below).

Six lines of the primary net were selected for the twin Laplace azimuths; these were:

Herstmonceux (481) — Fairlight Down (193)
Liddington Castle (35) — White Horse Hill (34)*
Tregonning Hill (181) — St. Agnes Beacon (175)
Inshanks (361) — Cairn Pat (360)
Spital Hill (398) — Warth Hill (399)†
Saxavord (463) — Fetlar (459)
* = Ridgeway Base   † = Caithness Base

Fig. 5.1 shows the distribution of the Laplace stations.

Astronomic latitude and longitude were determined at the following seven primary stations,

Herstmonceux (481)
White Horse Hill (34)
St. Agnes Beacon (175)
Cairn Pat (360)
Spital Hill (398)
Warth Hill (399)
Fetlar (459)

with longitude only at an eighth—Fairlight Down (193).

In addition, in order to find the personal equation in longitude of the observer, astronomic latitude and longitude were determined at the Royal Observatory, Greenwich, the observing station

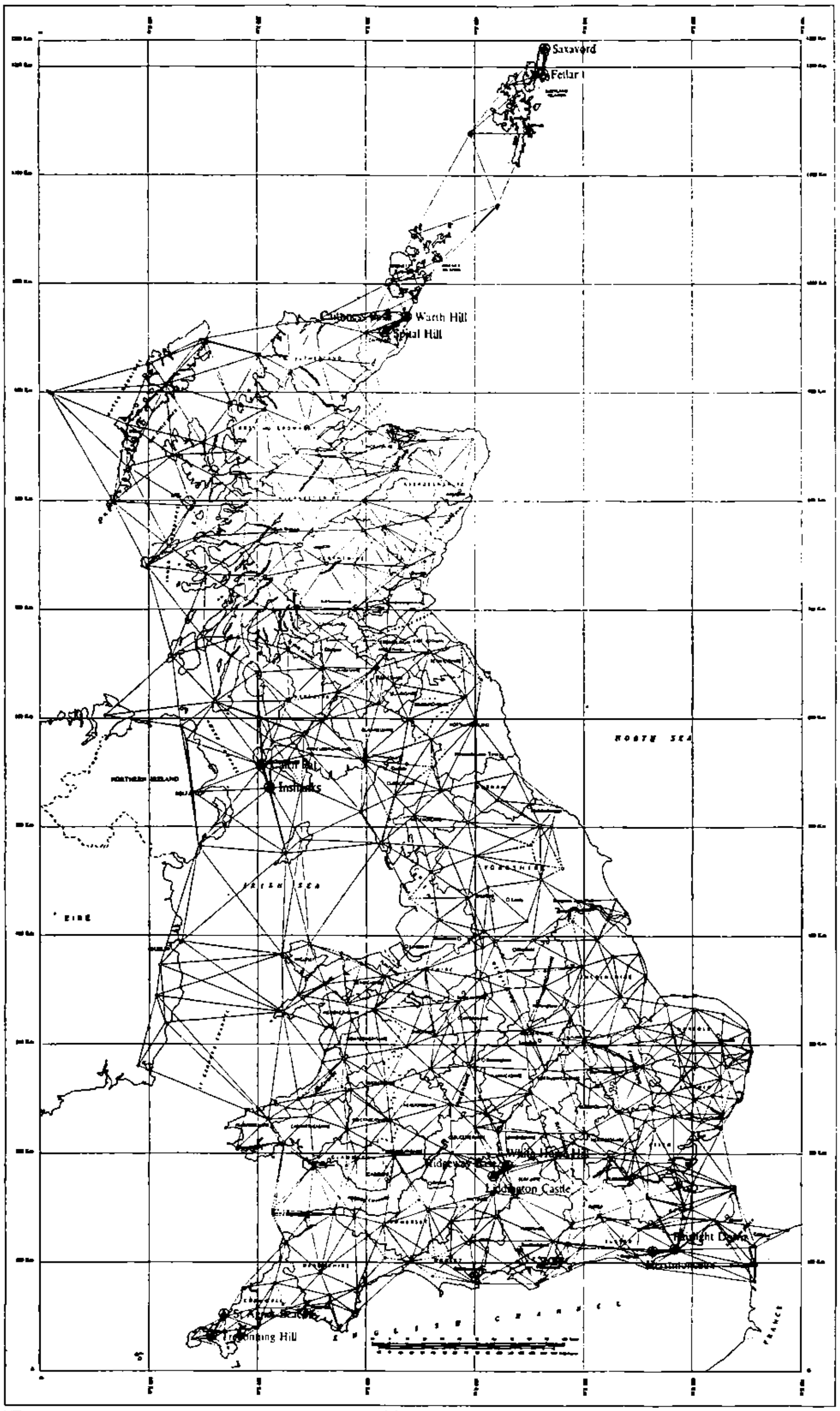


Fig. 5.1. Distribution of the Laplace stations

being an auxiliary to the primary triangulation station (see Fig. 5.11). The astronomic longitude of the auxiliary position was referred to the position of the 0° meridian, and any departure from zero was attributed to personal equation. See foot of Table 5.3, also § 5.044 (*f*) below.

To assess the observer's personal equation in azimuth, impersonal Laplace azimuths were derived from Polaris observations at Herstmonceux (481) and Fairlight Down (193) using the Laplace equation. See § 5.042 (*g*) below for details.

SUMMARY OF PROGRAMME

Station	*Observations for*	*Remarks*
Herstmonceux (481)	Azimuth and Position	Azimuth by Black's method* and Polaris
Fairlight Down (193)	Azimuth and Longitude	Azimuth by Black's method* and Polaris
White Horse Hill (34)	Azimuth and Position	Azimuth by Black's method*
Liddington Castle (35)	Azimuth	Azimuth by Black's method*
St. Agnes Beacon (175)	Azimuth and Position	Azimuth by Black's method*
Tregonning Hill (181)	Azimuth	Azimuth by Black's method*
Cairn Pat (360)	Azimuth and Position	Azimuth by Black's method*
Inshanks (361)	Azimuth	Azimuth by Black's method*
Greenwich Observatory (482)	Position	Of auxiliary to primary station
Spital Hill (398)	Azimuth and Position	Azimuth by Black's method*
Warth Hill (399)	Azimuth and Position	Azimuth by Black's method*
Saxavord (463)	Azimuth	Azimuth by Black's method*
Fetlar (459)	Azimuth and Position	Azimuth by Black's method*

* See § 5.041 below for details of Black's method.

5.01 Notation

Specific values are shown by suffixes or other indicators. Note that the quantity η has no connection at all with the η^2 of § 2.19.

φ = Latitude, north (+), or south (−).
λ = Longitude from Greenwich, east (+), or west (−).
A = Azimuth, measured 0°–360° clockwise from true north.
δ = Declination of a star, north (+), or south (−).
h = Altitude. (Also used as a superscript to indicate hours).
MT = Mean time.
ST = Sidereal time.
GST = Greenwich sidereal time.
RA = Right ascension of a star. Reckoned 0^h to 24^h eastwards.
t = Local hour angle, measured 0° to 360°, or 0^h to 24^h, clockwise from upper transit, $= \lambda - \text{RA} + \text{GST}$.
q = Parallactic angle, measured 0° to 360° clockwise from true north.
ξ = Meridional component of the deflection of the vertical, = Astronomic φ − Geodetic φ.

η = Prime vertical component of the deflection of the vertical, = (Astronomic λ – Geodetic λ) cos φ.
e = Chronometer error, fast (+), or slow (–).
r = Chronometer rate, gaining (+), or losing (–).

5.02 Methods Adopted

5.020 FOR AZIMUTH

The usual technique for finding Laplace azimuth is to observe for longitude and astronomic azimuth, using a circumpolar star, generally Polaris in the northern hemisphere, to obtain the azimuth. The Laplace equation is then used to deduce the Laplace geodetic azimuth, A_G, from the astronomic azimuth, A_A, the Laplace equation being:

$$\left.\begin{aligned} A_A - A_G &= (\lambda_A - \lambda_G) \sin \varphi \\ \text{or} \quad A_A - A_G &= \eta \tan \varphi \end{aligned}\right\} \tag{5.1}$$

hence

$$A_G = A_A - \eta \tan \varphi$$

suffixes A and G indicating astronomic and geodetic respectively.

An astronomic azimuth determined from a close circumpolar star, such as Polaris, is relatively insensitive to time errors. For Polaris, the maximum effect on azimuth of a time error is at transit, and is approximately:

$$\Delta A'' = \Delta t^s . \sec \varphi / 4$$

where $\Delta A''$ and Δt^s are the azimuth and time errors respectively. But equation (5.1) shows that this technique of using a close circumpolar star for Laplace azimuth is limited to moderate latitudes by the accuracy of the longitude determination.

A. N. Black(37) has described a method of obtaining Laplace azimuth wherein the error is proportional to tan h instead of tan φ, and no observations for longitude are necessary. He points out that a single azimuth observation of unit weight contributes information in the following proportions:

$$\left.\begin{array}{ll} \text{To azimuth} & : \cos^2 h \\ \text{To latitude} & : \sin^2 h \,.\, \sin^2 A \\ \text{To longitude} & : \sin^2 h \,.\, \cos^2 A \end{array}\right\} \quad \text{Total} = 1$$

Thus the major contribution of the observation is to the azimuth determination if low altitude stars are observed; this also minimizes any error since the latter is proportional to tan h.

Black's method of obtaining Laplace azimuth was adopted for the azimuth programme.

5.021 FOR LATITUDE AND LONGITUDE

Astronomic latitude and longitude were found by position lines using mid-quadrantal observations. The widely used Marc St. Hilaire method(38) of computing zenithal distance intercepts was not adopted, preference being given to the method of computing longitude cuts on an approximate parallel of latitude. The latter method requires approximate latitude only; the zenithal distance computation requires approximate latitude and longitude.

5.03 Equipment

5.030 THE THEODOLITE

All angular observations were made with a Wild T4 Universal Instrument, No. 33110, fitted with an impersonal eyepiece micrometer. The T4 is a broken-transit type of instrument with a 60 mm. (2⅜ in.) objective; it has a magnification of 65× and a focal length of 550 mm. (21⅝ in.). The horizontal circle has a diameter of 250 mm. (9⅞ in.), and can be read directly to 0″.1. The 145 mm. (5¾ in.) vertical circle reads directly to 0″.2.

A large, detachable, hanging level formed part of the equipment, and had a mean sensitivity of 1″.342 per division as found from tests at Messrs. Hilger and Watts. The makers' nominal value was 1″.22. The vertical circle level bubble is read by coincidence prisms, and the mean value of one division on the prism scale was found by practical tests to be 1″.21.

The impersonal, self-recording, eyepiece micrometer can be rotated through 90° for horizontal or vertical use. When using the impersonal micrometer the telescope is clamped, and a fine movable wire is set on the star which is thereafter kept continuously bisected by turning two knobs, using each hand in turn to maintain a smooth continuous movement of the wire. The moving wire and the stellar image thus traverse the telescope field together. The knobs also turn a drum in which are inset ten equally spaced platinum contact strips. These contacts successively complete an electrical circuit which automatically records on the chronograph when the contact is closed. As the chronometer is recording on the chronograph concurrently with the micrometer drum, the chronometer time of each contact position can be found. A correction to the times as taken from the chronograph is necessary because of the contact width; the correct time is at the centre of a contact whereas the electrical circuit is first actuated by the leading edge of the contact. The average width of a contact on T4 No. 33110 was found to be 0·01172 of a drum revolution.

Fig. 5.2 is a diagrammatic representation of the eyepiece micrometer diaphragm in the horizontal position, and shows the relative positions of the moving wire, reticule, and comb. One revolution of the micrometer contact drum moves the wire over one comb interval, thus 10 contact closures are recorded each comb interval. The particular contact which records the time at which the moving wire coincides with a comb division is flanked by two marker contacts, which thus distinguish the comb divisions on the chronograph record. The marker contacts are not used for time purposes. Fig. 5.3 shows how the chronograph trace appears over one comb interval.

When observing, it is essential that the contact pattern is symmetrical about the fixed centre wire in the eyepiece reticule, that is, the same number of times should be recorded before and after the star transits the centre wire. For azimuth observations in the Black method the stars were tracked across two comb intervals each side of the centre wire, from number 8 to number 12, or vice versa, on each face. For position observations one comb interval each side of the centre wire was taken, from number 9 to number 11 or vice versa, on each face. Thus on each face there were 40 times recorded for azimuth, and 20 times for position. Reducing these sets of times to the centre wire gave the mean moment of the star passage through the telescope axis. It should be noted that this reduction is not linear (see Curvature Corrections at § 5.042(*b*) and § 5.044(*c*) below).

A comprehensive description of the Wild T4 Universal Instrument and its accessories, and of the methods of calibration and adjustment is published by the makers of the instrument(39).

5.031 CHRONOMETERS

Three Mercer chronometer clocks were used, two measuring mean time, Nos. 19674 and 19666, and one measuring sidereal time, No. 19684. The sidereal time chronometer was accepted as the

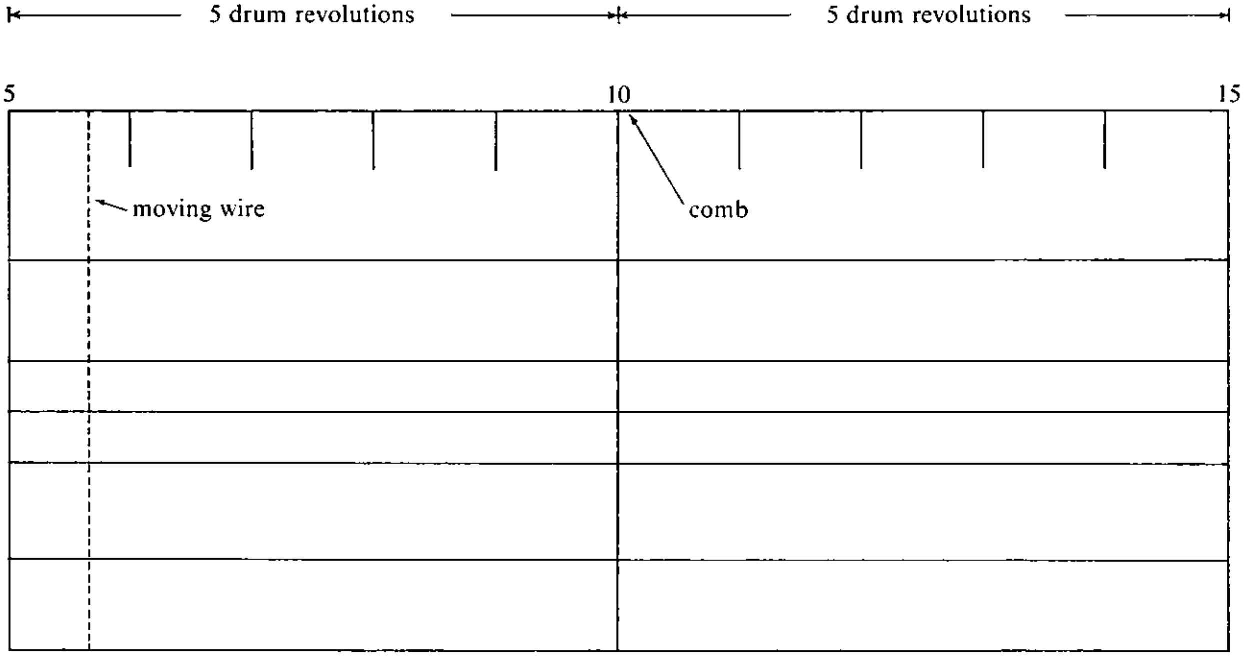


FIG. 5.2. Eyepiece micrometer diaphragm

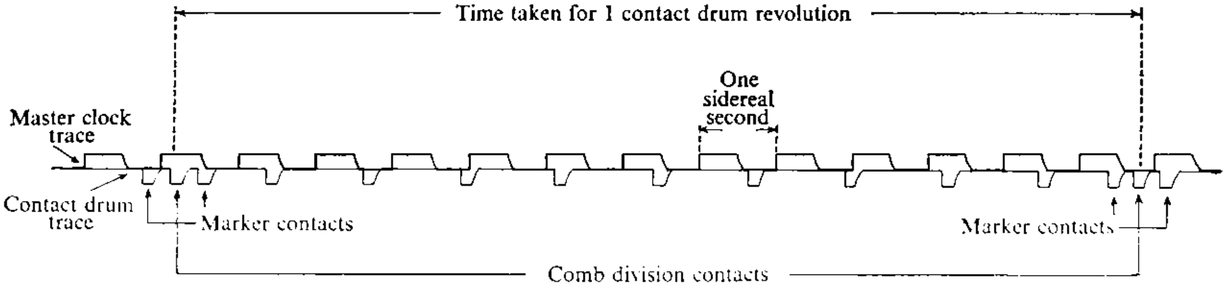


FIG. 5.3. Chronograph trace covering one comb interval

standard, or master, the other two being available to provide an auxiliary time signal should a rhythmic time signal be missed. See end of § 5.048. They were all fitted with contacts for recording alternate half-second beats automatically on the chronograph. To facilitate reading from the chronograph the master ST chronometer was modified so that the half-second beat occurring on the minute did not operate the contact, thus indicating the minutes automatically on the chronograph. The mean time chronometers were not modified in this way.

The error of the master was found from comparisons with rhythmic time signals, and the errors of the mean time chronometers by frequent comparisons with the master. In the event the master ST chronometer behaved very well—less erratically in fact than the mean time chronometers—and was accepted as the sole authority for time in the calculations. At one period two chronometer watches, one mean time and one sidereal time, were brought into use to supplement the mean time chronometers in the event of the latter giving any trouble, but their use was soon discontinued, mainly because they had to be recorded manually on the chronograph when making comparisons.

5.032 CHRONOGRAPH

All timed events were recorded on a Mercer weight-driven drum chronograph, which had two pens. One pen, pen No. 1, was used exclusively for recording the time trace of the master chronometer in blue ink. The other pen, No. 2, recorded in red ink the various events which required comparison with the master chronometer to assess time. Events consisted of mean time chronometer comparisons for clock errors, impersonal eyepiece micrometer contact patterns for star times, and rhythmic time signal coincidences for finding master chronometer error. In addition, the two pens were operated simultaneously on the master chronometer to find the pen equation. Pen equation is a correction to the times of all events recorded by pen No. 2, and is necessary because pen No. 2 lies behind pen No. 1 in the line of the trace (see § 5.045 for details).

5.033 WIRELESS RECEIVERS

A Marconi Receiver Type 730 was used for receiving rhythmic time signals during the first three months of the programme only, as it proved to be unsatisfactory on short wavebands. It gave admirable service on long wavebands. The receiver's failure to receive the higher frequencies was inconvenient as the Moscow short wave transmitters could not be used as planned during extended observing sessions. These transmitters operated for 5 minutes every 2 hours during the whole period from 22.01 hours to 06.01 hours inclusive, whereas the receivable long wave transmitters only covered 20.01 hours to 02.01 hours inclusive, 08.01 hours, and 09.31 hours, all GMT (see Table 5.2). This left the unduly long period from 02.06 hours to 08.01 hours without an accurate determination of the master chronometer error. At the beginning of August, about half way through the programme, a R.C.A. receiver type AR 88, for short and medium wavebands, was obtained. This proved satisfactory for receiving the Moscow short wave transmitters.

5.034 MISCELLANEOUS

Temperature and barometric pressure were recorded during the position line observations. This information was required for the calculation of refraction (see § 5.044 (*b*) below).

The three-armed brass spider on the standard Ordnance Survey pillar was too small to accommodate the foot-screws of the T4 theodolite. An adapter in the form of a large spider was specially made with a hemispherical boss under each arm. This adapter was emplaced on the pillar spider, the T4 theodolite being then set up on the adapter.

5.04 Principles and Methods of Calculation

5.040 FORMULAE

For convenience of reference various relationships in the astronomic triangle (Fig. 5.4), and some of the partial derivatives, are listed below.

$$\tan A = \sin t/(\sin\varphi \,.\, \cos t - \cos\varphi \,.\, \tan\delta) \tag{5.2}$$

$$\sin A = -\cos\delta \,.\, \sin t/\cos h \tag{5.3}$$

$$\cos A = (\sin\delta - \sin\varphi \,.\, \sin h)/\cos\varphi \,.\, \cos h \tag{5.4}$$

$$\cos A = (\cos\varphi \,.\, \sin\delta - \cos t \,.\, \cos\delta \,.\, \sin\varphi)/\cos h \tag{5.5}$$

$$\sin\delta = \sin\varphi \,.\, \sin h + \cos A \,.\, \cos\varphi \,.\, \cos h \tag{5.6}$$

$$\sin h = \sin\varphi \,.\, \sin\delta + \cos\varphi \,.\, \cos\delta \,.\, \cos t \tag{5.7}$$

$$\cos t = (\sin h - \sin\varphi \,.\, \sin\delta)/\cos\varphi \,.\, \cos\delta \tag{5.8}$$

$$\partial A/\partial t = (\sin\varphi - \tan h \,.\, \cos\varphi \,.\, \cos A) \tag{5.9}$$

$$\partial h/\partial t = \cos\varphi \,.\, \sin A \tag{5.10}$$

$$\partial t/\partial\varphi = -\cot A \,.\, \sec\varphi \tag{5.11}$$

5.041 BLACK'S METHOD FOR LAPLACE AZIMUTH

Black's method is essentially that of azimuth by hour angle using equation (5.2), but in calculating the azimuth of the star *geodetic* latitude and longitude are used. The azimuth of the star is thus calculated with reference to the spheroidal or geodetic zenith, and is the angle PZ_GS in Fig. 5.5. This Figure shows an instantaneous view of the celestial sphere as seen from a point near the observer's zenith. P is the celestial pole, and Z_G and Z_A are the projections on the celestial sphere of the observer's spheroidal or geodetic zenith and astronomic zenith respectively. S is a star, and ZR is the projection on the celestial sphere of the ray from the observing station to the azimuth referring object. To the stellar azimuth is applied the observed angle RZ_AS between the terrestrial referring object (R) and the star (S). This angle is, however, referred to the astronomic zenith because of the deflection of the vertical. To obtain the Laplace azimuth of R it is necessary to correct the observed angle from the astronomic zenith to the spheroidal zenith, that is, in Fig. 5.5, angle RZ_AS must be corrected to give angle RZ_GS. From (5.6):

$$\sin\delta = \sin\varphi \,.\, \sin h + \cos A \,.\, \cos\varphi \,.\, \cos h$$

Differentiating for small changes in latitude and hour angle caused by passing from geodetic to astronomic co-ordinates:

$$0 = \cos\varphi \,.\, \sin h \,.\, d\varphi + \sin\varphi \,.\, \cos h \,.\, dh - \sin\varphi \,.\, \cos h \,.\, \cos A \,.\, d\varphi - \cos\varphi \,.\, \sin h \,.\, \cos A \,.\, dh - \cos\varphi \,.\, \cos h \,.\, \sin A \,.\, dA \tag{5.12}$$

From (5.7):

$$\sin h = \sin\varphi \,.\, \sin\delta + \cos\varphi \,.\, \cos\delta \,.\, \cos t$$

Differentiating, and substituting from (5.5) and (5.3):

$$dh = \cos A \,.\, d\varphi + \cos\varphi \,.\, \sin A \,.\, dt$$

But:

$$\xi = \varphi_A - \varphi_G = d\varphi, \text{ and } \eta \sec\varphi = \lambda_A - \lambda_G = dt,$$

$$\therefore\ dh = \xi \cos A + \eta \sin A \tag{5.13}$$

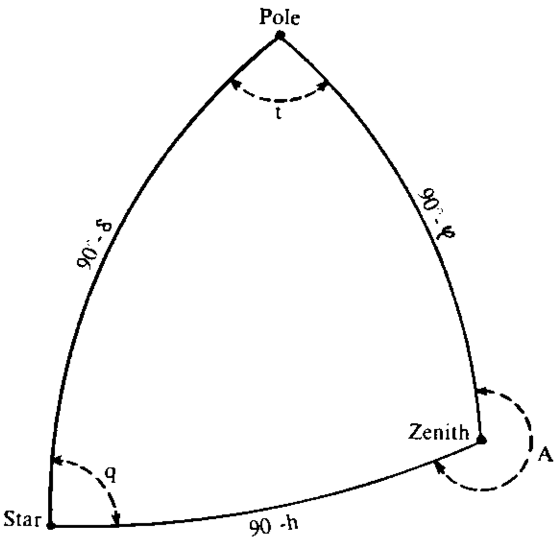


FIG. 5.4. The astronomic triangle

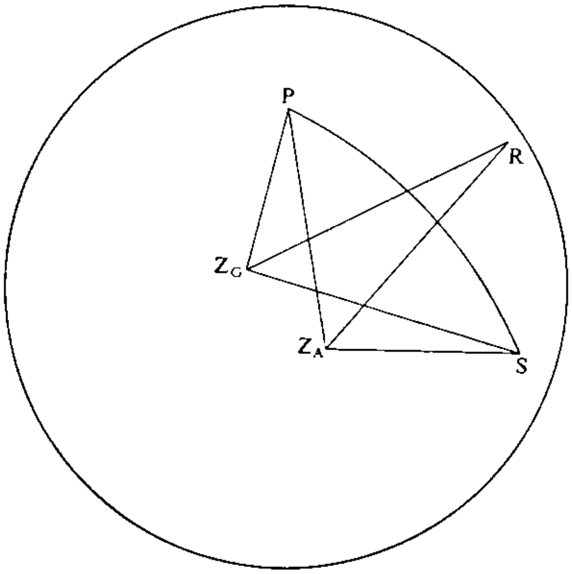


FIG. 5.5. Instantaneous view of the celestial sphere

In (5.12) substituting from (5.13) for dh, and ξ for dφ, simplifying, and re-arranging:

$$\mathrm{d}A = \eta \tan \varphi + \tan h\,(\xi \sin A - \eta \cos A) \tag{5.14}$$

where $\mathrm{d}A = A_A - A_G$.

Equation (5.14) gives the difference between PZ_AS and PZ_GS in Fig. 5.5. It will also give the difference between PZ_AR and PZ_GR if the values of h and A are those applicable to R. In this case, however, h is very small, and the term in $\tan h$ can be ignored, which gives:

$$\eta \tan \varphi \tag{5.15}$$

as the difference between PZ_AR and PZ_GR. This is equivalent to the Laplace equation; see (5.1).

Subtracting (5.14) from (5.15) gives the required difference between RZ_GS and RZ_AS, namely:

$$-\tan h_S\,(\xi \sin A_S - \eta \cos A_S) \tag{5.16}$$

where suffix S refers to the star. Equation (5.16) is the correction to the observed angle, measured clockwise from R to S, to transfer it from Z_A to Z_G. Let A_R denote the azimuth of R obtained by subtracting the *uncorrected* observed angle from the geodetic azimuth of the star, then:

$$A_G - \tan h_S\,(\xi \sin A_S - \eta \cos A_S) = A_R$$

or

$$A_G + \tan h_S \,.\, \cos A_S \,.\, \eta - \tan h_S \,.\, \sin A_S \,.\, \xi = A_R \tag{5.17}$$

where A_G is the Laplace azimuth of R.

If N stars are observed at a station, there will be N observation equations of the form of (5.17). Thus:

$$\left.\begin{array}{l} A_G + \tan h_{S.1} \,.\, \cos A_{S.1} \,.\, \eta - \tan h_{S.1} \,.\, \sin A_{S.1} \,.\, \xi = A_{R.1} \\ A_G + \tan h_{S.2} \,.\, \cos A_{S.2} \,.\, \eta - \tan h_{S.2} \,.\, \sin A_{S.2} \,.\, \xi = A_{R.2} \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ A_G + \tan h_{S.N} \,.\, \cos A_{S.N} \,.\, \eta - \tan h_{S.N} \,.\, \sin A_{S.N} \,.\, \xi = A_{R.N} \end{array}\right\} \tag{5.18}$$

Let $\bar{A}$ = Mean $A_R = (\Sigma A_R)/N$; then subtracting $\bar{A}$ from both sides in (5.18), and equating to the residual, v:

$$\left.\begin{array}{l} (A_G - \bar{A}) + \tan h_{S.1} \,.\, \cos A_{S.1} \,.\, \eta - \tan h_{S.1} \,.\, \sin A_{S.1} \,.\, \xi - (A_{R.1} - \bar{A}) = v_1 \\ (A_G - \bar{A}) + \tan h_{S.2} \,.\, \cos A_{S.2} \,.\, \eta - \tan h_{S.2} \,.\, \sin A_{S.2} \,.\, \xi - (A_{R.2} - \bar{A}) = v_2 \\ \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\ (A_G - \bar{A}) + \tan h_{S.N} \,.\, \cos A_{S.N} \,.\, \eta - \tan h_{S.N} \,.\, \sin A_{S.N} \,.\, \xi - (A_{R.N} - \bar{A}) = v_N \end{array}\right\} \tag{5.19}$$

From (5.19) the method of least squares is used to find the most probable values of $(A_G - \bar{A})$, η, and ξ. Then the Laplace azimuth, A_G, is given by:

$$A_G = \bar{A} + (A_G - \bar{A})$$

Because of the weight distribution given above in § 5.020, the lower the observed altitude of the star the less the accuracy with which η and ξ are determined. However, the lower the altitude the less is the accuracy to which they are needed to find the correction at (5.16), so the one factor exactly compensates the other.

Apart from confining the observations to low altitudes, it is very desirable to have them well-balanced and evenly distributed in azimuth; this ensures that $(A_G - \bar{A})$ is kept very small. To check on this, the observer should keep a vector diagram on which a vector is plotted for each star observed, the vector direction being the stellar azimuth, and vector length = tan h. If the observations were perfectly balanced in azimuth and altitude the vector plot would end where it started, and $(A_G - \bar{A})$ would be zero with $A_G = \bar{A}$. The results of the Ordnance Survey azimuth programme showed that with care $(A_G - \bar{A})$ will not exceed $0''.1$. See Table 5.4.

5.042 CORRECTIONS IN AZIMUTH CALCULATIONS

For simplicity, the description given above of the Black method takes no account of several necessary corrections. These will now be considered in the order in which they were calculated. Some of the corrections were negligible, but they are mentioned for the sake of completeness.

(*a*) *Corrections to circle readings for dislevelment.* These were corrections to the observations before finding the azimuth of R. They were computed from readings taken on the large hanging level. When observing stars, each end of the hanging level was read on the bubble scale twice on each face, thus giving four readings. When observing the referring object R each end of the hanging level was read once on each face, thus giving two readings. Corrections to the horizontal circle readings were:

To star pointing on Face Left: $+\frac{1}{4}(\Sigma_1^4\text{bubble readings} - 200) \,.\, d \,.\, \tan h_S$
To star pointing on Face Right: $-\frac{1}{4}(\Sigma_1^4\text{bubble readings} - 200) \,.\, d \,.\, \tan h_S$
To R pointing on Face Left: $+\frac{1}{2}(\Sigma_1^2\text{bubble readings} - 100) \,.\, d \,.\, \tan h_R$
To R pointing on Face Right: $-\frac{1}{2}(\Sigma_1^2\text{bubble readings} - 100) \,.\, d \,.\, \tan h_R$

An approximate altitude (h) was recorded by the observer, and d is the value of one division on the bubble scale, namely $1''.342$. (See § 5.030.)

(*b*) *Correction to azimuth for curvature.* Azimuth is not a linear function of time, so a stellar azimuth, A_0, computed from a mean GST, θ_0, requires a correction to give the mean azimuth, A_m, that would have been found had each separate time $\theta_1, \theta_2, \ldots \theta_n$, been used to compute the separate azimuths $A_1, A_2, \ldots A_n$, and the latter meaned. So $(A_m - A_0) = \Delta A$, and is the curvature correction. This correction is given by:

$$\Delta A'' = \cos\varphi \,.\, \sin A_S \,.\, \sec^2 h_S\,(\sin\delta_S \,.\, \cos h_S - 2 \,.\, \cos A_S \,.\, \cos\varphi) \,.\, m_0$$

where m_0 is the mean value of $m_1 \ldots m_n$, and

$$m_1 = 2 \,.\, \sin^2\tfrac{1}{2}(\theta_1 - \theta_0) \,.\, \operatorname{cosec} 1''$$
$$m_2 = 2 \,.\, \sin^2\tfrac{1}{2}(\theta_2 - \theta_0) \,.\, \operatorname{cosec} 1''$$
$$\cdots\cdots\cdots$$
$$m_n = 2 \,.\, \sin^2\tfrac{1}{2}(\theta_n - \theta_0)\operatorname{cosec} 1''$$

then:

$$m_0 = (\Sigma m)/n$$

Tables of $2 \,.\, \sin^2\frac{1}{2}(\theta - \theta_0) \,.\, \operatorname{cosec} 1''$ have been given by Roelofs(40) and Close and Winterbotham(41), among others.

The correction given here is strictly for the case where the star is timed over successive great circles, that is, over successive positions of the line of collimation. In the eyepiece micrometer, however, the successive times are over small circles, equally spaced about one position of the line of collimation, so another correction is theoretically necessary to allow for this. Over two comb intervals each side of the line of collimation, however, the correction is negligible. See Roelofs(40) pp. 96 *et seq.* for details.

(*c*) *Correction to azimuth for diurnal aberration.* Diurnal aberration is an apparent displacement of a star due to the observer moving with the rotation of the earth about its axis. Its effect is to make the apparent position of a star always east of its true position. The correction is:

$$\Delta A'' = 0{\cdot}320 \times \cos\varphi \,.\, \cos A_S \,.\, \sec h_S$$

It is added algebraically to the azimuth of R.

(*d*) *Correction to azimuth for skew normals.* The observed direction of the referring object is considered to lie in a plane containing the spheroidal normal at the observing station, and is projected to the spheroid as a curve of normal section. This curve requires correcting to the normal section curve passing through the spheroidal projection of the referring object. The correction is(42):

$$\Delta A'' = 0{\cdot}033 \times \sin 2A_R \,.\, \cos^2\varphi \,.\, H_R$$

where A_R is the azimuth of the referring object, and H_R is its height in thousands of feet above mean sea level. In the Ordnance Survey programme the maximum value of $\cos^2\varphi$ was about 0·41, giving a maximum correction of $0''\!.014$ per 1,000 feet of height when $\sin 2A_R = 1$. It was considered negligible in all cases, and was not applied.

(*e*) *Correction of azimuth to the geodesic.* The normal section curve obtained in (*d*) above requires reduction to the spheroidal geodesic. The correction is(42):

$$\Delta A'' = -0{\cdot}07 \times (L/100)^2 \,.\, \sin 2A_R \,.\, \cos^2\varphi$$

where L is the length of the line in miles. Taking appropriate maximum values, the correction for $L = 100$ is $0''\!.029$. It was completely negligible on all lines in the Ordnance Survey programme, where the longest line was about 15½ miles, and was not applied.

The five corrections § 5.042 (*a*) to (*e*) are all applied (where significant) before the least squares determinations of $(A_G - \bar{A})$, η, and ξ, are made. When the most probable value of A_G has been found from the least squares calculation, it is subject to two further corrections. These are described below at (*f*) and (*g*).

(*f*) *Correction to reduce azimuth to Mean Pole.* The earth's pole does not remain steady relative to the features on the surface of the earth; the motion varies with time, and is small and rather irregular. To reduce quantities referred to the pole at different instants of time to a common datum, or mean pole, x and y co-ordinates are found from special observations, and are published from time to time in the *Bulletin Horaire*. x is defined as positive measured southward along the Greenwich meridian, and y is positive measured southward along the meridian of 90° west. The correction to A_G is(42):

$$\Delta A_G'' = -(x \,.\, \sin\lambda + y \,.\, \cos\lambda)\sec\varphi$$

(*g*) *Correction to azimuth for personal equation.* Although the observer was using an impersonal eyepiece micrometer, his work might have contained a systematic error in time due to his personal tendency to lag or lead with the moving wire when tracking the star. This was his personal equation. Any attempt to correct for this must assume that the observer is fairly consistent in his behaviour.

It has been explained in § 5.020 that an astronomic azimuth obtained from Polaris observations is relatively insensitive to time errors, and therefore to personal equation, so a Laplace azimuth derived from a Polaris astronomic azimuth was accepted as being an impersonal azimuth, the λ_A in the Laplace equation having been corrected for personal equation in longitude. Laplace azimuths by Black's method were observed at Herstmonceux (481) and Fairlight Down (193) at the beginning and end of the programme, and compared with the impersonal Laplace azimuths from Polaris. The Fairlight Down (193) results were transferred to Herstmonceux (481) and the comparison was made at the latter station. (See § 5.08 for details). The discrepancies were attributed to errors from personal equation in the azimuths by Black's method.

From (5.9):

$$\frac{\partial A}{\partial t} = \sin\varphi - \cos\varphi \,.\, \tan h \,.\, \cos A$$

which gives the change in A for a change in t for each A_R in (5.18). In Black's method, however, the sum of the second terms of the right-hand side of this equation is made effectively zero, that is:

$$\Sigma(\cos\varphi \,.\, \tan h \,.\, \cos A) = 0$$

so the correction to the mean azimuth to eliminate personal equation is:

$$\Delta A = \Delta t \,.\, \sin\varphi$$

or

$$\text{Polaris } A_G - \text{Black } A_G = \Delta t \,.\, \sin\varphi$$

at the comparison station, Herstmonceux (481). A value for Δt is thus found at the beginning and end of the programme, and linear interpolation gives Δt for any intermediate date. Finally, the personal equation correction to the Black A_G at any other station x is:

$$\Delta A''_G = \Delta t'' \,.\, \sin\varphi_x$$

where Δt is the value obtained by interpolation for the date of the observations at x.

5.043 LATITUDE AND LONGITUDE BY POSITION LINES

As already indicated in § 5.021, the method chosen for the position lines was to compute longitude cuts on an approximate parallel of latitude.

Position can be found graphically or analytically, and both methods were used; a graphic plot was made first to show up any doubtful observations, and final values were found by the method of least squares.

(*a*) *The graphical method.* In Fig. 5.6, S_1, Pole, and P_1 form an astronomic triangle. The true position, P, lies on the position circle drawn with S_1 as centre and with an angular radius of $90°–h_1$. The displacement of P to P_1 results entirely from the approximate latitude, φ_0, and this displacement is for all practical purposes at right angles to the azimuth, A_1. So if the value of the longitude cut, λ'_1, at P_1 on φ_0 is computed, and a direction, or position line, equal to $A_1 \pm 90°$ is laid off through P_1, then the true position, P, lies somewhere along this position line. The data for finding λ'_1 are: δ_1, h_1 (an observed quantity), φ_0, RA_1, and GST_1 of observation. Then from (5.8):

$$\cos t'_1 = (\sin h_1 - \sin\varphi_0 \,.\, \sin\delta_1)/\cos\varphi_0 \,.\, \cos\delta_1$$

And from the definition of t in § 5.01:

$$\lambda'_1 = t'_1 + RA_1 - GST_1 \tag{5.20}$$

Finally, from (5.3):

$$\sin A_1 = -\cos\delta_1 \,.\, \sin t'_1/\cos h_1$$

This azimuth is only approximate, but it is adequate. A straight horizontal line is drawn on graph paper to represent φ_0. Adopting a suitable longitude scale λ'_1 is plotted and $A_1 \pm 90°$ laid off. The

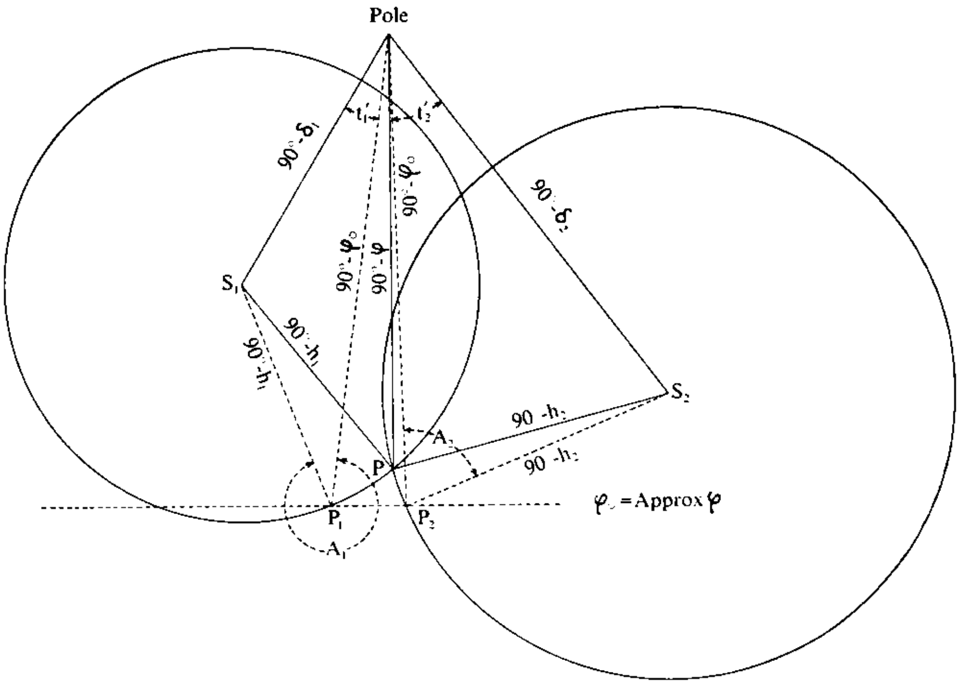


FIG. 5.6. Position lines

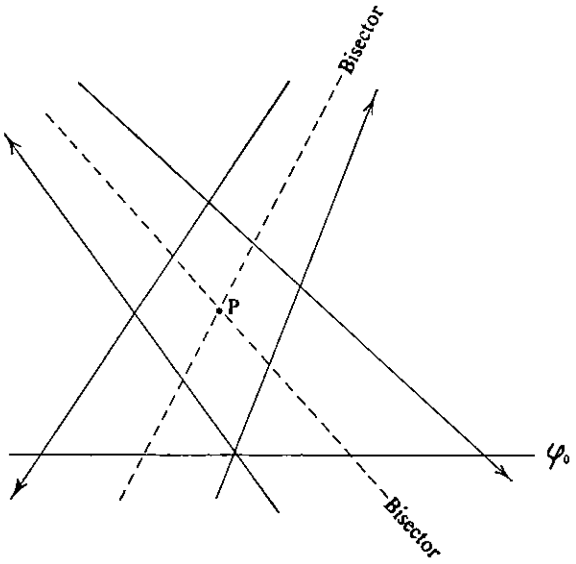


FIG. 5.7. Position of P

procedure is repeated for another star, S_2 in Fig. 5.6, thus giving the position line from P_2 to P, that is, the direction $A_2 \pm 90°$. The intersection of the directions $A_1 \pm 90°$ and $A_2 \pm 90°$ gives the position on the graph of P. The position circles intersect at two points, but the approximate latitude removes ambiguity. (See Fig. 5.6.) In practice, with mid-quadrantal observations it is usual to take a minimum of four stars, one in each quadrant. Due in the main to systematic errors in the assumed refraction and to the vertical collimation error in the case of single face observations, the graph plot usually forms a square box, and the position of P is taken as the centre of an inscribed circle which most nearly touches the four sides of the box. In fact the position of P is at the intersection of two lines found by bisecting the angle formed by each pair of tangent directions which are approximately 180° apart (see Fig. 5.7).

The final longitude of P is read directly from the graph, but as the scale of the graph is a longitude scale, the $\Delta\varphi$ read from the graph must be multiplied by $\cos\varphi$ before being applied to φ_0. We have then:

$$\lambda_P \text{ read from graph}$$
$$\varphi_P = \varphi_0 + \Delta\varphi \,.\, \cos\varphi$$

where $\Delta\varphi$ is the difference at the graph scale between P and the φ_0 axis. As $\Delta\varphi$ is small, $\cos\varphi_0$ is used here.

Since the graphical method was used principally as a check on the observations, all the stars observed at a station were plotted together on one graph.

(*b*) *The analytical method.* The equation for hour angle, t, is:

$$\cos t = (\sin h - \sin\varphi \,.\, \sin\delta)/\cos\varphi \,.\, \cos\delta$$

Differentiating, it can be shown that:

$$\mathrm{d}t = -\mathrm{d}\varphi \,.\, \cot A \,.\, \sec\varphi + \mathrm{d}h \,.\, \operatorname{cosec} A \,.\, \sec\varphi$$

Or

$$\mathrm{d}t + \mathrm{d}\varphi \,.\, \cot A \,.\, \sec\varphi - \mathrm{d}h \,.\, \operatorname{cosec} A \,.\, \sec\varphi = 0$$

From the equation for λ_1' in (5.20) above it can be seen that an error in t_1' will produce the same error in λ_1', and therefore

$$\Delta\lambda + \Delta\varphi \,.\, \cot A \,.\, \sec\varphi - \Delta h \,.\, \operatorname{cosec} A \,.\, \sec\varphi = 0$$

Using an approximate latitude φ_0, the longitude λ' and the approximate azimuth A are computed as described in (*a*) above. Let the correct longitude be λ_P, then $\lambda_P - \lambda' = \Delta\lambda$, and we have:

$$\lambda_P - \lambda' + \Delta\varphi \,.\, \cot A \,.\, \sec\varphi_0 - \Delta h \,.\, \operatorname{cosec} A \,.\, \sec\varphi_0 = 0$$

Or

$$\lambda_P + \Delta\varphi \,.\, \cot A \,.\, \sec\varphi_0 - \Delta h \,.\, \operatorname{cosec} A \,.\, \sec\varphi_0 - \lambda' = 0$$

To avoid handling large numbers in the computations it is convenient to reduce all the computed approximate longitudes by a constant amount; call it λ_0. We have then:

$$(\lambda_P - \lambda_0)'' + \Delta\varphi'' \,.\, \cot A \,.\, \sec\varphi_0 - \Delta h'' \,.\, \operatorname{cosec} A \,.\, \sec\varphi_0 - (\lambda' - \lambda_0)'' = 0$$

and this is the required observation equation. Each of the n stars gives an observation equation, thus:

$$\begin{aligned}
(\lambda_P - \lambda_0)'' + \Delta\varphi'' \,.\, \cot A_1 \,.\, \sec\varphi_0 - \Delta h'' \,.\, \operatorname{cosec} A_1 \,.\, \sec\varphi_0 - (\lambda_1' - \lambda_0)'' &= v_1 \\
(\lambda_P - \lambda_0)'' + \Delta\varphi'' \,.\, \cot A_2 \,.\, \sec\varphi_0 - \Delta h'' \,.\, \operatorname{cosec} A_2 \,.\, \sec\varphi_0 - (\lambda_2' - \lambda_0)'' &= v_2 \\
\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots & \\
(\lambda_P - \lambda_0)'' + \Delta\varphi'' \,.\, \cot A_n \,.\, \sec\varphi_0 - \Delta h'' \,.\, \operatorname{cosec} A_n \,.\, \sec\varphi_0 - (\lambda_n' - \lambda_0)'' &= v_n
\end{aligned}$$

There are three unknowns, $(\lambda_P - \lambda_0)''$, $\Delta\varphi''$, and $\Delta h''$ and since $n > 3$ it is necessary to equate each observation equation to its residual, v, instead of zero.

These equations are solved by the method of least squares to find the three unknowns.

Then:

$$\varphi_P = \varphi_0 + \Delta\varphi$$
$$\lambda_P = \lambda_0 + (\lambda_P - \lambda_0)$$

It should also be noted that $r = \Delta h \,.\, \sec\varphi_0$ where r is the radius of the circle of best fit; this can be compared with the circle determined by eye in the graphical method. (The prime requirements are, of course, φ_P and λ_P; r is incidental.) As can be seen the graphical method and the analytical method are identical up to the point of finding longitude cuts on φ_0, the difference lies in the subsequent stage of finding φ_P and λ_P.

5.044 CORRECTIONS IN POSITION LINES CALCULATIONS

The corrections (*a*) to (*d*) given below are to the observed quantity h, and correction (*e*) is to the computed longitude cuts on the approximate latitude. Correction (*f*) affects the longitude and corrections (*g*) and (*h*) affect the latitude found from the least squares calculation.

(*a*) *Correction to vertical circle readings for dislevelment.* The vertical bubble prism scale was read twice on each face. Let the left half of the bubble read L, and the right half read R. Then the *circle reading* on a face was corrected by the following amount which was added algebraically:

$$\tfrac{1}{2}(\Sigma_1^2 L - \Sigma_1^2 R) \times 1''.21$$

See § 5.030 for details of the constant 1″.21.

From the corrected circle readings the altitude, h, was found as follows:

On Face Left: $h = 90° - \text{Circle reading}$.
On Face Right: $h = \text{Circle reading} - 270°$.

(*b*) *Correction to altitude for refraction.* A table given by Roelofs⁽⁴⁰⁾, was used to calculate this correction. This table is based on the following formula (using Roelofs's notation):

$$\text{Refraction}'' = \frac{p'}{29{\cdot}92} \times \frac{486}{(454+t')}\,(60{\cdot}1 \times \cot h - 0{\cdot}072 \times \cot^3 h)$$

h = Altitude
p' = Barometric pressure in inches.
t' = Air temperature in degrees Fahrenheit.

The term $486(60{\cdot}1 \times \cot h - 0{\cdot}072 \times \cot^3 h)/29{\cdot}92$ is tabulated as R' in Roelofs's table, with h as argument.

Then:

$$\text{Refraction}'' = p'.\, R'/(454+t')$$

The refraction correction is invariably negative in sign.

(*c*) *Correction to altitude for curvature.* The correction to the altitude on a single face is: Curvature correction $= \cos\varphi \,.\, \cos A\ (\cos\varphi \,.\, \cos A \,.\, \tan h - \sin\varphi)m_0$, where m_0 is as defined in § 5.042 (*b*).

If altitudes on different faces are meaned, a further correction is necessary⁽⁴⁰⁾. Faces were calculated separately in the Ordnance Survey programme.

(*d*) *Correction to altitude for small-circle projection of eyepiece micrometer comb.* § 5.042 (*b*) explains the reasons for this correction, which is significant with position lines. The following formula is applicable⁽⁴⁰⁾.

Small circle correction $= -\sin h \,.\, \cos h \,.\, \cos^2\varphi \,.\, K^2 \,.\, m_0$, where:

$$K = \cos A \,.\, \tan h - \tan \varphi$$

and m_0 is the same as in the curvature correction. This correction is applied to each face separately.

(*e*) *Correction to the computed λ′ for diurnal aberration.* For a description of this phenomenon see § 5.042(*c*). It has no effect on approximate latitude. Each computed λ' (see § 5.043) was corrected as follows:

$$\text{Correction to } \lambda' \text{ for diurnal aberration} = +0\overset{''}{.}320 \times \sin h = 0\overset{s}{.}0213 \times \sin h$$

The corrections described so far affect the data before the least squares solution is done for φ_P and λ_P. The following corrections relate to φ_P and λ_P.

(*f*) *Correction to longitude for personal equation.* This was discussed in § 5.00. It has no effect on φ_P, but does affect λ_P. Astronomic observations were made to find the observed astronomic longitude of the Airy Transit instrument at the Royal Observatory, Greenwich. Any difference between zero and the observed value was attributed to personal equation. (But see Chapter 3, § 3.09.) Let the observed value be λ_{ob}, then:

$$\lambda_{ob} = 0^\circ + \Delta\lambda$$

and the personal equation correction is $-\Delta\lambda$, that is, all observed position line longitudes are corrected, irrespective of latitude, by $-\Delta\lambda$ as found at Greenwich.

(*g*) *Correction to latitude for height above mean sea level.* This is due to the spheroidal shape of the earth and does not affect longitude. The correction is⁽⁴²⁾:

$$\Delta\varphi'' = -0{\cdot}000\,052 \,.\, H \,.\, \sin 2\varphi$$

where H is height in feet above mean sea level. It was hardly significant at any of the stations, so it was ignored.

(*h*) *Correction to Mean Pole.* As indicated in § 5.042 (*f*) the reduction to mean pole requires special observations. The correction to longitude was not considered to be worth while. The correction for latitude is⁽⁴²⁾:

$$\Delta\varphi'' = y \,.\, \sin\lambda - x \,.\, \cos\lambda$$

The quantities are as defined in § 5.042 (*f*) in connection with azimuth.

5.045 PEN EQUATION

This has already been referred to in § 5.032, and is defined as the amount (in time) by which pen No. 2 is displaced behind pen No. 1. Fig. 5.8 shows how the minute is indicated by suppressing the half second mark occurring on each minute. As pen No. 1 was invariably connected to the master chronometer, all recordings made by pen No. 2 were stepped forward by the pen equation before they were read relative to the trace of the master chronometer. Operational details are given below in § 5.054.

The pen equation was applied to all chronograph recordings made by pen No. 2 before relating them to the record of pen No. 1. For this reason it will be assumed in the subsequent discussion that the pen equation has been applied where necessary.

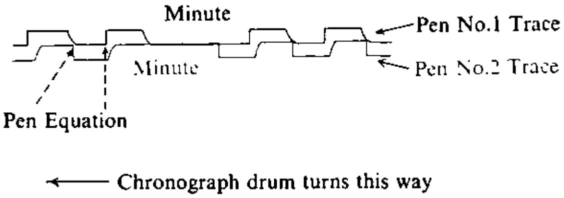
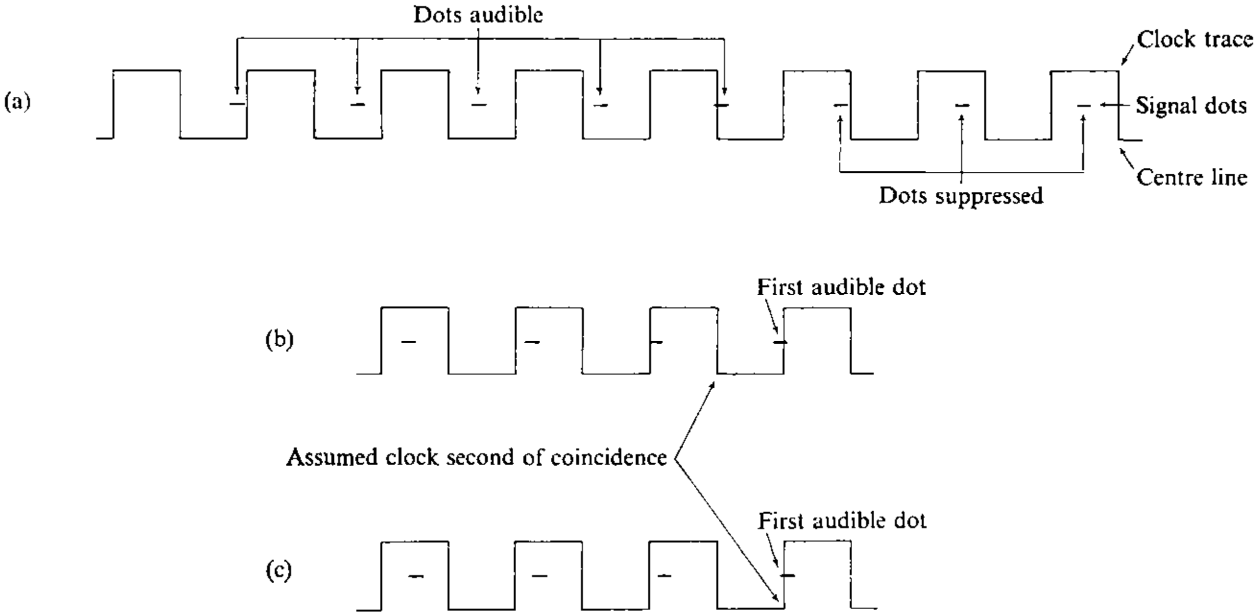


FIG. 5.8. Pen equation



NOTE: Time scale increases from left to right

FIG. 5.9. Time signal coincidences

5.046 CHRONOMETER ERROR AND RATE

Before the azimuth and position lines observations can be used, it is necessary to know the correct time of observation. As the star time trace is recorded on the chronograph in juxtaposition with the master chronometer trace, simple scaling gives the time of observation according to the master chronometer. The chronometer error is found by comparing the chronometer with rhythmic wireless time signals; knowing the error of the chronometer, then the correct instant of observation is known.

Rhythmic time signals. A time signal consists of 306 equally spaced signals, starting at a reputed instant of Greenwich Mean Time, transmitted over a period of 5 mean time minutes, that is in 300 mean time seconds. The signals are dots of $0^{\mathrm{s}}{\cdot}1$ duration with the exception of the 1st, 62nd, 123rd, 184th, 245th, and 306th; these are dashes of $0^{\mathrm{s}}{\cdot}4$ duration, and indicate the beginning of the 1st, 2nd, . . . , 6th minutes. The last dash ends the transmission.

As the signals divide each mean time minute into 61 intervals, the dot interval is 60/61 mean time seconds, and for a mean time chronometer the whole time signal provides five consecutive verniers, with coincidence at intervals of one minute of mean time.

For the sidereal time chronometer, which was used as the master clock, the dot interval, k, is:

$$k = 60/61 \times 366{\cdot}2422/365{\cdot}2422 \text{ sidereal time seconds.}$$

So the difference between a sidereal time second and a dot interval is:

$$\left(1-\frac{60}{61}\times\frac{366{\cdot}2422}{365{\cdot}2422}\right)^{\mathrm{s}} = 0^{\mathrm{s}}{\cdot}01370 = 1/72{\cdot}99 \text{ ST seconds.}$$

This means that on a sidereal time chronometer coincidences are separated by approximately 73 dot intervals, that is, they occur about every 72 sidereal time seconds.

Let the first coincidence fall on dot number p, then the fifth coincidence will fall on dot number $p+4\times 73 = p+292$. As there are 306 dots, five coincidences will be obtained only if $p<14$. If $p>14$ then four coincidences will be found. This assumes that the whole time signal is used. Let:

$$k = \frac{60}{61}\times\frac{366{\cdot}2422}{365{\cdot}2422} = 0{\cdot}986\,2996$$

$n_1, n_2, \ldots n_5$ = the signal dot number of the 1st . . . 5th coincidence *counting the first dash at the start of transmission as zero.*

T_0 = ST chronometer time of the start of transmission.

$T_1, T_2, \ldots T_5$ = ST chronometer time of the 1st . . . 5th coincidence, by definition an exact clock second.

Then:

$$\begin{aligned} T_0 &= T_1 - k\,.\,n_1 \\ &= T_2 - k\,.\,n_2 \\ &= T_3 - k\,.\,n_3 \\ &= T_4 - k\,.\,n_4 \\ &= T_5 - k\,.\,n_5 \end{aligned}$$

In practice the method of finding T_0 differs slightly from that given above.

The wireless receiver, a pair of earphones, and the chronograph, are connected via a switchbox in such a way that each time the clock makes the circuit to mark the chronograph sheet, it cuts out the wireless signals. This means that the signals are suppressed for about $0^{s}5$ at the beginning of each second of the clock. As long as the time signals alternate with the cut-out periods, they are heard unimpaired.

The dot interval is less then a clock second however, so the two gradually converge until the dots are falling entirely in the cut-out periods; this gives a silence of approximately 36 seconds. At the instant of the first dot heard after the silence, the beginning of the dot and the beginning of the clock second obviously coincide very closely. This is shown in Fig. 5.9 (a), (b), and (c). (Not to scale). Fig. 5.9 (b) and (c) represent two extreme cases of coincidence. In (b) the dot of exact coincidence is the one preceding the first one heard, and the assumed clock second of coincidence is therefore wrong by one second which means an error in T_0 of $0^{s}014$ as a maximum. Putting it in algebraic form, the assumed clock second of coincidence gives:

$$T_0 = T - k \,.\, n$$

whereas the correct value is:

$$T_0 = (T-1) - k \,.\, (n-1)$$

the difference being $(1-k)$ which is $0^{s}014$.

In (c) the audible part of the first dot is very small, and the assumed clock second of coincidence is almost exactly right, giving a minimum error in T_0. To keep errors from this source as small as possible, the assumed clock time of coincidence is taken to be $0^{s}5$ before the whole second corresponding to the first audible dot. This halves the maximum error given above, making the greatest possible error $0^{s}007$.

Accepting the definitions above, the practical equations for T_0 are:

$$\left.\begin{aligned} T_0 &= (T_1-\tfrac{1}{2}) - k \,.\, (n_1-\tfrac{1}{2}) \\ &= (T_2-\tfrac{1}{2}) - k \,.\, (n_2-\tfrac{1}{2}) \\ &= (T_3-\tfrac{1}{2}) - k \,.\, (n_3-\tfrac{1}{2}) \\ &= (T_4-\tfrac{1}{2}) - k \,.\, (n_4-\tfrac{1}{2}) \\ &= (T_5-\tfrac{1}{2}) - k \,.\, (n_5-\tfrac{1}{2}) \end{aligned}\right\} \tag{5.21}$$

This assumes for the moment that the chronometer has no rate; this is discussed below.

It is hardly convenient in practice to count the signal dots to find $(n-\frac{1}{2})$, so the operator records the first minute dash of the transmission and as many of the following minute dashes as he can, subject to recording the coincidences, which is the prime task. The minute dashes enable an approximate value of T_0 to be scaled from the chronograph record to an accuracy of about $0^{s}2$; obviously the first dash is the most convenient, the remainder serving as checks, or as an insurance against faulty recording of the first.

Let this approximate scaled value be T_0', then approximate values of $k \,.\, (n_1-\frac{1}{2})$, $k \,.\, (n_2-\frac{1}{2})$, . . . , $k \,.\, (n_5-\frac{1}{2})$ are given by:

$$\left.\begin{aligned} (T_1-\tfrac{1}{2}) - T_0' &= k \,.\, (n_1-\tfrac{1}{2}) \\ (T_2-\tfrac{1}{2}) - T_0' &= k \,.\, (n_2-\tfrac{1}{2}) \\ (T_3-\tfrac{1}{2}) - T_0' &= k \,.\, (n_3-\tfrac{1}{2}) \\ (T_4-\tfrac{1}{2}) - T_0' &= k \,.\, (n_4-\tfrac{1}{2}) \\ (T_5-\tfrac{1}{2}) - T_0' &= k \,.\, (n_5-\tfrac{1}{2}) \end{aligned}\right\} \tag{5.22}$$

A table of $k \,.\, (n-\frac{1}{2})$, to three decimal places, is prepared taking every integral value of n from 1 to 305, and since the approximate values of $k \,.\, (n-\frac{1}{2})$ found from (5.22) will be correct to about $0^{s}2$, inspection in the table will show what the correct value of $k \,.\, (n-\frac{1}{2})$ should be. Take the value from the table which is nearest the approximation.

Rate. The chronometer will usually have a rate, defined as gaining (+), or losing (−). As the rate is generally appreciable, the five values of T_0 found in (5.21) must each have a correction made for it; the amount to apply will clearly be a function of n. The differences between determinations of the chronometer error at successive time signals give the rate; it is given as so much per hour, and should be constant within $0^{s}.01$ between two successive time signals unless they are from different transmitters. The rate will also vary over a short period if the clock has been wound between time signals, a practice which must be avoided.

For the purpose of correcting the values of T_0 at (5.21), a rate table is prepared for half-minute intervals from 0 to 5 minutes, – the duration of the time signal. The rate correction to any particular value of T_0 is then taken from the rate table with $k \,.\, (n-\frac{1}{2})$ as argument. Putting it another way:

$$r = \text{Rate for } 5^{m} \times k \,.\, (n-\tfrac{1}{2})/5$$

where r is the correction for rate.

With a losing rate the assumed time of coincidence in (5.21) is *deficient* by r. With a gaining rate the assumed time of coincidence in (5.21) is *too great* by r.

Then we get finally for T_0:

$$\left.\begin{aligned} T_0 &= (T_1-\tfrac{1}{2})-k \,.\, (n_1-\tfrac{1}{2})-r_1 \\ &= (T_2-\tfrac{1}{2})-k \,.\, (n_2-\tfrac{1}{2})-r_2 \\ &= (T_3-\tfrac{1}{2})-k \,.\, (n_3-\tfrac{1}{2})-r_3 \\ &= (T_4-\tfrac{1}{2})-k \,.\, (n_4-\tfrac{1}{2})-r_4 \\ &= (T_5-\tfrac{1}{2})-k \,.\, (n_5-\tfrac{1}{2})-r_5 \end{aligned}\right\} \tag{5.23}$$

where r is subtracted algebraically.

If the rate is unknown, it is assumed provisionally to be zero, and the equations at (5.21) are used; these are then corrected when a reliable estimate of the rate has been made.

The mean of all the values in (5.23) is accepted for T_0.

Correction to T_0 for travel time of signal. It takes a finite time for the rhythmic time signals to travel from the transmitter to the receiver, the result being that T_0 requires reduction by the time of travel to obtain the chronometer time of the start of emission. *Bulletin Horaire* (1950) gives the velocity of long waves as 252,000 km./sec. The velocity of short waves will be different, but not enough to make any significant change in the correction for travel time.

Let the velocity be V, and the transmitter-receiver distance D, then the correction is:

$$D/V \text{ seconds of time } (D \text{ in the same units as } V).$$

This gives the correction as:

$$D \times 0{\cdot}000\,00\,639 \text{ seconds of time, where } D \text{ is in miles.}$$

or

$$D \times 0{\cdot}000\,00\,397 \text{ seconds of time, where } D \text{ is in km.}$$

or

$$D \times 0{\cdot}000\,441 \text{ seconds of time, where } D \text{ is in } \textit{degrees} \text{ of arc.}$$

To a sufficient accuracy:

$$\cos D = \sin \varphi_T \,.\, \sin \varphi_R + \cos \varphi_T \,.\, \cos \varphi_R \,.\, \cos \Delta\lambda$$

The suffixes refer to the transmitter (T) and receiver (R). Typical values in the Ordnance Survey programme were: Moscow (Russia) $0^{s}.010$; Pontoise (France) $0^{s}.002$; Norddeich (Germany) $0^{s}.003$.

The correction is invariably subtracted from T_0.

Correction for error in time of emission. The time signals are alleged to start on an exact instant of GMT. In practice, however, there is usually a small and variable error in the time of emission. The Bureau International de l'Heure publishes the correct time of emission in the *Bulletin Horaire* for a large number of transmitters. These correct times are published in two forms, *Bulletin Horaire Série 3* (demi-définitif), and *Bulletin Horaire Série E* (définitif). The 'heure demi-définitive' is issued about two months after the date of the signals and contains the GMT at which the signals were received by the Bureau International de l'Heure, the time being checked by Paris Observatory. By subtracting from the heure demi-définitive the time of travel from the transmitter to Paris, the GMT of emission is found. This time is correct to about $0\overset{s}{.}02$.

The 'heure définitive' is published about six months after the date of the signals, and is the GMT of *emission* as calculated from the time of reception at several observatories. It is, of course, a more accurate assessment of the GMT of emission than that deduced from the heure demi-définitive. The definitive values only were used by the Ordnance Survey.

Assuming that T_0 has been found, corrected for rate and travel time, and that the definitive time of the start of the rhythmic time signal is available, then the chronometer error, e, is given by:

$$T_0 - \text{GST}_s = e$$

where GST_s is the Greenwich Sidereal Time equivalent of the definitive GMT of the start of the time signal. The error, e, is defined as fast (+), or slow (−).

5.047 CLOCK COMPARISONS IMMEDIATELY AFTER TIME SIGNALS

The purpose of these comparisons was to find the errors of the two auxiliary mean time chronometers. Briefly, the procedure was to relate the mean time chronometers to the rhythmic time signal via the master sidereal time chronometer.

The master was connected to pen No. 1 and an auxiliary to pen No. 2, and the chronograph was run for a minute. Seven comparisons were read at exact 10^s intervals of the mean time clock. Let $S_0, S_1, \ldots, S_6$ be the seven master clock readings coinciding with the M_0, $(M_0+10^s), \ldots, (M_0+60^s)$, readings on the mean time clock. Then each comparison gave a value of S_0, the ST clock time of the first comparison, thus:

$$\begin{aligned} S_0 &= S_0 \\ &= S_1 - 10\overset{s}{.}03 \\ &= S_2 - 20\overset{s}{.}05 \\ &= S_3 - 30\overset{s}{.}08 \\ &= S_4 - 40\overset{s}{.}11 \\ &= S_5 - 50\overset{s}{.}14 \\ &= S_6 - 60\overset{s}{.}16 \end{aligned}$$

where $10\overset{s}{.}03$, $20\overset{s}{.}05$, etc, are the sidereal time equivalents of the mean time intervals from M_0. The mean of the seven was accepted as the reading of the master clock when the mean time clock read M_0. Let this mean value be $\bar{S}_0$.

At a time signal at GMT_s the master read T_0, so:

$$(\bar{S}_0 - T_0 - r) \times 0{\cdot}997\,2696$$

is the *mean time interval* from GMT_s to $\bar{S}_0$, r being the master rate for the interval $(\bar{S}_0 - T_0)$. Thus the GMT of comparison, GMT_c, is:

$$\text{GMT}_c = \text{GMT}_s + (\bar{S}_0 - T_0 - r) \times 0{\cdot}997\,2696$$

and the error of the auxiliary mean time clock is:

$$\begin{aligned} e &= M_0 - \mathrm{GMT}_c \\ &= M_0 - \mathrm{GMT}_s - (\bar{S}_0 - T_0 - r) \times 0{\cdot}997\,2696 \end{aligned}$$

e has the usual sign, fast (+), or slow (−).

This was carried out for both auxiliary mean time clocks. The clock errors were plotted in graphic form to simplify interpolation.

5.048 CLOCK COMPARISONS BETWEEN TIME SIGNALS

Let the two auxiliary mean time clocks be indicated by suffixes a and b. Compare each mean time clock with the master as described in § 5.047 above to find: $\bar{S}_{0.a}$, $M_{0.a}$; $\bar{S}_{0.b}$, $M_{0.b}$; *where b was compared after a.*

Then:

$$M_{0.b} - (\bar{S}_{0.b} - \bar{S}_{0.a} - r) \times 0{\cdot}997\,2696 = M'_{0.b}$$

where $M'_{0.b}$ is the clock b time corresponding to the same master time, $\bar{S}_{0.a}$, as $M_{0.a}$ on clock a. The master rate, r, should be applied unless the interval $(\bar{S}_{0.b} - \bar{S}_{0.a})$ is small enough to make r negligible. From curves of the clock errors find e_{master}, e_a, and e_b, for the clock comparison times $\bar{S}_{0.a}$, $M_{0.a}$, and $M'_{0.b}$ respectively.

Then:

$$\begin{aligned} \bar{S}_{0.a} - e_{\text{master}} &= \text{GST of comparison according to master.} \\ M_{0.a} - e_a &= \text{GMT of comparison according to clock } a. \\ M'_{0.b} - e_b &= \text{GMT of comparison according to clock } b. \end{aligned}$$

Converting the two GMT to GST gives three values for the GST of comparison. The scatter should not exceed $0^{\mathrm{s}}1$.

The purpose of the comparisons between time signals was to check on the behaviour of the clocks. In an emergency the mean of the three GST could also be used in lieu of a time signal to obtain a mean estimate of the error of the master clock.

5.049 CORRECTION TO MEAN STAR TIME FOR MICROMETER CONTACT WIDTH

During observations for position lines, and azimuth by Black's method, the contact drum in the eyepiece micrometer of the T4 Theodolite automatically recorded a trace on the chronograph, as described in § 5.030. All such recordings, called blips, were too early by the time taken for the drum to move from the leading edge of the contact to the middle. As a consequence, each of the mean star times taken from the chronograph sheet required a correction. The average width of a contact was found to be 0·01172 of a drum revolution (see § 5.030). The correction for a star was:

+0·00586 × average time taken on the star for one contact drum revolution.

The chronograph record of one drum revolution is shown in Fig. 5.3. Four of these were recorded on each face on an azimuth star, and the mean of the four equivalent clock time intervals gave the average time for a drum revolution. Two drum revolutions were recorded on each face of a position line star, and in this case the mean of the two equivalent clock time intervals gave the average time for a drum revolution.

This correction did not apply, of course, to the chronograph recordings made for azimuth from Polaris, as the latter were recorded manually. See § 5.050(k) below.

5.05 Field Procedure

5.050 PROCEDURE FOR AZIMUTH OBSERVATIONS

(*a*) On each star the observing routine was:

Referring Object→Star→Change Face→Star→Referring Object.

(*b*) The collimation error of the vertical wire was kept down to about 10 seconds of arc. The exact 'vertical wire' which was thus limited was the position of the moving wire when recording on the chronograph the centre of the contact closure nominally corresponding to comb division number 10. See Fig. 5.2. (As already described in § 5.030, the contact which nominally coincides with the comb divisions is flanked by two marker contacts, which serve to distinguish the comb divisions on the chronograph record). The purpose of minimizing the collimation in this way was to ensure that intervals of time from the centre wire were true intervals from the line of collimation.

The verticality of the moving wire was checked. This was important as observations were made at considerable distances from the horizontal wire, and although the procedure given in (*c*) below did much to eliminate possible error, the procedure could not be followed perfectly.

(*c*) With the eyepiece micrometer in the horizontal position each star was followed from comb division number 8 to comb division number 12, or vice versa, on each face. The star's 'vertical' position on comb division number 12 (or 8) was noted, and after changing face, the observer started back from the same position, which was then below the cross wire instead of above it, or vice versa.

(*d*) The hanging level was read before and after each eyepiece micrometer run on each face, that is, four times per star.

(*e*) The horizontal plate micrometer was read three times on each face.

(*f*) The vertical circle was read on both faces to the star to the nearest minute or so.

(*g*) The hanging level was read once on each face to the referring object.

(*h*) The horizontal plate micrometer was read three times on each face to the referring object. If the mark showed any tendency to drift about when watched for half a minute or so, more pointings were taken to it, either with the horizontal plate slow motion screw, or with the eyepiece micrometer. At night horizontal refraction is apt to take the form of irregular, slow drift over a period of several seconds, particularly over flat ground.

(*i*) The vertical circle was read on both faces to the referring object once only at each station.

(*j*) At each station the programme was 16 stars on both faces, with the following zero settings on the referring object. These zeros need only be set to the nearest 10″ or 15″.

Approximate Stellar Azimuth	*R.O. Setting*	*Approximate Stellar Azimuth*	*R.O. Setting*
00°	00° 00′ 00″	180°	90° 00′ 03″
22½	11 15 07	202½	101 15 11
45	22 30 15	225	112 30 18
67½	33 45 23	247½	123 45 27
90	45 00 30	270	135 00 33
112½	56 15 37	292½	146 15 41
135	67 30 45	315	157 30 48
157½	78 45 53	337½	168 45 57

The zero settings were spaced at 11¼°, corresponding to 16 stellar azimuths at 22½° intervals. If the scheme is followed exactly, neither the R.O. reading nor the star reading will repeat itself, although, as stated below, an azimuth tolerance of 10° or so may be allowed, and a repetition may then result.

(*k*) For observations for azimuth from Polaris the routine was as given above with the exceptions of (*c*) and (*d*). The star was taken as it crossed the centre vertical wire, and the instant of transit was recorded manually on the chronograph by means of a hand switch, or hand tappet. The hanging level was read once only on each face on the star, that is, twice on each zero.

In the Black method stars may be observed in any convenient order, but there is possibly some merit in keeping to a balanced programme, that is, taking stars in each quadrant in pairs 180° apart in azimuth. A star can be accepted if within 10°, or even 15°, of the preferred azimuth. The slight lack of balance makes only a very trivial difference to the strength of the least squares solution. Normal altitude limits are about 10° to 20°, but 5° can be accepted if the star is otherwise acceptable. Similarly a suitable star should not be ignored if it is slightly above 20°. Magnitude limits should be such as to satisfy the observer that they provide good marks, of a quality that he would accept for observing primary horizontal angles, and remembering that he is making a large number of intersections on each star which will tend to cancel out random errors of observation. Provided it looks circular, a star which is too bright is preferable to one which is too faint.

5.051 PROCEDURE FOR POSITION LINE OBSERVATIONS

(*a*) With the eyepiece micrometer in the vertical position each star was followed from comb division number 9 to comb division number 11 (or vice versa) on each face.

(*b*) The vertical circle bubble was read before and after each eyepiece micrometer run on each face.

(*c*) The vertical circle micrometer was read three times on each face.

(*d*) The horizontal plate was read to the nearest second or so. This was for the benefit of the observer, who used it with a Polaris pointing to obtain the approximate azimuths of the stars for balancing purposes. See (*f*) below.

(*e*) Barometric pressure and air temperature were recorded.

(*f*) The programme at a station was at least four sets of stars, two sets on face left and two sets on face right. A set comprised four stars observed on the same face, with the stars disposed one in the middle of each quadrant, that is, at azimuths of approximately 45°, 135°, 225°, and 315°. Altitudes were between 30° and 60°, and where possible over 40°. When convenient the stars were observed in pairs 180° apart in azimuth. When possible the minimum programme was exceeded, and each star observed on both faces.

5.052 PROCEDURE FOR RECEIVING RHYTHMIC TIME SIGNALS

(*a*) Pen No. 1 was connected to the master clock, and pen No. 2 to a hand tappet through the switchbox.

(*b*) The switchbox was set so that the time signal was heard continuously in the headphones, and the first minute dash was awaited. Picking up the rhythm of the dots the hand tappet was pressed on the *third* dot after the minute dash. This recorded the start of the signal; every minute dash of the signal was recorded similarly if convenient. See (*e*) below. As there was silence after the last dash, which ended the signal, the observer counted three dots from memory of the dot rhythm when recording the last dash.

(*c*) After a minute dash had been recorded, a switch was thrown immediately; this routed the time signal so that it was cut out during the half-second clock beats recorded by pen No. 1. Throwing the switch produced either immediate silence, or the dots were heard unimpaired. If the dots were heard, however, they eventually converged with the clock beat, resulting in silence. The first dot heard after the silence was a coincidence. Counting this as dot number one, the hand tappet was pressed on dot number three (see § 5.046 for basic principles).

(*d*) After recording a coincidence the cut-out switch was returned to normal, and the sequence repeated from (*b*). Thus ideally each minute and each coincidence of the whole signal was recorded.

(*e*) As a mean time signal was being recorded against a sidereal time clock, the observer had to choose occasionally between recording a minute and recording a coincidence, the two occurring very close together, or even coinciding. The coincidence always took priority. Two minute dashes and two coincidences were considered a minimum.

(*f*) Time signals were recorded before, during, and after observing, as often as possible.

5.053 PROCEDURE FOR CLOCK COMPARISONS

(*a*) Pen No. 1 was connected to the master clock, and pen No. 2 to the auxiliary clock via a hand tappet.

(*b*) The chronograph was run for at least a minute. As the auxiliary clocks were not modified to distinguish minutes automatically, the hand tappet was used as follows. On the exact minute of the auxiliary clock the hand tappet was pressed smartly to mutilate the second mark on the pen No. 2 trace. This was sufficient to indicate the minute mark on the auxiliary clock trace.

(*c*) The sequence was repeated from (*a*) for the second auxiliary clock.

(*d*) Clock comparisons were made after each time signal, every 1½–2 hours as convenient during observing, and at the end of work.

5.054 PROCEDURE FOR RECORDING THE PEN EQUATION

(*a*) Both pens were connected to the master clock and the chronograph was run for at least a minute. (At least one minute mark must be recorded.)

(*b*) A pen equation was recorded at the beginning and end of each chronograph sheet, and additionally if the pens had been disturbed for any reason, e.g. re-filling.

5.055 ANNOTATION OF CHRONOGRAPH SHEETS

This covered the following items. Station; date; sheet number, e.g. sheet 1 of 3; GMT of at least two minute marks each time a fresh start was made with the master clock trace; pen equation; clock comparisons, with clock numbers and minutes marked; time signals, with transmitter details; star names. If time signals were only received in part, full details were given, especially time of start, and faulty or missed coincidences. In all cases during time signals each clock minute was marked with its time.

5.06 Field Observations

Table 5.1 shows details of all the observations made at the various stations. Not all of these observations were used however, because of such troubles as misidentification and non-identification, stars not listed in the ephemeris, single-face pointings, and failure to close on the referring object (in azimuth programme). Occasional pointings gave discordant results, and were rejected. In most cases the observer was aware of his faulty results, and took steps to provide suitable replacements; this was essential where balance was important, as in the Black method for azimuth. With position lines some rejections were made to avoid asymmetry.

TABLE 5.1

CHRONOLOGY OF FIELD OBSERVATIONS

P = Polaris; B = Black Method; * = On different zeros; F.L. = Face Left; F.R. = Face Right; D.F. = F.L. and F.R.

Station and date of arrival	*Night of Obs'ns*	*Observations for*	*Total Number of Stars Observed*			*Rejections*									*Finally Accepted Observations*		
						Misidentified			*Not Listed in Ephemeris*			*Faulty or Deficient Observations*					
	1953		F.L.	F.R.	D.F.	F.L.	F.R.	D.F.	F.L.	F.R.	D.F.	F.L.	F.R.	D.F.	F.L.	F.R.	D.F.
Herstmonceux	3 May	Azimuth (B)			8												8
(481)	5 May	Azimuth (B)			4									1			3
2 May	6 May	Azimuth (B)			4												4
	7 May	Azimuth (B)			7									1			6 (*a*)
	8 May	Azimuth (B)			6									4			2 (*b*)
	9 May	Azimuth (P)			*14												*14
	10 May	Azimuth (P)			*2												*2
	10 May	Azimuth (B)			8						1			1			6
Fairlight Down	12 May	Azimuth (B)			12			1			1						10
(193)	13 May	Azimuth (B)		1	2								1				2
11 May	16 May	Azimuth (B)	1	1	2							1	1				2
	17 May	Azimuth (B)			1												1
	18 May	Azimuth (B)	1		5			1				1		2			2 (*c*)
	19 May	Azimuth (B)			12									2			10
White Horse Hill	23 May	Azimuth (B)			1												1
(34)	24 May	Azimuth (B)			12									3			9
21 May	26 May	Azimuth (B)			1												1
	27 May	Azimuth (B)			1												1
	27 May	Position			1												1
	28 May	Position	4	4	1	1							1		3	3	1
	30 May	Azimuth (B)			3									1			2
	30 May	Position			2	One D.F. rejected to avoid unbalanced sets.											1
Liddington Castle	1 June	Azimuth (B)			6												6
(35)	6 June	Azimuth (B)			6												6
1 June	8 June	Azimuth (B)			7						1						6
St. Agnes Beacon	13 June	Azimuth (B)	1		3							1					3
(175)	17 June	Azimuth (B)			1						1						0
11 June	17 June	Position	2	2	6										2	2	6
	22 June	Azimuth (B)	1		11			1				1		3			7 (*d*)
	23 June	Azimuth (B)	1		9			2			1	1		1			5
Tregonning Hill	3 July	Azimuth (B)			13			1			1						11
(181)	4 July	Azimuth (B)		1	4								1				4
25 June	6 July	Azimuth (B)		1									1				—
	7 July	Azimuth (B)			5												5

TABLE 5.1 *continued*

CHRONOLOGY OF FIELD OBSERVATIONS

P = Polaris; B = Black Method; * = On different zeros; F.L. = Face Left; F.R. = Face Right; D.F. = F.L. and F.R.

Station and date of arrival	*Night of Obs'ns*	*Observations for*	*Total Number of Stars Observed*			*Rejections*									*Finally Accepted Observations*		
						Misidentified			*Not Listed in Ephemeris*			*Faulty or Deficient Observations*					
	1953		F.L.	F.R.	D.F.	F.L.	F.R.	D.F.	F.L.	F.R.	D.F.	F.L.	F.R.	D.F.	F.L.	F.R.	D.F.
Cairn Pat	15 July	Azimuth (B)			9			2									7
(360)	20 July	Position		2	8			1									6 (*e*)
13 July	21 July	Position			2												2
	21 July	Azimuth (B)			4			1									3
	27 July	Azimuth (B)			3									1			2
	28 July	Azimuth (B)		1	4						3		1				1
	29 July	Azimuth (B)			1												1
Inshanks	31 July	Azimuth (B)		1	14						2		1	1			11
(361)	1 Aug.	Azimuth (B)			6						1			1			4
31 July																	
Greenwich	17 Aug.	Position			12												4
Observatory (482)	18 Aug.	Position		2	13												10
(Auxiliary)	21 Aug.	Position			2												2
17 Aug.	22 Aug.	Position	1	2	19			3									14 (*f*)
	23 Aug.	Position			3												2
	24 Aug.	Position	4	3	9									1			4
Herstmonceux	26 Aug.	Position	5	4	19	All single-faced pointings were rejected, together with D.F. pointings which produced unbalanced sets.											12
(481)	27 Aug.	Position	1	1	13												6
26 Aug.	28 Aug.	Position	1														— (*g*)
	30 Aug.	Position	4	3	23												18
	31 Aug.	Position			18												18
Spital Hill	7 Sept.	Azimuth (B)			14			1									13
(398)	9 Sept.	Azimuth (B)			2												2
7 Sept.	11 Sept.	Position			8	All single-faced pointings were rejected, together with D. F. pointings which gave unbalanced sets.											5
	12 Sept.	Position	2		4												3
Warth Hill	14 Sept.	Position			6												4
(399)	15 Sept.	Position			5									1			2 (*h*)
14 Sept.	18 Sept.	Position			6												6
Fetlar	28 Sept.	Position	1	1	11	All single-faced pointings were rejected, together with D. F. pointings which produced unbalanced sets.											10
(459)	29 Sept.	Position		1	2												0
25 Sept.																	
	1 Oct.	Position			6												6
	1 Oct.	Azimuth (B)	1		8							1					8
	2 Oct.	Azimuth (B)			2												2
	3 Oct.	Azimuth (B)			5			1									4
	5 Oct.	Azimuth (B)			3			1			1						1

TABLE 5.1 *continued*

CHRONOLOGY OF FIELD OBSERVATIONS

P = Polaris; B = Black Method; * = On different zeros; F.L. = Face Left; F.R. = Face Right; D.F. = F.L. and F.R.

Station and date of arrival	*Night of Obs'ns*	*Observations for*	*Total Number of Stars Observed*			*Rejections*									*Finally Accepted Observations*			
						Misidentified			*Not Listed in Ephemeris*			*Faulty or Deficient Observations*						
	1953		F.L.	F.R.	D.F.	F.L.	F.R.	D.F.	F.L.	F.R.	D.F.	F.L.	F.R.	D.F.	F.L.	F.R.	D.F.	
Saxavord (463) 7 Oct.	13 Oct.	Azimuth (B)			18			1						1			16	
Warth Hill (399) 18 Oct.	18 Oct.	Position			4												4	(*i*)
	18 Oct.	Azimuth (B)			3												3	
	19 Oct.	Azimuth (B)			3												3	
	23 Oct.	Azimuth (B)			12			1									11	
Herstmonceux (481) 30 Oct.	30 Oct.	Azimuth (B)			17									1			16	
	1 Nov.	Azimuth (B)			1												1	
	1 Nov.	Azimuth (P)			*9												*9	
	2 Nov.	Azimuth (P)			*23												*23	
	2 Nov.	Azimuth (B)			17												17	
Fairlight Down (193) 4 Nov.	4 Nov.	Azimuth (B)			7												7	
	6 Nov.	Azimuth (B)			2												2	
	6 Nov.	Azimuth (P)			*16												*16	
	9 Nov.	Azimuth (B)			13												13	
	9 Nov.	Longitude		1	9	Single-faced pointing rejected.											9	(*j*)
	10 Nov.	Longitude			10									1			9	
	10 Nov.	Azimuth (B)			3			1						1			1	
	11 Nov.	Azimuth (B)			4			2									2	
	11 Nov.	Azimuth (P)			*16												*16	
	12 Nov.	Azimuth (B)			5												5	

KEY TO REMARKS

(*a*) One D.F. not closed on R.O.
(*b*) T4 telescope lighting failed.
(*c*) Much cloud and mist.
(*d*) Two D.F. did not record on the chronograph.
(*e*) Single faces rejected, together with D.F. which produced unbalanced sets.
(*f*) All single faces rejected. Also such D.F. as were necessary to produce balance. Includes 16 D.F. stars near the prime vertical, as longitude was the main requirement.
(*g*) Includes 18 D.F. stars near the prime vertical to strengthen the longitude determination.
(*h*) Certain D.F. rejected to get balanced sets.
(*i*) Combined with observations of the 14–18 Sept., above.
(*j*) Only east and west stars observed.

At the first station the observer prepared a star programme for Laplace azimuth by computing the times of suitably disposed stars all round the horizon, and taking the stars as they reached the required position. The effort involved was considerable, and because of normal English weather, much of it was wasted when selected stars were obscured. A more simple practical method was adopted using an astronomic globe showing the constellations, and a stellar atlas. This enabled constellations to be selected at the required altitudes, and in the required azimuths. The globe was 6 inches in diameter, and was made by Cary & Co., London. Its equator was graduated in time, 0^h to 24^h eastward from the vernal equinox, and it could be rotated about its polar axis, this motion taking place inside a brass meridian circle which held the polar pivots. The brass meridian circle was graduated 0°–90° north and south from the globe's equator, and this circle, together with the pivoted globe, could be rotated about the globe's equatorial diameter. A horizontal plane through this equatorial diameter formed the horizon, or azimuth plane. Knowing the approximate latitude and longitude of the station, a model of the celestial sphere could be set up on the astronomic globe for any given GST, the latter being known. The stellar atlas provided details of star names and magnitudes, and was also used to identify the selected stars. The globe also had a detachable altitude quadrant, which proved useful in restricting the choice of constellations to the prescribed altitude limits.

The ephemeris used for all calculations was *Apparent Places of Fundamental Stars, 1953*, and it was the observer's practice to check where possible that his selected stars were listed in the ephemeris. Occasionally, unlisted stars were taken when a quick change in the selected programme had to be made because of cloud or haze. These deficiencies were invariably made good later.

The observer kept a vector diagram (see § 5.041), and any lack of balance was virtually eliminated by carefully selecting the azimuth and altitude of the last three or four stars in the programme. This meant more flexibility in the earlier part of the programme, enabling the observer to take advantage of gaps in the clouds as they occurred without being unduly concerned with meticulous balancing.

Position lines hardly needed a programme. Because of the higher altitude limits it was always easier to get position line stars than azimuth stars, in spite of the mid-quadrantal azimuth restriction on the former. A minimum, balanced, position line programme, as defined in § 5.051 (*f*), was not quite achieved at White Horse Hill (34), but the result was considered satisfactory.

One unsatisfactory feature of the azimuth work was the time taken on occasions to observe the full, balanced, programme of stars required for Black's method. This was not of course the fault of the method, but resulted from the capricious weather peculiar to this country which is too well known to require comment. This affected particularly the azimuth programme, with its fairly stringent requirements of balance in altitude and azimuth. A striking example occurred at Saxavord (463), where for five consecutive nights no stellar observations were possible, but on the sixth night a complete Black programme was observed. Table 5.1 shows quite a number of completely blank nights at stations after the dates of arrival. It also shows restricted observations on other nights, together with the exasperating occasions when fleeting cloud prevented the completion of a double-faced pointing for azimuth; this was almost invariably the cause of single-faced azimuth pointings.

Table 5.2 shows the transmitters from which rhythmic time signals were received. As stated in § 5.033, the short wave transmitters could not be received until the second wireless set was obtained on the 6th August. However, the reliability of the sidereal master chronometer made this shortcoming more of a nuisance than a problem.

TABLE 5.2

RHYTHMIC TIME SIGNAL TRANSMITTERS

Transmitter	*Country*	*Call Sign*	*Wavelength (metres)*	*Reputed GMT of Emission*		*Remarks*
				h	*m*	
Pontoise	France	FYP	3300·3	20	01	
				21	01	
				22	31	
				08	01	
				09	01	
				09	31	
Norddeich	Germany	DAN 1	2290·0	00	01	
Rugby	England	GBR	18750·0	10	01	Received once only
Moscow	Russia	RES	3333·0	22	01	
				02	01	
Moscow	Russia	RWM 1	29·85	22	01	
				00	01	
				02	01	
				04	01	
Moscow	Russia	RWM 2	55·76	22	01	
				04	01	
Moscow	Russia	RWM 3	24·47	06	01	
Moscow	Russia	RWM 4	39·01	22	01	
				00	01	
				02	01	
				04	01	

5.07 Office Work

The principles and methods of calculation, and all necessary corrections, are given in § 5.04. Eight-figure natural trigonometrical functions were used for the major spherical calculations.

Provisional values were found by accepting the reputed times of emission of the time signals, but no least squares computations were carried out until definitive times of emission were available. The main reason for computing provisional results was to verify the field work.

In azimuth computations for the Black method, each face on each star was calculated separately, and the resulting F.L. and F.R. azimuths meaned. A check calculation was then done from the mean of the observations on the two faces, thus computing the mean azimuth directly. In the latter case the curvature correction at § 5.042 (*b*) had to be applied for the whole period of the two faces, and was appreciable. Over the period of a single face it was very small, rarely affecting the first decimal place of the seconds in the azimuth. Meaned faces were used for computing azimuth from Polaris. When calculating astronomic azimuth from Polaris observations the astronomic latitude and longitude were used; the azimuth in the Black method is, of course, computed with geodetic latitude and longitude. As stated at the end of § 5.041 the least squares correction, $(A_G - \bar{A})$, to the mean azimuth, $\bar{A}$, did not exceed $0\overset{''}{.}1$, and was usually smaller. (see the last two columns of Table 5.4 below.) For this reason it was found more convenient when carrying out the least squares solutions to take $\bar{A}$ rounded down to a convenient whole number of seconds, instead of taking $\bar{A}$ itself, in the observation equations at (5.19). The azimuth correction found from the solution of the normal equations was then applied to the rounded down mean value. It was purely an arithmetic convenience which ensured that the azimuth correction was of some size; it was also always positive in sign.

In position lines computations each face on each star was calculated separately, and checked by duplicate calculation. Care was taken to obtain balanced sets before computing the final values. For convenience longitudes were calculated in time units.

Because of small mechanical imperfections in the chronograph and in the chronometer make-and-break mechanism, the length of a clock second on the chronograph sheet varied a little. To allow for this when scaling times, the transparent scaling implement shown in Fig. 5.10 was used for reading off subdivisions of a second. It spans two clock seconds, and can be read directly to $0\overset{s}{.}1$, with estimation to $0\overset{s}{.}01$. In use it is laid vertically across the clock trace and moved up or down until it fits the seconds exactly.

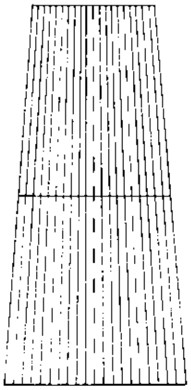


FIG. 5.10. Scaling implement

For the reason given in § 5.030 the sequence of star times on a face, recorded on the chronograph by the T4 micrometer contact drum, must be symmetrical about the centre wire of the eyepiece comb. Therefore if a blip on the chronograph sheet was rejected for any reason, its symmetrical counterpart on the other side of the centre wire was also rejected automatically. Scaling the multiple star times from the chronograph sheets was the most time-consuming part of the office work.

When scaling the clock times from the pen No. 2 record made at time signals, it was important to remember that time signal minute dashes were manually recorded 3 seconds (or dots) after they occurred, and that coincidences were recorded 2½ seconds (or 2 dots plus $0\overset{s}{.}5$) after they occurred. (see § 5.046 and § 5.052.)

The published values of x and y for mean pole corrections (see § 5.042 (*f*) and § 5.044 (*h*)) were given at intervals of 1/20 of a year in the *Bulletin Horaire*. The values were plotted as curves to simplify interpolation for specific dates.

5.08 Results

Table 5.3 shows the astronomic latitudes and longitudes obtained from the accepted position lines observations. The geodetic co-ordinates are also given for comparison. All astronomic positions are referred to the mean pole.

Fig. 5.11 shows the relative positions of the primary station at the Royal Observatory, Greenwich, and the auxiliary station at which the position lines observations were taken.

Table 5.4 shows the various azimuth results at the individual stations. All are referred to the mean pole, but are uncorrected for personal equation. Geodetic azimuths from the triangulation are also given. Transferring reverse Laplace azimuths by the geodetic difference of azimuth from the triangulation, gives the comparable values and means shown in Table 5.5. The second value of each pair is the transferred value. As described in § 5.042 (*g*), the personal equation in azimuth is found by a comparison with an impersonal Laplace azimuth derived from Polaris observations. The original intention was to use the position lines longitude of Herstmonceux (481), namely $+00^\circ\ 20'\ 41''\!.85$, to find the Laplace equation correction to the Polaris astronomic azimuth. However, in 1962, the Astronomer Royal supplied the following definitive value for the astronomic longitude of the Cooke Transit Circle at the Royal Greenwich Observatory, Herstmonceux:

Cooke Transit Circle = $+00^\circ\ 20'\ 15''\!.630$ (see Chapter 3, § 3.09 and § 3.103).

The geodetic longitude difference, from the triangulation, between the Transit Circle and Herstmonceux (481) primary station is $+26''\!.609$. (See Appendix 10 for the co-ordinates of the primary station, and § 3.08 of Chapter 3 for the co-ordinates of the Transit Circle.)

Assuming that this difference differs negligibly from the astronomic difference, the astronomic longitude of the primary station derived from the Transit Circle is $+00^\circ\ 20'\ 42''\!.239$. This was accepted for finding the impersonal Laplace azimuth at Herstmonceux (481). The Laplace equation corrections were as follows:

At Herstmonceux (481)	*At Fairlight Down (193)*
$\lambda_G = +00^\circ\ 20'\ 45''\!.88$ (From Table 5.3)	$\lambda_G = +00^\circ\ 37'\ 14''\!.06$ (From Table 5.3)
$\lambda_A = +00\ \ 20\ \ 42{\cdot}24$	$\lambda_A = +00\ \ 37\ \ 07{\cdot}74$ (From Table 5.3)
$\Delta\lambda = +\ 3{\cdot}64$	$\Delta\lambda = +\ 6{\cdot}32$
$\Delta\lambda\,.\,\sin\varphi = +\ 2{\cdot}83$	$\Delta\lambda\,.\,\sin\varphi = +\ 4{\cdot}90$

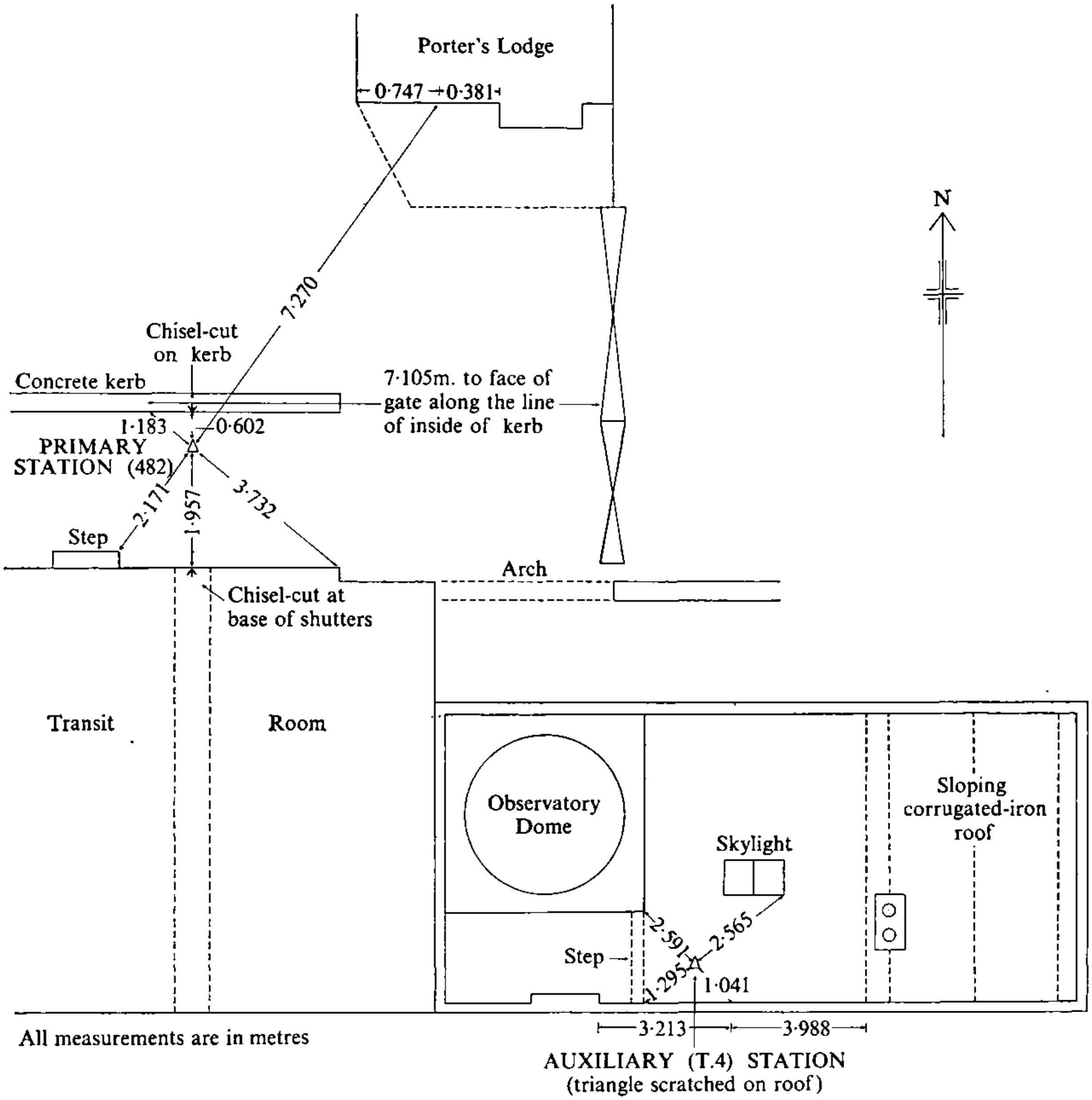
See § 5.020 for details of the Laplace equation.

So the personal equation correction $\Delta A''$, at Herstmonceux (481) at the beginning of the astronomy programme was:

Impersonal A_G from Table 5.5 = $86^\circ\ 07'\ 12''\!.90$
Mean Laplace Azimuth from Table 5.5 = $-86\ \ 07\ \ 12{\cdot}60$
ΔA = $+00{\cdot}30$
$\therefore \Delta t'' = \Delta A''\,.\,\operatorname{cosec}\varphi$ = $+00''\!.39$

And at the end of the programme:

Impersonal A_G from Table 5.5 = $86^\circ\ 07'\ 12''\!.90$
Mean Laplace Azimuth from Table 5.5 = $-86\ \ 07\ \ 11{\cdot}13$
ΔA = $+\ 1{\cdot}77$
$\therefore \Delta t'' = \Delta A''\,.\,\operatorname{cosec}\varphi$ = $+\ 2''\!.28$



All measurements are in metres

The Primary Station is emplaced on the Greenwich Meridian
and is marked by a brass bolt in a hydrant box

The T.4 position is 4·298 metres South of the Airy Transit Instrument
and 9·601 metres East of the Greenwich Meridian

FIG. 5.11. Relative positions of the primary station and the auxiliary station at the Royal Observatory, Greenwich

Taking these two $\Delta t''$ as ordinates, and the dates as abscissae, a straight line was drawn on graph paper. This gave $\Delta t''$ by linear interpolation for any other station according to the mean date of the Black programme observations made there. Let the interpolated value for station x be $\Delta t''_x$, then the personal equation correction, $\Delta A''_x$, to the mean Laplace azimuth in Table 5.5 is:

$$\Delta A''_x = \Delta t''_x \,.\, \sin \varphi_x$$

See § 5.042 (*g*) for further details.

The corrected azimuths are shown in Table 5.6, where they are compared with the geodetic azimuths from the triangulation.

The personal equation in azimuth behaved rather unexpectedly at the end of the programme. The makers of the T4 theodolite claim that the impersonal eyepiece micrometer reduces personal equation to about $0^{s}.03$, which is smaller than the amount found at the end of the programme. Again, an experienced observer using an unfamiliar instrument might be expected to reduce his personal equation with practice, not to increase it, as was apparently the case. If the forward Laplace azimuths only are used from Table 5.5, the personal equation in azimuth at Herstmonceux (481) becomes $-1''.29$ at the beginning and $+0''.17$ at the end, and at Fairlight Down (193) $+2''.50$ at the beginning and $+3''.99$ at the end. The amounts differ considerably at the two stations, but the increase is remarkably consistent. The apparent illogicality in the behaviour of the accepted personal equation is probably due to unknown station errors which have not been eliminated by combining the twin azimuths.

The aim of the azimuth programme was to produce Laplace azimuths at field stations with a standard error of about $0''.5$–$0''.6$. This was achieved, and the final results, and comparisons, in Table 5.6 are considered to be satisfactory.

Appendix 9 gives details of the individual azimuths and position lines at each station.

5.09 Acknowledgements

The geodetic astronomy programme described above was made possible through the generosity of the Department of Surveying, Oxford University, who loaned to the Ordnance Survey all the major equipment described in § 5.03. In addition, the staff of the Department of Surveying gave much practical advice on methods and procedures, all of which played a major part in the successful implementation of the programme.

TABLE 5.3

ASTRONOMIC POSITIONS REFERRED TO MEAN POLE

Station	*Geodetic Position*		*Astronomic Position*					
	φ	λ	φ	λ	*Personal Equation (P.E.)*	λ *(Corrected for P.E.)*	*Standard Error* $\pm\sigma_\varphi$	*Standard Error* $\pm\sigma_\lambda$
Cairn Pat (360)	54° 51′ 48″·797	−05° 02′ 51″·966	54° 51′ 46″·61	−05° 02′ 50″·34	−0″·54	−05° 02′ 50″·88	0″·33	0″·58
Fairlight Down (193)	50 52 36·505	+00 37 14·059	Not observed	+00 37 08·28	−0·54	+00 37 07·74	—	0·41
Fetlar (459)	60 37 14·933	−00 51 46·174	60 37 08·76	−00 51 41·74	−0·54	−00 51 42·28	0·31	0·65
Greenwich Observatory (482) (Auxiliary)	51 28 38·130	+00 00 00·916	51 28 37·88	+00 00 01·04	−0·54	+00 00 00·50	0·24	0·28
Greenwich Observatory (Airy Transit, International Longitude Datum)	51 28 38·265	+00 00 00·418						
Herstmonceux (481)	50 51 55·271	+00 20 45·882	50 51 54·13	+00 20 42·39	−0·54	+00 20 41·85	0·16	0·22
St. Agnes Beacon (175)	50 18 24·241	−05 12 58·655	50 18 25·26	−05 13 01·65	−0·54	−05 13 02·19	0·61	0·99
Spital Hill (398)	58 28 53·026	−03 25 38·513	58 28 49·56	−03 25 24·79	−0·54	−03 25 25·33	0·45	0·94
Warth Hill (399)	58 36 45·089	−03 04 57·013	58 36 38·98	−03 04 43·50	−0·54	−03 04 44·04	0·26	0·50
White Horse Hill (34)	51 34 29·872	−01 33 57·031	51 34 31·59	−01 33 53·32	−0·54	−01 33 53·86	0·71	1·22

Standard errors were derived from the least squares calculations, with the exception of σ_λ for Fairlight Down (193). At this station east–west stars only were observed, and the arithmetic mean of all the longitudes was accepted, the standard error σ_λ being derived from the arithmetic mean. The geodetic latitude of Fairlight Down (193) was used in the calculations instead of the astronomic latitude. Error in the accepted mean longitude will be negligible provided the difference between the geodetic and astronomic latitudes does not exceed about 10″; such a large difference is very unlikely according to the astro-geodetic latitude differences at the other stations.

The astronomic longitude of Greenwich Observatory (482) (Auxiliary) was referred to the international longitude datum by applying the geodetic difference of longitude, thus:

λ_A Greenwich Observatory (482) (Auxiliary)	+00° 00′ 01″·04
Geodetic $\Delta\lambda$	− 0·50
Deduced λ_A International Datum	+00 00 00·54

By definition this meridian is zero*, so the correction for personal equation was −0″·54, to be applied to all position lines longitudes.

* But see Chapter 3, § 3.09.

TABLE 5.4

CALCULATED AZIMUTH RESULTS REFERRED TO MEAN POLE

σ_m = Standard error of the tabulated azimuth

From	*To*	*Geodetic Azimuth from Triangulation*		A_G*, Laplace Azimuth (Black's Method)*				A_A*, Astronomic Azimuth (from Polaris)*				$(A_G-\bar{A})$ *in Black's Method*	
		Forward	*Reverse*	*Forward*	$\pm\sigma_m$	*Reverse*	$\pm\sigma_m$	*Forward*	$\pm\sigma_m$	*Reverse*	$\pm\sigma_m$	*Forward*	*Reverse*
Herstmonceux (481)	Fairlight Down (193)	86° 07′ 14″·12	266° 20′ 00″·68	86° 07′ 13″·55 (3–10 May)	0″·41	266° 19′ 58″·22 (12–19 May)	0″·35	86° 07′ 09″·33 (9–10 May)	0″·42	—	—	−0″·04	+0″·06
Herstmonceux (481)	Fairlight Down (193)			86 07 12·09 (30 Oct.–3 Nov.)	0·35	266 19 56·73 (4–13 Nov.)	0·34	86 07 09·54 (1–2 Nov.)	0·30	266° 19′ 55″·82 (6–11 Nov.)	0″·33	0	−0·04
White Horse Hill (34)	Liddington Castle (35)	234 18 04·25	54 11 52·83	234 18 05·22 (23–30 May)	0·36	54 11 54·07 (1–8 June)	0·39	—		—		+0·01	+0·02
St. Agnes Beacon (175)	Tregonning Hill (181)	206 19 08·97	26 12 34·20	206 19 11·07 (13–23 June)	0·34	26 12 34·46 (3–7 July)	0·28	—		—		−0·06	−0·07
Cairn Pat (360)	Inshanks (361)	159 00 00·83	339 05 59·66	159 00 02·20 (15–29 July)	0·33	339 06 00·66 (31 July–1 Aug.)	0·38	—		—		−0·02	+0·02
Spital Hill (398)	Warth Hill (399)	53 49 19·04	234 06 58·13	53 49 17·50 (7–9 Sept.)	0·38	234 06 56·40 (18–23 Oct.)	0·35	—		—		+0·10	+0·09
Fetlar (459)	Saxavord (463)	03 12 11·62	183 13 26·22	03 12 08·64 (1–5 Oct.)	0·28	183 13 22·78 (13 Oct.)	0·24	—		—		−0·01	−0·02

The last two columns of this table show that the least squares corrections, $(A_G-\bar{A})$, to the arithmetic means, $\bar{A}$, were all completely negligible relative to the standard errors, and that careful balancing of the Black programme in the field can make the least squares calculation unnecessary, even on first order work. However, as the least squares calculations had been done, the values from them were accepted.

TABLE 5.5

MEAN AZIMUTHS

From	*To*	*Laplace Azimuth (Black's Method)*	*Impersonal Laplace Azimuth (From Polaris Observations)*
		86° 07′ 13″55	86° 07′ 12″16
Herstmonceux (481)	Fairlight Down (193)	86 07 11·66	
(Beginning of programme)		86 07 12·60 Mean	
		86 07 12·09	86 07 12·37
Herstmonceux (481)	Fairlight Down (193)	86 07 10·17	86 07 14·16
(End of programme)		86 07 11·13 Mean	86 07 12·90 Mean of 3.
		234 18 05·22	
White Horse Hill (34)	Liddington Castle (35)	234 18 05·49	
		234 18 05·36 Mean	
		206 19 11·07	
St. Agnes Beacon (175)	Tregonning Hill (181)	206 19 09·23	
		206 19 10·15 Mean	
		159 00 02·20	
Cairn Pat (360)	Inshanks (361)	159 00 01·83	
		159 00 02·02 Mean	
		53 49 17·50	
Spital Hill (398)	Warth Hill (399)	53 49 17·31	
		53 49 17·40 Mean	
		03 12 08·64	
Fetlar (459)	Saxavord (463)	03 12 08·18	
		03 12 08·41 Mean	

TABLE 5.6

FINAL LAPLACE AZIMUTHS AND COMPARISONS WITH AZIMUTHS FROM TRIANGULATION

From	*To*	*Mean Laplace Azimuth from Table 5.5*	*Personal Equation*	*Laplace Azimuth Final Value*	*Geodetic Azimuth from Triangulation*	*Final Laplace Azimuth minus Geodetic Azimuth*
Herstmonceux (481)	Fairlight Down (193)	86° 07′ 12″60	+0″30	86° 07′ 12″90	86° 07′ 14″12	−1″22
Herstmonceux (481)	Fairlight Down (193)	86 07 11·13	+1·77			
White Horse Hill (34)	Liddington Castle (35)	234 18 05·36	+0·48	234 18 05·84	234 18 04·25	+1·59
St. Agnes Beacon (175)	Tregonning Hill (181)	206 19 10·15	+0·68	206 19 10·83	206 19 08·97	+1·86
Cairn Pat (360)	Inshanks (361)	159 00 02·02	+0·99	159 00 03·01	159 00 00·83	+2·18
Spital Hill (398)	Warth Hill (399)	53 49 17·40	+1·62	53 49 19·02	53 49 19·04	−0·02
Fetlar (459)	Saxavord (463)	03 12 08·41	+1·73	03 12 10·14	03 12 11·62	−1·48

CHAPTER SIX

Discussion of the Results of the Primary Retriangulation

6.00 Introduction

The Primary Retriangulation of which the methods and results have been described in previous chapters, constitutes a great undertaking and will provide much material for study and for discussion in the future. Such a study will probably demand additional field work as well as much mathematical and statistical analysis. So far there has been little time or opportunity for work of this nature, and in general it is impossible at this stage to do more than indicate some of the salient points of interest and importance, to draw some tentative conclusions and to suggest further investigations.

6.01 Comparison between the Principal Triangulation and the Retriangulation

Great Britain is the first country of any size to have been completely triangulated twice to first order standards. A comparison of these two triangulations therefore provides an unique opportunity for the practical study both of the accuracy of triangulation and of its stability over a long period—in this case about a century.

Diagram 17 shows by vectors the discrepancies between the two triangulations determined at a number of stations identified with reasonable certainty as being common to both systems. All of these stations were first order stations of the earlier triangulation; the majority, however, are lower order stations of the Retriangulation. This does not invalidate the comparison since the absolute accuracy of these lower order points differs but little from that of the higher order points from which they were fixed. Lower order stations of the earlier triangulation have been excluded as the method of computing then adopted has left their co-ordinates in doubt often by as much as a metre or more. As described in Chapter 2, the Retriangulation was fitted by least squares for position, scale and azimuth to the Principal Triangulation at 11 common points along the backbone of England. Understandably therefore little discrepancy between the two systems is to be found in this area. Elsewhere the discrepancy vectors are of varying length and display a marked regional correlation such as one would expect in a comparison of any two independent frameworks covering the same area. Over the greater part of the country and across to the Irish coast the vectors are small, two metres or less, and in no way remarkable. Along the Orkney and Shetland chain they increase rapidly in length reaching a maximum of 18 m. at the northern extremity. This, however, is an inherently weak chain with poorly conditioned figures and rays evidently subject to unusual lateral refraction (see Chapter 2, § 2.153 and § 2.161). In spite of these unavoidable handicaps scale and azimuth checks (see § 6.04 and § 6.05) indicate that the Retriangulation in this area is reasonably sound. The most likely explanation for this large discrepancy—roughly 60 p.p.m. in scale—is the presence of comparatively large error in the earlier triangulation, most of it probably occurring in the weak and inadequately observed figures connecting Orkney to the mainland. Vectors of up to 5 m. also occur at the north coast of Ireland and in western Scotland and the Hebrides, but here again there is no need to look for any unusual explanation. The pattern of the vectors with its marked local correlation is typical of discrepancy due to differential errors of triangulation, and their size is not remarkable having regard to the remoteness of the area from the 11 points.

It is not, however, quite so easy to account for a third group of large discrepancy vectors, that which occurs in East Anglia and Kent. Here the vectors radiate from an area in the centre of southern England and increase progressively reaching a maximum of about 6 m. at the coast in East Anglia. At first sight it is surprising to find such large and rapidly increasing vectors in an area immediately adjacent to the 11 basic points. In accounting for them therefore one might be tempted to discount triangulation error and to ascribe them to the only possible alternative cause, namely land movement since the days of the Principal Triangulation. However, a closer examination of the facts shows that this inherently unlikely alternative need not be resorted to.

It is clear that the scale of Clarke's triangulation at the southern end of the 11-points area is appreciably smaller than in the northern part of this area. As explained in Chapter 2 the Retriangulation Figure 1 was first scaled on the Clarke value for the side Beacon Hill (15)–Dunnose (10). The scale thus derived was found to be 15 p.p.m. too small when the combined Figures 1 and 2 were adjusted to give a best fit at all the 11 points (§ 2.27). The pattern of the vectors at White Horse Hill (34) and southwards clearly indicates the relative smallness of Clarke's scale in this area. The figures into which Clarke divided his triangulation are shown on Diagram 1 and the numbers indicate the order in which the figures were adjusted(1). It is clear that the area of relatively small scale falls in Clarke's Figures 14 and 15 and that this must have been communicated virtually in full to the East Coast Figures 18, 19 and 20 via Figure 16. It must also have considerably influenced the southern portion of Figure 21. One would therefore expect the scale of Clarke's Figures 14, 15, 16, 18, 19 and 20 and the southern portion of Figure 21 to be about 15 p.p.m. smaller than the Clarke scale for the 11 points as a whole. In the Retriangulation on the other hand, the scale of the East Anglian Figure 5 is derived from the overall Clarke scale for the 11 points since it is directly adjusted to the eastern edge of Retriangulation Figures 1 and 2. Scale checks confirm that little if any scale change has occurred. At Beacon Hill (15) there is close coincidence between the two systems. The largest discrepancies amounting to about 6 m. are at the coast about 300 km. distant. This indicates an overall scale discrepancy (Clarke smaller than Retriangulation) of 20 p.p.m. which is much the same as the 15 p.p.m. discrepancy one would expect.

It seems reasonably certain therefore that the larger systematic discrepancies between the two triangulations are all due to differential errors and that they provide no evidence of widespread land movement during the intervening period. Nor does a closer scrutiny of individual vectors suggest that appreciable local movement has taken place. The greatest rate of change from station to station is that between Hart Fell (320) and Dunrig (313) where the apparent distance between the two stations has increased by about 50 p.p.m. A survey of the other vectors in the neighbourhood shows that about 15–20 p.p.m. of this may be ascribed to the difference in scale between the two systems in this area. One is therefore left with about 30–35 p.p.m. or about ¾ m. to account for. Local movement of this order is quite possible, but equally the discrepancy may be due to slight local weakness in the earlier triangulation. There are one or two other instances of comparatively rapidly changing vectors in East Anglia; for example between Keysoe Church Spire and Ely Cathedral (430) where the apparent distance changes by about 40 p.p.m. Again allowing 15 p.p.m. for overall scale difference one is left with 25 p.p.m. or about 1 m. to account for. Quite possibly in this and other similar cases in the area progressive tilting of towers and spires may have contributed. The general reversal of the vectors between the northern and southern group of stations of the Irish connection is striking. This tendency is evidently the result of the inclusion by Clarke of the two groups of points in different adjustment figures, Figure 1 to the north and Figure 2 to the south.

To sum up, the comparison between the two systems shows a remarkable degree of agreement and consistency which establishes the general soundness of both. In the two regions of most striking disagreement, the Orkney–Shetland chain and East Anglia, the discrepancies are probably due in the main to errors in the earlier triangulation. There is no evidence of major land movement. Neither is there substantial evidence of local movement although it is possible that some stations, especially those on towers and spires, may have moved by about a metre or less since the earlier triangulation.

These conclusions are based on a reasonably large number of stations of comparison, which nevertheless represent but a small fraction of the total that would have been available if during the earlier triangulation more care had been taken to preserve the stations and if when pillars were built for the Retriangulation more attention had been given to ensuring exact identity of the new marks with the old or to recording their exact relative positions. A large number of stations identical at first sight, have perforce been excluded from the comparison because the station records lack unequivocal evidence of their identity. In many cases the recorded statement of the pillar constructor is in vague terms and could apply to a pillar built anywhere in the immediate neighbourhood of the old mark. It is evident that when new marks are built on old stations strict instructions to the builders are required which should amongst other things provide for careful recording of the exact method of obtaining identity together with details of measurements made for the purpose.

6.02 Origin of the Retriangulation

The Retriangulation has no origin in the accepted sense, since at no single geodetically coordinated point in it have the relationships between the geodetic and astronomic values of latitude and either longitude or azimuth been specifically defined or the separation between geoid and spheroid stated. The triangulation as a whole was fitted for position, scale and azimuth to the earlier Principal Triangulation and in a sense therefore shares the origin of that triangulation, that is the site of the Pond Transit Instrument at the Royal Observatory at Greenwich (see § 3.060). This assumption, however, is not strictly correct, since the coincidence between the two triangulations at this point is not exact. The lack of a formal origin, however, need cause no practical difficulty because it may, if required, be established at any point at which the necessary observations for astronomical latitude, longitude and/or azimuth have been made. The obvious place for such an origin is either the site of the Cooke Transit Circle at the Royal Greenwich Observatory at Herstmonceux or the Airy Transit Circle at the Royal Observatory at Greenwich, where rigorous connections to the Retriangulation have been made for geodetic position and azimuth, and where astronomic values of high accuracy for latitude, longitude and azimuth are available (see § 3.067 and § 3.103 in Chapter 3). The question of an origin is of importance when considering a readjustment of the Retriangulation (see § 6.03 below).

6.03 The Readjustment of the Retriangulation

Although the adjustment described in Chapter 2 has produced National Grid co-ordinate values which are amply adequate as regards accuracy and consistency for their intended purpose of controlling large scale surveys throughout the country, it is clear that it has not produced the best attainable absolute accuracy. This requirement was deliberately subordinated to the much greater practical need of obtaining early results and a good fit between the old and the new work. A readjustment in fewer figures and the incorporation of later scale, azimuth and geoidal data will certainly improve the absolute accuracy of the system. Such a readjustment would not only be of scientific interest but would also be of practical importance, since modern developments, especially in the field of missile guidance, have greatly increased the need for absolute accuracy in triangulation.

One such readjustment has in fact already been made by Brigadier Bomford, Reader in Surveying and Geodesy at Oxford University, who used a simple graphical method similar to that employed by him for the readjustment of the primary triangulation of India(46). The adjustment was based on an origin at Herstmonceux (481) and incorporated all the base measurements then completed (Ridgeway, Lossiemouth, Caithness, and a primary side in the Shetlands measured by Geodimeter—see Chapter 4) as well as the Laplace stations (see Chapter 5). It also made use of a geoidal section based on deviation observations by Dr. A. R. Robbins in 1950–52 at 43 stations between Dover and Cape Wrath. The readjustment was done in two stages, first to harmonise the results with the new data and second to convert the harmonised co-ordinates from the Airy spheroid to the International based on the European Datum. The first stage of the adjustment produced a shift in the Shetlands relative to Herstmonceux (481) of +10·1 m. in Eastings and −10·9 m. in Northings.

This readjustment, made to meet an urgent military requirement for determining the European Datum values of the Retriangulation co-ordinates, was necessarily provisional. It could be greatly improved by the incorporation of later work, notably the numerous Tellurometer scale checks (see Chapter 4) and by the recomputation of the triangulation in a single figure. This last, although a major work, would nowadays be perfectly practicable with the aid of a suitable electronic computer. Such a readjustment should certainly be carried out at some future convenient date. Consideration might also be given to the inclusion of a readjustment of the primary triangulations of Northern and Southern Ireland so that the triangulation systems of the entire British Isles may form a single homogeneous block.

6.04 Scale Errors of the Retriangulation

The Retriangulation and the supplementary work connected with it provides valuable material for the study of scale error generation in a triangulation. As has been described in an earlier chapter the general scale was determined by securing the best fit with Clarke's triangulation at 11 selected common points. Subsequently a number of scale checks were carried out by invar tape, Geodimeter and Tellurometer measurements. These measurements have been described in Chapter 4 and a summary of the scale errors revealed is shown on Diagram 16. From these it is clear that in general the scale of the Retriangulation is too great and that its error varies from place to place. We can thus consider the scale error at any point as consisting of two parts:

(*a*) A constant error arising from the method of adjustment to fit Clarke's triangulation.

(*b*) A varying error due to the accidental errors of observation.

6.040 CONSTANT ERROR

The constant error may be regarded either as one of the following or as some combination of them.

(*a*) An error of scale deliberately adopted and capable of elimination by a simple change of scale.

(*b*) The result of the adoption of a spheroid whose dimensions are greater than the nominal value.

(*c*) An incorrect assumption regarding the separation of geoid and spheroid at a measured base.

It was originally Hotine's intention(4) to regard the error as in (*b*) above and to eliminate it by means of a change in the major axis of the Airy spheroid. He intended at the same time to change the scale factor of the projection and hoped thus to preserve the existing values of the published National Grid co-ordinates. However, after the publication of projection tables for the National Grid in 1950(16), based upon the existing Airy spheroid, this course of action became impracticable and the presence of a constant error must be accepted until such time as a readjustment is carried out. The magnitude of this error depends on the method used for its definition. It is probably most convenient to regard it as the scale error deduced from the various check measurements of the Ridgeway Base, i.e. +7 p.p.m.

6.041 SCALE ERROR DUE TO ACCIDENTAL ERRORS OF OBSERVATION

Although observation errors may be accidental in character their interaction in a simultaneously adjusted network gives rise to a regional or pseudo-systematic error varying gradually from place to place. It is possible to study the behaviour of this varying error in the Retriangulation network by comparison of the numerous directly measured bases and sides with their triangulated lengths. The first direct measurements to be available were the invar tape check measurements of the bases at Ridgeway (see § 4.01 and § 4.12), Lossiemouth (see § 4.07) and Caithness (see § 4.17). These were followed by Geodimeter measurements of the Ridgeway and Caithness Bases and of the side Saxavord (463)–Fetlar (459) at the northern tip of the Shetland Islands (see § 4.25, § 4.26). From the scale discrepancies revealed by these measurements it was first supposed that the scale error of the triangulation increased gradually from +7 p.p.m. at Ridgeway in southern England to +20 p.p.m. at Lossiemouth on the Moray coast of Scotland, decreasing slightly to +17 p.p.m. in the extreme north of Scotland before falling rather sharply to +6 p.p.m. at the northern end of the Shetlands. Brigadier Bomford made his readjustment (see § 6.03) on this assumption. Later, however, a number of scale checks by Tellurometer were made (see § 4.28 *et seq.* and Diagram 16) which indicated that in fact the scale error was more or less constant at about 7–10 p.p.m. over most of southern and central England and Wales—roughly south of a line joining The Wash to Cardigan Bay and that it then rose sharply to 20–30 p.p.m. in northern England and to 30–40 p.p.m.

in southern and central Scotland before again falling sharply to 15 p.p.m. on the north coast. Not all of these Tellurometer checks were of geodetic standard—some consisting only of single measurements of single sides instead of six measurements or more of a complete figure taken on at least two days, the standard now recommended for first order work by the International Association of Geodesy(43). Nevertheless, taken as a whole the results appear remarkably consistent, and provide strong evidence of unexpectedly rapid variations of scale.

The area in which the largest and most sudden apparent change of scale occurs is that extending from the Border country to Kirkcudbrightshire and Wigtownshire in south-west Scotland. Here the Tellurometer shows a maximum scale error of +41 p.p.m. for a series of checks in Wigtownshire confirmed by an error of +38 p.p.m. in a secondary block in neighbouring Kirkcudbrightshire (see § 4.301 and Table 4.7). Independent evidence is available from a comparison with Clarke's Principal Triangulation in this area. Consideration of the relative vectors for the group of common stations roughly centred on Cairn Pat (360) near the Mull of Galloway (Knocklayd, Trostan, Divis, Slieve Donard, South Barrule (469), Merrick (301)) shows that the general scale of the Retriangulation here is about 20 p.p.m. greater than the scale of Clarke's Figure 4. The vectors at Knocklayd and Slieve Snaght on the north coast of Ireland (confirmed by that at Beinn Tart a' Mhill (383)) indicate that the Retriangulation scale is about 33 p.p.m. greater than the scale of Clarke's Figure 1, which in turn is known to be about 5 p.p.m. too great at the Lough Foyle Base(1). One could thus deduce the scale error of the Retriangulation in the Wigtownshire area in two ways:

(*a*) By direct comparison with Clarke's Figure 1 on the assumption that the Retriangulation scale in the area under consideration is constant +38 p.p.m.

(*b*) By comparison with Clarke's Figure 4 on the assumption that Clarke's Figures 1 and 4 have the same scale +25 p.p.m.

Obviously neither of the above assumptions can be entirely valid. Estimates of errors naturally vary with the assumptions made and exact conclusions cannot be drawn. Nevertheless the comparison confirms the likelihood of a large positive scale error in the Retriangulation in Wigtownshire and tends to confirm that the errors derived from the Tellurometer checks are reliable although possibly slightly too great.

It is in theory possible to test the validity of the apparent scale variations by comparing them with the probable errors of scale deduced from the internal evidence of the triangulation. Formulae, such as that given on page 117 of Bomford's *Geodesy* (First Edition), exist for this purpose, but these are invariably based on relatively simple chains of polygons and can only be applied to networks if a number of questionable assumptions are made, even when, as in the case of the Retriangulation, computations are broken down into blocks or figures of limited size. One may, for example, consider Figures 1, 2 and 3 as a single chain having the strength of a double chain of hexagons.

On this assumption and using mean values for the errors of the figures concerned (see Table 2.2 at the end of Chapter 2) we may put the following values into the formula:

Length of chain = 100S miles	$S = 3$
Strength factor (double chain of hexagons)	$A = 27 \div \sqrt{2}$
Ratio of breadth to length of component figures	$B = 1$
Average triangular misclosure	$E_m = 1''.1$
p.e. of observed angle	$e = 0{\cdot}48E_m = 0''.53$
Number of figures per 100 miles of chain	$f = 4$

The probable change of scale between the Ridgeway Base and Cairn Pat (360) would thus be: (in the 7th decimal of the logarithm)

$$\sqrt{3} \times \frac{27}{\sqrt{2}} \times 0{\cdot}53 \times \sqrt{4} = 35 \text{ or } 8 \text{ p.p.m.}$$

By a similar calculation the probable scale change between the Caithness Base and Cairn Pat (360) would be about ± 6 p.p.m. The scale changes from Ridgeway and Caithness respectively are, according to the Tellurometer checks, $41 - 7 = 34$ p.p.m. and $41 - 15 = 26$ p.p.m., or more than 4 × p.e. assessed as above in each case. The possibility that both p.e.'s and scale changes are valid is thus too small for acceptance. One is left therefore with the following possible explanations:

(*a*) The Tellurometer scale checks are not valid.
(*b*) Serious error has occurred in the triangulation adjustment.
(*c*) The error assessment formula is not valid in the circumstances.

It seems unlikely that (*a*) can provide a complete explanation although it is very possible that say 5 p.p.m. might be accounted for thus. There is no other evidence for (*b*) which can therefore be discounted. As regards (*c*) it seems that the matter needs further study. Apart from the inherent difficulty of applying formulae of this type to a network, there seems some possibility that such formulae may depend on assumptions which are not always valid in practice. In the past the difficulty of frequently checking scale in triangulation networks has of necessity tended to obscure the behaviour of scale error. The advent of electronic distance measuring equipment has, however, now ensured that scale checks will be much more common in future. It will be interesting to see to what extent this increasing volume of evidence will modify the existing theory.

It is noteworthy that the rate of scale variation seems to increase as the triangulation reaches the mountainous areas of northern England and the Southern Uplands of Scotland, remaining high as the triangulation passes across the Highlands. If this tendency has any significance, which is doubtful, it could be due to either:

(*a*) The inferior layout of the triangulation in this area. There seems to be some possibility of this. In the area of the junction between Figure 2 and Figure 3 the belt of triangulation is rather narrow (ignoring figures connected subsequently) and the actual junction between the two figures is relatively long and contorted. It is possible that in the process of being adjusted to fit Figure 2 along this junction Figure 3 has suffered some distortion, although examination of the adjustment corrections reveals no obvious sign of strain. Readjustment in a single figure with the whole strengthened by the inclusion of Figure 7 and the Irish connecting figure would eliminate any distortion arising from this cause.

(*b*) More difficult conditions of observation resulting from high winds, cold, and possibly lateral refraction. This may well be the most likely explanation, although the statistics of the observations (see § 6.06, also Table 2.2 at the end of Chapter 2) do not suggest that observations in the figures concerned were inferior.

(*c*) Geoidal anomalies. These might take the form of more or less random variations in the direction of the vertical due to local attraction. Again such variations, if significant, would have shown up in the statistics of the observations. Moreover Robbins' geoidal section (see § 6.03) has shown that throughout the country geoidal anomalies are small.

It is noteworthy that there is no evidence of unusual scale error in East Anglia or Kent. Thus, neglecting the possibility of land movement since the days of the Principal Triangulation, the likelihood that the latter must have been somewhat defective in this area is confirmed (see § 6.01).

In the narrow Orkney/Shetland chain a negative scale change of about 7 p.p.m. in about 260 km. has occurred. This is not unduly large and, with the evidence of the Laplace azimuth errors, which change by an equivalent amount ($1''\!.46$), suggests that in the Retriangulation this figure is sound. Most of the accumulated difference between this and the Principal Triangulation along the chain (about 17 metres or 64 p.p.m.) is therefore probably attributable to the earlier triangulation.

To sum up, there is a need for further scale checks, but from results already obtained it is clear that the scale of the Retriangulation is everywhere too large, probably varying from about +7 p.p.m. in southern England to about +35 p.p.m. in south central Scotland. Some of the scale variations seem larger than might be expected on the basis of existing theory having regard to the standard of the observations; this requires further investigation. The average error for the whole triangulation in the north and south direction is probably about +15 to +20 p.p.m. indicating that the northern tip of the Shetland Islands probably in fact lies about 15 to 20 m. south of its National Grid position.

6.05 Azimuth Errors in the Retriangulation

The results of the programme of Laplace azimuths (see Chapter 5) provide data for study of the azimuth errors of the Retriangulation. Unfortunately the number of azimuth checks available is by no means as great as the number of scale checks; the behaviour of the azimuth errors is therefore much more conjectural. Moreover the results themselves are of somewhat uncertain accuracy, partly because of the unavoidable effects of lateral refraction which are clearly indicated by the discrepancies between forward and back azimuths, and partly because of doubt as to the validity of the correction for personal equation (see below).

Taking the results corrected for personal equation given in Table 5.6, it would appear that there is a sharp change in the azimuth error between Herstmonceux (481) and White Horse Hill (34) (2″81 or 14 p.p.m. in about 150 km.), but that the rate of change between the other azimuth stations is much less. The next greatest changes are between Cairn Pat (360) and Spital Hill (398) (2″20 in 430 km.) and between Spital Hill (398) and Fetlar (459) (1″46 in 290 km.), that is a rate of about 2·5 p.p.m. per 100 km. in each case. There is no apparent reason why azimuth should have held so much better than scale and it seems very possible that, at least in the case of the long line from Cairn Pat (360) to Spital Hill (398), the apparently small rate of error change is fortuitous, the actual errors (both of the azimuth determinations and of the triangulation) having been greater than this but with a self-cancelling tendency. Similarly the good agreement between White Horse Hill (34) and Cairn Pat (360) (0″59 in 440 km.) seems, in the light of the evident scale variations along this line, to be fortuitous. The remarkably good agreement between White Horse Hill (34) and St. Agnes Beacon (175) (0″27 in 310 km.) may also be fortuitous but it has a better claim to validity since scale also seems to have held well along this line.

Unfortunately no Laplace azimuth was observed in East Anglia where the short-sided triangulation might have been expected to give rise to relatively great azimuth error variation. Scale, however, seems to have held well in this area (see Diagram 16) and one might therefore expect azimuth to have held also.

The question of personal equation has been mentioned in Chapter 5, § 5.08. It is clear that the correction applied was open to doubt in a number of respects, and it is questionable whether its use has improved the results. If it is ignored the magnitude of the discrepancies between the geodetic azimuths and the Laplace azimuths change, but their average value and their spread do not increase significantly, as can be seen from Table 5.6. This uncertainty necessarily results in the general orientation of the Retriangulation remaining in doubt within about 1″0 of arc.

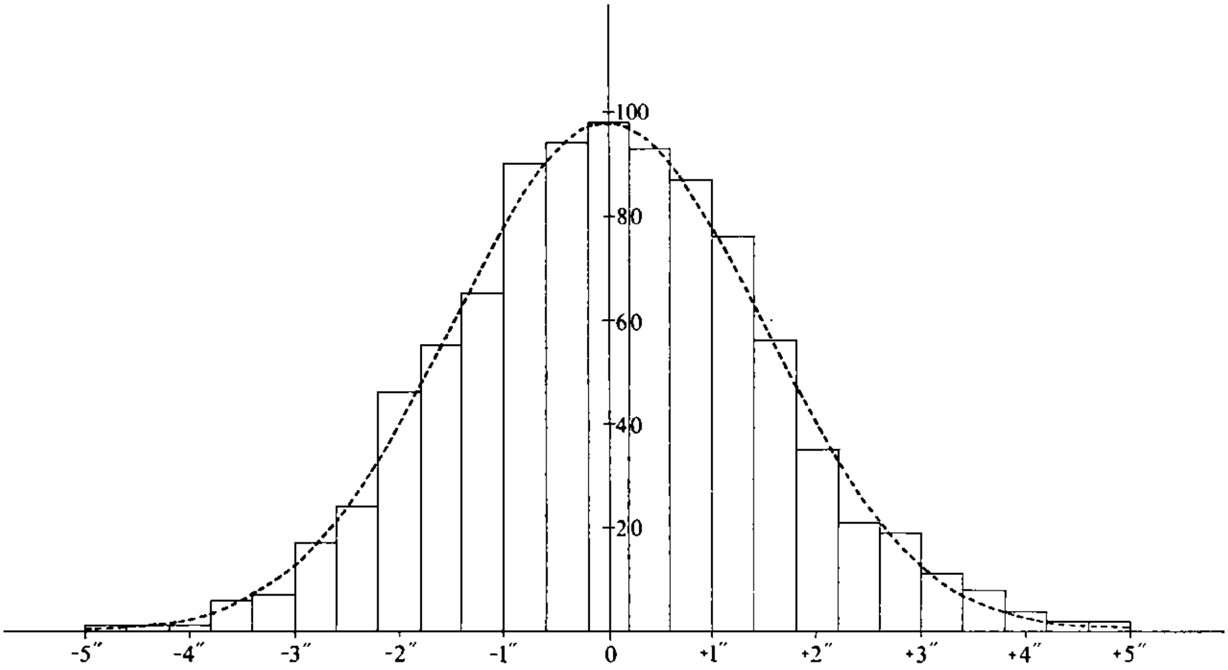


FIG. 6.1. Distribution of the triangle misclosures of the primary Retriangulation

Summarising, it can be said that the azimuth of the Retriangulation has evidently been held throughout the network within about 4″0 of arc (20 p.p.m.)—possibly rather better than scale. As regards general orientation, subject to the doubts expressed in the paragraphs above, it seems likely that the azimuth of the triangulation as adjusted over the greater part of England, Wales and southern Scotland is too small by about 1″ to 2″. North of this area the tendency is progressively reversed. The effect of these tendencies would be to make the National Grid eastings of northern Scotland, the Orkneys, and the Shetlands too small by about 5–10 m.

6.06 Statistical Analysis of Observations

Table 2.2 at the end of Chapter 2 gives a summary of statistical data relating to the angular observations of the Primary Retriangulation. This shows that the work is remarkably consistent. As regards the main triangulation, only in Figure 5 is there any significant falling off of directional accuracy; a result to be expected owing to the comparatively large proportion of short-sided triangles in the figure and to the extensive use of steel towers for observation. There is nothing in these statistics to indicate why Figures 3 and 6 should have apparently varied in scale so much more than Figures 1, 2, 4 and 5.

An analysis of the 919 triangle misclosures shows that they are random in nature and normally distributed (see Table 6.1 and Fig. 6.1). Of these misclosures 469 were positive and 450 negative in sign. There is no evidence therefore to suggest that the observations as a whole are biassed in any way or suffer from error having any overall systematic tendency such as might explain an unusual accumulation of scale error.

TABLE 6.1

COMPARISON OF THE ACTUAL AND THEORETICAL FREQUENCIES OF THE PRIMARY TRIANGLE MISCLOSURES

Class-Interval	*Theoretical No. of Errors*	*Actual No. of Errors*	*Class-Interval*	*Theoretical No. of Errors*	*Actual No. of Errors*
Over −5″0	0·4	0	+0″2 to +0″6	94·3	93
−5″0 to −4·6	0·6	1	+0·6 to +1·0	84·8	87
−4·6 to −4·2	1·3	1	+1·0 to +1·4	70·9	76
−4·2 to −3·8	2·8	1	+1·4 to +1·8	55·4	56
−3·8 to −3·4	5·5	6	+1·8 to +2·2	40·2	35
−3·4 to −3·0	10·1	7	+2·2 to +2·6	27·3	21
−3·0 to −2·6	17·1	17	+2·6 to +3·0	17·1	19
−2·6 to −2·2	27·3	24	+3·0 to +3·4	10·1	11
−2·2 to −1·8	40·2	46	+3·4 to +3·8	5·5	8
−1·8 to −1·4	55·4	55	+3·8 to +4·2	2·8	4
−1·4 to −1·0	70·9	65	+4·2 to +4·6	1·3	2
−1·0 to −0·6	84·8	90	+4·6 to +5·0	0·6	2
−0·6 to −0·2	94·3	94	Over +5·0	0·4	0
−0·2 to +0·2	97·6	98			
			Total	919·0	919

6.07 A Tribute

The word 'error' has been used not infrequently throughout the foregoing chapter. In normal conversation such a word carries with it some connotation of human shortcoming; of work left undone, or of work imperfectly completed. Used as it is in this volume in a scientific context the word of course has a very different significance, and certainly no one who has studied the earlier chapters could be excused for interpreting it as implying criticism of those who carried out the Retriangulation of Great Britain, and the Principal Triangulation before it. Night after night, in conditions of rain and snow, frustration, boredom and continual hardship, which would have defeated all but the stoutest hearts, the work was carried enthusiastically forward to a triumphant conclusion. References to 'errors' must therefore be regarded rather as reminders of the natural difficulties with which all concerned had to contend; physical difficulties calling for muscle and endurance as well as those more subtly frustrating which arise from the wayward behaviour of the atmosphere and the other elements involved when man sets out to make measurements of the highest precision upon the Earth's surface. It is surely fitting, at the conclusion of this, the last chapter dealing with the Primary Retriangulation of Great Britain, to pay a tribute to the men who helped to carry through this great work. Of these only a few have been named in this volume. Many more must remain anonymous although their actual contribution to the work has been great. Yet whether or not their names find a place in this book all will surely have a truly great and enduring memorial, the Primary Retriangulation itself.

CHAPTER SEVEN

Secondary and Lower Order Triangulation

FIELDWORK

7.00 Introduction

The old secondary triangulation was never rigorously adjusted to the Principal Triangulation, a fact which constituted one of the main reasons for the complete retriangulation considered essential in 1936 (see § 1.05). Secondary triangulation was therefore planned as an integral part of the Retriangulation which was to consist of the primary framework with a secondary triangulation rigorously adjusted to it. The ruling side length of the new secondary triangulation was laid down as 4 miles (7 km.), a relatively short length designed to avoid the need for a subsequent conventional tertiary triangulation covering the whole country. It was considered that in rural areas a density of one point in 4 miles would provide adequate triangulation control, whereas in urban areas a denser control could be provided as and when required for the large scale surveys of the Department or for any other reason. It was intended that this minor control would consist of a system of points, known as 'town control', intersected or resected from the stations of the secondary net and computed by minor trigonometrical methods.

In practice this original plan had to be modified and a tertiary triangulation introduced as the work progressed, but the principle of adjusting the secondary triangulation rigorously to the primary framework was retained, and all secondary and tertiary work can be considered as an integral part of the Retriangulation.

7.01 Layout of the Secondary and Tertiary Triangulations

As the secondary work was to be rigorously adjusted, it was necessary in order to keep the computations down to a manageable size to sub-divide it into areas or 'blocks' each of which could be treated as one unit. The boundaries of blocks were defined by lines joining primary stations and they are shown on Diagram 18.

Observation of the first blocks started in 1936 concurrently with primary work and by the end of 1938 21 blocks had been completed. It then became clear that the observation and rigorous adjustment of a network of 7 km. side length had serious disadvantages. In many areas, particularly in the heavily wooded parts of southern England, it proved very difficult to establish the network of intervisible points at intervals of 7 km., but a more serious difficulty was the computation of the network once it was established. In block SU 15 for example, with the machines and methods then available, it would have taken two men no less than 8 months to solve the condition equations needed to co-ordinate the 59 new stations. The size of the blocks could not conveniently be reduced so it was decided instead to extend the side length of the secondary triangulation from 7 to 13 km., and to fill in between these stations with a tertiary triangulation at a 7 km. density. The secondary network would be rigorously adjusted but the tertiary stations were to be co-ordinated by simpler methods.

This modification of the original plan increased the field output and also resulted in a better conditioned secondary network. The accuracy of the whole network was preserved, and if there was any loss of internal consistency in the tertiary triangulation, this was more than compensated by the considerable saving of computing effort.

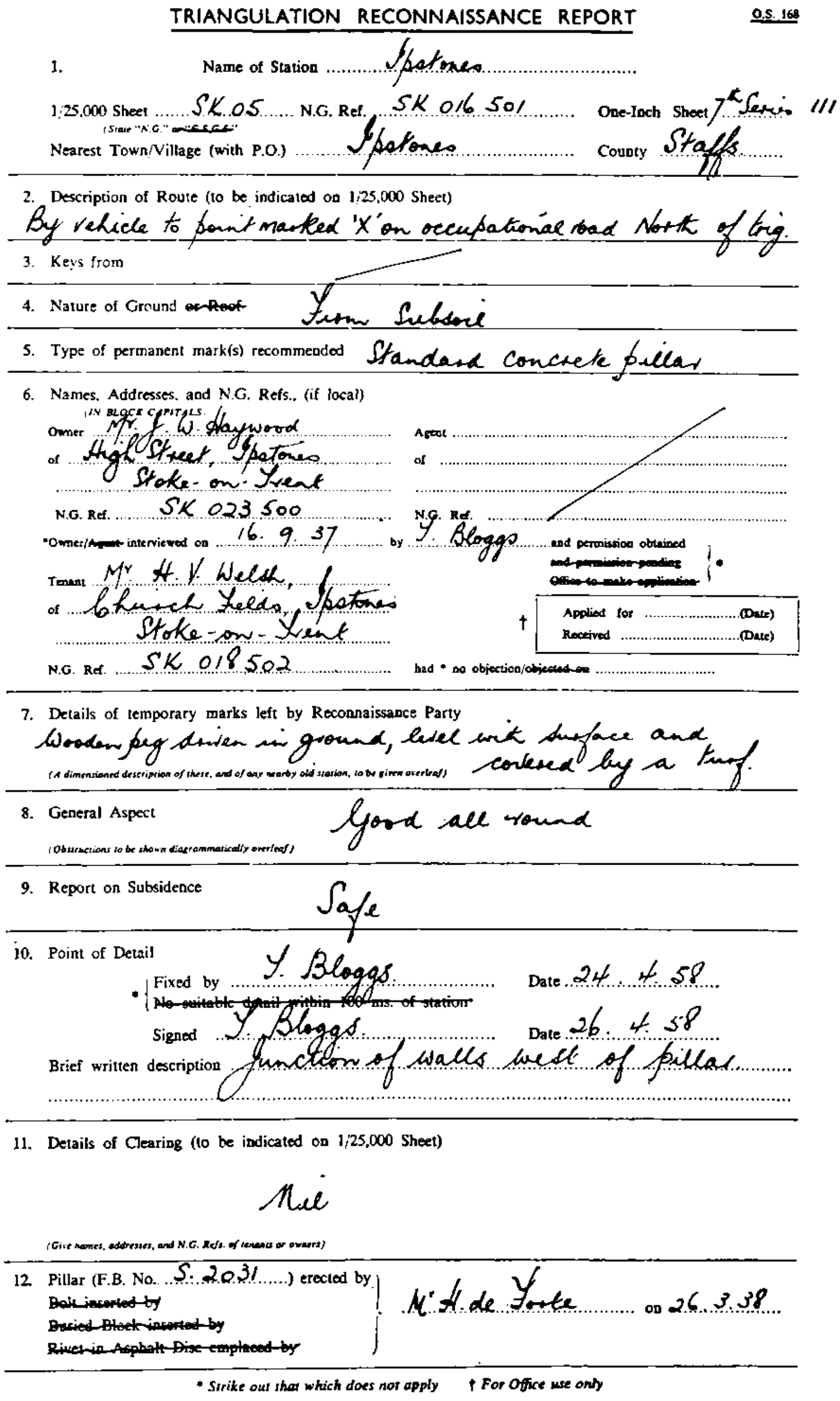
The modified plan was used for all secondary and tertiary triangulation up to 1950 by which time the more developed parts of the country had been completed and work in the mountain areas was about to start. Here there seemed little point in providing a tertiary triangulation; something more than the primary framework was, however, required to control the six inches to one mile mapping planned for these areas. It was decided therefore to omit the tertiary but to retain the secondary triangulation.

In 1958 the I.T.C.–Jerie Analogue Computer was introduced to assist in the aerial triangulation adjustment. With this computer it was possible to produce adequate control for a six inches to one mile aerial survey of an area 48 km. × 48 km. with ground control points spaced about 13 km. apart around the perimeter only. At about the same time the Tellurometer electronic distance measuring equipment became available and enabled the perimeter control to be supplied very economically by Tellurometer traverse. The Retriangulation was, however, then nearing completion and this economical method could only be used for three secondary blocks.

7.02 Reconnaissance

Secondary and tertiary triangulation schemes were reconnoitred in one operation. The method of reconnaissance was the same as for the primary except that because of the density of points required it was not possible in most cases to draw up a paper scheme from maps. The reconnaissance had as its object the selection of suitable sites for the establishment of secondary and tertiary stations to the required density. In addition suitable prominent objects up to a density of one point per 2 km. were to be selected as intersected points. For these it was laid down that a minimum of five intersecting rays would be required. For normal tertiary points fixation from six directions was stipulated, not less than two of these being inwards and not less than two outwards from the point. These rules were not introduced at the beginning of the Retriangulation, but were found to be necessary as the work proceeded in order to maintain a high standard of accuracy and consistency.

The reconnaissance was carried out by blocks whose boundaries were nominally defined by straight lines joining primary stations. To avoid any discontinuity between work in neighbouring blocks it was the practice to carry the reconnaissance over these boundaries to ensure that the accepted schemes provided an adequate connection between the secondary and tertiary triangulations in the two blocks.



TRIANGULATION RECONNAISSANCE REPORT

O.S. 168

1\. Name of Station Ipstones

1/25,000 Sheet SK.05 (State "N.G." ~~or "G.S.G.S."~~) N.G. Ref. SK 016 501 One-Inch Sheet 7th Series 111

Nearest Town/Village (with P.O.) Ipstones County Staffs

2\. Description of Route (to be indicated on 1/25,000 Sheet)
By vehicle to point marked 'X' on occupational road North of trig.

3\. Keys from

4\. Nature of Ground ~~or Roof~~ Firm Subsoil

5\. Type of permanent mark(s) recommended Standard Concrete pillar

6\. Names, Addresses, and N.G. Refs., (if local)
(IN BLOCK CAPITALS)

Owner Mr. J. W. Haywood
of High Street, Ipstones
Stoke-on-Trent
N.G. Ref. SK 023 500

Agent
of
N.G. Ref.

*Owner/~~Agent~~ interviewed on 16. 9. 37 by J. Bloggs and permission obtained ~~and permission pending~~ ~~Office to make application~~ *

Tenant Mr H. V. Welsh,
of Church Fields, Ipstones
Stoke-on-Trent
N.G. Ref. SK 018 502 had * no objection/~~objected on~~

† Applied for ....(Date)
Received ....(Date)

7\. Details of temporary marks left by Reconnaissance Party
Wooden peg driven in ground, level with surface and covered by a turf.
(A dimensioned description of these, and of any nearby old station, to be given overleaf)

8\. General Aspect Good all round
(Obstructions to be shown diagrammatically overleaf)

9\. Report on Subsidence Safe

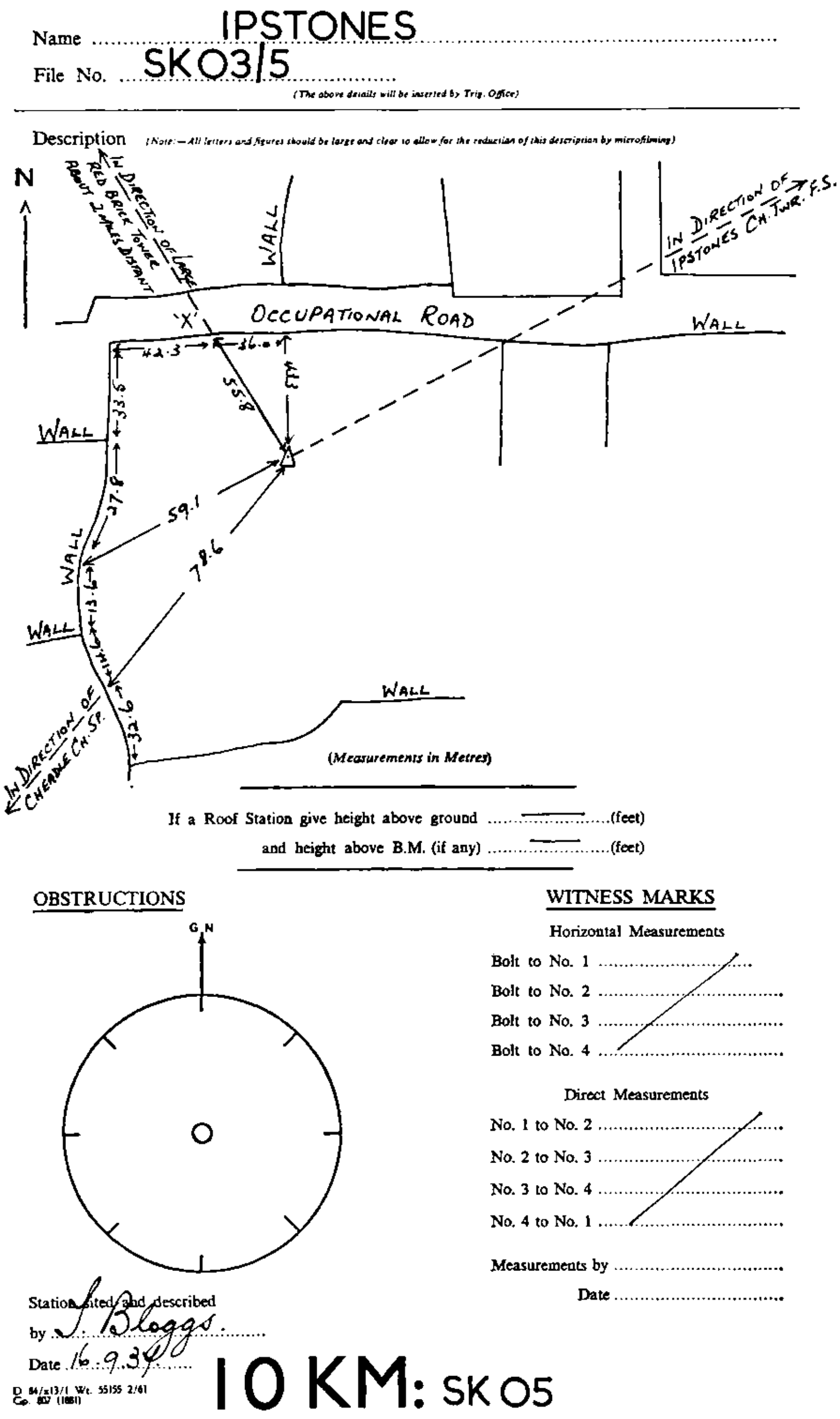
10\. Point of Detail
* Fixed by J. Bloggs Date 24. 4. 58
~~No suitable detail within 100 ms. of station~~
Signed J. Bloggs Date 26. 4. 58
Brief written description Junction of walls west of pillar

11\. Details of Clearing (to be indicated on 1/25,000 Sheet)
Nil
(Give names, addresses, and N.G. Refs. of tenants or owners)

12\. Pillar (F.B. No. S. 2031) erected by
~~Bolt inserted by~~
~~Buried Block inserted by~~
~~Rivet in Asphalt Disc emplaced by~~
Mr H. de Yorke on 26. 3. 38

* Strike out that which does not apply † For Office use only

FIG. 7.1. Front of reconnaissance report for a ground station



Name ....IPSTONES....

File No. ...SK03/5....

(The above details will be inserted by Trig. Office)

Description *(Note:—All letters and figures should be large and clear to allow for the reduction of this description by microfilming)*

(Measurements in Metres)

If a Roof Station give height above ground ....—........(feet)

and height above B.M. (if any) ....—........(feet)

OBSTRUCTIONS

WITNESS MARKS

Horizontal Measurements

Bolt to No. 1 ....

Bolt to No. 2 ....

Bolt to No. 3 ....

Bolt to No. 4 ....

Direct Measurements

No. 1 to No. 2 ....

No. 2 to No. 3 ....

No. 3 to No. 4 ....

No. 4 to No. 1 ....

Measurements by ....

Date ....

Station sited and described

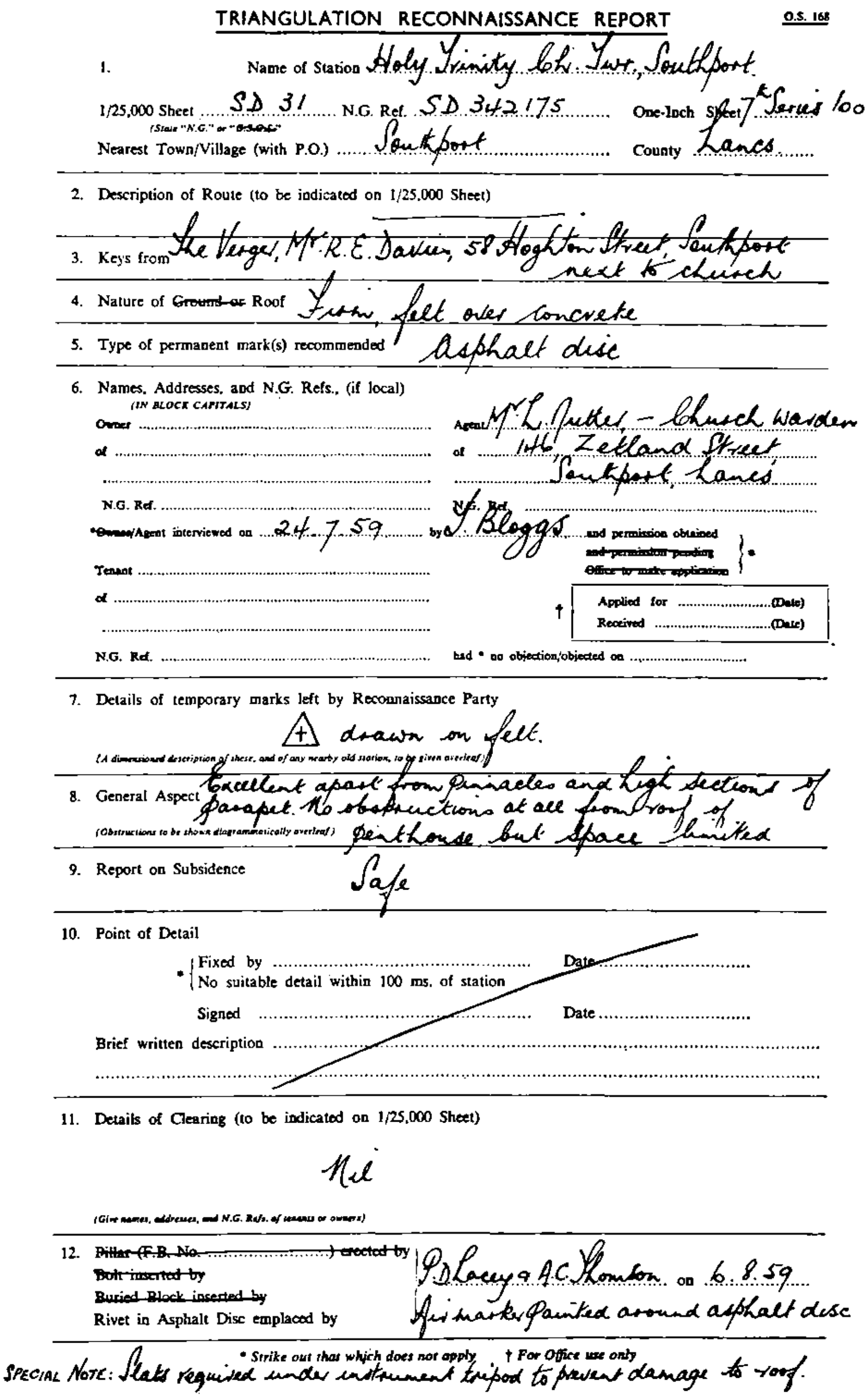
by ...J. Bloggs....

Date ...16.9.34....

D 84/x13/1 Wt. 55155 2/61
Gp. 807 (1881)

10 KM: SK05

FIG. 7.2. Back of reconnaissance report for a ground station



TRIANGULATION RECONNAISSANCE REPORT

O.S. 168

1. Name of Station Holy Trinity Ch. Twr, Southport

1/25,000 Sheet SD 31 N.G. Ref. SD 342175 One-Inch Sheet 7th Series 100

(State "N.G." or "~~O.S.G.~~")

Nearest Town/Village (with P.O.) Southport County Lancs

2. Description of Route (to be indicated on 1/25,000 Sheet)

3. Keys from The Verger, Mr. R. E. Davies, 58 Hoghton Street, Southport next to church

4. Nature of ~~Ground or~~ Roof Iron, felt over concrete

5. Type of permanent mark(s) recommended Asphalt disc

6. Names, Addresses, and N.G. Refs., (if local)
(IN BLOCK CAPITALS)

Owner ....
of ....
....
N.G. Ref. ....

Agent Mr. L. Butler – Church Warden
of 146, Zetland Street
Southport, Lancs
N.G. Ref. ....

*~~Owner~~/Agent interviewed on 24.7.59 by J. Bloggs and permission obtained
~~and permission pending~~ } *
~~Office to make application~~

Tenant ....
of ....
....
N.G. Ref. .... had * no objection/objected on ....

† Applied for ....(Date)
Received ....(Date)

7. Details of temporary marks left by Reconnaissance Party

$\triangle$ drawn on felt.

(A dimensioned description of these, and of any nearby old station, to be given overleaf)

8. General Aspect Excellent apart from pinnacles and high sections of parapet. No obstructions at all from roof of penthouse but space limited

(Obstructions to be shown diagrammatically overleaf)

9. Report on Subsidence Safe

10. Point of Detail

* { Fixed by .... Date ....
No suitable detail within 100 ms. of station

Signed .... Date ....

Brief written description ....

....

11. Details of Clearing (to be indicated on 1/25,000 Sheet)

Nil

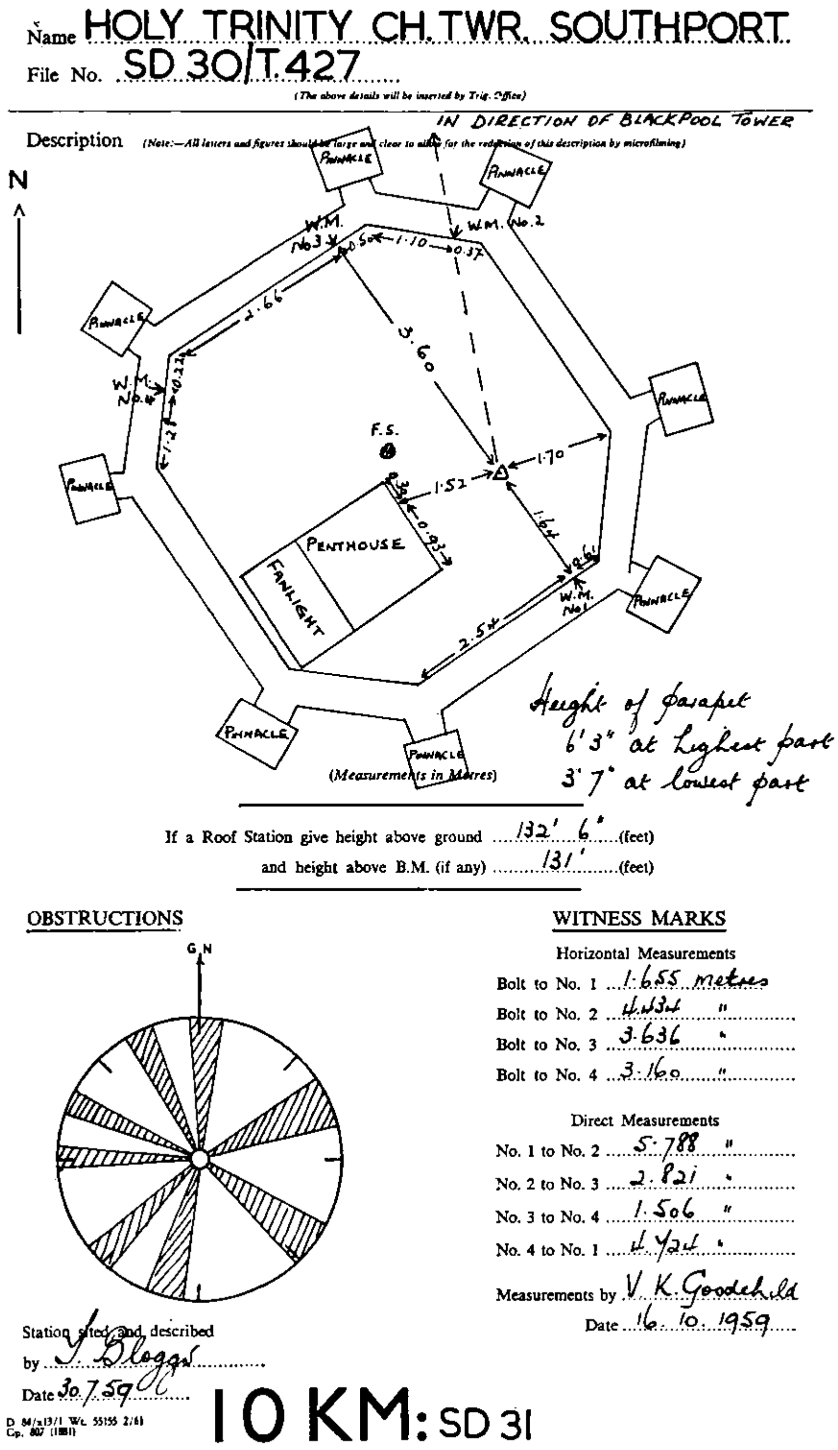
(Give names, addresses, and N.G. Refs. of tenants or owners)

12. ~~Pillar (F.B. No. ....) erected by~~
~~Bolt inserted by~~
~~Buried Block inserted by~~
Rivet in Asphalt Disc emplaced by P. D. Lacey & A. C. Thomson on 6.8.59
Air marks painted around asphalt disc

* Strike out that which does not apply † For Office use only

SPECIAL NOTE: Slabs required under instrument tripod to prevent damage to roof.

FIG. 7.3. Front of reconnaissance report for a roof station



Name HOLY TRINITY CH. TWR. SOUTHPORT.
File No. SD 30/T.427
(The above details will be inserted by Trig. Office)

Description (Note:—All letters and figures should be large and clear to allow for the reduction of this description by microfilming)

(Measurements in Metres)

Height of parapet
6' 3" at highest part
3' 7" at lowest part

If a Roof Station give height above ground 132' 6" (feet)
and height above B.M. (if any) 131' (feet)

OBSTRUCTIONS

WITNESS MARKS

Horizontal Measurements

Bolt to No. 1 1.655 metres
Bolt to No. 2 4.434 "
Bolt to No. 3 3.636 "
Bolt to No. 4 3.160 "

Direct Measurements

No. 1 to No. 2 5.788 "
No. 2 to No. 3 2.821 "
No. 3 to No. 4 1.506 "
No. 4 to No. 1 4.424 "

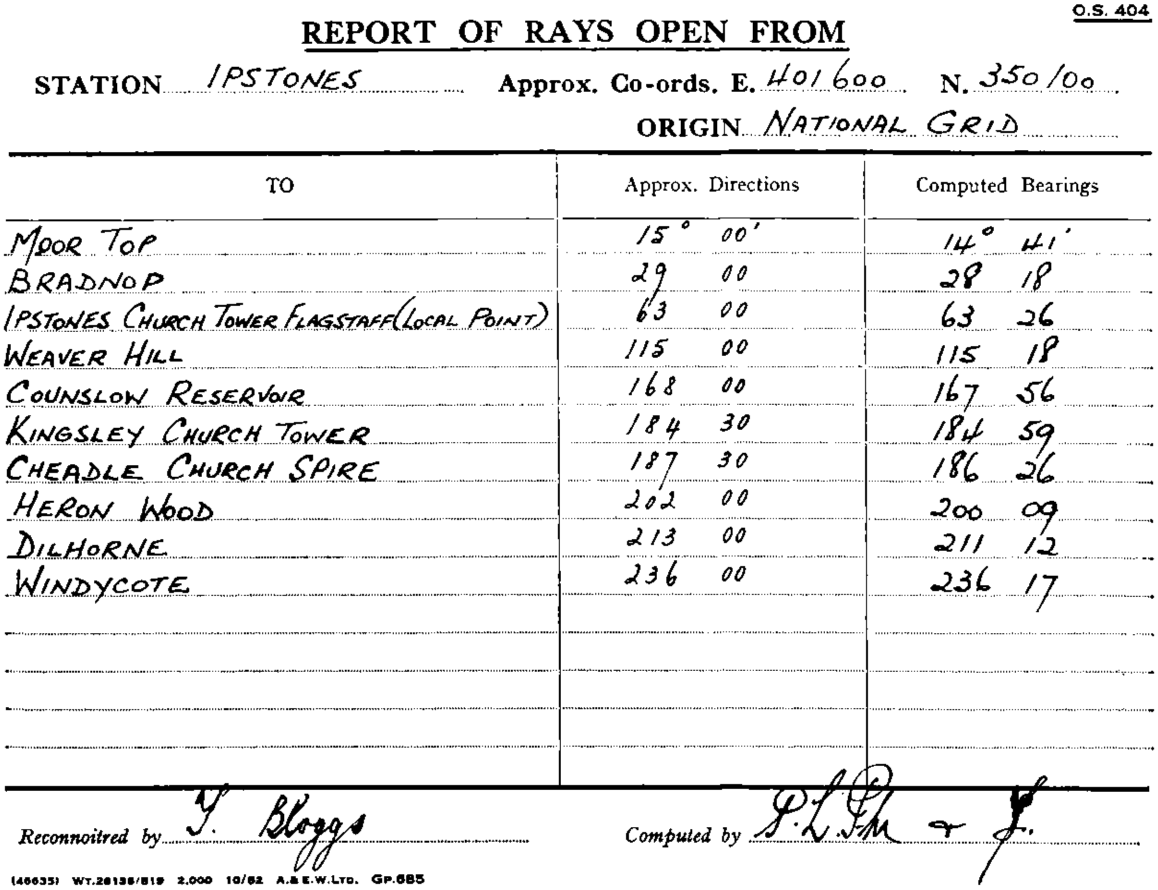
Measurements by V. K. Goodchild
Date 16. 10. 1959

Station sited and described
by J. Bloggs
Date 30.7.59

D 84/x13/1 Wt. 55155 2/61
Cp. 807 (1881)

10 KM: SD 31

FIG. 7.4. Back of reconnaissance report for a roof station



O.S. 404

REPORT OF RAYS OPEN FROM

STATION IPSTONES **Approx. Co-ords. E.** 401 600 **N.** 350 100

ORIGIN NATIONAL GRID

TO	Approx. Directions	Computed Bearings
MOOR TOP	15° 00′	14° 41′
BRADNOP	29 00	28 18
IPSTONES CHURCH TOWER FLAGSTAFF (LOCAL POINT)	63 00	63 26
WEAVER HILL	115 00	115 18
COUNSLOW RESERVOIR	168 00	167 56
KINGSLEY CHURCH TOWER	184 30	184 59
CHEADLE CHURCH SPIRE	187 30	186 26
HERON WOOD	202 00	200 09
DILHORNE	213 00	211 12
WINDYCOTE	236 00	236 17

Reconnoitred by J. Bloggs *Computed by* S. L. Sm & J.

(46635) Wt.28136/819 2,000 10/62 A.&E.W.Ltd. Gp.685

FIG. 7.5. Report of rays open

Except in open hilly country the reconnaissance was normally carried out in two stages:

Stage I: A preliminary reconnaissance of the whole block during which the locations of likely stations were noted. During this stage the stations of the old triangulation were visited and a search made for the centre mark. Whenever possible a new station was established at such primary and secondary stations of the old triangulation as were recovered.

Stage II: A final point-by-point reconnaissance during which doubtful rays were checked if necessary by beacon lamps and the exact locations of the stations chosen, marked and described. Owners and tenants were interviewed at this stage and permission for the final station mark and for visits by subsequent parties was obtained. Stations scheduled for intersection were photographed.

It is worth remarking that much care and attention was given to interviewing landlords and tenants and obtaining permission to enter and to mark stations on private property. The reconnaissance party was quite likely to be followed later by pillar constructors, lightkeepers, tower erectors, observers and maintenance parties, and the reception accorded to all these parties was often conditioned by the impression created during the reconnaissance.

When the reconnaissance was completed the following documents were submitted to headquarters:

(*a*) *The Reconnaissance Diagram*

This was a diagram, usually at half-inch to one mile scale, showing all the stations reconnoitred and all rays open. It also showed a suggested secondary scheme. A specimen diagram is at Diagram 19.

(*b*) *A Reconnaissance Report for each station*

This report recorded all information acquired by the reconnaissance party which might be useful to subsequent parties. Two sample reports are shown in Figs. 7.1/7.2 and 7.3/7.4.

These reports were modified in the light of experience gained as the work proceeded and the specimens shown are in the final version.

(*c*) *A list of directions verified at each station*

All rays open from each station and their approximate grid bearings were listed. A specimen list is at Fig. 7.5.

(*d*) *A map for each station*

Initially each station was plotted on a six inches to one mile map and access routes were marked. In remote areas these six-inch maps were at too large a scale and were cumbersome to use, so the 1/25,000 maps were adopted instead.

(*e*) *A photograph of each intersected point*

To ensure the correct identification intersected points were all photographed and the exact point of intersection marked by an arrow on the photograph.

7.03 Station Marking

7.030 GENERAL

In the old triangulation secondary and tertiary ground stations were usually buried. As a result many stations were lost and even when a station was intact the tedious and expensive task

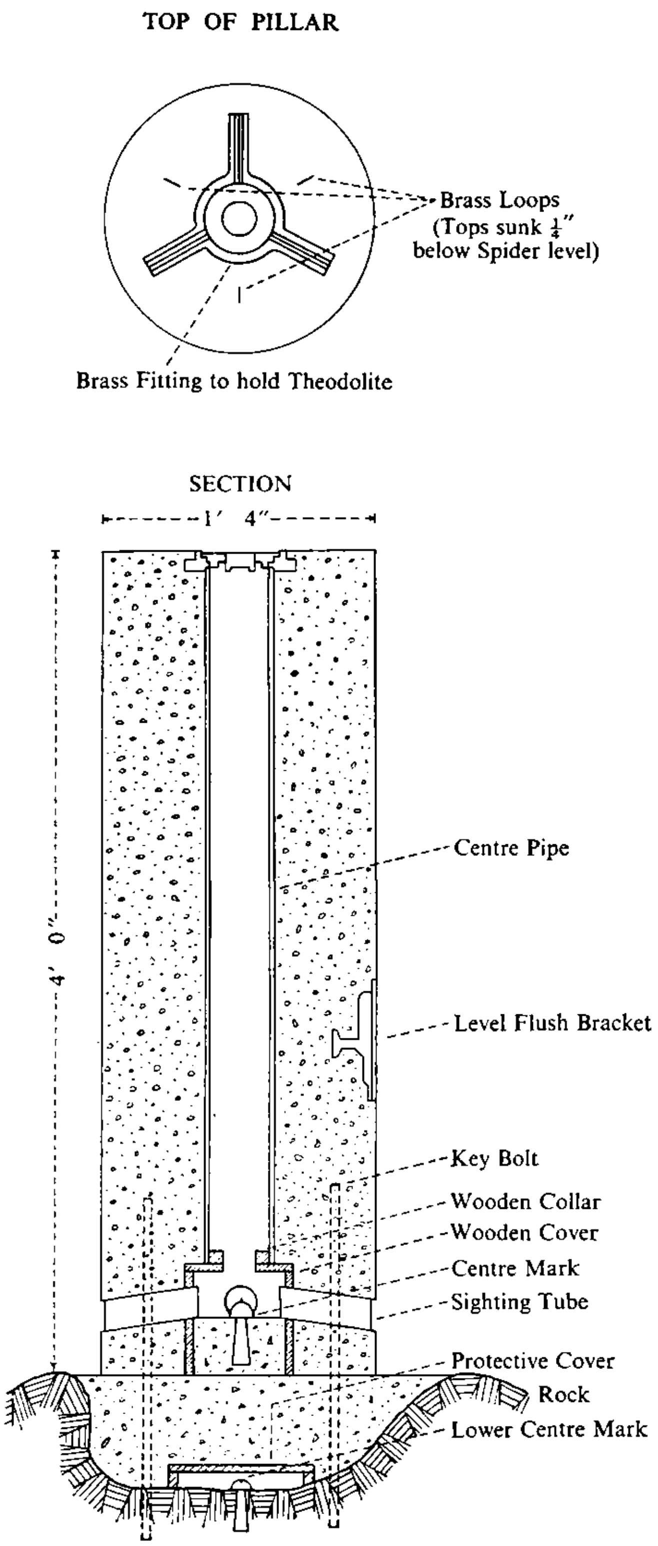


FIG. 7.6. Design of circular triangulation pillar

of uncovering it had to be undertaken before it could be occupied. The lesson was learnt and it was decided to mark secondary and tertiary ground stations of the Retriangulation with visible marks which could be occupied with a minimum of trouble. Since such stations would often be in relatively populous areas and therefore liable to damage by vandalism they had to be substantially constructed. It was decided therefore as a general rule to use the standard primary type pillar for secondary and tertiary points rather than a cheaper and less massive alternative. The standard pillar has been fully described in § 2.061.

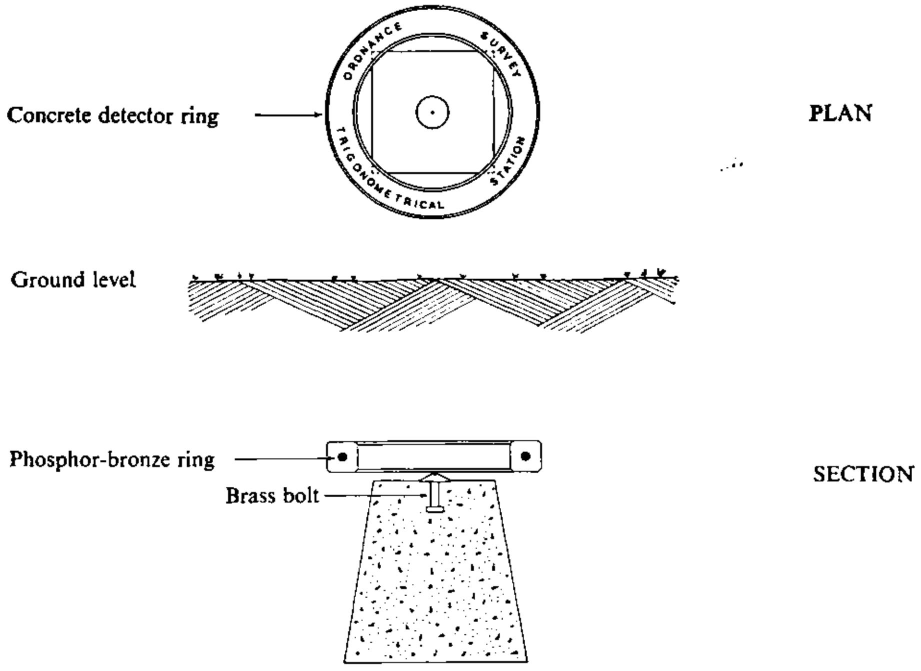
Before the war the building of pillars did not create many difficulties as it was easy to hire casual labour and transport of all forms. The work then was in the more developed areas of the country which were readily accessible to wheeled vehicles. After the war casual labour was almost unobtainable, animal transport was a thing of the past and mechanical transport was very scarce and expensive. These factors, coupled with the fact that the triangulation was now reaching the more remote areas, complicated the work and caused an alarming rise in costs. Permanent labour had to be engaged and complete pillar construction parties formed. These parties were provided with mechanical transport, but in the more remote areas the material for the pillars often had to be manhandled over long distances. As a standard pillar with a 3-foot base requires about two tons of material, transportation to site was often a long and expensive job. In 1950 two ex-army tracked vehicles, known as Carden-Loyd carriers, were bought and for four years they did valuable work in transporting the pillar materials much closer to the sites. But the vehicles were old and became progressively less reliable. Spares were difficult to obtain and running and maintenance costs became excessive, while the vehicles were frequently unserviceable. To try to cut down costs it was decided in 1954 to omit pillars at sites which were remote and inaccessible provided a bolt could be put directly into living rock at ground level. It was soon discovered that the difficulty of making satisfactory observations from an ordinary tripod in the high winds of Northern Scotland was such that the cost of the delays and re-observation thus occasioned heavily outweighed the cost of providing pillars. This decision was therefore reversed.

In 1955 another tracked vehicle, an ex-U.S. Army 'Weasel', was hired. This vehicle, designed for Arctic travel, was a great improvement on the Carden-Loyd and could often haul materials right up to the pillar site. But in 1957 it was planned to mark the stations in a block in the Western Highlands and the reconnaissance reports showed that even the Weasel would be of little help. A wide-awake pillar builder forwarded to headquarters a newspaper cutting showing a small helicopter working in the Highlands for the Forestry Commission carrying timber down from the hilltops to the roadside, and suggested that helicopters might help in transporting pillar materials. This was followed up and a scheme was prepared and costed and proved to be practicable and economic. The ensuing operation was most successful and 41 tons of material were landed at 46 sites in 40 hours of flying time spread over ten days. The success of this first scheme led to further use of helicopters in 1958 and again in 1959, 1960 and 1961. These operations led to a number of new developments including the use of special moisture resistant cement, a new circular pillar and a lighter type of shuttering both for the standard and for the new pillar.

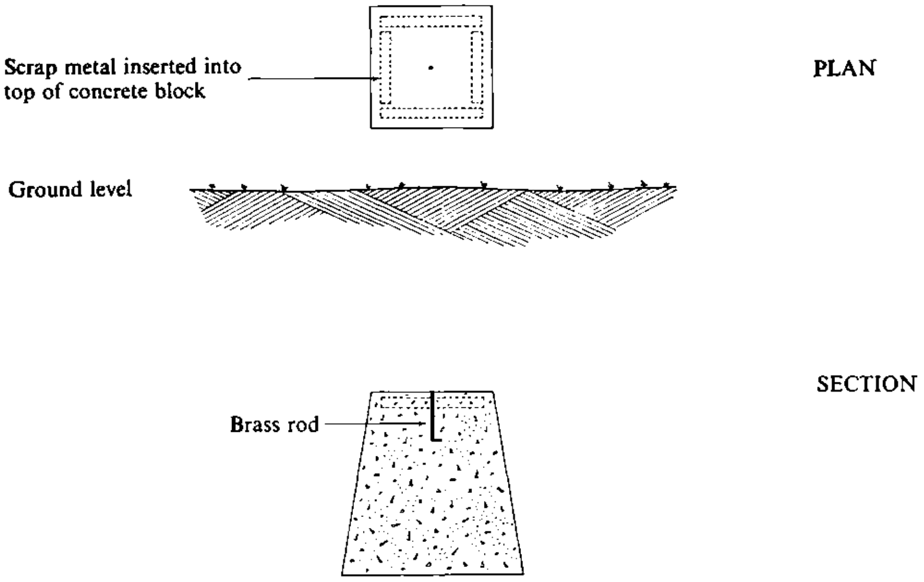
7.031 THE CIRCULAR PILLAR

This pillar is illustrated in Fig. 7.6. It is less massive than the standard pillar, weighing 6½ cwt. as against 14 cwt., and since it was almost always constructed on inaccessible rocky hilltops where a deep foundation was not required, this represented a great saving in transport cost.

The pillar was built with a lower centre mark where possible, and over this was emplaced a



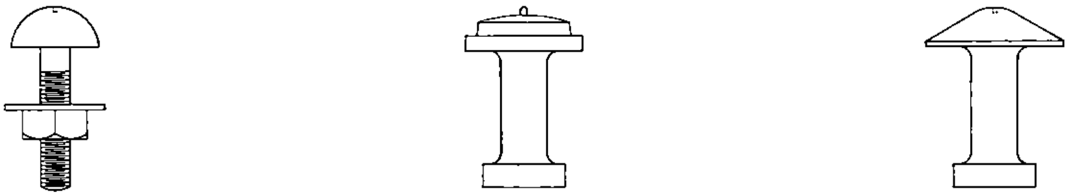
SECONDARY STATIONS



TERTIARY AND ADDITIONAL CONTROL STATIONS

0 1 2 3 Feet

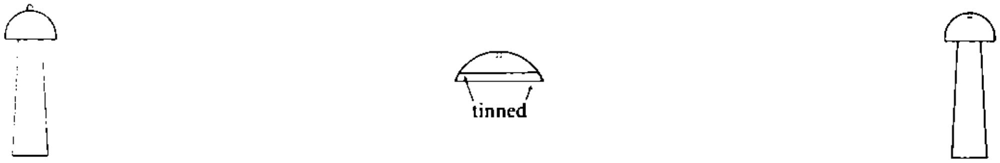
FIG. 7.7. Concrete blocks



Used on plain and felt-covered wooden roofs

Upper centre mark for standard pillars

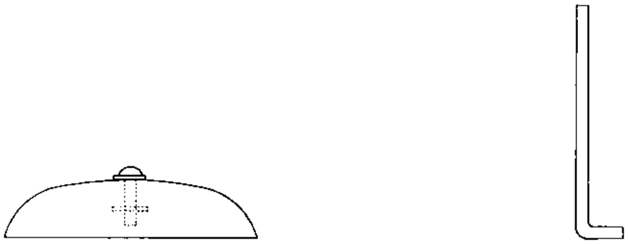
Lower centre mark for standard pillars



Upper centre mark for circular pillars

Used on lead-covered roofs

Lower centre mark for circular pillars



Rivet
Used as centre mark on iron and steel roofs and as witness marks

Rivet in asphalt disc
Used on plain concrete and bitumen-covered roofs

Brass rod
Used as centre mark in concrete blocks at minor triangulation stations

All centre marks are constructed of brass

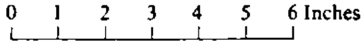


FIG. 7.8. Types of station marks

bottom bolt as with the standard pillar. The centre and sighting tubes were of cardboard and the whole pillar was cast in a shuttering consisting of a cardboard tube lined with aluminium foil.

The aluminium foil helped to give a good surface to the pillar and protected it from the weather for some time thus obviating the need for a subsequent early visit to complete the surfacing.

Using rapid-hardening cement the whole base and pillar could be built in one day, and when this was done the cardboard shuttering was left *in situ*. The next surveyor to visit the station in the normal course of the survey removed the tube and carried out such little facing as was normally found to be necessary.

The design of the circular pillar is not considered by some to be as aesthetically satisfying as that of the standard pillar. However, it was decided that the inaccessibility of those places at which it would be erected reduced the risk of criticism from the more sensitive element of the population to an acceptable level.

7.032 OTHER MARKS

(*a*) From 1941 to 1949 ground stations for minor triangulation and Town Control were also marked by standard pillars. As a result, by 1948 there were too many pillars in and around the larger towns, and it was decided to introduce a simple less obtrusive, and incidentally less expensive, type of ground mark consisting of a concrete block cast *in situ*. This mark is illustrated in Fig. 7.7. The same mark was used for secondary and tertiary stations where for any reason pillars could not be constructed. Most of the concrete blocks were buried, but at places where it was difficult to make an accurate locating description—for example on common land—the blocks were inserted with their upper surface flush with ground level.

When used as a secondary station a phosphor-bronze ring was buried with the block so as to enable it to be detected by mine detectors if the locating description should prove insufficient. With the lower order stations these rings were replaced by scrap iron incorporated in the block itself.

(*b*) Not all stations were ground stations. Some had perforce to be located on the roofs of buildings, and a variety of marks was used when this was necessary. These marks, each of which was designed for a particular type of roof surface, are illustrated at Fig. 7.8 which also gives an indication of the circumstances in which they were used.

7.04 Maintenance

To avoid the mistake of the past and to ensure that the stations of the Retriangulation should not, like so many of those of the earlier triangulation, be lost, all stations were not only marked clearly and substantially, but a regular maintenance programme was also initiated. This programme began in 1951 when it was decided that each station should be inspected, and repaired if necessary, every four years. As a result of the experience gained during the first four-year period it was decided to lengthen the cycle to 10 years with the exception of stations much frequented by the public. These stations, known as ‘popular pillars’, are inspected annually.

It occasionally happens that a station of the Retriangulation is entirely destroyed or removed. If prior information of the removal is received, an alternative station is sited before the removal of the old station. But if no prior information is received, a replacement station is sited as close as possible to the original station as soon as the loss is discovered. When permission to erect a pillar or emplace a mark is requested from the owner of land or property, he is requested to notify the Department if at any time he wishes to remove or disturb the mark. Owners and tenants have proved most co-operative and the occasions when a mark has been lost without prior notification having been given, have been very few.

7.05 Observations

The observing schemes were worked out at headquarters from the documents sent in by the reconnaissance parties. The schemes were drawn up on observing diagrams which showed all the secondary and tertiary rays to be observed, and these diagrams were sent out to the observing sections (see Diagram 20).

The standard signals used in secondary and tertiary work were beacon lamps or 'lights', and an observing section originally consisted of one observer and about nine lightkeepers, one of whom acted as booker. With this organisation in areas where communications were good it was often possible to observe at two or even three stations in a day. But when the triangulation reached more remote areas it became necessary to increase the size of the sections in some cases to as many as 20, including two or three observers.

Most observations were taken with the first-order theodolites used for the primary work, and it was found that the high accuracy of these instruments reduced the number of observations needed and more than offset the increased weight as against a normal second-order instrument. Moreover the weight itself helped to ensure steadiness when observing in strong winds.

The observing procedures were similar to those adopted for primary triangulation (see Appendix 12) except that for secondary work eight zeros were used—the first eight zeros listed in Appendix 12—and only one reading of the micrometer was taken on each pointing. No limit was set for the range of the eight observations, and it was left to the discretion of the observer to decide if additional zeros were required. As a guide observers were instructed to examine closely all series of observations where the range of eight readings exceeded 8″. For tertiary work four zeros were usually taken, but for some tertiary stations the small (3½″) Tavistock theodolite was employed and in these cases eight zeros were taken. No auxiliary or satellite stations were permitted on secondary work and their use on tertiary work was restricted to a minimum.

Special precautions were taken when using steel towers. When used originally for primary work (see § 2.07) it had been found that they were not stable in winds stronger than a light breeze, and that in direct sunshine there was sometimes appreciable twist of the inner tower. Observations in wind were therefore avoided, and in sunshine they were carried out quickly but at an even speed, the number of stations in a round being limited to three or four and each face being closed on the R.O. With these precautions it was found that a double-faced round could be accepted if the misclosure on the two faces were equal and opposite to within 2″. The same precautions were adopted on certain roof stations which were unstable.

At most steel tower stations the station mark was a pillar and observations were repeated from the pillar to all visible stations. Where it was necessary 'bearing blocks' were put in near the station so as to provide orientation for any subsequent work without having to re-erect the tower.

Where old triangulation stations existed, if for any reason it was impossible to site the new station over the old, it was sited a short distance away and the observer was required to take observations and measurements sufficient to fix the old station relative to the new.

When roofs or towers are used as stations it is often helpful to the subsequent survey if the centre of the roof or some prominent object such as a flagstaff or vane on it is co-ordinated. On

TABLE 7.1

SECONDARY BLOCK REFERENCES

HU 46	Shetlands
HY 52	Orkneys
NC 20	Ullapool
NC 35	Cape Wrath
NC 60	Lairg
NC 85	Tongue
ND 25	Wick
NG 24	Skye
NG 80	Fort William
NH 16	Wester Ross
NH 40	Loch Ness
NH 54	Inverness
NH 90	Cairngorms
NJ 06	Nairn
NJ 40	Ballater
NJ 45	Elgin
NJ 75	Banff
NK 03	Peterhead
NM 52	Mull
NN 44	Rannoch Muir
NN 61	Callander
NN 96	Pitlochry
NO 02	Perth
NO 42	Dundee
NO 46	Forfar
NO 89	Aberdeen
NR 55	Islay
NS 04	Isle of Arran
NS 29	Greenock
NS 44	Ayr and Kilmarnock
NS 77	Glasgow
NS 83	Lanark
NT 18	Dunfermline
NT 33	Peebles
NT 48	Edinburgh
NT 72	Jedburgh
NT 86	Berwick
NX 29	Girvan
NX 56	Kirkcudbright
NX 99	Dumfries
NY 11	Workington
NY 29	Lockerbie
NY 45	Carlisle
NY 71	Appleby
NY 86	Hexham
NZ 29	Newcastle
NZ 42	Middlesbrough
SC 38	Isle of Man
SD 30	Liverpool
SD 47	Barrow-in-Furness
SD 74	Blackburn
SD 80	Manchester
SD 96	Skipton
SE 02	Rochdale
SE 22	Huddersfield
SE 38	Harrogate
SE 44	Leeds
SE 73	Doncaster
SE 91	Scunthorpe
SE 97	Bridlington
SH 57	Anglesey
SH 62	Dolgelley
SH 95	Bala
SJ 44	Shrewsbury
SJ 82	Stafford
SJ 90	Wolverhampton
SJ 95	Stoke
SK 03	Stone
SK 08	Macclesfield
SK 10	Birmingham
SK 22	Burton-on-Trent
SK 26	Dovedale
SK 34	Derby
SK 39	Sheffield
SK 54	Nottingham
SK 56	Mansfield
SK 58	Chesterfield
SK 73	Melton Mowbray
SK 98	East Retford
SN 02	Haverfordwest
SN 41	Carmarthen
SN 55	Lampeter
SN 61	Llanelly
SN 80	Swansea
SN 94	Brecon
SO 11	Aberdare
SO 19	Newtown
SO 45	Hereford
SO 52	Monmouth
SO 59	Church Stretton
SO 79	Bridgnorth
SO 92	Gloucester
SO 96	Kidderminster
SP 26	Stratford-on-Avon
SP 38	Coventry
SP 42	Oxford
SP 69	Leicester
SP 74	Northampton
SP 91	Wendover
SS 30	Launceston
SS 72	Barnstaple
ST 07	Cardiff
ST 11	Exeter
ST 53	Yeovil
ST 57	Bristol
ST 70	Dorchester
ST 86	Bath
SU 15	Warminster
SU 19	Swindon
SU 31	Southampton
SU 54	Andover
SU 59	Abingdon
SU 77	Reading
SV 91	Isles of Scilly
SW 63	Falmouth
SX 06	Bodmin
SX 76	Plymouth
TA 12	Hull
TA 30	Grimsby
TA 43	Withernsea
TF 05	Newark-on-Trent
TF 22	Spalding
TF 47	Skegness
TF 71	Kings Lynn
TF 92	Fakenham
TG 12	Norwich
TG 41	Yarmouth
TL 05	Bedford
TL 08	Kettering
TL 32	Hertford
TL 36	Cambridge
TL 49	Peterborough
TL 51	Epping
TL 74	Haverhill
TL 78	Mildenhall
TL 81	Chelmsford
TM 17	Stowmarket
TM 23	Ipswich
TM 47	Saxmundham
TQ 00	Brighton
TQ 08	Windsor
TQ 13	Horsham
TQ 47	London
TQ 53	Tunbridge Wells
TQ 87	Thames Estuary
TQ 94	Ashford
TR 26	Canterbury

such occasions the observer took the necessary observations and measurements. Observations were also taken to fix witness marks whose function was to make possible the restoration of the station mark if this were destroyed during roof repairs.

SECONDARY COMPUTATIONS

7.06 Introduction

The blocks into which secondary work was divided are shown in Diagram 18. Each block has been given an unique number corresponding to the National Grid Reference of a point near the centre of the block, and a name indicating the main town or district covered by the block. A list of these block numbers and names is at Table 7.1.

7.07 Standard Methods of Computation

All secondary blocks were rigorously adjusted by the method of least squares. Unlike the primary, the secondary observations were not 'processed', the arithmetic means of the observed directions being used in the calculations, each having unit weight. Where a block was computed before any of the adjacent blocks had been completed it was adjusted to the primary control on or within the perimeter of the block. During the computation of subsequent adjacent blocks the adjusted secondary along the edge of the previous block was also used as fixed control.

Initially the method of condition equations was adopted as the standard method of adjustment, and up to about 1944 the computations were usually made in spheroidal terms, transverse Mercator co-ordinates of the adjusted secondary stations being calculated by the formulae given in § 2.29. From 1944 onwards the computations were carried out in plane terms using approximate co-ordinates to compute the transverse Mercator $(t-T)$ corrections (see § 2.224).

In 1943 two blocks were experimentally adjusted by the method of variation of rectangular co-ordinates, but it was not until 1948 that this method was adopted as standard. By that date the amount of adjusted secondary work was considerable and each new block usually had to satisfy a number of fixed conditions. These fixed conditions added greatly to the labour of adjustment by condition equations, but they could be incorporated in an adjustment by the method of variation of rectangular co-ordinates with comparatively little extra work. For details of the method see § 2.241.

Up to 1952 the solution of all normal equations had been effected by the widely used Gauss–Doolittle routine, but in July 1952 the Cholesky method of solution was adopted as standard practice. This change was made because the Cholesky method gives the more compact and more convenient layout.

All this work was carried out with desk machines until, in 1956, the Ordnance Survey installed a punched card calculator. The capacity of the installation was small, but it was programmed to calculate all stages of the secondary computations up to the formation of the observation equations. The formation and solving of the normal equations, however, continued to be carried out with desk machines.

7.08 Grouping of Blocks for Adjustment

In general each block was adjusted as a separate unit, but where the progress of the work allowed and the number of stations involved was not too large, pairs of adjacent blocks were adjusted as single units. Conversely, some blocks were sub-divided and adjusted in parts. In the main this was done to meet urgent demands for large scale survey control in priority town areas. But in one large block, NG 24, triangulation fieldwork was done in three stages and each stage was computed separately. Block SU 15 was a special case and is discussed below. Details of the grouped and partitioned blocks are given in Table 7.2.

TABLE 7.2

Grouped Blocks		*Partitioned Blocks*	
SW 63 and SX 06	SP 74 and TL 05	SU 15	3 parts
NH 54 and NJ 06	SK 34 and SK 54	NT 48	2 ,,
SO 45 and SO 52	SK 39 and SE 22	SX 76	2 ,,
SO 79 and SJ 90	TL 32 and TL 51	ST 57	2 ,,
SJ 82 and SK 03	TM 17 and TM 47	NS 77	2 ,,
NO 46 and NJ 40	TF 22 and TL 49	NG 24	3 ,,

Block SU 15 was observed in 1938 when the policy was to provide a dense network of secondary control (see § 7.01). To compute it as a single unit would have been a very heavy task as it contained 59 secondary stations and involved a very large number of condition equations. It was therefore computed in three parts. The other blocks which were observed when the same policy was in force were not computed until later. Some of them were small enough to be computed as a single unit, for the others a secondary scheme with longer ruling side lengths was picked out from the observations already made, so reducing the number of stations and enabling the computation to be carried out in one operation. The surplus secondary stations were then computed by simpler methods as tertiaries (see § 7.12).

7.09 Blocks Treated in a Special Manner

7.090 BLOCKS SN 41 AND SN 61

These are adjacent blocks with a common edge, SN 41 lying to the west of SN 61. In computing them SN 61 was first adjusted as a free figure and then it was scaled to a length obtained from the side joining two stations common to it and block SN 80 to the east. Block SN 41 was adjusted holding fixed the common edge with SN 61 but with no other fixed conditions. At the completion of this stage the two blocks formed a consistent whole but were not in any way related to fixed control. Starting with the two fixed stations in SN 80, co-ordinates were computed for all stations in the two blocks. There was an additional secondary station common to SN 61 and SN 80 and two primary stations on the perimeter of SN 41 and SN 61, and misclosures were of course revealed at these three stations. These misclosures were equated across the two blocks by a graphical method of drawing 'error contours' and the 'contoured' values were accepted.

This work was carried out in 1944 and the reasons for it are not now clear. It is possible that the computation of SN 41 and SN 61 was started before the computation of SN 80 had been completed and this method was chosen as a way of ensuring that there would be no discrepancy on the common edge.

7.091 BLOCK SK 56

This block was adjusted by the standard methods to the primary stations on its perimeter, but in the adjustment no account was taken of the common edges of secondary work with SK 58 to the north and SK 54 to the south, both of which had been computed earlier. There were, of course, discrepancies at the common stations along these edges and these discrepancies were distributed across SK 56 by graphical 'contouring'.

Here again there is some doubt about the reasons for this procedure.

7.092 BLOCK SU 54

This block was the first secondary block to be taken up and the original observation of it was completed at the beginning of 1938. An ill-conditioned layout and bad triangle misclosures produced an unsatisfactory result and the co-ordinate values were never published. In 1953 the block was redesigned and reobserved. It was calculated by the standard methods with satisfactory results, and these results were published.

7.093 BLOCKS NX 56, NG 80, HU 46, NH 40, NH 90 AND NN 44

All these blocks were observed by special methods involving the measurement of some of the distances between the stations with the Tellurometer equipment. Blocks NX 56, NG 80 and HU 46 were covered with a combination of triangulation and traverse, and further details of the observations and computations are given in § 7.15.

Blocks NH 40, NH 90 and NN 44 were designed to meet special requirements and this work is described in § 7.16.

7.10 Accuracy of the Secondary Triangulation

Average triangle misclosures for each block were computed and the routine calculations included the evaluation of the standard errors of observed and of adjusted directions in each block. Nearly all the triangle misclosures were less than $5''$ but in a few cases, for special reasons, values up to about $7''$ were accepted. The average triangle misclosure from all blocks was $1''.8$. The average standard error of an observed direction of unit weight was $\pm 1''.4$ and of an adjusted direction $\pm 1''.0$.

Of the 140 triangulated blocks 97 were adjusted by the variation of co-ordinates method and the standard errors of position were calculated for the stations in seven of these blocks. The seven blocks were selected in contrasting types of terrain and were adjusted to varying amounts of fixed control, so that the results should be representative of all the blocks. The results are tabulated in Table 7.3, in which also are included results for two blocks where Tellurometer traverses were used (see § 7.15).

Taking into account all the results in Table 7.3 it may be accepted that the standard error of position of a secondary station is about ±0·06 m.

TABLE 7.3

Quantity	*Secondary Blocks*								
	Triangulation							*Traverses*	
	ND 25	NJ 75	NN 61	SE 38	SH 95	ST 53	SU 54	NX 56	NG 80
Average Triangle misclosure	1″6	1″7	1″6	1″4	1″9	1″3	1″7	—	—
Maximum Triangle misclosure	4·6	3·8	3·9	4·6	5·3	3·6	4·1	—	—
No. of new stations	22	18	16	21	18	8	13	13*	20*
No. of fixed stations	8	4	9	23	16	13	17	10	16
No. of independent unknowns	74	58	57	86	70	37	56	50	75
No. of observation equations	142	98	116	210	162	94	140	125	166
Standard error of an observed direction of unit weight	1″01	1″03	1″02	1″22	1″35	1″80	2″33	1″50	1″62
Standard error of an adjusted direction	0″73	0″79	0″72	0″78	0″89	1″13	1″47	1″16	1″09
Standard errors of position for secondary stations These are vectors, or total displacements, and are in metres	0·059	0·066	0·077	0·040	0·058	0·072	0·074	0·066	0·076
	0·039	0·075	0·095	0·043	0·058	0·081	0·084	0·084	0·086
	0·073	0·065	0·075	0·058	0·045	0·082	0·074	0·077	0·078
	0·051	0·063	0·052	0·053	0·052	0·073	0·089	0·101	0·078
	0·060	0·053	0·045	0·056	0·038	0·075	0·084	0·083	0·073
	0·043	0·052	0·040	0·061	0·034	0·069	0·086	0·075	0·079
	0·039	0·076	0·050	0·062	0·044	0·067	0·090	0·060	0·062
	0·072	0·064	0·060	0·054	0·046	0·080	0·096	0·055	0·053
	0·046	0·069	0·059	0·045	0·045		0·094	0·054	0·076
	0·037	0·076	0·093	0·053	0·042		0·096	0·078	0·081
	0·056	0·084	0·049	0·059	0·035		0·093	0·090	0·068
	0·065	0·064	0·052	0·058	0·035		0·099	0·103	0·087
	0·046	0·082	0·039	0·047	0·046		0·087	0·062	0·087
	0·045	0·088	0·040	0·056	0·044				0·067
	0·045	0·070	0·060	0·055	0·045				0·088
	0·068	0·083	0·065	0·047	0·044				0·093
	0·073	0·088		0·046	0·045				0·088
	0·054	0·086		0·045	0·039				0·063
	0·045			0·050					0·072
	0·062			0·057					0·102
	0·064			0·037					
	0·084								
Average	0·056	0·072	0·059	0·052	0·044	0·075	0·088	0·076	0·078

* Junction points

TERTIARY COMPUTATIONS

7.11 Introduction

This section deals with the method of computation of the tertiary network which covers the greater part of the country. The methods here described also apply to the lower order triangulation which is carried out as required to provide control for surveys of urban areas.

7.12 Standard Methods of Computation

The tertiary work was observed concurrently with the secondary and consisted of a comparatively dense and complex network. In earlier work it was the standard practice to examine the network and to decide on a sequence of computations which would enable each point to be fixed from two or more triangles each of which contained two points whose co-ordinates were known (i.e. primary or secondary points, or tertiary points previously fixed). For each triangle the misclosures were distributed equally among the three angles and then the co-ordinates at unknown points were computed using standard plane formulae—in most cases the transverse Mercator $(t-T)$ correction was negligible but it was applied where it was significant. The finally accepted co-ordinates were the mean of the results of all the triangles into the point. The same procedure was adopted for intersected points but, of course, no distribution of triangular error could be made.

After some experience it became clear that often the majority of the triangle misclosure was caused by a bad pointing in one direction only in the triangle, and the practice of distributing the misclosure equally to the three angles was resulting in errors and inconsistencies. As a result it was decided about 1944 to adopt semi-graphic methods for the computation of all orders of work below secondary.

With this semi-graphic method it was the practice to examine the network as before and to decide upon a sequence of computation so that where possible each point had a number of rays into it from stations whose co-ordinates were known and also had a number of rays out from it to known stations. With this arrangement it was possible to compute the co-ordinates of the point in two independent ways, first as a semi-graphic intersection and then as a semi-graphic resection. It was found to be convenient to plot both these semi-graphic fixations on one graph and then to assess the graph to find the most likely value for the co-ordinates of the point. No hard and fast rules were laid down as it was found that experience and common sense were the best guides, but in practice the method was simple, quick and very accurate as bad pointings could be recognised at once and a general picture of the fixation was seen at a glance. Another great advantage of the method was that local consistency among the points was ensured; this was of major importance when they were used as the basis for control of the large scale surveys.

This semi-graphic method is somewhat unusual and a worked example is given at Appendix 14.

7.13 Accuracy of the Tertiary Triangulation

The accuracy of the tertiary triangulation was assessed in terms of standard errors of position. A proportion of the tertiary points, about 10%, in each secondary block were selected at random and computed individually by least squares using a method which was essentially that of variation of co-ordinates applied to a single fixation. The accepted semi-graphic values were used as the approximate values in the computation and the small co-ordinate changes dE and dN were computed. The method also allowed the standard errors of the dE and dN to be calculated and in nearly all cases dE and dN were considerably smaller than their standard errors, indicating that the semi-graphic value could not be improved upon. The standard errors of position derived from the least squares computation can be accepted as satisfactory estimates of the accuracy of the semi-graphic fixations. The average calculated from about 1,000 points was $\pm 0{\cdot}05$ m. This figure, of course, gives a measure of the internal consistency only; it is not a measure of absolute accuracy.

THE USE OF THE TELLUROMETER ON SECOND AND LOWER ORDERS OF TRIANGULATION

7.14 Introduction

In 1957 the Tellurometer equipment became available for the first time. A set was purchased and trials were carried out to see how best it could be used to assist in the completion of the Retriangulation. At that time only 29 of the 147 secondary blocks remained to be completed and the majority of these were in the mountainous parts of Scotland. A series of trials were carried out which showed that in this type of country no advantage in accuracy or speed was likely to be realised by using the Tellurometer for trilateration, but that if the equipment was used to run traverses, the required second-order and third-order control could be provided with a substantial saving in time and cost. Therefore maximum use was made of this method; the results are described below.

7.15 Tellurometer Traversing in normal Secondary Blocks

In 1958 block NX 56 was in the observing programme. It had already been reconnoitred and marked for normal triangulation but the proposed scheme was replaced by a network of main traverses to fix second-order points at a density of about 13 km. and subsidiary two- or three-legged traverses to fix tertiary points. This was satisfactory but it was still necessary to occupy all stations and the full potential of the Tellurometer was not realised. Consequently in a subsequent block, NG 80, which had also been reconnoitred and marked for normal triangulation, the secondary stations were fixed in the same way as in block NX 56 but where possible the tertiary stations were intersected. This resulted in greater economies being achieved. Fig. 7.9 shows part of the observing diagram for block NG 80.

Because both the above blocks were reconnoitred for normal triangulation, the stations were sited on the highest, and therefore the most inaccessible, mountains. Had the stations been sited specifically for Tellurometer traverses much lower hills near to roads could have been used as main stations and the more remote stations could have been intersected.

In both blocks angular measurements were made with geodetic theodolites; eight zeros were used on the main traverses and four on the subsidiary traverses. The Tellurometer measurements were normally made in one direction only with 20 fine readings on secondary lines and 10 on tertiary lines. In each block the main traverse scheme was adjusted as one unit by the usual variation of co-ordinates method with the measured distances and directions given equal weight. In Table 7.3 the statistical results of these adjustments are shown beside those for some of the triangulated blocks. The average values of the standard errors of junction points in the traverse blocks were within the range of values for the triangulated blocks, and it is reasonable to conclude that the accuracy of position is about the same for both methods. An examination of the standard errors shows a greater range for the traverse blocks which suggests that they may be internally slightly less consistent than the triangulated blocks.

In addition to the above two blocks the Tellurometer was also used in block HU 46. This block covered the Shetland Islands and many rays were across water, and this, combined with bad weather, made Tellurometer measurements difficult. The majority of the block was therefore covered by triangulation, the Tellurometer measurements being supplementary. The computations were carried out as for a standard secondary block, introducing the Tellurometer measurements as additional observations.

7.16 The Tellurometer and the I.T.C.–Jerie Analogue Computer

In some blocks the subsequent survey was to be by aerial methods at a scale of six inches to one mile. In these blocks it had already been decided to omit the tertiary work as the secondary triangulation was sufficient to control the aerial survey (see § 7.01). At about the same time as the Tellurometer was purchased a new equipment called the I.T.C.–Jerie Analogue Computer was introduced for the adjustment of blocks of aerial triangulation. With this computer the control required was reduced still further so that in a block 48 km. × 48 km. points in the corners only and at intervals of about 13 km. along each side were required. The provision of such control was an ideal task for the Tellurometer. By this time the Retriangulation had been nearly completed, and only three blocks remained where these techniques could be applied. Even so, substantial savings were made. By running Tellurometer traverses between existing primary or secondary stations, the necessary control for the three blocks was produced with 30 new stations compared with an estimate of about 140 new stations by earlier methods.

In order to exploit the speed of the Tellurometer to the full helicopters were used to carry the men and equipment to the traverse stations. All the traverses were completed in 17 flying days spread over two short periods in the summers of 1959 and 1960. Fig. 7.10 shows the layout of two of the blocks.

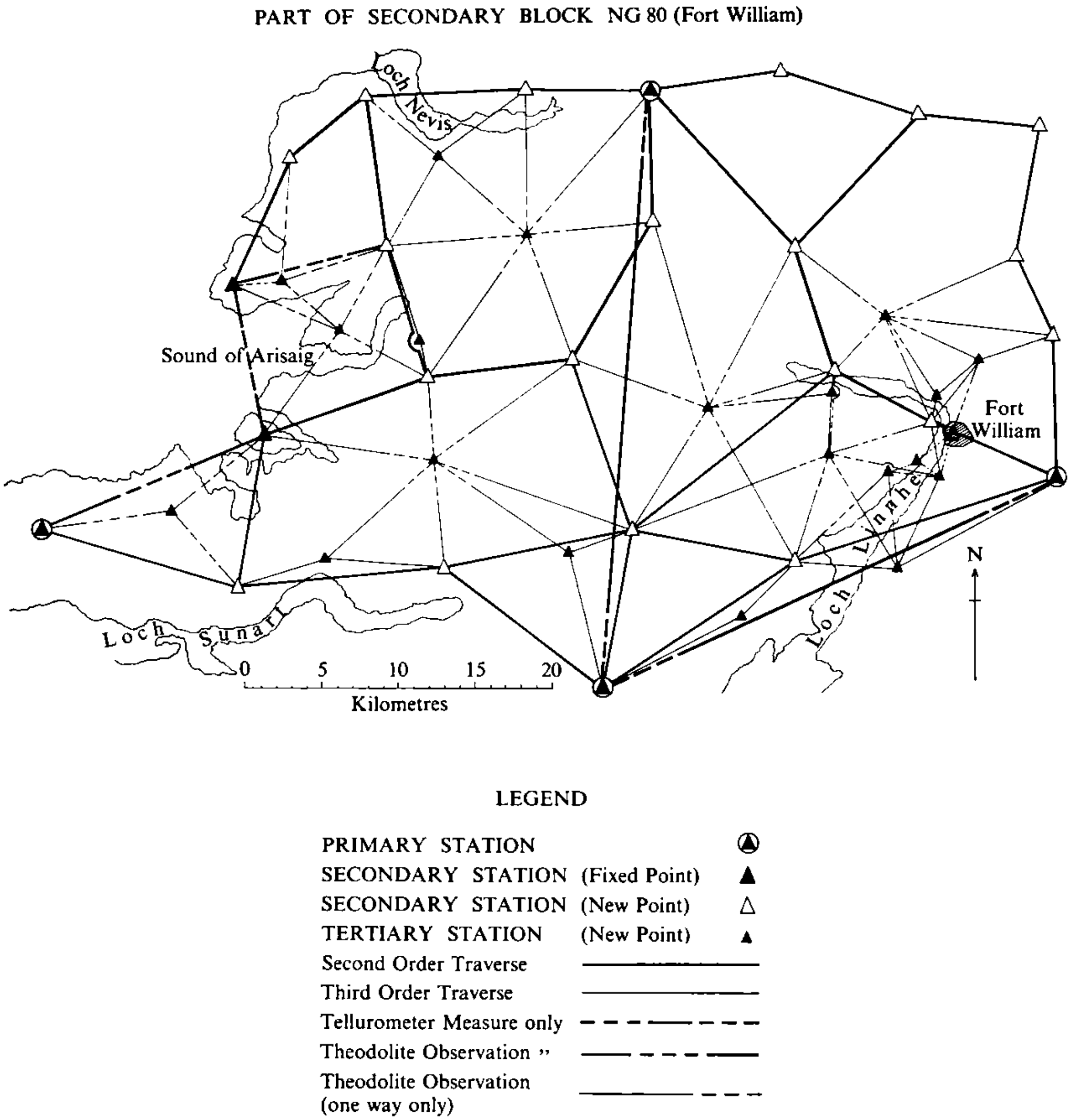
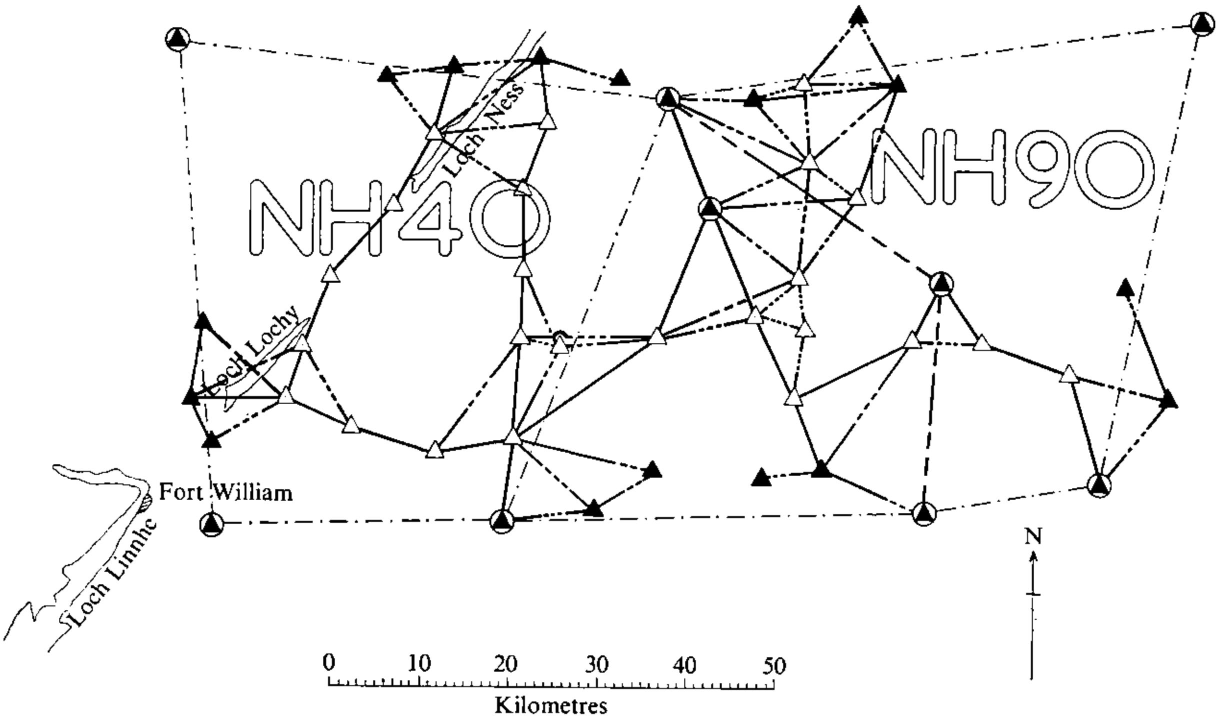


FIG. 7.9. Layout of secondary and tertiary control

SECONDARY BLOCKS NH 40 (Loch Ness) and NH 90 (Cairngorms)



LEGEND

PRIMARY STATION		◉
SECONDARY STATION	(Fixed Point)	▲
SECONDARY STATION	(New Point)	△
Traverse Leg		————
Tellurometer Measure only		– – – —— – – –
Theodolite Observation ”		—— – – – ——
Theodolite Observation (one way only)		———— – – –
Secondary block boundary		·—·—·—·—

FIG. 7.10. Layout of skeleton traverse network to provide ground control for aerial surveys at 1/10,560 scale

TRIGONOMETRICAL HEIGHTS

7.17 Observations

Great Britain is plentifully supplied with lines of spirit levelling and the altitudes of most of the triangulation stations in the flatter and more accessible parts of the country have been determined by this method. In the mountains and more remote areas the lines of spirit levelling are more widely spaced and in general run along the valleys. But heights were needed in the mountains to provide control for contouring, and the most economical way of providing them was to fix the heights of the triangulation stations by trigonometrical methods.

The necessary vertical angles were taken during the course of normal secondary and tertiary observations. Not less than four sets were observed. Observations were in general reciprocal but not in most cases simultaneously carried out at each end of a line. To reduce the effects of refraction observations were taken between the hours of noon and 15.00 hours GMT.

7.18 Computations

For the purpose of calculating trigonometrical heights a number of blocks were selected and computed as separate units. These blocks did not necessarily correspond with secondary blocks but were designed so as to make the best use of the fixed control, consisting mainly of spirit-levelled heights but including also some previously adjusted trigonometrical heights. Generally speaking the acceptable minimum number of fixed control heights was about 15% of the number of new stations to be heighted. In nearly all cases this percentage was exceeded; see Table 7.4 where some typical blocks are listed. Such control should be well distributed throughout the block.

The height blocks were adjusted by the method of least squares using observation equations, the observation equation for a reciprocally observed line being given a weight of $1/S$ where S is the length of the line. Lines observed in one direction only were not normally used, but when used were corrected for curvature and refraction by the standard formulae before being put into the computations as observations of weight $1/3\,S$. The computations were carried out in the usual way except that before starting a block the height differences round the various triangles or polygons were checked to make sure there were no gross errors; from these height differences approximate heights were computed for all new points. The approximate heights were in general correct to better than ± 1 foot and putting them into the observation equations enabled the small corrections to the approximate heights to be calculated and so reduced the arithmetic involved.

7.19 Accuracy

It would be a straightforward task to calculate the standard errors of the adjusted heights, but it is a long calculation, and so a simpler method was adopted to check the accuracy. A test area was selected in which there were more spirit-levelled heights than required in the adjustment.

The test area was hilly with some large height differences and could be considered a typical area. It was adjusted treating some of the spirit-levelled heights as unknown, and a comparison was made between the calculated values and the spirit-levelled values. The block chosen was the first one in Table 7.4 and contained 57 stations of which the heights of 24 were known. Only 12 of these were used in the adjustment and a comparison was made at the other 12 known points. The results are shown in Table 7.5 and it will be seen that the average difference (without regard to sign) between the calculated and the spirit-levelled heights was 0·404 feet.

TABLE 7.4

Area Covered by Height Block in Terms of Triangulation Secondary Blocks	*No. of New Heights*	*No. of Fixed Control Heights*	*Fixed Control as a Percentage of New Points*
NC 35, NC 85, ND 25 (part)	45	12	27%
NC 20, NC 60, NH 54 (part)	46	41	89%
NH 54, NJ 06	34	39	115%
NH 16, NH 54 (part)	48	33	69%
NS 04, NS 29 (part)	69	29	42%
NR 55	28	12	43%
NT 72	84	91	108%
NX 29, NX 56, NX 99, NS 44, NS 83	179	67	37%
NJ 40, NO 46, NO 89, NK 03, NJ 06, NJ 45, NJ 75	99	90	91%
NG 24 West	36	12	33%
NG 24 East	70	24	34%
NC 20 West	16	9	56%
NS 29	98	35	36%
NG 80	60	9	15%

TABLE 7.5

Adjusted Trig. Height (feet)	*Spirit Levelled Height (feet)*	*Difference*
969·245	968·998	+0·247
862·152	862·122	+0·030
577·324	577·883	−0·559
508·392	508·490	−0·098
530·845	531·591	−0·746
507·094	506·987	+0·107
658·344	658·529	−0·185
295·049	296·241	−1·192
962·817	963·608	−0·791
616·450	617·299	−0·849
966·069	966·067	+0·002
555·264	555·223	+0·041

APPENDIX 1

FIGURE 1 (see DIAGRAM 5)

1.1 Mean observed directions, adjustment corrections, and adjusted directions

From	*To*		*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Bardon Hill	Cold Ashby	(S)	00° 00′ 00″·00	−0″·262	359° 59′ 59″·738
(V)	Charwelton	(R)	21 21 54·75	+0·228	21 21 54·978
	Walton Hill	(T)	83 52 42·82	−0·049	83 52 42·771
	Castle Ring	(B₁)	116 17 48·79	+0·084	116 17 48·874
Beacon Hill	Martinsell	(I)	00 00 00·00	+0·437	00 00 00·437
(G)	Inkpen	(J)	47 53 52·50	−0·530	47 53 51·970
	Butser	(E)	117 45 38·37	+0·171	117 45 38·541
	Dunnose	(B)	153 43 24·74	+0·009	153 43 24·749
	Coringdon	(A)	201 22 13·24	+0·158	201 22 13·398
	Wingreen	(D)	235 11 25·47	−0·419	235 11 25·051
	Bradley Knoll	(F)	267 23 41·21	+0·068	267 23 41·278
	Westbury Down	(F₁)	290 25 41·24	+0·106	290 25 41·346
Bradley Knoll	Wingreen	(D)	00 00 00·00	−0·538	359 59 59·462
(F)	Bulbarrow	(C)	40 45 33·68	−0·116	40 45 33·564
	Pen Hill	(H)	155 57 07·51	+0·191	155 57 07·701
	Westbury Down	(F₁)	259 46 09·98	−0·029	259 46 09·951
	Martinsell	(I)	275 31 43·81	+0·186	275 31 43·996
	Beacon Hill	(G)	302 10 15·41	+0·307	302 10 15·717
Broadway Tower	Cleeve Hill	(P)	00 00 00·00	−0·796	359 59 59·204
(Q)	Malvern	(O)	59 30 18·61	−0·322	59 30 18·288
	Titterstone Clee	(U)	83 30 07·60	−0·162	83 30 07·438
	Walton Hill	(T)	113 26 31·59	+0·709	113 26 32·299
	Cold Ashby	(S)	187 36 24·19	+0·387	187 36 24·577
	Charwelton	(R)	198 24 05·71	−0·524	198 24 05·186
	White Horse Hill	(M)	294 17 38·31	+0·709	294 17 39·019
Bulbarrow	Wingreen	(D)	00 00 00·00	−0·155	359 59 59·845
(C)	Coringdon	(A)	92 14 32·88	+0·280	92 14 33·160
	Gore Hill	(G₁)	218 40 21·20	+0·012	218 40 21·212
	Pen Hill	(H)	289 22 03·10	−0·479	289 22 02·621
	Bradley Knoll	(F)	317 07 12·64	+0·343	317 07 12·983
Butser	Dunnose	(B)	150 46 11·80	+0·126	150 46 11·926
(E)	Coringdon	(A)	191 30 25·00	+0·103	191 30 25·103
	Wingreen	(D)	220 39 27·37	−0·329	220 39 27·041
	Beacon Hill	(G)	243 40 55·05	+0·345	243 40 55·395
	Inkpen	(J)	270 41 12·95	−0·245	270 41 12·705

1.1 *continued*

From	*To*		*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Castle Ring	Walton Hill	(T)	00° 00′ 00″·00	+0″·458	00° 00′ 00″·458
(B$_1$)	Titterstone Clee	(U)	35 23 15·58	−0·524	35 23 15·056
	Wrekin	(A$_1$)	66 31 10·94	+0·312	66 31 11·252
	Bardon Hill	(V)	252 40 21·09	−0·246	252 40 20·844
Charwelton	Broadway Tower	(Q)	00 00 00·00	−0·170	359 59 59·830
(R)	Malvern	(O)	18 08 27·63	+0·215	18 08 27·845
	Walton Hill	(T)	48 59 46·27	−0·189	48 59 46·081
	Bardon Hill	(V)	111 07 07·93	+0·386	111 07 08·316
	Cold Ashby	(S)	149 03 28·58	−0·242	149 03 28·338
Cleeve Hill	Broadway Tower	(Q)	00 00 00·00	+0·765	00 00 00·765
(P)	White Horse Hill	(M)	96 23 06·46	+0·397	96 23 06·857
	Liddington Castle	(E$_1$)	109 28 47·15	−0·025	109 28 47·125
	Peglers Tump	(L)	175 02 42·55	−0·511	175 02 42·039
	Malvern	(O)	266 59 41·64	−0·625	266 59 41·015
Cold Ashby	Charwelton	(R)	00 00 00·00	−0·281	359 59 59·719
(S)	Broadway Tower	(Q)	20 08 51·52	+0·496	20 08 52·016
	Bardon Hill	(V)	120 41 46·84	−0·215	120 41 46·625
Coringdon	Bulbarrow	(C)	00 00 00·00	−0·229	359 59 59·771
(A)	Wingreen	(D)	31 28 54·92	+0·440	31 28 55·360
	Beacon Hill	(G)	60 16 01·58	+0·351	60 16 01·931
	Butser	(E)	104 29 06·58	−0·596	104 29 05·984
	Dunnose	(B)	134 27 35·66	−0·016	134 27 35·644
	Gore Hill	(G$_1$)	344 50 04·45	+0·050	344 50 04·500
Dunnose	Butser	(E)	00 00 00·00	−0·017	359 59 59·983
(B)	Coringdon	(A)	250 42 36·86	+0·225	250 42 37·085
	Wingreen	(D)	281 51 39·85	−0·109	281 51 39·741
	Beacon Hill	(G)	308 52 23·60	−0·099	308 52 23·501
Gwynydd Bach	Mynydd Maen	(K)	00 00 00·00	+0·736	00 00 00·736
(N)	Radnor Forest	(C$_1$)	182 17 26·14	−0·073	182 17 26·067
	Titterstone Clee	(U)	225 58 08·32	+0·509	225 58 08·829
	Malvern	(O)	262 29 04·36	−0·817	262 29 03·543
	Peglers Tump	(L)	305 25 42·47	−0·355	305 25 42·115
Inkpen	White Horse Hill	(M)	00 00 00·00	−0·553	359 59 59·447
(J)	Butser	(E)	156 36 46·18	−0·044	156 36 46·136
	Beacon Hill	(G)	239 44 45·30	+0·464	239 44 45·764
	Martinsell	(I)	292 54 35·08	+0·643	292 54 35·723
	Liddington Castle	(E$_1$)	334 16 50·64	−0·510	334 16 50·130

1.1 *continued*

From	*To*		*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Malvern (O)	Walton Hill	(T)	00° 00′ 00″00	−0″182	359° 59′ 59″818
	Charwelton	(R)	54 58 58·70	−0·215	54 58 58·485
	Broadway Tower	(Q)	77 56 45·65	+0·575	77 56 46·225
	Cleeve Hill	(P)	105 26 08·18	+0·493	105 26 08·673
	Peglers Tump	(L)	150 41 09·23	−0·833	150 41 08·397
	Mynydd Maen	(K)	200 20 45·37	−0·539	200 20 44·831
	Gwynydd Bach	(N)	227 54 59·04	+0·516	227 54 59·556
	Radnor Forest	(C_1)	260 58 38·90	+0·215	260 58 39·115
	Titterstone Clee	(U)	304 51 16·23	−0·030	304 51 16·200
Martinsell (I)	Inkpen	(J)	00 00 00·00	−0·449	359 59 59·551
	Beacon Hill	(G)	78 56 19·43	−0·337	78 56 19·093
	Wingreen	(D)	113 48 53·05	+0·123	113 48 53·173
	Bradley Knoll	(F)	139 41 30·68	−0·255	139 41 30·425
	Pen Hill	(H)	159 38 02·96	+0·822	159 38 03·782
	Peglers Tump	(L)	216 22 11·98	−0·055	216 22 11·925
	Liddington Castle	(E_1)	274 38 30·59	+0·071	274 38 30·661
	White Horse Hill	(M)	291 59 38·55	+0·079	291 59 38·629
Mynydd Maen (K)	Gwynydd Bach	(N)	87 06 56·57	−0·437	87 06 56·133
	Malvern	(O)	142 01 48·46	+0·436	142 01 48·896
	Peglers Tump	(L)	182 36 20·38	−0·187	182 36 20·193
	Pen Hill	(H)	243 10 02·43	+0·188	243 10 02·618
Peglers Tump (L)	Cleeve Hill	(P)	00 00 00·00	+0·477	00 00 00·477
	White Horse Hill	(M)	64 47 07·56	+0·240	64 47 07·800
	Liddington Castle	(E_1)	75 35 24·87	−0·914	75 35 23·956
	Martinsell	(I)	92 45 34·89	+0·340	92 45 35·230
	Pen Hill	(H)	163 33 04·26	−0·424	163 33 03·836
	Mynydd Maen	(K)	227 25 58·28	+0·049	227 25 58·329
	Gwynydd Bach	(N)	257 22 19·96	+0·028	257 22 19·988
	Malvern	(O)	317 11 56·47	+0·205	317 11 56·675
Pen Hill (H)	Bradley Knoll	(F)	00 00 00·00	−0·192	359 59 59·808
	Bulbarrow	(C)	37 03 16·85	+0·284	37 03 17·134
	Gore Hill	(G_1)	54 06 09·73	+0·066	54 06 09·796
	Mynydd Maen	(K)	211 29 06·33	+0·354	211 29 06·684
	Peglers Tump	(L)	267 02 36·85	−0·333	267 02 36·517
	Martinsell	(I)	319 31 07·21	−0·329	319 31 06·881
	Westbury Down	(F_1)	329 19 26·86	+0·149	329 19 27·009
Titterstone Clee (U)	Wrekin	(A_1)	00 00 00·00	−0·471	359 59 59·529
	Castle Ring	(B_1)	45 20 54·49	+0·416	45 20 54·906
	Walton Hill	(T)	80 03 20·46	+0·786	80 03 21·246
	Broadway Tower	(Q)	121 41 24·88	−0·768	121 41 24·112
	Malvern	(O)	144 36 07·50	−0·108	144 36 07·392
	Gwynydd Bach	(N)	211 09 01·02	+0·288	211 09 01·308
	Radnor Forest	(C_1)	244 06 35·50	−0·143	244 06 35·357

1.1 *continued*

From	*To*		*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Walton Hill (T)	Wrekin	(A$_1$)	00° 00′ 00″·00	+0″·103	00° 00′ 00″·103
	Castle Ring	(B$_1$)	64 55 22·91	−0·405	64 55 22·505
	Bardon Hill	(V)	105 10 40·25	+0·024	105 10 40·274
	Cold Ashby	(S)	140 38 25·33	+0·443	140 38 25·773
	Charwelton	(R)	160 32 37·90	+0·277	160 32 38·177
	Broadway Tower	(Q)	206 35 24·18	+0·136	206 35 24·316
	Malvern	(O)	254 42 27·79	−0·474	254 42 27·316
	Titterstone Clee	(U)	315 01 00·65	−0·103	315 01 00·547
White Horse Hill (M)	Martinsell	(I)	00 00 00·00	+0·024	00 00 00·024
	Liddington Castle	(E$_1$)	25 24 35·34	−0·042	25 24 35·298
	Peglers Tump	(L)	76 24 09·40	−0·172	76 24 09·228
	Cleeve Hill	(P)	112 57 31·20	−0·579	112 57 30·621
	Broadway Tower	(Q)	130 52 06·38	−0·009	130 52 06·371
	Inkpen	(J)	315 05 42·71	+0·778	315 05 43·488
Wingreen (D)	Bulbarrow	(C)	00 00 00·00	+0·089	00 00 00·089
	Gore Hill	(G$_1$)	15 22 46·67	−0·061	15 22 46·609
	Bradley Knoll	(F)	96 21 39·98	+0·310	96 21 40·290
	Martinsell	(I)	166 00 50·04	+0·147	166 00 50·187
	Beacon Hill	(G)	186 19 42·25	+0·010	186 19 42·260
	Butser	(E)	225 52 31·56	+0·293	225 52 31·853
	Dunnose	(B)	257 51 04·66	−0·091	257 51 04·569
	Coringdon	(A)	303 43 27·90	−0·698	303 43 27·202
Wrekin (A$_1$)	Castle Ring	(B$_1$)	193 21 04·46	−0·431	193 21 04·029
	Walton Hill	(T)	241 54 34·15	+0·040	241 54 34·190
	Titterstone Clee	(U)	296 52 15·19	+0·391	296 52 15·581

1.2 Triangle misclosures and spherical excesses

Triangle	*Spherical Excess* (ϵ)	*Triangle Misclosure*	*Triangle*	*Spherical Excess* (ϵ)	*Triangle Misclosure*
BAD	5″573	+1″397	BAG	8″778	+0″542
BAE	5·735	−0·315	BDG	6·371	+0·519
BDE	8·072	+0·748	BGE	6·158	−0·138
ADG	3·166	+1·374	ADE	7·910	+2·460
AGE	9·202	+0·718	DGE	4·457	−0·367
CDA	1·792	−1·892	CFD	1·166	−0·146
CHF	1·824	−1·604	HFI	2·578	−1·208
FIG	2·210	−0·570	FID	2·615	+1·265
FGD	1·942	+0·658	DIG	1·538	−1·178
EGJ	3·510	−0·620	GIJ	1·034	+0·676
HKL	6·751	−0·161	HLI	7·112	+1·638
IMJ	1·182	+2·478	ILM	3·339	−0·039
LPM	3·899	+1·551	LOP	2·503	+1·167
LKN	4·341	−1·321	LKO	6·076	+0·174
LNO	6·416	−1·986	KNO	4·681	−3·481
MPQ	2·026	+1·304	POQ	1·282	−1·782
QOU	2·455	−1·425	QOT	3·417	−1·177
QOR	2·651	−0·971	QTR	5·277	+1·393
QSR	1·413	+0·207	OTR	6·042	+1·188
ONU	5·274	+1·476	OUT	2·995	+0·675
QUT	3·957	+0·923	UA_1T	2·665	−1·815
UA_1B_1	3·125	−2·545	UB_1T	2·897	+0·913
A_1B_1T	3·357	+0·183	B_1VT	3·485	−1·265
TVR	7·931	−0·551	VSR	2·168	+0·072

Unclosed Triangles

Triangle	*Spherical Excess* (ϵ)	*Triangle*	*Spherical Excess* (ϵ)
TVS	6″356	OUC_1	4″025
TQS	7·607	ONC_1	4·884
TRS	3·743	UNC_1	3·635
GFF_1	1·249	LPE_1	3·681
FHF_1	1·082	PME_1	1·391
HCG_1	1·629	MJE_1	0·693
DCG_1	0·472	MIE_1	0·314
ACG_1	0·968	JIE_1	0·803
DAG_1	3·231	ILE_1	1·853
		LME_1	1·173

1.3 Symbolic statement of condition equations

Angle Closure	*Side Closure*	*Remarks*	*Angle Closure*	*Side Closure*	*Remarks*
ABD			HLK		
ABE			KOL		
DEB			LOP		
	B(ADE)		LPM		
ABG				L(PMIHKO)	
DBG				E_1(LPM)	
	D(GBA)		PQM		
GEB			POQ		
	B(DGE)			P(OQML)	
ACD			LKN		
	G_1(DCA)		NOL		
FDG				N(OLK)	
CDF			OUN		
	D(FGAC)			C_1(UON)	
GEJ			UQT		
JIG				O(UQPLN)	
DIG			UOT		
	G(DIJE)		OTQ		
FIG				O(UTQ)	
	x(FIGD)	Pole at intersection of diagonals	TOR		
			TQR		
CFH					
	C(DFHG_1)			Q(OTR)	
HIF			TVSQ		Polygon Closure
	F(HIDC)			R(QTS)	
	F(HF_1GI)		RSV		
JIM			RTV		
MIL				S(RTV)	
HIL			TVB_1		
	I(LMJGFH)		UTB_1		
	E_1(MIJ)			T(UB_1VRO)	
	E_1(LIM)		B_1A_1T		
			UTA_1		
				A_1(B_1TU)	

APPENDIX 2

FIGURE 2 (see DIAGRAM 6)

2.1 Mean observed directions, adjustment corrections, and adjusted directions

From	*To*		*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Acre	Lincoln Minster	(R)	00° 00′ 00″00	+0″045	00° 00′ 00″045
(V)	Clifton	(U)	59 21 56·55	−0·668	59 21 55·882
	Normanby Gashldr	(Z)	96 30 09·54	−0·019	96 30 09·521
	Cave Wold	(E$_1$)	124 08 37·07	+0·643	124 08 37·713
Alport Heights	Harland South	(P)	00 00 00·00	−0·268	359 59 59·732
(J)	Loath Hill	(K)	87 55 14·25	+0·436	87 55 14·686
	Bardon Hill	(F)	159 43 33·23	−0·136	159 43 33·094
	Weaver Hill	(I)	257 48 07·90	−0·139	257 48 07·761
	Blake Mere	(O)	291 14 25·27	+0·107	291 14 25·377
Bardon Hill	Walton Hill	(B)	83 52 42·771(1)	—	83 52 42·771
(F)	Castle Ring	(E)	116 17 48·762(2)	−0·041	116 17 48·721
	Weaver Hill	(I)	158 58 54·30	−0·620	158 58 53·680
	Alport Heights	(J)	184 49 32·32	+0·427	184 49 32·747
	Loath Hill	(K)	230 05 26·66	+0·013	230 05 26·673
	Belvoir Castle	(L)	267 02 11·28	+0·026	267 02 11·306
Blake Mere	Weaver Hill	(I)	00 00 00·00	+0·108	00 00 00·108
(O)	Hanchurch Wtr Twr	(H)	63 29 28·43	−0·682	63 29 27·748
	Delamere	(N)	119 50 11·51	−0·399	119 50 11·111
	The Edge	(S)	207 10 16·17	+1·086	207 10 17·256
	Margery	(T)	223 04 50·66	−0·369	223 04 50·291
	Harland South	(P)	274 35 30·23	+0·293	274 35 30·523
	Alport Heights	(J)	309 37 22·25	−0·037	309 37 22·213
Botton Head	Hambleton Down	(H$_1$)	00 00 00·00	+0·192	00 00 00·192
(L$_1$)	Great Whernside	(G$_1$)	39 57 32·43	−0·002	39 57 32·428
	Water Crag	(N$_1$)	67 37 00·73	−0·383	67 37 00·347
	Royal Oak	(O$_1$)	95 56 20·78	+0·835	95 56 21·615
	Collier Law	(R$_1$)	99 49 41·08	+0·043	99 49 41·123
	Warden Law	(S$_1$)	130 25 21·30	−0·339	130 25 20·961
	Leavening Brow	(I$_1$)	307 54 22·39	−0·106	307 54 22·284
	York Minster	(D$_1$)	333 58 50·96	−0·240	333 58 50·720

(1) Fixed direction from Figure 1.

(2) Mean observed direction plus overlap correction from Figure 1.

2.1 *continued*

From	*To*	*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Boulsworth (B_1)	Rombalds Moor (C_1)	00° 00′ 00″00	−0″201	359° 59′ 59″799
	Holme Moss (X)	91 47 09·28	+0·152	91 47 09·432
	The Edge (S)	99 45 12·32	−0·643	99 45 11·677
	Rivington (W)	169 55 33·25	+0·477	169 55 33·727
	Weeton Res'r (A_1)	206 03 14·99	−0·496	206 03 14·494
	Mallowdale Pike (F_1)	241 49 03·05	+0·137	241 49 03·187
	Little Whernside (K_1)	274 44 18·76	+0·517	274 44 19·277
	Great Whernside (G_1)	308 06 21·14	+0·057	308 06 21·197
Castle Ring (E)	Walton Hill (B)	00 00 00·00	+0·297	00 00 00·297
	Titterstone Clee (A)	35 23 15·58	−0·471	35 23 15·109
	Wrekin (D)	66 31 10·94	+0·357	66 31 11·297
	Hanchurch Wtr Twr (H)	125 58 43·86	+0·219	125 58 44·079
	Weaver Hill (I)	171 49 34·87	−0·080	171 49 34·790
	Bardon Hill (F)	252 40 21·09	−0·322	252 40 20·768
Cave Wold (E_1)	Normanby Gashldr (Z)	36 18 31·50	+0·047	36 18 31·547
	Clifton (U)	68 04 47·25	+0·170	68 04 47·420
	York Minster (D_1)	138 13 06·35	+0·266	138 13 06·616
	Acre (V)	352 21 38·34	−0·483	352 21 37·857
Clifton (U)	Harland South (P)	00 00 00·00	−0·071	359 59 59·929
	Margery (T)	51 26 51·40	+0·267	51 26 51·667
	York Minster (D_1)	150 24 11·55	−0·300	150 24 11·250
	Cave Wold (E_1)	191 52 31·40	−0·449	191 52 30·951
	Normanby Gashldr (Z)	206 13 25·68	+0·049	206 13 25·729
	Acre (V)	231 22 44·53	+0·480	231 22 45·010
	Lincoln Minster (R)	259 36 39·91	+0·024	259 36 39·934
Collier Law (R_1)	Royal Oak (O_1)	00 00 00·00	+0·282	00 00 00·282
	Water Crag (N_1)	61 26 35·61	−0·101	61 26 35·509
	Cross Fell (Q_1)	125 22 32·53	+0·258	125 22 32·788
	Warden Law (S_1)	304 09 13·01	−0·283	304 09 12·727
	Botton Head (L_1)	352 58 52·26	−0·157	352 58 52·103
Cross Fell (Q_1)	Cold Fell Pike (T_1)	00 00 00·00	+0·131	00 00 00·131
	Collier Law (R_1)	98 09 42·08	−0·044	98 09 42·036
	Water Crag (N_1)	161 52 48·42	+0·124	161 52 48·544
	High Street (M_1)	247 36 45·16	−0·247	247 36 44·913
	Skiddaw (P_1)	283 57 53·51	+0·036	283 57 53·546
Delamere (N)	Cader Berwyn (G)	53 25 19·88	−0·261	53 25 19·619
	Moel Fammau (M)	81 08 52·94	−0·080	81 08 52·860
	Rivington (W)	195 56 06·61	−0·335	195 56 06·275
	Holme Moss (X)	238 15 52·22	−0·483	238 15 51·737
	The Edge (S)	251 12 14·04	+0·063	251 12 14·103
	Blake Mere (O)	281 19 48·10	+0·008	281 19 48·108
	Hanchurch Wtr Twr (H)	316 43 09·58	−0·104	316 43 09·476
	Wrekin (D)	353 39 14·69	+1·192	353 39 15·882

2.1 *continued*

From	*To*		*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Great Whernside	Rombalds Moor	(C_1)	00° 00′ 00″00	−0″568	359° 59′ 59″432
(G_1)	Boulsworth	(B_1)	32 08 30·38	−0·068	32 08 30·312
	Mallowdale Pike	(F_1)	91 06 40·35	−0·053	91 06 40·297
	Little Whernside	(K_1)	127 21 05·65	+0·396	127 21 06·046
	Water Crag	(N_1)	187 59 26·06	−0·022	187 59 26·038
	Botton Head	(L_1)	266 23 25·16	−0·403	266 23 24·757
	Hambleton Down	(H_1)	280 34 05·90	+0·102	280 34 06·002
	Leavening Brow	(I_1)	299 45 51·77	+0·326	299 45 52·096
	York Minster	(D_1)	311 18 40·63	+0·288	311 18 40·918
Hanchurch Wtr Twr	Wrekin	(D)	00 00 00·00	−0·368	359 59 59·632
(H)	Stiperstones	(C)	15 11 19·98	+0·308	15 11 20·288
	Cader Berwyn	(G)	50 59 18·23	+0·610	50 59 18·840
	Moel Fammau	(M)	74 53 07·18	+0·096	74 53 07·276
	Delamere	(N)	101 26 06·18	−0·475	101 26 05·705
	Blake Mere	(O)	189 42 03·82	+0·275	189 42 04·095
	Weaver Hill	(I)	221 36 19·93	+0·311	221 36 20·241
	Castle Ring	(E)	289 05 55·61	−0·300	289 05 55·310
	Walton Hill	(B)	316 29 22·73	+0·094	316 29 22·824
	Titterstone Clee	(A)	348 07 15·11	−0·550	348 07 14·560
Harland South	Alport Heights	(J)	00 00 00·00	+0·379	00 00 00·379
(P)	Blake Mere	(O)	76 12 36·00	−0·569	76 12 35·431
	Margery	(T)	159 32 17·00	+0·448	159 32 17·448
	Clifton	(U)	219 45 11·35	−0·151	219 45 11·199
	Lincoln Minster	(R)	268 32 34·93	−0·270	268 32 34·660
	Thoresby Wtr Twr	(Q)	272 25 48·24	+0·032	272 25 48·272
	Loath Hill	(K)	295 00 12·49	+0·131	295 00 12·621
High Street	Skiddaw	(P_1)	54 39 10·90	−0·087	54 39 10·813
(M_1)	Cold Fell Pike	(T_1)	119 57 06·05	−0·493	119 57 05·557
	Cross Fell	(Q_1)	146 18 12·64	+0·808	146 18 13·448
	Water Crag	(N_1)	197 09 29·66	−0·649	197 09 29·011
	Little Whernside	(K_1)	234 30 54·29	+0·543	234 30 54·833
	Mallowdale Pike	(F_1)	263 28 31·35	−0·241	263 28 31·109
	Black Combe	(J_1)	329 42 36·59	+0·120	329 42 36·710
Holme Moss	The Edge	(S)	00 00 00·00	−0·726	359 59 59·274
(X)	Delamere	(N)	56 19 08·23	+0·169	56 19 08·399
	Rivington	(W)	103 19 22·45	+1·456	103 19 23·906
	Boulsworth	(B_1)	153 56 01·16	−0·394	153 56 00·766
	Rombalds Moor	(C_1)	184 42 16·76	−0·361	184 42 16·399
	Upton Beacon	(Y)	256 24 07·69	+0·290	256 24 07·980
	Margery	(T)	308 34 17·67	−0·433	308 34 17·237
Lincoln Minster	Loath Hill	(K)	00 00 00·00	−0·128	359 59 59·872
(R)	Thoresby Wtr Twr	(Q)	20 54 19·13	+0·033	20 54 19·163
	Harland South	(P)	24 43 28·35	+0·116	24 43 28·466
	Clifton	(U)	55 32 49·75	−0·177	55 32 49·573
	Normanby Gashldr	(Z)	105 50 37·73	+0·065	105 50 37·795
	Acre	(V)	147 57 02·49	+0·063	147 57 02·553
	Belvoir Castle	(L)	320 21 11·48	+0·028	320 21 11·508

2.1 *continued*

From	*To*		*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Little Whernside	Mallowdale Pike	(F_1)	00° 00′ 00″·00	−0″·239	359° 59′ 59″·761
(K_1)	Black Combe	(J_1)	61 06 15·78	−0·191	61 06 15·589
	High Street	(M_1)	102 06 13·25	+0·509	102 06 13·759
	Water Crag	(N_1)	186 33 21·60	−0·335	186 33 21·265
	Great Whernside	(G_1)	253 08 44·54	−0·386	253 08 44·154
	Boulsworth	(B_1)	304 34 08·55	+0·642	304 34 09·192
Loath Hill	Belvoir Castle	(L)	00 00 00·00	−0·011	359 59 59·989
(K)	Bardon Hill	(F)	66 06 05·97	+0·094	66 06 06·064
	Alport Heights	(J)	129 01 57·27	−0·254	129 01 57·016
	Harland South	(P)	156 06 55·88	−0·190	156 06 55·690
	Thoresby Wtr Twr	(Q)	223 50 10·41	+0·002	223 50 10·412
	Lincoln Minster	(R)	284 55 51·56	+0·359	284 55 51·919
Mallowdale Pike	Little Whernside	(K_1)	00 00 00·00	+0·567	00 00 00·567
(F_1)	Great Whernside	(G_1)	36 54 21·01	−0·007	36 54 21·003
	Boulsworth	(B_1)	91 38 57·00	−0·300	91 38 56·700
	Rivington	(W)	138 30 19·38	−0·110	138 30 19·270
	Weeton Res'r	(A_1)	185 56 51·19	+0·179	185 56 51·369
	Black Combe	(J_1)	267 35 22·15	+0·077	267 35 22·227
	High Street	(M_1)	311 03 48·44	−0·405	311 03 48·035
Margery	Holme Moss	(X)	00 00 00·00	+0·440	00 00 00·440
(T)	Rombalds Moor	(C_1)	42 26 22·87	−0·350	42 26 22·520
	Upton Beacon	(Y)	108 22 48·87	−0·213	108 22 48·657
	Clifton	(U)	140 33 24·10	−0·436	140 33 23·664
	Harland South	(P)	208 53 40·66	−0·178	208 53 40·482
	Blake Mere	(O)	254 03 20·59	−0·344	254 03 20·246
	The Edge	(S)	291 33 39·44	+1·080	291 33 40·520
Rivington	Boulsworth	(B_1)	00 00 00·00	−0·684	359 59 59·316
(W)	Holme Moss	(X)	51 15 00·83	+0·225	51 15 01·055
	The Edge	(S)	68 59 01·47	−0·323	68 59 01·147
	Delamere	(N)	141 55 04·69	+0·499	141 55 05·189
	Moel Fammau	(M)	171 06 14·77	+0·080	171 06 14·850
	Weeton Res'r	(A_1)	254 00 25·97	+0·233	254 00 26·203
	Mallowdale Pike	(F_1)	298 44 48·03	−0·029	298 44 48·001
Royal Oak	Collier Law	(R_1)	00 00 00·00	+0·123	00 00 00·123
(O_1)	Warden Law	(S_1)	81 01 02·79	+0·276	81 01 03·066
	Botton Head	(L_1)	169 05 32·89	+0·097	169 05 32·987
	Water Crag	(N_1)	281 45 03·95	−0·496	281 45 03·454
Rombalds Moor	Boulsworth	(B_1)	00 00 00·00	−0·094	359 59 59·906
(C_1)	Great Whernside	(G_1)	95 57 52·46	−0·420	95 57 52·040
	York Minster	(D_1)	199 18 15·73	+0·794	199 18 16·524
	Upton Beacon	(Y)	248 24 51·70	−0·460	248 24 51·240
	Margery	(T)	288 51 44·96	+0·099	288 51 45·059
	Holme Moss	(X)	302 33 23·28	+0·081	302 33 23·361

2.1 *continued*

From	*To*	*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Skiddaw (P_1)	Cold Fell Pike (T_1)	95° 12′ 30″13	+0″311	95° 12′ 30″441
	Cross Fell (Q_1)	125 46 18·50	−0·493	125 46 18·007
	High Street (M_1)	177 46 08·68	+0·249	177 46 08·929
	Black Combe (J_1)	238 45 12·76	−0·067	238 45 12·693
The Edge (S)	Holme Moss (X)	00 00 00·00	+0·813	00 00 00·813
	Margery (T)	60 08 00·46	−1·169	60 07 59·291
	Blake Mere (O)	186 43 07·69	−0·958	186 43 06·732
	Delamere (N)	249 15 29·55	+0·689	249 15 30·239
	Rivington (W)	301 03 25·09	−1·179	301 03 23·911
	Boulsworth (B_1)	341 54 02·16	+1·804	341 54 03·964
Titterstone Clee (A)	Wrekin (D)	00 00 00·011(2)	−0·221	359 59 59·790
	Hanchurch Wtr Twr (H)	14 57 38·26	−0·032	14 57 38·228
	Castle Ring (E)	45 20 54·501(2)	+0·380	45 20 54·881
	Walton Hill (B)	80 03 21·246(1)	—	80 03 21·246
	Stiperstones (C)	305 48 46·34	+0·061	305 48 46·401
Upton Beacon (Y)	Margery (T)	00 00 00·00	−0·272	359 59 59·728
	Holme Moss (X)	19 27 02·41	+1·006	19 27 03·416
	Rombalds Moor (C_1)	73 36 44·17	−0·474	73 36 43·696
	York Minster (D_1)	141 13 27·90	−0·260	141 13 27·640
Walton Hill (B)	Wrekin (D)	00 00 00·050(2)	+0·069	00 00 00·119
	Hanchurch Wtr Twr (H)	38 17 31·41	−0·235	38 17 31·175
	Castle Ring (E)	64 55 22·960(2)	−0·693	64 55 22·267
	Bardon Hill (F)	105 10 40·274(1)	—	105 10 40·274
	Titterstone Clee (A)	315 01 00·547(1)	—	315 01 00·547
Water Crag (N_1)	Royal Oak (O_1)	00 00 00·00	+0·945	00 00 00·945
	Botton Head (L_1)	39 01 13·35	−0·500	39 01 12·850
	Hambleton Down (H_1)	56 11 39·78	+0·342	56 11 40·122
	Great Whernside (G_1)	112 57 52·17	−0·835	112 57 51·335
	Little Whernside (K_1)	165 44 09·47	+0·894	165 44 10·364
	High Street (M_1)	223 55 40·36	−0·149	223 55 40·211
	Cross Fell (Q_1)	267 20 31·84	−0·285	267 20 31·555
	Collier Law (R_1)	319 41 31·11	−0·412	319 41 30·698
Weaver Hill (I)	Castle Ring (E)	00 00 00·00	−0·279	359 59 59·721
	Wrekin (D)	41 53 27·88	−0·131	41 53 27·749
	Hanchurch Wtr Twr (H)	66 39 36·20	−0·182	66 39 36·018
	Blake Mere (O)	151 15 53·50	−0·236	151 15 53·264
	Alport Heights (J)	247 26 58·34	+0·263	247 26 58·603
	Bardon Hill (F)	303 31 46·56	+0·565	303 31 47·125

(1) Fixed direction from Figure 1.

(2) Mean observed direction plus overlap correction from Figure 1.

2.1 *continued*

From	*To*	*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Wrekin (D)	Stiperstones (C)	00° 00′ 00″·00	−0″·152	359° 59′ 59″·848
	Cader Berwyn (G)	43 49 25·11	−0·296	43 49 24·814
	Delamere (N)	102 04 46·43	+0·027	102 04 46·457
	Hanchurch Wtr Twr (H)	143 42 38·04	−0·083	143 42 37·957
	Weaver Hill (I)	160 32 51·51	+0·475	160 32 51·985
	Castle Ring (E)	193 21 04·46	−0·536	193 21 03·924
	Walton Hill (B)	241 54 34·15	−0·016	241 54 34·134
	Titterstone Clee (A)	296 52 15·19	+0·581	296 52 15·771
York Minster (D_1)	Hambleton Down (H_1)	00 00 00·00	+0·353	00 00 00·353
	Botton Head (L_1)	15 18 42·48	+0·202	15 18 42·682
	Leavening Brow (I_1)	78 34 28·16	−0·072	78 34 28·088
	Cave Wold (E_1)	136 28 24·22	−0·044	136 28 24·176
	Normanby Gashldr (Z)	159 43 01·96	−0·043	159 43 01·917
	Clifton (U)	204 51 50·55	+0·090	204 51 50·640
	Upton Beacon (Y)	214 57 25·70	+0·787	214 57 26·487
	Rombalds Moor (C_1)	278 14 13·59	−1·261	278 14 12·329
	Great Whernside (G_1)	306 12 33·09	−0·012	306 12 33·078

2.2 Triangle misclosures and spherical excesses

Triangle	*Spherical Excess* (ϵ)	*Triangle Misclosure*	*Triangle*	*Spherical Excess* (ϵ)	*Triangle Misclosure*	*Triangle*	*Spherical Excess* (ϵ)	*Triangle Misclosure*
ADH	1″323	−1″023	$\mathrm{XB_1C_1}$	1″810	−0″210	$\mathrm{UD_1E_1}$	5″361	−0″081
ADB	2·665	−1·815	$\mathrm{B_1F_1K_1}$	2·791	+1·369	TSX	0·433	+2·917
AHB	5·382	−0·042	$\mathrm{B_1K_1G_1}$	2·691	−1·031	TXY	1·162	+0·098
BDH	4·040	+0·750	$\mathrm{C_1G_1D_1}$	3·747	−1·607	$\mathrm{SWB_1}$	3·934	−4·464
BHE	2·388	+0·092	$\mathrm{D_1G_1L_1}$	7·474	−1·144	$\mathrm{SB_1X}$	0·585	+1·455
FEI	3·534	+1·636	$\mathrm{F_1K_1G_1}$	1·791	−0·021	$\mathrm{WB_1X}$	2·894	+0·616
FJK	3·286	+1·334	ADE	3·125	−2·545	$\mathrm{XC_1Y}$	3·983	+0·287
EDI	3·460	+1·300	AHE	4·873	−0·863	$\mathrm{B_1F_1G_1}$	3·691	+0·359
DNH	3·980	−1·080	AEB	2·897	+0·913	$\mathrm{B_1G_1C_1}$	1·617	+0·083
HNO	3·122	−0·922	BDE	3·357	+0·183	$\mathrm{C_1D_1Y}$	4·503	+3·087
IOJ	0·851	−0·891	BEF	3·486	−1·266	$\mathrm{F_1M_1K_1}$	2·806	−0·936
JPK	1·385	−1·015	FIJ	2·255	−1·345	$\mathrm{G_1K_1N_1}$	1·910	−1·260
OST	0·749	−0·179	EDH	3·071	+0·659	$\mathrm{G_1N_1L_1}$	5·122	+1·098
NWX	5·103	−1·413	EHI	2·076	+0·814	$\mathrm{L_1N_1O_1}$	3·640	+0·820
NXS	2·064	−1·564	DHI	1·686	+0·174	$\mathrm{L_1O_1R_1}$	0·550	+0·380
PUR	4·563	+0·507	HOI	1·032	+0·808	$\mathrm{N_1Q_1R_1}$	2·930	−0·400
RUV	3·741	+0·929	JOP	1·098	+1·652	$\mathrm{M_1P_1Q_1}$	2·189	−1·919
$\mathrm{UE_1V}$	5·453	−2·893	ONS	3·657	−3·077	$\mathrm{K_1M_1N_1}$	3·176	+0·694
$\mathrm{TXC_1}$	1·222	+0·878	OTP	2·013	−1·513	$\mathrm{L_1N_1R_1}$	6·334	−0·394
$\mathrm{TC_1Y}$	3·923	−0·493	NWS	5·542	+0·648	$\mathrm{N_1M_1Q_1}$	3·276	+1·964
SWX	1·626	−3·626	PTU	2·307	+0·003	$\mathrm{N_1R_1O_1}$	2·143	−1·593
$\mathrm{WF_1B_1}$	3·346	+0·804	PRK	2·785	−1·195			

Unclosed Triangles

Triangle	*Spherical Excess* (ϵ)	*Triangle*	*Spherical Excess* (ϵ)
ADC	1″902	AHC	4″806
DHC	1·582	DHG	5·777
DNG	8·134	HNG	6·337
FKL	2·784	KRL	2·584
KPQ	1·192	RKQ	1·201
PRQ	0·392	RUZ	4·329
$\mathrm{UD_1Z}$	4·890	$\mathrm{D_1E_1Z}$	1·918
$\mathrm{UE_1Z}$	1·449	$\mathrm{E_1VZ}$	1·343
VRZ	2·074	VUZ	2·662
$\mathrm{D_1G_1I_1}$	2·598	$\mathrm{G_1L_1I_1}$	7·303
$\mathrm{D_1L_1I_1}$	2·428	$\mathrm{D_1G_1H_1}$	4·288
$\mathrm{G_1N_1H_1}$	4·140	$\mathrm{N_1L_1H_1}$	3·084
$\mathrm{G_1L_1H_1}$	2·101	$\mathrm{L_1D_1H_1}$	1·085
NHM	3·412	WNM	4·172
$\mathrm{B_1WA_1}$	2·705	$\mathrm{F_1B_1A_1}$	3·221
$\mathrm{F_1WA_1}$	2·580	$\mathrm{K_1F_1J_1}$	3·614
$\mathrm{M_1K_1J_1}$	4·217	$\mathrm{M_1F_1J_1}$	5·026
$\mathrm{P_1M_1J_1}$	2·561	$\mathrm{M_1P_1T_1}$	2·789
$\mathrm{Q_1M_1T_1}$	1·812	$\mathrm{Q_1P_1T_1}$	2·412
$\mathrm{L_1O_1S_1}$	3·514	$\mathrm{O_1R_1S_1}$	1·928
$\mathrm{L_1R_1S_1}$	4·892		

2.3 Symbolic statement of condition equations

Angle Closure	*Side Closure*	*Remarks*	*Angle Closure*	*Side Closure*	*Remarks*
ABE			C_1XY		
BFE			TXY		
	B(AEF)	Fixed Sides		$Y(TXC_1)$	
ABD			C_1YD_1		
BDE			YD_1UT		Polygon closure.
	A(BED)			$Y(C_1D_1UT)$	Contains artificial direction Y–U
HDE					
BDH				T(UPOSXY)	Eliminator for artificial direction Y–U
	E(BDH)				
ABH			UD_1E_1		
	D(ABH)		UE_1V		
	C(HAD)		RUV		
HIE				$U(TYD_1E_1VRP)$	Contains artificial direction Y–U
DIE					
	H(DIE)			T(UPOSXY)	Eliminator for artificial direction Y–U
FIE					
	E(FBDI)			Z(URV)	
FIJ				$Z(D_1UE_1)$	
IJO				$Z(VUE_1)$	
HIO			B_1WF_1		
	I(FEHOJ)			$A_1(WB_1F_1)$	
HNO			$B_1F_1G_1$		
DHN			$B_1C_1G_1$		
	H(NOIED)			$B_1(F_1G_1C_1XW)$	
	D(GNH)		$C_1G_1D_1$		
OJP				$C_1(G_1D_1YXB_1)$	
PJK			$B_1F_1K_1$		
FJK			$B_1G_1K_1$		
	J(FIOPK)			$F_1(K_1G_1B_1)$	
PRK			$G_1D_1L_1$		
	K(RLFJP)		$G_1L_1N_1$		
	K(PRQ)		$G_1K_1N_1$		
PRU				$G_1(K_1N_1L_1D_1C_1B_1)$	
PUT			$K_1N_1M_1$		
TPO			$F_1K_1M_1$		
	P(RUTOJK)			$K_1(F_1M_1N_1G_1)$	
TOS				$J_1(M_1K_1F_1)$	
NOS				$I_1(L_1G_1D_1)$	
	O(NSTPJIH)			$G_1(N_1H_1D_1L_1)$	
NSW				$H_1(L_1D_1G_1)$	
	N(MWSOH)		$N_1L_1R_1$		
NWX			$N_1L_1O_1$		
WXS			$N_1R_1O_1$		
	W(XSN)			$N_1(R_1L_1O_1)$	
XST			$R_1N_1Q_1$		
	S(XTON)		$M_1N_1Q_1$		
WXB_1				$N_1(G_1K_1M_1Q_1R_1L_1)$	
B_1SW				$S_1(R_1O_1L_1)$	
	$X(B_1SW)$		$M_1Q_1P_1$		
XB_1C_1				$M_1(P_1Q_1N_1K_1J_1)$	
XTC_1				$T_1(P_1M_1Q_1)$	
	$X(C_1TSB_1)$				

APPENDIX 3

FIGURE 3 (see DIAGRAM 7)

3.1 Mean observed directions, adjustment corrections, and adjusted directions

From	*To*		*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Ben Aigan (W_2)	Knock	(U_2)	00° 00′ 00″·00	−0″·408	359° 59′ 59″·592
	Bennachie	(S_2)	51 46 56·42	−1·088	51 46 55·332
	Corryhabbie	(R_2)	115 36 16·66	+0·195	115 36 16·855
	Findlays Seat	(T_2)	249 26 19·97	+0·498	249 26 20·468
	Lossiemouth Base West Terminal	(A_3)	267 35 20·28	+0·299	267 35 20·579
	Lossiemouth Base East Terminal	(B_3)	282 45 46·38	+0·190	282 45 46·570
	Bin of Cullen	(Z_2)	333 40 27·37	+0·314	333 40 27·684
Ben Cleugh (Y_1)	West Lomond	(Z_1)	00 00 00·00	+0·391	00 00 00·391
	Scald Law	(T_1)	65 22 03·53	+0·211	65 22 03·741
	Black Mount	(O_1)	83 33 27·31	−0·204	83 33 27·106
	Corse Hill	(S_1)	130 50 46·62	−0·004	130 50 46·616
	Earls Seat	(X_1)	164 40 52·49	−0·380	164 40 52·110
	Ben Lomond	(B_2)	193 53 33·39	−0·020	193 53 33·370
	Ben Lawers	(F_2)	248 16 36·68	+0·058	248 16 36·738
	Meall Dearg	(G_2)	279 14 57·96	−0·065	279 14 57·895
	Kings Seat	(C_2)	326 52 57·72	−0·316	326 52 57·404
	Craigowl	(D_2)	331 49 25·01	+0·328	331 49 25·338
Beneraird (D_1)	Cairn Pat	(Z)	00 00 00·00	+0·093	00 00 00·093
	Cnoc Moy	(J_1)	102 39 06·75	+0·366	102 39 07·116
	Ailsa Craig Lighthouse	(K_1)	130 11 00·48	−0·017	130 11 00·463
	Goat Fell	(R_1)	144 45 18·68	+0·113	144 45 18·793
	Brown Carrick	(L_1)	179 13 21·66	−0·089	179 13 21·571
	Merrick	(E_1)	234 06 16·01	+0·099	234 06 16·109
	Cairnsmore of Fleet	(A_1)	264 59 45·09	−0·375	264 59 44·715
	Carleton Fell	(W)	304 20 22·09	+0·058	304 20 22·148
	Inshanks	(V)	340 30 04·56	−0·249	340 30 04·311
Ben Lawers (F_2)	Meall Dearg	(G_2)	00 00 00·00	−0·152	359 59 59·848
	Ben Cleugh	(Y_1)	56 56 02·44	+0·003	56 56 02·443
	Ben Lomond	(B_2)	125 01 00·15	+0·121	125 01 00·271
	Ben Alder	(I_2)	245 33 26·69	+0·040	245 33 26·730
	Carn Gower	(J_2)	316 40 18·58	−0·013	316 40 18·567

3.1 *continued*

From	*To*	*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Ben Lomond (B_2)	Hill of Stake (W_1)	00° 00′ 00″·00	+0″·309	00° 00′ 00″·309
	Ben Lawers (F_2)	201 38 30·40	+0·253	201 38 30·653
	Ben Cleugh (Y_1)	259 10 35·20	−0·366	259 10 34·834
	Earls Seat (X_1)	300 01 37·65	−0·196	300 01 37·454
Ben Macdhui (O_2)	Glas Maol (K_2)	00 00 00·00	−0·044	359 59 59·956
	Carn Gower (J_2)	42 37 52·00	+0·290	42 37 52·290
	Ben Alder (I_2)	99 43 16·40	+0·137	99 43 16·537
	Beinn Bhreac Mhor (N_2)	162 26 29·76	+0·148	162 26 29·908
	Carn nan-tri-tighearnan (Q_2)	196 02 04·96	−0·069	196 02 04·891
	Corryhabbie (R_2)	262 48 10·42	−0·466	262 48 09·954
	Mount Battock (L_2)	322 59 23·53	+0·004	322 59 23·534
Bennachie (S_2)	Brimmond (P_2)	00 00 00·00	−0·152	359 59 59·848
	Trusta (M_2)	37 02 06·92	−0·293	37 02 06·627
	Mount Battock (L_2)	71 55 37·90	−0·423	71 55 37·477
	Corryhabbie (R_2)	151 48 53·96	−0·310	151 48 53·650
	Ben Aigan (W_2)	177 21 59·64	+0·517	177 22 00·157
	Knock (U_2)	208 46 28·37	+0·580	208 46 28·950
	Mormond (V_2)	273 30 32·48	+0·081	273 30 32·561
Bin of Cullen (Z_2)	Knock (U_2)	00 00 00·00	+0·012	00 00 00·012
	Corryhabbie (R_2)	61 31 54·04	+1·298	61 31 55·338
	Ben Aigan (W_2)	78 47 28·67	+0·163	78 47 28·833
	Findlays Seat (T_2)	99 19 34·80	−0·316	99 19 34·484
	Cutties Hillock East (Y_2)	120 19 07·95	−0·482	120 19 07·468
	Lossiemouth Base East Terminal (B_3)	128 52 19·42	−0·486	128 52 18·934
	Lossiemouth Base West Terminal (A_3)	134 57 23·45	−0·190	134 57 23·260
Black Combe (I)	Rottington (M)	00 00 00·00	−0·386	359 59 59·614
	Skiddaw (R)	49 18 38·39	+0·008	49 18 38·398
	Sca Fell (N)	53 30 55·74	+0·377	53 30 56·117
	High Street (O)	83 23 03·91	−0·739	83 23 03·171
	Little Whernside (J)	127 11 26·71	−0·317	127 11 26·393
	Mallowdale Pike (F)	153 40 35·06	+0·301	153 40 35·361
	Weeton Reservoir (E)	186 12 11·98	+0·756	186 12 12·736
Black Mount (O_1)	Scald Law (T_1)	00 00 00·00	+0·118	00 00 00·118
	Sayers Law (U_1)	36 03 46·41	+0·023	36 03 46·433
	Dunrig (P_1)	93 08 25·34	−0·598	93 08 24·742
	Hart Fell (G_1)	137 35 18·98	+0·510	137 35 19·490
	Tinto (N_1)	191 09 36·56	+0·036	191 09 36·596
	Cairn Table (M_1)	202 05 27·38	−0·718	202 05 26·662
	Corse Hill (S_1)	234 07 20·95	+0·695	234 07 21·645
	Earls Seat (X_1)	270 05 19·58	−0·066	270 05 19·514

3.1 *continued*

From	*To*		*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Botton Head (K)	Hambleton Down	(H)	00° 00′ 00″·192 (1)	—	00° 00′ 00″·192
	Great Whernside	(G)	39 57 32·428 (1)	—	39 57 32·428
	Royal Oak	(P)	95 56 21·615 (1)	—	95 56 21·615
	Warden Law	(U)	130 25 21·374 (2)	−0″·695	130 25 20·679
	Easington	(Q)	196 03 13·634 (2)	−0·041	196 03 13·593
	Loose Howe	(L)	247 10 00·814 (2)	−0·578	247 10 00·236
	Leavening Brow	(D)	307 54 22·464 (2)	−0·135	307 54 22·329
	York Minster	(C)	333 58 50·720 (1)	—	333 58 50·720
Boulsworth (B)	Rivington	(A)	169 55 33·727 (1)	—	169 55 33·727
	Weeton Reservoir	(E)	206 03 15·061 (2)	−0·487	206 03 14·574
	Mallowdale Pike	(F)	241 49 03·187 (1)	—	241 49 03·187
Brimmond (P$_2$)	Bennachie	(S$_2$)	00 00 00·00	+0·100	00 00 00·100
	Mormond	(V$_2$)	67 14 03·91	−0·067	67 14 03·843
	Trusta	(M$_2$)	251 17 41·35	−0·043	251 17 41·307
	Mount Battock	(L$_2$)	283 56 11·98	+0·010	283 56 11·990
Cairn Pat (Z)	Inshanks	(V)	00 00 00·00	+0·080	00 00 00·080
	Cnoc Moy	(J$_1$)	162 10 28·15	−0·196	162 10 27·954
	Goat Fell	(R$_1$)	194 56 21·85	−0·490	194 56 21·360
	Ailsa Craig Lighthouse	(K$_1$)	195 59 50·00	+0·023	195 59 50·023
	Beneraird	(D$_1$)	220 51 32·21	+0·249	220 51 32·459
	Merrick	(E$_1$)	251 12 13·22	+0·414	251 12 13·634
	Cairnsmore of Fleet	(A$_1$)	275 18 25·99	−0·022	275 18 25·968
	Carleton Fell	(W)	315 46 33·24	−0·059	315 46 33·181
Cairnsmore of Deugh (F$_1$)	Cairnsmore of Fleet	(A$_1$)	00 00 00·00	−0·677	359 59 59·323
	Merrick	(E$_1$)	36 34 04·78	+0·578	36 34 05·358
	Brown Carrick	(L$_1$)	103 17 48·76	+0·174	103 17 48·934
	Corse Hill	(S$_1$)	163 44 33·35	+0·374	163 44 33·724
	Cairn Table	(M$_1$)	189 36 43·11	+0·352	189 36 43·462
	Tinto	(N$_1$)	207 50 47·44	−2·030	207 50 45·410
	Hart Fell	(G$_1$)	236 34 25·25	+0·581	236 34 25·831
	Criffell	(X)	298 09 25·50	+0·648	298 09 26·148
Cairnsmore of Fleet (A$_1$)	Carleton Fell	(W)	00 00 00·00	−0·072	359 59 59·928
	Cairn Pat	(Z)	58 00 51·88	−0·202	58 00 51·678
	Beneraird	(D$_1$)	88 33 45·00	+0·108	88 33 45·108
	Merrick	(E$_1$)	139 22 47·92	−0·114	139 22 47·806
	Cairnsmore of Deugh	(F$_1$)	177 55 25·15	+0·206	177 55 25·356
	Criffell	(X)	257 43 47·66	+0·198	257 43 47·858
	Rottington	(M)	301 13 02·58	−0·124	301 13 02·456

(1) Fixed direction from Figure 2.

(2) Mean observed direction plus overlap correction from Figure 2.

3.1 *continued*

From	*To*		*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Cairn Table (M_1)	Corse Hill	(S_1)	00° 00′ 00″·00	−0″·054	359° 59′ 59″·946
	Black Mount	(O_1)	88 04 25·73	+0·191	88 04 25·921
	Tinto	(N_1)	95 34 22·42	+0·925	95 34 23·345
	Hart Fell	(G_1)	134 48 57·67	−0·484	134 48 57·186
	Criffell	(X)	189 02 00·60	−0·343	189 02 00·257
	Cairnsmore of Deugh	(F_1)	235 51 03·72	−0·078	235 51 03·642
	Merrick	(E_1)	247 00 38·86	−0·089	247 00 38·771
	Brown Carrick	(L_1)	288 52 58·11	−0·067	288 52 58·043
Carn Gower (J_2)	Kings Seat	(C_2)	00 00 00·00	+0·089	00 00 00·089
	Meall Dearg	(G_2)	47 43 53·04	−0·097	47 43 52·943
	Ben Lawers	(F_2)	79 25 35·04	+0·448	79 25 35·488
	Ben Alder	(I_2)	121 17 40·76	−0·126	121 17 40·634
	Beinn Bhreac Mhor	(N_2)	180 50 13·09	−0·160	180 50 12·930
	Ben Macdhui	(O_2)	217 01 34·38	−0·284	217 01 34·096
	Glas Maol	(K_2)	293 09 05·56	+0·176	293 09 05·736
	Craigowl	(D_2)	342 09 22·30	−0·048	342 09 22·252
Cheviot (I_1)	Tosson Hill	(C_1)	00 00 00·00	−0·254	359 59 59·746
	Whitelyne Common	(B_1)	61 06 17·59	+0·551	61 06 18·141
	Wisp Hill	(H_1)	91 12 09·81	−0·161	91 12 09·649
	Dunrig	(P_1)	122 50 32·36	−0·031	122 50 32·329
	Sayers Law	(U_1)	164 45 37·01	+0·168	164 45 37·178
	Lumsdaine	(V_1)	198 52 10·45	−0·288	198 52 10·162
	Greensheen Hill	(Q_1)	247 17 28·70	+0·016	247 17 28·716
Collier Law (T)	Royal Oak	(P)	00 00 00·282⁽¹⁾	—	00 00 00·282
	Cross Fell	(S)	125 22 32·788⁽¹⁾	—	125 22 32·788
	Tosson Hill	(C_1)	227 00 51·951⁽²⁾	−0·370	227 00 51·581
	Warden Law	(U)	304 09 13·081⁽²⁾	−0·737	304 09 12·344
Corryhabbie (R_2)	Carn nan-tri-tighearnan	(Q_2)	00 00 00·00	+0·138	00 00 00·138
	Findlays Seat	(T_2)	72 28 31·71	−0·255	72 28 31·455
	Ben Aigan	(W_2)	86 01 07·34	+0·207	86 01 07·547
	Bin of Cullen	(Z_2)	106 49 46·51	−0·917	106 49 45·593
	Knock	(U_2)	121 44 21·55	−0·630	121 44 20·920
	Bennachie	(S_2)	176 38 40·82	+0·706	176 38 41·526
	Mount Battock	(L_2)	226 17 46·24	−0·192	226 17 46·048
	Glas Maol	(K_2)	269 46 18·44	+0·379	269 46 18·819
	Ben Macdhui	(O_2)	301 46 16·01	+0·671	301 46 16·681
	Beinn Bhreac Mhor	(N_2)	338 58 52·10	−0·107	338 58 51·993

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(1) Fixed direction from Figure 2.
(2) Mean observed direction plus overlap correction from Figure 2.

3.1 *continued*

From	*To*		*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Corse Hill (S$_1$)	Cairn Table	(M$_1$)	00° 00′ 00″00	+0″229	00° 00′ 00″229
	Cairnsmore of Deugh	(F$_1$)	29 58 55·79	−0·037	29 58 55·753
	Brown Carrick	(L$_1$)	75 24 22·05	−0·211	75 24 21·839
	Goat Fell	(R$_1$)	114 52 04·59	+0·358	114 52 04·948
	Hill of Stake	(W$_1$)	146 29 11·50	−0·272	146 29 11·228
	Earls Seat	(X$_1$)	205 08 14·46	+0·613	205 08 15·073
	Ben Cleugh	(Y$_1$)	238 50 03·78	+0·013	238 50 03·793
	Black Mount	(O$_1$)	300 06 18·96	−0·468	300 06 18·492
	Tinto	(N$_1$)	318 21 23·45	−0·223	318 21 23·227
Craigowl (D$_2$)	Meall Dearg	(G$_2$)	00 00 00·00	−0·017	359 59 59·983
	Carn Gower	(J$_2$)	37 29 06·09	+0·296	37 29 06·386
	Glas Maol	(K$_2$)	58 23 24·50	−0·179	58 23 24·321
	Mount Battock	(L$_2$)	109 27 40·38	+0·576	109 27 40·956
	Wuddy Law	(H$_2$)	152 13 56·36	+0·269	152 13 56·629
	West Hills	(E$_2$)	161 33 12·38	−0·162	161 33 12·218
	Kellie Law	(A$_2$)	245 31 00·37	−0·078	245 31 00·292
	West Lomond	(Z$_1$)	296 33 20·43	−0·376	296 33 20·054
	Ben Cleugh	(Y$_1$)	318 33 38·80	−0·339	318 33 38·461
	Kings Seat	(C$_2$)	332 43 23·23	+0·009	332 43 23·239
Criffell (X)	Cairnsmore of Deugh	(F$_1$)	00 00 00·00	−0·140	359 59 59·860
	Cairn Table	(M$_1$)	24 38 17·49	−0·106	24 38 17·384
	Hart Fell	(G$_1$)	61 57 13·50	+0·604	61 57 14·104
	Wisp Hill	(H$_1$)	93 59 51·32	−0·171	93 59 51·149
	Whitelyne Common	(B$_1$)	118 38 53·54	+0·371	118 38 53·911
	Cold Fell Pike	(Y)	140 37 13·99	−1·028	140 37 12·962
	Cross Fell	(S)	155 48 06·68	+0·950	155 48 07·630
	Skiddaw	(R)	182 22 12·10	−0·206	182 22 11·894
	Sca Fell	(N)	199 50 56·09	+0·282	199 50 56·372
	Rottington	(M)	225 44 00·86	−0·535	225 44 00·325
	Cairnsmore of Fleet	(A$_1$)	321 38 52·15	−0·301	321 38 51·849
	Merrick	(E$_1$)	339 12 56·52	+0·280	339 12 56·800
Cross Fell (S)	Cold Fell Pike	(Y)	359 59 59·944(2)	+0·030	359 59 59·974
	Whitelyne Common	(B$_1$)	10 30 18·884(2)	+0·679	10 30 19·563
	Tosson Hill	(C$_1$)	47 21 59·864(2)	−0·116	47 21 59·748
	Collier Law	(T)	98 09 42·036(1)	—	98 09 42·036
	High Street	(O)	247 36 44·913(1)	—	247 36 44·913
	Skiddaw	(R)	283 57 53·454(2)	+0·556	283 57 54·010
	Criffell	(X)	311 37 34·494(2)	+0·939	311 37 35·433
Cutties Hillock (X$_2$)	Findlays Seat	(T$_2$)	00 00 00·00	−0·450	359 59 59·550
	Lossiemouth Base West Terminal	(A$_3$)	262 06 07·55	+0·450	262 06 08·000

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(1) Fixed direction from Figure 2.

(2) Mean observed direction plus overlap correction from Figure 2.

3.1 *continued*

From	*To*	*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Cutties Hillock East (Y_2)	Findlays Seat (T_2)	00° 00′ 00″·00	+0″·474	00° 00′ 00″·474
	Lossiemouth Base West Terminal (A_3)	257 20 10·69	−0·334	257 20 10·356
	Lossiemouth Base East Terminal (B_3)	295 20 49·30	−0·100	295 20 49·200
	Bin of Cullen (Z_2)	309 08 43·92	−0·114	309 08 43·806
	Knock (U_2)	324 01 25·33	+0·074	324 01 25·404
Dunrig (P_1)	Black Mount (O_1)	00 00 00·00	+0·502	00 00 00·502
	Scald Law (T_1)	38 30 44·82	+0·108	38 30 44·928
	Sayers Law (U_1)	97 45 16·24	−0·100	97 45 16·140
	Cheviot (I_1)	149 58 15·38	−0·073	149 58 15·307
	Wisp Hill (H_1)	208 00 39·14	−0·405	208 00 38·735
	Hart Fell (G_1)	268 15 21·31	−0·032	268 15 21·278
Earls Seat (X_1)	Corse Hill (S_1)	00 00 00·00	−0·300	359 59 59·700
	Hill of Stake (W_1)	59 18 16·52	−0·260	59 18 16·260
	Ben Lomond (B_2)	137 35 31·73	+0·328	137 35 32·058
	Ben Cleugh (Y_1)	247 31 50·33	+0·319	247 31 50·649
	Black Mount (O_1)	310 55 56·53	−0·087	310 55 56·443
Findlays Seat (T_2)	Knock (U_2)	00 00 00·00	+0·169	00 00 00·169
	Ben Aigan (W_2)	52 53 46·32	−0·666	52 53 45·654
	Carn nan-tri-tighearnan (Q_2)	160 28 02·16	−0·111	160 28 02·049
	Cutties Hillock (X_2)	228 39 40·01	+0·450	228 39 40·460
	Cutties Hillock East (Y_2)	229 30 47·24	−0·212	229 30 47·028
	Lossiemouth Base West Terminal (A_3)	261 05 54·98	−0·079	261 05 54·901
	Lossiemouth Base East Terminal (B_3)	288 50 44·25	+0·220	288 50 44·470
	Bin of Cullen (Z_2)	337 39 57·79	+0·229	337 39 58·019
Glas Maol (K_2)	Craigowl (D_2)	00 00 00·00	+0·070	00 00 00·070
	Kings Seat (C_2)	21 32 48·29	−0·149	21 32 48·141
	Meall Dearg (G_2)	68 29 14·34	+0·274	68 29 14·614
	Carn Gower (J_2)	110 05 27·91	−0·293	110 05 27·617
	Ben Macdhui (O_2)	171 20 04·75	+0·157	171 20 04·907
	Corryhabbie (R_2)	222 08 20·58	−0·537	222 08 20·043
	Mount Battock (L_2)	288 11 45·32	+0·098	288 11 45·418
	Wuddy Law (H_2)	327 29 12·85	+0·246	327 29 13·096
	West Hills (E_2)	340 42 03·94	+0·133	340 42 04·073
Great Whernside (G)	Botton Head (K)	266 23 24·757(1)	—	266 23 24·757
	Hambleton Down (H)	280 34 06·002(1)	—	280 34 06·002
	Leavening Brow (D)	299 45 51·729(2)	+0·382	299 45 52·111
	York Minster (C)	311 18 40·918(1)	—	311 18 40·918

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(1) Fixed direction from Figure 2.

(2) Mean observed direction plus overlap correction from Figure 2.

3.1 *continued*

From	*To*		*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Hart Fell (G$_1$)	Dunrig	(P$_1$)	00° 00′ 00″00	+0″448	00° 00′ 00″448
	Wisp Hill	(H$_1$)	79 40 37·76	+0·793	79 40 38·553
	Criffell	(X)	158 58 03·32	−0·594	158 58 02·726
	Cairnsmore of Deugh	(F$_1$)	215 25 53·80	+0·542	215 25 54·342
	Cairn Table	(M$_1$)	247 26 08·14	+0·312	247 26 08·452
	Tinto	(N$_1$)	284 24 58·00	+0·203	284 24 58·203
	Black Mount	(O$_1$)	316 11 33·54	−0·424	316 11 33·116
	Scald Law	(T$_1$)	331 27 35·84	−1·279	331 27 34·561
High Street (O)	Sca Fell	(N)	00 00 00·115(2)	+0·128	00 00 00·243
	Skiddaw	(R)	54 39 11·015(2)	−0·239	54 39 10·776
	Cold Fell Pike	(Y)	119 57 06·165(2)	−0·678	119 57 05·487
	Cross Fell	(S)	146 18 13·448(1)	—	146 18 13·448
	Little Whernside	(J)	234 30 54·833(1)	—	234 30 54·833
	Mallowdale Pike	(F)	263 28 31·109(1)	—	263 28 31·109
	Black Combe	(I)	329 42 36·705(2)	+0·701	329 42 37·406
Hill of Stake (W$_1$)	Brown Carrick	(L$_1$)	00 00 00·00	+0·188	00 00 00·188
	Goat Fell	(R$_1$)	53 57 26·55	−0·262	53 57 26·288
	Ben Lomond	(B$_2$)	194 24 31·19	−0·158	194 24 31·032
	Earls Seat	(X$_1$)	236 08 54·89	−0·015	236 08 54·875
	Corse Hill	(S$_1$)	298 11 37·17	+0·248	298 11 37·418
Kellie Law (A$_2$)	West Lomond	(Z$_1$)	00 00 00·00	−0·591	359 59 59·409
	Craigowl	(D$_2$)	66 56 39·42	+0·491	66 56 39·911
	West Hills	(E$_2$)	92 19 57·38	+0·082	92 19 57·462
	Wuddy Law	(H$_2$)	103 27 06·63	+0·027	103 27 06·657
	Lumsdaine	(V$_1$)	226 43 29·69	−0·023	226 43 29·667
	Sayers Law	(U$_1$)	261 33 39·01	+0·014	261 33 39·024
	Scald Law	(T$_1$)	305 21 17·31	0·0	305 21 17·310
Knock (U$_2$)	Ben Aigan	(W$_2$)	00 00 00·00	+0·038	00 00 00·038
	Findlays Seat	(T$_2$)	16 32 35·10	+0·808	16 32 35·908
	Bin of Cullen	(Z$_2$)	74 53 00·18	−0·246	74 52 59·934
	Mormond	(V$_2$)	194 45 13·57	−0·036	194 45 13·534
	Bennachie	(S$_2$)	263 11 22·82	−0·390	263 11 22·430
	Corryhabbie	(R$_2$)	331 19 29·79	−0·174	331 19 29·616
Little Whernside (J)	Mallowdale Pike	(F)	359 59 59·761(1)	—	359 59 59·761
	Black Combe	(I)	61 06 15·818(2)	−0·482	61 06 15·336
	High Street	(O)	102 06 13·759(1)	—	102 06 13·759
Loose Howe (L)	Easington	(Q)	00 00 00·00	+0·006	00 00 00·006
	Leavening Brow	(D)	151 42 08·63	+0·010	151 42 08·640
	York Minster	(C)	176 36 53·06	−0·659	176 36 52·401
	Hambleton Down	(H)	212 43 33·22	+0·084	212 43 33·304
	Botton Head	(K)	257 23 44·22	+0·558	257 23 44·778

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(1) Fixed direction from Figure 2.

(2) Mean observed direction plus overlap correction from Figure 2.

3.1 *continued*

From	*To*	*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Lossiemouth Base East Terminal (B_3)	Findlays Seat (T_2)	00° 00′ 00″00	−0″546	359° 59′ 59″454
	Cutties Hillock East (Y_2)	56 00 51·10	−0·068	56 00 51·032
	Lossiemouth Base West Terminal (A_3)	100 19 33·86	+0·350	100 19 34·210
	Bin of Cullen (Z_2)	258 21 56·99	−0·091	258 21 56·899
	Ben Aigan (W_2)	337 22 26·17	+0·354	337 22 26·524
Lossiemouth Base West Terminal (A_3)	Findlays Seat (T_2)	00 00 00·00	−0·189	359 59 59·811
	Cutties Hillock East (Y_2)	45 45 01·48	+0·564	45 45 02·044
	Cutties Hillock (X_2)	49 39 54·51	−0·450	49 39 54·060
	Bin of Cullen (Z_2)	292 11 50·01	+0·798	292 11 50·808
	Knock (U_2)	305 13 12·56	+0·226	305 13 12·786
	Lossiemouth Base East Terminal (B_3)	308 04 24·32	−0·401	308 04 23·919
	Ben Aigan (W_2)	349 56 51·07	−0·549	349 56 50·521
Lumsdaine (V_1)	Sayers Law (U_1)	00 00 00·00	−0·217	359 59 59·783
	Greensheen Hill (Q_1)	253 18 38·45	+0·217	253 18 38·667
Mallowdale Pike (F)	Little Whernside (J)	00 00 00·568(1)	—	00 00 00·568
	Boulsworth (B)	91 38 56·700(1)	—	91 38 56·700
	Rivington (A)	138 30 19·270(1)	—	138 30 19·270
	Weeton Reservoir (E)	185 56 51·139(2)	−0·007	185 56 51·132
	Black Combe (I)	267 35 22·099(2)	−0·602	267 35 21·497
	High Street (O)	311 03 48·035(1)	—	311 03 48·035
Meall Dearg (G_2)	West Lomond (Z_1)	00 00 00·00	+0·003	00 00 00·003
	Ben Cleugh (Y_1)	39 27 16·61	+0·016	39 27 16·626
	Ben Lawers (F_2)	131 32 55·38	+0·090	131 32 55·470
	Carn Gower (J_2)	236 31 34·04	−0·384	236 31 33·656
	Glas Maol (K_2)	260 20 35·01	−0·059	260 20 34·951
	Craigowl (D_2)	313 28 00·35	+0·177	313 28 00·527
	Kings Seat (C_2)	325 34 32·26	+0·158	325 34 32·418
Merrick (E_1)	Cairnsmore of Deugh (F_1)	00 00 00·00	−0·494	359 59 59·506
	Criffell (X)	60 48 20·52	−0·616	60 48 19·904
	Cairnsmore of Fleet (A_1)	104 53 16·58	+0·354	104 53 16·934
	Carleton Fell (W)	129 45 12·53	+0·089	129 45 12·619
	Inshanks (V)	158 48 05·86	+0·161	158 48 06·021
	Cairn Pat (Z)	179 25 11·38	−0·570	179 25 10·810
	Beneraird (D_1)	203 10 46·71	+0·417	203 10 47·127
	Ailsa Craig Lighthouse (K_1)	236 06 20·96	+0·014	236 06 20·974
	Cnoc Moy (J_1)	236 41 46·58	−0·287	236 41 46·293
	Goat Fell (R_1)	268 47 09·30	+0·001	268 47 09·301
	Cairn Table (M_1)	344 12 11·11	+0·931	344 12 12·041

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(1) Fixed direction from Figure 2.

(2) Mean observed direction plus overlap correction from Figure 2.

3.1 *continued*

From	*To*		*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Mount Battock (L_2)	Craigowl	(D_2)	00° 00′ 00″·00	−0″·502	359° 59′ 59″·498
	Glas Maol	(K_2)	57 07 32·34	−0·166	57 07 32·174
	Ben Macdhui	(O_2)	83 15 17·83	−0·068	83 15 17·762
	Corryhabbie	(R_2)	127 35 38·56	+0·303	127 35 38·863
	Bennachie	(S_2)	178 03 21·92	+0·318	178 03 22·238
	Brimmond	(P_2)	210 03 58·46	+0·161	210 03 58·621
	Trusta	(M_2)	242 57 07·27	−0·218	242 57 07·052
	Wuddy Law	(H_2)	324 44 51·86	+0·036	324 44 51·896
	West Hills	(E_2)	340 51 35·52	+0·136	340 51 35·656
Rivington (A)	Boulsworth	(B)	359 59 59·316(1)	—	359 59 59·316
	Weeton Reservoir	(E)	254 00 25·931(2)	+0·621	254 00 26·552
	Mallowdale Pike	(F)	298 44 48·001(1)	—	298 44 48·001
Royal Oak (P)	Collier Law	(T)	00 00 00·123(1)	—	00 00 00·123
	Warden Law	(U)	81 01 02·698(2)	−0·378	81 01 02·320
	Easington	(Q)	143 54 39·538(2)	+0·143	143 54 39·681
	Botton Head	(K)	169 05 32·987(1)	—	169 05 32·987
Sayers Law (U_1)	Lumsdaine	(V_1)	00 00 00·00	+0·300	00 00 00·300
	Greensheen Hill	(Q_1)	41 26 20·73	−0·229	41 26 20·501
	Cheviot	(I_1)	64 15 57·61	−0·307	64 15 57·303
	Wisp Hill	(H_1)	120 06 14·15	+0·632	120 06 14·782
	Dunrig	(P_1)	150 07 58·42	+0·786	150 07 59·206
	Black Mount	(O_1)	175 18 08·10	−0·323	175 18 07·777
	Scald Law	(T_1)	191 48 07·92	−0·613	191 48 07·307
	West Lomond	(Z_1)	242 13 57·31	−0·553	242 13 56·757
	Kellie Law	(A_2)	274 38 54·41	+0·306	274 38 54·716
Scald Law (T_1)	Black Mount	(O_1)	00 00 00·00	−0·090	359 59 59·910
	Tinto	(N_1)	05 19 58·84	−0·322	05 19 58·518
	Ben Cleugh	(Y_1)	107 22 33·03	−0·304	107 22 32·726
	West Lomond	(Z_1)	144 13 57·14	−0·616	144 13 56·524
	Kellie Law	(A_2)	179 12 05·14	+0·418	179 12 05·558
	Sayers Law	(U_1)	232 33 44·17	+0·113	232 33 44·283
	Dunrig	(P_1)	311 39 07·88	+0·009	311 39 07·889
	Hart Fell	(G_1)	332 51 18·89	+0·791	332 51 19·681
Skiddaw (R)	Criffell	(X)	00 00 00·00	−0·207	359 59 59·793
	Wisp Hill	(H_1)	52 55 49·39	−0·803	52 55 48·587
	Whitelyne Common	(B_1)	76 05 52·86	+0·660	76 05 53·520
	Cold Fell Pike	(Y)	95 12 30·13	+0·440	95 12 30·570
	Cross Fell	(S)	125 46 18·50	−0·449	125 46 18·051
	High Street	(O)	177 46 08·68	−0·208	177 46 08·472
	Sca Fell	(N)	234 23 35·60	+0·231	234 23 35·831
	Black Combe	(I)	238 45 12·76	+0·130	238 45 12·890
	Rottington	(M)	285 45 01·04	+0·207	285 45 01·247

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(1) Fixed direction from Figure 2.

(2) Mean observed direction plus overlap correction from Figure 2.

3.1 *continued*

From	*To*		*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Tosson Hill (C_1)	Greensheen Hill	(Q_1)	00° 00′ 00″·00	+0″·096	00° 00′ 00″·096
	Warden Law	(U)	134 42 44·27	−0·081	134 42 44·189
	Collier Law	(T)	171 01 44·38	+0·609	171 01 44·989
	Cross Fell	(S)	198 35 47·98	+0·651	198 35 48·631
	Cold Fell Pike	(Y)	215 18 38·99	−1·107	215 18 37·883
	Whitelyne Common	(B_1)	238 56 49·54	−0·031	238 56 49·509
	Cheviot	(I_1)	328 55 24·43	−0·136	328 55 24·294
Trusta (M_2)	Mount Battock	(L_2)	00 00 00·00	+0·152	00 00 00·152
	Bennachie	(S_2)	80 12 46·17	+0·465	80 12 46·635
	Brimmond	(P_2)	114 28 22·39	−0·092	114 28 22·298
	Wuddy Law	(H_2)	299 33 48·17	−0·525	299 33 47·645
Warden Law (U)	Royal Oak	(P)	00 00 00·00	+0·587	00 00 00·587
	Collier Law	(T)	43 08 11·76	+0·620	43 08 12·380
	Tosson Hill	(C_1)	109 40 56·75	−0·853	109 40 55·897
	Easington	(Q)	276 29 20·72	−0·098	276 29 20·622
	Botton Head	(K)	302 33 27·06	−0·256	302 33 26·804
West Lomond (Z_1)	Kellie Law	(A_2)	00 00 00·00	+0·337	00 00 00·337
	Sayers Law	(U_1)	49 08 44·91	+0·704	49 08 45·614
	Scald Law	(T_1)	90 23 13·01	−0·114	90 23 12·896
	Ben Cleugh	(Y_1)	168 09 49·55	−0·413	168 09 49·137
	Meall Dearg	(G_2)	227 57 33·74	−0·652	227 57 33·088
	Kings Seat	(C_2)	276 52 23·82	+0·117	276 52 23·937
	Craigowl	(D_2)	297 58 57·87	+0·021	297 58 57·891
Whitelyne Common (B_1)	Cold Fell Pike	(Y)	00 00 00·00	+0·237	00 00 00·237
	Skiddaw	(R)	34 19 05·72	−0·156	34 19 05·564
	Criffell	(X)	74 30 01·05	−0·388	74 30 00·662
	Wisp Hill	(H_1)	131 34 49·85	−0·040	131 34 49·810
	Cheviot	(I_1)	218 49 16·73	+0·178	218 49 16·908
	Tosson Hill	(C_1)	247 44 26·06	+0·363	247 44 26·423
	Cross Fell	(S)	350 31 50·69	−0·193	350 31 50·497
Wisp Hill (H_1)	Hart Fell	(G_1)	00 00 00·00	−0·404	359 59 59·596
	Dunrig	(P_1)	40 04 40·39	+0·306	40 04 40·696
	Sayers Law	(U_1)	79 47 37·64	−0·278	79 47 37·362
	Cheviot	(I_1)	130 24 00·08	−0·515	130 23 59·565
	Whitelyne Common	(B_1)	193 03 44·00	+0·549	193 03 44·549
	Skiddaw	(R)	252 37 59·20	+0·581	252 37 59·781
	Criffell	(X)	291 19 56·92	−0·240	291 19 56·680
Wuddy Law (H_2)	Craigowl	(D_2)	00 00 00·00	−0·081	359 59 59·919
	Glas Maol	(K_2)	53 38 43·78	−0·144	53 38 43·636
	Mount Battock	(L_2)	101 58 39·09	−0·138	101 58 38·952
	Trusta	(M_2)	139 44 43·04	+0·496	139 44 43·536
	Kellie Law	(A_2)	309 47 27·76	−0·018	309 47 27·742
	West Hills	(E_2)	347 16 17·78	−0·115	347 16 17·665

3.1 *continued*

From	*To*		*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
York Minster (C)	Hambleton Down	(H)	00° 00′ 00″353 (1)	—	00° 00′ 00″353
	Botton Head	(K)	15 18 42·682 (1)	—	15 18 42·682
	Loose Howe	(L)	27 43 00·619 (2)	+0″546	27 43 01·165
	Leavening Brow	(D)	78 34 28·169 (2)	−0·083	78 34 28·086
	Great Whernside	(G)	306 12 33·078 (1)	—	306 12 33·078

(1) Fixed direction from Figure 2.

(2) Mean observed direction plus overlap correction from Figure 2.

3.2 Triangle misclosures and spherical excesses

Triangle	*Spherical Excess* (ε)	*Triangle Misclosure*	*Triangle*	*Spherical Excess* (ε)	*Triangle Misclosure*	*Triangle*	*Spherical Excess* (ε)	*Triangle Misclosure*
AFB	3″346	+0″804	FIO	5″026	−2″346	$T_1Y_1Z_1$	3″388	+0″792
FIJ	3·614	−1·634	FOJ	2·806	−0·936	$A_2Z_1D_2$	2·711	−1·101
JIO	4·217	−1·647	OIR	2·561	+1·349	$Z_1Y_1G_2$	3·070	−0·230
ORS	2·189	−1·919	SRX	3·945	+1·015	$Z_1G_2D_2$	4·207	−0·857
SRB_1	5·151	+0·949	SXB_1	8·014	−0·124	$Y_1B_2F_2$	5·377	+0·423
SB_1C_1	5·137	+2·033	SC_1T	4·722	+0·388	$Y_1G_2D_2$	5·064	−0·554
TC_1U	5·081	+1·149	TUP	1·928	−0·388	$G_2J_2K_2$	1·505	+0·515
PUK	3·514	+0·046	KLC	1·344	−1·834	$G_2K_2D_2$	4·460	−0·280
KCG	7·474	−1·144	$C_1B_1I_1$	2·696	−0·886	$D_2K_2L_2$	3·962	−1·062
B_1RX	6·808	−0·058	B_1RH_1	4·411	−1·611	$D_2L_2H_2$	2·308	+0·902
B_1XH_1	4·041	−0·101	$B_1H_1I_1$	3·590	−0·570	$H_2L_2M_2$	1·935	−1·565
RXH_1	6·438	+1·452	XA_1E_1	2·033	−1·863	$M_2L_2P_2$	1·261	+0·569
XA_1F_1	3·688	+1·172	XE_1M_1	6·962	+1·678	$L_2K_2R_2$	4·836	−1·676
XE_1F_1	2·668	+0·612	XF_1M_1	3·596	−0·596	$L_2O_2R_2$	5·314	−1·704
XF_1G_1	6·178	−1·948	XM_1G_1	5·517	−1·757	$L_2S_2P_2$	2·121	+0·339
XG_1H_1	4·134	+2·326	$H_1G_1P_1$	1·748	−1·428	$K_2O_2R_2$	3·000	−0·020
$H_1P_1U_1$	3·686	+0·734	$H_1P_1I_1$	4·977	+1·023	$R_2W_2Z_2$	0·713	+2·377
$H_1U_1I_1$	7·210	−1·030	$I_1P_1U_1$	5·919	−1·319	$R_2W_2S_2$	2·010	−2·610
$U_1P_1O_1$	2·518	+2·332	$U_1P_1T_1$	2·919	+1·711	$S_2W_2U_2$	2·141	+0·189
$U_1O_1T_1$	1·472	+0·588	$U_1T_1Z_1$	4·490	+0·030	$U_2W_2Z_2$	0·624	+0·856
$U_1T_1A_2$	4·420	−0·600	$U_1Z_1A_2$	3·621	−0·621	$W_2T_2A_3$	0·154	+0·426
$P_1G_1O_1$	1·302	−2·512	$P_1G_1T_1$	1·328	−2·648	$W_2T_2Z_2$	0·500	+1·560
$P_1O_1T_1$	1·072	+1·208	$G_1F_1M_1$	2·935	−0·405	$W_2A_3Z_2$	1·244	+1·686
$G_1M_1O_1$	3·102	+2·638	$G_1O_1T_1$	1·045	+1·345	$T_2X_2A_3$	0·240	+1·690
$F_1A_1E_1$	1·013	−2·423	$F_1E_1M_1$	0·699	+1·661	$T_2Y_2B_3$	0·293	−1·483
$F_1S_1M_1$	1·567	+0·263	A_1ZD_1	2·317	−0·507	$T_2A_3B_3$	0·216	−1·406
A_1ZE_1	2·338	+1·272	$A_1D_1E_1$	1·497	+0·633	$T_2B_3Z_2$	0·556	+0·614
E_1ZD_1	1·476	−1·146	$W_1B_2X_1$	2·496	−1·236	$Z_2Y_2B_3$	0·205	−0·005
$W_1X_1S_1$	2·949	−1·189	$S_1X_1Y_1$	3·266	+1·594	$B_3Y_2A_3$	0·146	−1·616
$S_1X_1O_1$	4·545	+2·055	$S_1O_1M_1$	2·695	−2·355	$T_1Z_1A_2$	3·691	+0·009

3.2 *continued*

Triangle	*Spherical Excess (ε)*	*Triangle Misclosure*	*Triangle*	*Spherical Excess (ε)*	*Triangle Misclosure*	*Triangle*	*Spherical Excess (ε)*	*Triangle Misclosure*
$A_2D_2H_2$	2″586	+0″874	$M_2L_2S_2$	2″147	+0″353	$U_2T_2Z_2$	0″646	+1″444
$Z_1Y_1D_2$	2·214	−0·534	$M_2P_2S_2$	1·235	+0·555	$W_2T_2B_3$	0·216	+2·094
$Y_1X_1B_2$	2·470	−0·520	$L_2K_2O_2$	2·521	+0·009	$W_2A_3B_3$	0·278	+0·262
$Y_1F_2G_2$	2·596	−0·106	$L_2R_2S_2$	4·069	+0·771	$W_2B_3Z_2$	0·840	+0·080
$F_2J_2G_2$	2·012	+0·068	$K_2J_2O_2$	1·264	−1·244	$T_2Y_2A_3$	0·223	−1·693
$G_2J_2D_2$	3·965	−0·825	$R_2U_2S_2$	3·091	−2·441	$T_2Y_2Z_2$	0·644	−0·864
$D_2J_2K_2$	1·999	+1·061	$R_2W_2U_2$	1·060	+0·020	$T_2A_3Z_2$	0·898	+0·552
$D_2K_2H_2$	2·998	−0·208	$R_2Z_2U_2$	0·971	−1·501	$Z_2Y_2A_3$	0·477	−0·277
$H_2K_2L_2$	3·272	+0·048	$U_2W_2T_2$	0·478	+0·972	$Z_2B_3A_3$	0·126	+1·344

Unclosed Triangles

Triangle	*Spherical Excess (ε)*	*Triangle*	*Spherical Excess (ε)*	*Triangle*	*Spherical Excess (ε)*	*Triangle*	*Spherical Excess (ε)*
AEB	2″705	AEF	2″580	BEF	3″222	IEF	4″141
CHG	4·288	GHK	2·101	KHL	0·496	KHC	1·085
LHC	1·933	CDG	2·598	GDK	7·303	KDL	1·049
CDK	2·428	CDL	2·133	LQK	0·502	KQP	2·687
KQU	2·948	PQU	3·776	XNR	2·052	RNO	1·204
ONI	1·162	RNI	0·195	A_1MX	5·600	XMR	3·764
RMI	2·905	SYO	1·812	SYR	2·412	OYR	2·789
RYX	4·903	SYX	3·370	XYB_1	4·145	RYB_1	2·240
B_1YC_1	2·602	B_1YS	0·499	C_1YS	3·034	ZVD_1	0·871
D_1VE_1	3·139	ZVE_1	2·534	ZWD_1	2·433	D_1WE_1	3·477
E_1WA_1	1·010	ZWE_1	4·434	ZWA_1	3·107	D_1WA_1	2·990
ZJ_1D_1	3·784	$D_1J_1E_1$	3·645	ZJ_1E_1	8·905	ZK_1D_1	1·106
$D_1K_1E_1$	1·762	ZK_1E_1	4·344	$W_1L_1S_1$	3·825	$S_1L_1M_1$	2·744
$M_1L_1F_1$	2·654	$S_1L_1F_1$	3·831	$G_1N_1F_1$	3·367	$F_1N_1M_1$	1·184
$G_1N_1M_1$	1·616	$F_1N_1S_1$	4·360	$M_1N_1S_1$	1·609	$S_1N_1O_1$	1·429
$M_1N_1O_1$	0·344	$T_1N_1O_1$	0·158	$O_1N_1G_1$	1·141	$T_1N_1G_1$	2·345
$X_1O_1Y_1$	5·361	$S_1O_1Y_1$	6·640	$Y_1O_1T_1$	2·224	$C_1Q_1I_1$	1·200
$I_1Q_1U_1$	2·801	$U_1Q_1V_1$	2·708	$I_1Q_1V_1$	1·924	ZR_1D_1	2·262
$R_1D_1E_1$	4·914	ZR_1E_1	8·651	$S_1R_1W_1$	2·944	$A_2V_1U_1$	3·402
$I_1V_1U_1$	3·585	$Z_1C_2Y_1$	1·915	$Y_1C_2G_2$	3·522	$Z_1C_2G_2$	2·367
$G_2C_2J_2$	2·940	$J_2C_2K_2$	2·221	$G_2C_2K_2$	3·656	$K_2C_2D_2$	1·726
$G_2C_2D_2$	0·923	$J_2C_2D_2$	1·948	$D_2C_2Z_1$	0·918	$D_2C_2Y_1$	0·619
$A_2E_2D_2$	1·512	$D_2E_2K_2$	1·717	$K_2E_2L_2$	3·819	$D_2E_2L_2$	1·574
$L_2E_2H_2$	0·925	$K_2E_2H_2$	1·472	$D_2E_2H_2$	0·191	$H_2E_2A_2$	0·884
$F_2I_2J_2$	3·699	$J_2I_2O_2$	3·083	$J_2N_2O_2$	2·124	$O_2N_2R_2$	3·899
$O_2Q_2R_2$	4·217	$R_2Q_2T_2$	2·958	$R_2T_2U_2$	1·841	$R_2T_2W_2$	0·303
$R_2T_2Z_2$	1·516	$T_2U_2Y_2$	0·597	$T_2U_2A_3$	1·057	$Y_2U_2A_3$	0·683
$W_2U_2A_3$	1·381	$A_3U_2Z_2$	0·487	$Y_2U_2Z_2$	0·692	$P_2V_2S_2$	2·533
$S_2V_2U_2$	3·748						

3.3 Symbolic statement of condition equations

Angle Closure	*Side Closure*	*Remarks*	*Angle Closure*	*Side Closure*	*Remarks*
	E(ABF)		$F_1E_1A_1$		
	F(BJIE)	Fixed sides		$E_1(F_1XA_1)$	
FJI			$A_1E_1D_1$		
JOI			A_1D_1Z		
	x(JFIO)	Fixed sides. Pole at intersection of diagonals	A_1E_1Z		
				$D_1(A_1E_1Z)$	
				$W(ZE_1A_1)$	
OSR				$x(ZD_1E_1W)$	Pole at intersection of diagonals
ROI					
	O(JIRS)	Fixed sides		$V(ZD_1E_1)$	
	O(INR)			$D_1(J_1E_1Z)$	
	Y(SOR)			$K_1(ZD_1E_1)$	
RB_1S			$G_1F_1M_1$		
	$Y(B_1RS)$		XG_1M_1		
SB_1C_1				$F_1(M_1G_1X)$	
STC_1			E_1XM_1		
	$S(ORB_1C_1T)$	Fixed sides		$F_1(M_1E_1X)$	
	$C_1(B_1YS)$		$F_1M_1S_1$		
C_1TU				$x(L_1S_1M_1F_1)$	Pole at intersection of diagonals
TPU					
	$T(SC_1UP)$	Fixed sides		$E_1(D_1L_1F_1A_1)$	Contains artificial direction E_1 to L_1
KPU					
	P(TUK)	Fixed sides		$L_1(M_1F_1E_1)$	Eliminator for artificial direction
	x(UPKQ)	Pole at intersection of diagonals			
				$D_1(ZR_1E_1)$	
KCL				$E_1(R_1S_1F_1A_1D_1)$	Contains artificial direction E_1 to S_1
	K(PQLC)	Fixed sides			
	H(KLC)	Fixed sides		$S_1(M_1F_1E_1)$	Eliminator for artificial direction
	D(LKC)				
	D(CGK)		$G_1P_1O_1$		
B_1RX			$M_1G_1O_1$		
SB_1X				$G_1(M_1O_1P_1H_1X)$	
	$x(SRXB_1)$	Pole at intersection of diagonals	$S_1M_1O_1$		
				$M_1(S_1O_1G_1F_1)$	
	$Y(B_1RX)$			$N_1(G_1F_1M_1)$	
	R(XNOS)			$M_1(N_1F_1S_1)$	
	R(XMIOS)			$N_1(O_1G_1M_1S_1)$	
XB_1H_1			$O_1S_1X_1$		
XRH_1			$X_1W_1S_1$		
	$B_1(XRH_1)$			$S_1(W_1L_1M_1O_1X_1)$	
$C_1B_1I_1$				$S_1(W_1R_1E_1F_1L_1)$	Contains artificial direction E_1 to S_1
$B_1H_1I_1$					
	$B_1(H_1I_1C_1SR)$			$S_1(M_1F_1E_1)$	Eliminator for artificial direction
$I_1H_1P_1$					
$P_1H_1G_1$			$W_1X_1B_2$		
XH_1G_1			$X_1Y_1B_2$		
	$H_1(XG_1P_1I_1B_1)$		$X_1Y_1S_1$		
XG_1F_1				$X_1(Y_1S_1W_1B_2)$	
F_1XA_1				$S_1(X_1Y_1O_1)$	
	$X(MA_1F_1G_1H_1R)$		$X_1Y_1T_1O_1$		Polygon closure
A_1E_1X			$T_1G_1O_1$		

3.3 *continued*

Angle Closure	*Side Closure*	*Remarks*	*Angle Closure*	*Side Closure*	*Remarks*
	$O_1(Y_1T_1G_1M_1S_1)$			$E_2(D_2L_2H_2)$	
$T_1P_1O_1$				$E_2(D_2K_2L_2)$	
	$P_1(U_1H_1G_1T_1)$		$H_2L_2M_2$		
$P_1U_1O_1$			$L_2M_2P_2$		
	$x(T_1U_1P_1O_1)$	Pole at intersection of diagonals	$L_2P_2S_2$		
			$M_2L_2S_2$		
$U_1H_1I_1$				$P_2(M_2L_2S_2)$	
	$x(P_1H_1I_1U_1)$	Pole at intersection of diagonals	$L_2R_2S_2$		
			$K_2L_2R_2$		
	$I_1(C_1B_1H_1U_1Q_1)$			$L_2(R_2S_2M_2H_2K_2)$	
	$U_1(I_1Q_1V_1)$		$K_2R_2O_2$		
$Y_1T_1Z_1$			$L_2O_2R_2$		
$T_1Z_1U_1$				$O_2(R_2L_2K_2)$	
	$T_1(Y_1Z_1U_1O_1)$		$O_2J_2K_2$		
$Z_1U_1A_2$				$K_2(J_2O_2L_2D_2)$	
$T_1U_1A_2$				$J_2(I_2O_2K_2G_2F_2)$	
	$x(T_1U_1A_2Z_1)$	Pole at intersection of diagonals		$O_2(N_2R_2K_2J_2)$	
			$R_2S_2U_2$		
	$U_1(V_1I_1P_1T_1A_2)$			$S_2(U_2V_2P_2L_2R_2)$	
$A_2D_2Z_1$				$R_2(T_2U_2S_2L_2O_2Q_2)$	
$Z_1D_2G_2$			$R_2S_2W_2$		
$G_2Y_1Z_1$			$S_2U_2W_2$		
	$Z_1(Y_1G_2D_2A_2T_1)$			$x(W_2R_2S_2U_2)$	Pole at intersection of diagonals
$G_2Y_1D_2$					
	$Z_1(Y_1G_2D_2)$		$T_2W_2U_2$		
$G_2Y_1F_2$				$W_2(T_2U_2R_2)$	
$F_2Y_1B_2$			$W_2U_2Z_2$		
	$Y_1(B_2F_2G_2Z_1T_1O_1X_1)$		$R_2U_2Z_2$		
	$C_2(Z_1Y_1G_2)$			$W_2(R_2U_2Z_2)$	
	$C_2(G_2D_2Z_1)$		$T_2U_2Z_2$		
$A_2D_2H_2$				$W_2(T_2Z_2U_2)$	
$H_2D_2K_2$			$T_2Z_2B_3$		
$G_2D_2K_2$			$Z_2B_3W_2$		
	$D_2(H_2A_2Z_1G_2K_2)$			$W_2(T_2Z_2B_3)$	
$K_2J_2G_2$			$T_2Z_2Y_2$		
$J_2G_2D_2$			$T_2B_3Y_2$		
	$J_2(K_2D_2G_2)$			$B_3(Y_2T_2Z_2)$	
$J_2F_2G_2$				$U_2(T_2Y_2Z_2)$	
	$G_2(J_2D_2Y_1F_2)$		$B_3Y_2A_3$		
	$D_2(C_2G_2K_2)$		$T_2B_3A_3$		
	$G_2(J_2K_2C_2)$			$Y_2(T_2B_3A_3)$	
	$P_1(G_1O_1T_1)$		$A_3B_3W_2$		
	$G_1(N_1O_1T_1)$			$T_2(A_3B_3W_2)$	
$T_1P_1U_1$			$A_3T_2Z_2$		
$U_1H_1P_1$				$B_3(A_3Z_2T_2)$	
$K_2H_2L_2$				$U_2(Y_2A_3Z_2)$	
	$E_2(A_2D_2H_2)$		$A_3T_2X_2$		
$D_2H_2L_2$					
	$x(L_2H_2D_2K_2)$	Pole at intersection of diagonals			

APPENDIX 4

FIGURE 4 (see DIAGRAM 8)

4.1 Mean observed directions, adjustment corrections, and adjusted directions

From	*To*		*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Aberystwyth	Talsarn	(Y_1)	00° 00′ 00″·00	−0″·349	359° 59′ 59″·651
(E_2)	Cader Idris	(J_2)	189 39 13·19	+0·003	189 39 13·193
	Plynlimon	(F_2)	247 32 27·19	+0·854	247 32 28·044
	Llyn Du	(Z_1)	309 22 11·73	−0·508	309 22 11·222
Aran Fawddwy	Cader Berwyn	(L_2)	00 00 00·00	−0·620	359 59 59·380
(K_2)	Radnor Forest	(B_2)	87 36 43·24	+1·075	87 36 44·315
	Plynlimon	(F_2)	127 53 45·71	−0·208	127 53 45·502
	Cader Idris	(J_2)	174 35 40·24	−0·301	174 35 39·939
	Arenig	(P_2)	282 29 52·32	+0·054	282 29 52·374
Arenig	Cader Berwyn	(L_2)	00 00 00·00	+0·100	00 00 00·100
(P_2)	Aran Fawddwy	(K_2)	66 27 12·70	+0·052	66 27 12·752
	Cader Idris	(J_2)	106 04 22·02	−0·845	106 04 21·175
	Yr Eifl	(N_2)	179 47 12·02	+0·344	179 47 12·364
	Garnedd Ugain	(O_2)	210 18 24·72	+0·167	210 18 24·887
	Great Ormes Head	(U_2)	252 53 29·37	−0·039	252 53 29·331
	Moelfre Isaf	(V_2)	279 03 28·16	+0·035	279 03 28·195
	Moel Fammau	(W_2)	312 40 23·44	−0·360	312 40 23·080
	Cyrn-y-Brain	(Q_2)	332 01 16·78	+0·546	332 01 17·326
Bagborough	Dunkery	(D_1)	00 00 00·00	−0·500	359 59 59·500
(E_1)	Mynydd Maen	(O_1)	85 22 06·42	+0·387	85 22 06·807
	Blagdon	(F_1)	132 04 51·61	+0·056	132 04 51·666
	Pen Hill	(G_1)	147 56 04·70	−0·046	147 56 04·654
	Gore Hill	(X)	200 19 34·72	+0·308	200 19 35·028
	Pilsdon	(U)	220 39 50·94	+0·123	220 39 51·063
	Dumpdon	(T)	254 49 09·72	+0·414	254 49 10·134
	Little Haldon	(R)	279 18 54·53	−0·229	279 18 54·301
	Yes Tor	(S)	309 11 06·18	−0·513	309 11 05·667
Bartinney	Trendrine Hill	(E)	00 00 00·00	−0·244	359 59 59·756
(A)	Carnmenellis	(F)	34 55 19·15	+0·048	34 55 19·198
	Tregonning Hill	(B)	46 13 10·54	+0·535	46 13 11·075
	Goonhilly Down	(C)	62 09 33·79	−0·483	62 09 33·307
	Carn Galver	(D)	340 27 57·75	+0·143	340 27 57·893

4.1 *continued*

From	*To*		*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Bin Down (J)	Wembury	(K)	00° 00′ 00″·00	−0″·001	359° 59′ 59″·999
	Dodman	(H)	132 20 33·76	+0·544	132 20 34·304
	Hensbarrow	(I)	165 51 15·45	−0·341	165 51 15·109
	Brown Willy	(N)	228 29 13·27	−0·177	228 29 13·093
	Yes Tor	(S)	299 17 01·32	+0·013	299 17 01·333
	Ryders Hill	(O)	329 30 51·67	−0·376	329 30 51·294
	Three Barrows	(P)	338 37 34·72	+0·338	338 37 35·058
Brown Willy (N)	Hensbarrow	(I)	00 00 00·00	−0·326	359 59 59·674
	Trevose Head	(M)	47 38 36·42	+0·202	47 38 36·622
	Hendon Moor	(A_1)	159 06 23·40	+0·779	159 06 24·179
	Yes Tor	(S)	220 41 21·34	−0·767	220 41 20·573
	Ryders Hill	(O)	246 30 59·03	+0·076	246 30 59·106
	Bin Down	(J)	296 38 12·91	+0·418	296 38 13·328
	Dodman	(H)	345 19 27·97	−0·382	345 19 27·588
Bradley Knoll (H_1)	Bulbarrow	(Y)	40 45 33·564(1)	—	40 45 33·564
	Pilsdon	(U)	84 51 12·920(2)	−1·639	84 51 11·281
	Pen Hill	(G_1)	155 57 07·701(1)	—	155 57 07·701
Bulbarrow (Y)	Wingreen	(Z)	359 59 59·845(1)	—	359 59 59·845
	Coringdon	(W)	92 14 33·160(1)	—	92 14 33·160
	Blackdown	(V)	178 08 36·587(2)	−0·194	178 08 36·393
	Gore Hill	(X)	218 40 21·197(2)	+0·009	218 40 21·206
	Pen Hill	(G_1)	289 22 02·621(1)	—	289 22 02·621
	Bradley Knoll	(H_1)	317 07 12·983(1)	—	317 07 12·983
Cader Berwyn (L_2)	Cyrn-y-Brain	(Q_2)	00 00 00·00	−0·001	359 59 59·999
	Delamere	(X_2)	12 02 12·07	−1·007	12 02 11·063
	Hanchurch Wtr Twr	(M_2)	44 53 20·42	+0·344	44 53 20·764
	Wrekin	(H_2)	74 00 53·72	+0·670	74 00 54·390
	Stiperstones	(G_2)	99 12 22·68	+1·020	99 12 23·700
	Radnor Forest	(B_2)	131 02 51·09	−0·572	131 02 50·518
	Aran Fawddwy	(K_2)	203 52 08·08	+0·328	203 52 08·408
	Arenig	(P_2)	239 54 49·80	−0·185	239 54 49·615
	Moel Fammau	(W_2)	336 45 16·85	−0·597	336 45 16·253
Cader Idris (J_2)	Plynlimon	(F_2)	00 00 00·00	+0·206	00 00 00·206
	Aberystwyth	(E_2)	38 31 31·80	−1·092	38 31 30·708
	Rhiw	(I_2)	125 28 16·22	−0·085	125 28 16·135
	Yr Eifl	(N_2)	149 14 35·44	+0·409	149 14 35·849
	Garnedd Ugain	(O_2)	183 21 15·10	+0·304	183 21 15·404
	Arenig	(P_2)	222 36 58·04	−0·019	222 36 58·021
	Aran Fawddwy	(K_2)	255 05 37·53	+0·277	255 05 37·807

(1) Fixed direction from previous Figures.

(2) Mean observed direction plus overlap correction from previous Figures.

4.1 *continued*

From	*To*	*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Capel Cynon	Mynydd Rhos-Wen (W_1)	00° 00′ 00″·00	−0″·235	359° 59′ 59″·765
(X_1)	Prescelly (V_1)	90 31 56·27	+0·055	90 31 56·325
	Garn Fawr (U_1)	111 16 14·78	−0·414	111 16 14·366
	Talsarn (Y_1)	271 50 02·01	−0·410	271 50 01·600
	Llyn Du (Z_1)	288 06 28·39	+0·739	288 06 29·129
	Drygarn (A_2)	293 17 52·84	+0·266	293 17 53·106
Carnmenellis	St. Agnes Beacon (G)	00 00 00·00	−0·109	359 59 59·891
(F)	Hensbarrow (I)	48 57 29·69	+0·094	48 57 29·784
	Dodman (H)	78 28 39·91	−0·167	78 28 39·743
	Goonhilly Down (C)	162 58 04·99	−0·094	162 58 04·896
	Tregonning Hill (B)	230 23 17·84	+0·206	230 23 18·046
	Bartinney (A)	250 41 35·41	−0·130	250 41 35·280
	Carn Galver (D)	262 28 19·64	+0·188	262 28 19·828
	Trendrine Hill (E)	270 05 01·06	+0·012	270 05 01·072
Cefn Bryn	Pendine (R_1)	00 00 00·00	−0·375	359 59 59·625
(L_1)	Mynydd Rhos-Wen (W_1)	48 43 21·08	−0·408	48 43 20·672
	Trecastle (S_1)	96 49 10·94	−0·626	96 49 10·314
	Mynydd Margam (M_1)	143 55 09·34	+0·474	143 55 09·814
	Dunkery (D_1)	195 28 29·37	−0·230	195 28 29·140
	Parracombe (C_1)	211 54 46·64	+0·969	211 54 47·609
	Eastacott Hill (B_1)	239 53 54·84	−0·110	239 53 54·730
	Rat Island Lighth'se (I_1)	273 15 49·46	+0·326	273 15 49·786
	Lundy Island N.W. Point Lighthouse (K_1)	277 12 42·34	+0·537	277 12 42·877
	Marros Beacon (Q_1)	354 56 13·85	−0·557	354 56 13·293
Cyrn-y-Brain	Moel Fammau (W_2)	00 00 00·00	−0·148	359 59 59·852
(Q_2)	Delamere (X_2)	80 34 42·77	+1·052	80 34 43·822
	Wirswall (R_2)	121 35 55·19	−0·042	121 35 55·148
	Cader Berwyn (L_2)	241 41 07·34	+0·527	241 41 07·867
	Arenig (P_2)	273 37 17·30	−1·389	273 37 15·911
Coringdon	Bulbarrow (Y)	359 59 59·771(1)	—	359 59 59·771
(W)	Wingreen (Z)	31 28 55·360(1)	—	31 28 55·360
	Blackdown (V)	322 36 57·660(2)	+0·748	322 36 58·408
	Pilsdon (U)	331 56 45·450(2)	+0·204	331 56 45·654
	Gore Hill (X)	344 50 04·440(2)	+0·136	344 50 04·576
Dodman	Bin Down (J)	00 00 00·00	−0·118	359 59 59·882
(H)	Goonhilly Down (C)	180 15 02·50	−0·036	180 15 02·464
	Carnmenellis (F)	208 13 13·31	−0·146	208 13 13·164
	St. Agnes Beacon (G)	234 04 38·45	+0·525	234 04 38·975
	Hensbarrow (I)	302 06 58·85	+0·281	302 06 59·131
	Brown Willy (N)	324 49 51·35	−0·507	324 49 50·843

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(1) Fixed direction from previous Figures.

(2) Mean observed direction plus overlap correction from previous Figures.

4.1 *continued*

From	*To*		*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Drygarn (A_2)	Radnor Forest	(B_2)	00° 00′ 00″·00	+0″·408	00° 00′ 00″·408
	Gwynydd Bach	(T_1)	48 36 16·21	−0·818	48 36 15·392
	Trecastle	(S_1)	105 33 35·62	−0·158	105 33 35·462
	Mynydd Rhos-Wen	(W_1)	156 45 54·31	+0·371	156 45 54·681
	Capel Cynon	(X_1)	179 23 15·09	+0·618	179 23 15·708
	Llyn Du	(Z_1)	203 45 21·49	−0·015	203 45 21·475
	Plynlimon	(F_2)	265 31 59·42	−0·407	265 31 59·013
Dunkery (D_1)	Parracombe	(C_1)	00 00 00·00	−0·378	359 59 59·622
	Cefn Bryn	(L_1)	46 07 43·22	−0·856	46 07 42·364
	Mynydd Margam	(M_1)	75 40 35·14	+0·228	75 40 35·368
	Llangeinor	(N_1)	86 39 59·51	+0·083	86 39 59·593
	Mynydd Maen	(O_1)	117 36 50·68	+0·432	117 36 51·112
	Bagborough	(E_1)	187 35 30·05	+0·411	187 35 30·461
	Dumpdon	(T)	227 14 26·36	−0·570	227 14 25·790
	Little Haldon	(R)	262 11 45·21	+0·462	262 11 45·672
	Yes Tor	(S)	295 29 45·30	+0·188	295 29 45·488
Delamere (X_2)	Wirswall	(R_2)	00 00 00·057(2)	+0·190	00 00 00·247
	Cader Berwyn	(L_2)	53 25 19·937(2)	−1·407	53 25 18·530
	Cyrn-y-Brain	(Q_2)	60 16 45·117(2)	−0·998	60 16 44·119
	Moel Fammau	(W_2)	81 08 52·997(2)	−0·161	81 08 52·836
	Rivington	(Y_2)	195 56 06·275(1)	—	195 56 06·275
	Hanchurch Wtr Twr	(M_2)	316 43 09·476(1)	—	316 43 09·476
	Wrekin	(H_2)	353 39 15·882(1)	—	353 39 15·882
Eastacott Hill (B_1)	Hendon Moor	(A_1)	00 00 00·00	−0·956	359 59 59·044
	Rat Island Ligth'se	(I_1)	51 15 17·86	+0·517	51 15 18·377
	Lundy Island Lighthouse	(J_1)	52 15 24·31	+0·458	52 15 24·768
	Lundy Island N.W. Point Lighthouse	(K_1)	58 42 34·36	+0·465	58 42 34·825
	Cefn Bryn	(L_1)	146 03 16·00	−1·091	146 03 14·909
	Parracombe	(C_1)	228 25 41·55	+0·368	228 25 41·918
	Yes Tor	(S)	307 48 49·91	+0·240	307 48 50·150
Furland (Q)	Little Haldon	(R)	00 00 00·00	+0·249	00 00 00·249
	Pilsdon	(U)	41 32 23·40	+0·317	41 32 23·717
	Portlemouth	(L)	217 53 28·37	−0·393	217 53 27·977
	Three Barrows	(P)	285 32 24·96	−0·354	285 32 24·606
	Ryders Hill	(O)	298 20 32·12	+0·180	298 20 32·300
Garnedd Ugain (O_2)	Yr Eifl	(N_2)	00 00 00·00	−0·675	359 59 59·325
	Holyhead	(S_2)	58 15 44·02	+0·035	58 15 44·055
	Llaneilian	(T_2)	92 16 33·42	−0·176	92 16 33·244
	Moelfre Isaf	(V_2)	174 48 50·89	−0·034	174 48 50·856
	Moel Fammau	(W_2)	195 12 03·37	+1·054	195 12 04·424
	Arenig	(P_2)	243 03 12·19	−0·827	243 03 11·363
	Cader Idris	(J_2)	279 33 26·54	+0·338	279 33 26·878
	Rhiw	(I_2)	348 58 22·68	+0·285	348 58 22·965

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(1) Fixed direction from previous Figures.

(2) Mean observed direction plus overlap correction from previous Figures.

4.1 *continued*

From	*To*	*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Gwynydd Bach (T$_1$)	Mynydd Maen (O$_1$)	00° 00′ 00″·736(1)	—	00° 00′ 00″·736
	Llangeinor (N$_1$)	48 34 23·958(2)	+1″·412	48 34 25·370
	Trecastle (S$_1$)	86 02 25·378(2)	+0·380	86 02 25·758
	Llyn Du (Z$_1$)	132 36 13·198(2)	+0·537	132 36 13·735
	Drygarn (A$_2$)	136 41 37·608(2)	+1·330	136 41 38·938
	Radnor Forest (B$_2$)	182 17 26·158(2)	+0·798	182 17 26·956
	Titterstone Clee (D$_2$)	225 58 08·829(1)	—	225 58 08·829
	Malvern (C$_2$)	262 29 03·543(1)	—	262 29 03·543
Hendon Moor (A$_1$)	Eastacott Hill (B$_1$)	00 00 00·00	−0·007	359 59 59·993
	Parracombe (C$_1$)	20 01 41·85	+0·865	20 01 42·715
	Yes Tor (S)	91 08 39·49	−0·123	91 08 39·367
	Brown Willy (N)	154 47 45·86	−0·850	154 47 45·010
	Trevose Head (M)	184 13 30·34	+0·114	184 13 30·454
	Lundy Island Lighthouse (J$_1$)	293 35 38·39	−0·012	293 35 38·378
	Rat Island Lighth'se (I$_1$)	295 15 00·76	+0·013	295 15 00·773
Hensbarrow (I)	Bin Down (J)	00 00 00·00	−0·056	359 59 59·944
	Dodman (H)	88 36 19·86	−0·190	88 36 19·670
	Carnmenellis (F)	145 11 24·67	+0·486	145 11 25·156
	St. Agnes Beacon (G)	165 51 54·44	−0·446	165 51 53·994
	Trevose Head (M)	232 51 27·58	−0·509	232 51 27·071
	Brown Willy (N)	305 59 41·98	+0·715	305 59 42·695
Hanchurch Wtr Twr (M$_2$)	Wrekin (H$_2$)	359 59 59·632(1)	—	359 59 59·632
	Stiperstones (G$_2$)	15 11 19·835(2)	+0·242	15 11 20·077
	Cader Berwyn (L$_2$)	50 59 18·085(2)	+0·038	50 59 18·123
	Moel Fammau (W$_2$)	74 53 07·035(2)	−0·500	74 53 06·535
	Delamere (X$_2$)	101 26 05·705(1)	—	101 26 05·705
Little Haldon (R)	Furland (Q)	00 00 00·00	−0·207	359 59 59·793
	Three Barrows (P)	58 38 37·94	+0·096	58 38 38·036
	Ryders Hill (O)	70 48 25·18	+0·200	70 48 25·380
	Yes Tor (S)	108 14 37·52	+0·141	108 14 37·661
	Dunkery (D$_1$)	172 05 52·17	−0·091	172 05 52·079
	Bagborough (E$_1$)	196 48 36·39	−0·158	196 48 36·232
	Dumpdon (T)	216 17 37·74	+0·209	216 17 37·949
	Pilsdon (U)	236 43 56·64	−0·191	236 43 56·449
Llaneilian (T$_2$)	Holyhead (S$_2$)	00 00 00·00	−0·011	359 59 59·989
	Great Ormes Head (U$_2$)	214 55 22·39	+0·103	214 55 22·493
	Moelfre Isaf (V$_2$)	220 00 52·10	−0·089	220 00 52·011
	Garnedd Ugain (O$_2$)	268 22 08·93	+0·036	268 22 08·966
	Yr Eifl (N$_2$)	301 58 42·69	−0·039	301 58 42·651

(1) Fixed direction from previous Figures.

(2) Mean observed direction plus overlap correction from previous Figures.

4.1 *continued*

From	*To*	*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Llangeinor (N₁)	Mynydd Maen (O₁)	00° 00′ 00″00	+0″233	00° 00′ 00″233
	Dunkery (D₁)	97 18 20·35	−0·207	97 18 20·143
	Parracombe (C₁)	117 31 58·49	−0·223	117 31 58·267
	Mynydd Margam (M₁)	152 43 28·14	+0·235	152 43 28·375
	Trecastle (S₁)	257 04 36·43	−0·854	257 04 35·576
	Gwynydd Bach (T₁)	315 41 18·16	+0·816	315 41 18·976
Moel Fammau (W₂)	Cyrn-y-Brain (Q₂)	00 00 00·00	+0·313	00 00 00·313
	Cader Berwyn (L₂)	38 26 24·97	+0·301	38 26 25·271
	Arenig (P₂)	74 16 23·40	+0·167	74 16 23·567
	Garnedd Ugain (O₂)	104 03 22·02	−0·634	104 03 21·386
	Moelfre Isaf (V₂)	138 49 03·81	−0·056	138 49 03·754
	Great Ormes Head (U₂)	139 29 20·39	+0·169	139 29 20·559
	Rivington (Y₂)	245 25 12·33	−0·885	245 25 11·445
	Delamere (X₂)	281 26 51·71	−0·058	281 26 51·652
	Hanchurch Wtr Twr (M₂)	310 28 11·85	+0·684	310 28 12·534
Mynydd Rhos-Wen (W₁)	Talsarn (Y₁)	00 00 00·00	−0·232	359 59 59·768
	Drygarn (A₂)	43 33 20·31	−0·114	43 33 20·196
	Trecastle (S₁)	95 14 10·41	+0·107	95 14 10·517
	Cefn Bryn (L₁)	161 36 24·61	−0·195	161 36 24·415
	Pendine (R₁)	212 34 24·24	−0·183	212 34 24·057
	Prescelly (V₁)	253 08 41·18	+0·730	253 08 41·910
	Capel Cynon (X₁)	312 52 45·78	−0·112	312 52 45·668
Malvern (C₂)	Gwynydd Bach (T₁)	227 54 59·556(1)	—	227 54 59·556
	Radnor Forest (B₂)	260 58 38·873(2)	+0·045	260 58 38·918
	Titterstone Clee (D₂)	304 51 16·200(1)	—	304 51 16·200
Mynydd Maen (O₁)	Llangeinor (N₁)	00 00 00·000(2)	−0·839	359 59 59·161
	Trecastle (S₁)	33 53 32·640(2)	−1·156	33 53 31·484
	Gwynydd Bach (T₁)	87 06 56·133(1)	—	87 06 56·133
	Pen Hill (G₁)	243 10 02·618(1)	—	243 10 02·618
	Blagdon (F₁)	246 00 59·140(2)	+0·270	246 00 59·410
	Bagborough (E₁)	283 35 47·430(2)	+0·651	283 35 48·081
	Dunkery (D₁)	308 15 06·150(2)	−0·222	308 15 05·928
Parracombe (C₁)	Eastacott Hill (B₁)	00 00 00·00	+0·244	00 00 00·244
	Rat Island Lighth'se (I₁)	01 38 53·47	−1·067	01 38 52·403
	Lundy Island Lighthouse (J₁)	02 15 59·63	−0·762	02 15 58·868
	Lundy Island N.W. Point Lighthouse (K₁)	06 07 32·84	−1·085	06 07 31·755
	Cefn Bryn (L₁)	69 38 29·03	−0·217	69 38 28·813
	Mynydd Margam (M₁)	106 09 05·30	+0·558	106 09 05·858
	Llangeinor (N₁)	113 58 04·64	+0·587	113 58 05·227
	Dunkery (D₁)	187 04 29·26	+0·461	187 04 29·721
	Yes Tor (S)	284 02 47·74	+0·616	284 02 48·356
	Hendon Moor (A₁)	331 35 58·01	+0·666	331 35 58·676

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(1) Fixed direction from previous Figures.

(2) Mean observed direction plus overlap correction from previous Figures.

4.1 *continued*

From	*To*	*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Pendine (R$_1$)	St. Anns Hill (P$_1$)	00° 00′ 00″·00	+0″·144	00° 00′ 00″·144
	Prescelly (V$_1$)	64 17 42·31	−0·401	64 17 41·909
	Mynydd Rhos-Wen (W$_1$)	143 33 58·83	−0·107	143 33 58·723
	Cefn Bryn (L$_1$)	223 52 40·04	+0·987	223 52 41·027
	Rat Island Lighth'se (I$_1$)	285 21 55·80	−0·239	285 21 55·561
	Lundy Island N.W. Point Lighthouse (K$_1$)	287 08 04·39	−0·404	287 08 03·986
	Marros Beacon (Q$_1$)	337 17 34·16	+0·021	337 17 34·181
Pilsdon (U)	Blackdown (V)	00 00 00·00	+0·380	00 00 00·380
	Furland (Q)	103 04 55·82	−0·610	103 04 55·210
	Little Haldon (R)	118 16 30·85	+0·177	118 16 31·027
	Dumpdon (T)	152 42 27·24	−0·078	152 42 27·162
	Bagborough (E$_1$)	199 42 13·48	+0·004	199 42 13·484
	Pen Hill (G$_1$)	253 24 01·02	+0·347	253 24 01·367
	Bradley Knoll (H$_1$)	281 23 53·65	+0·224	281 23 53·874
	Gore Hill (X)	318 54 07·15	−0·064	318 54 07·086
	Coringdon (W)	344 22 28·06	−0·381	344 22 27·679
Plynlimon (F$_2$)	Talsarn (Y$_1$)	00 00 00·00	−0·602	359 59 59·398
	Capel Cynon (X$_1$)	05 30 17·65	−1·218	05 30 16·432
	Aberystwyth (E$_2$)	37 10 39·84	−0·438	37 10 39·402
	Cader Idris (J$_2$)	120 45 55·18	+0·243	120 45 55·423
	Aran Fawddwy (K$_2$)	149 09 40·09	−0·316	149 09 39·774
	Stiperstones (G$_2$)	216 04 25·18	+1·007	216 04 26·187
	Radnor Forest (B$_2$)	257 56 41·74	+0·329	257 56 42·069
	Drygarn (A$_2$)	303 16 14·47	+0·619	303 16 15·089
	Llyn Du (Z$_1$)	320 59 08·29	+0·374	320 59 08·664
Portlemouth (L)	Furland (Q)	00 00 00·00	+0·349	00 00 00·349
	Wembury (K)	256 49 37·30	−0·001	256 49 37·299
	Three Barrows (P)	294 28 31·93	−0·622	294 28 31·308
	Ryders Hill (O)	299 51 38·21	+0·274	299 51 38·484
Prescelly (V$_1$)	Pendine (R$_1$)	00 00 00·00	+0·676	00 00 00·676
	Marros Beacon (Q$_1$)	07 55 31·84	−0·314	07 55 31·526
	St. Anns Hill (P$_1$)	79 37 45·19	−0·197	79 37 44·993
	Garn Fawr (U$_1$)	144 33 00·75	+0·163	144 33 00·913
	Capel Cynon (X$_1$)	270 06 31·97	−0·087	270 06 31·883
	Mynydd Rhos-Wen (W$_1$)	299 50 33·42	−0·241	299 50 33·179
Pen Hill (G$_1$)	Bradley Knoll (H$_1$)	359 59 59·808(1)	—	359 59 59·808
	Bulbarrow (Y)	37 03 17·134(1)	—	37 03 17·134
	Gore Hill (X)	54 06 09·719(2)	−0·038	54 06 09·681
	Pilsdon (U)	80 54 13·159(2)	+0·822	80 54 13·981
	Bagborough (E$_1$)	134 28 44·879(2)	−0·894	134 28 43·985
	Blagdon (F$_1$)	200 04 41·969(2)	+0·068	200 04 42·037
	Mynydd Maen (O$_1$)	211 29 06·684(1)	—	211 29 06·684

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(1) Fixed direction from previous Figures.

(2) Mean observed direction plus overlap correction from previous Figures.

4.1 *continued*

From	*To*		*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Radnor Forest	Stiperstones	(G_2)	00° 00′ 00″·00	−0″·210	359° 59′ 59″·790
(B_2)	Titterstone Clee	(D_2)	42 58 16·26	+1·066	42 58 17·326
	Malvern	(C_2)	79 35 16·70	+0·158	79 35 16·858
	Trecastle	(S_1)	192 12 37·82	−0·609	192 12 37·211
	Drygarn	(A_2)	232 08 05·56	+0·026	232 08 05·586
	Plynlimon	(F_2)	272 20 33·69	−0·104	272 20 33·586
	Aran Fawddwy	(K_2)	303 16 33·90	+0·154	303 16 34·054
	Cader Berwyn	(L_2)	322 50 35·65	−0·480	322 50 35·170
Ryders Hill	Furland	(Q)	00 00 00·00	−0·192	359 59 59·808
(O)	Portlemouth	(L)	39 24 34·71	+0·381	39 24 35·091
	Three Barrows	(P)	61 40 25·25	−0·135	61 40 25·115
	Bin Down	(J)	129 24 06·78	+0·750	129 24 07·530
	Brown Willy	(N)	158 15 18·04	−0·419	158 15 17·621
	Yes Tor	(S)	215 25 49·70	−0·068	215 25 49·632
	Little Haldon	(R)	312 27 52·26	−0·316	312 27 51·944
Rivington	Delamere	(X_2)	141 55 05·189(1)	—	141 55 05·189
(Y_2)	Moel Fammau	(W_2)	171 06 14·708(2)	+1·008	171 06 15·716
St. Agnes Beacon	Carnmenellis	(F)	00 00 00·00	+0·044	00 00 00·044
(G)	Tregonning Hill	(B)	22 46 14·02	−0·399	22 46 13·621
	Trendrine Hill	(E)	57 37 53·14	+0·182	57 37 53·322
	Trevose Head	(M)	202 24 48·82	+0·310	202 24 49·130
	Hensbarrow	(I)	249 37 57·89	−0·087	249 37 57·803
	Dodman	(H)	284 20 04·70	−0·050	284 20 04·650
St. Anns Hill	Garn Fawr	(U_1)	00 00 00·00	−0·170	359 59 59·830
(P_1)	Prescelly	(V_1)	32 38 41·40	+0·485	32 38 41·885
	Pendine	(R_1)	68 43 18·50	−0·210	68 43 18·290
	Marros Beacon	(Q_1)	70 42 18·03	−0·105	70 42 17·925
Stiperstones	Wrekin	(H_2)	00 00 00·00	+0·755	00 00 00·755
(G_2)	Titterstone Clee	(D_2)	62 41 05·07	+0·380	62 41 05·450
	Radnor Forest	(B_2)	138 00 38·70	+0·645	138 00 39·345
	Plynlimon	(F_2)	188 29 03·06	−1·266	188 29 01·794
	Cader Berwyn	(L_2)	249 00 52·76	−0·655	249 00 52·105
	Wirswall	(R_2)	311 54 55·19	+0·218	311 54 55·408
	Hanchurch Wtr Twr	(M_2)	338 53 58·35	−0·077	338 53 58·273
Three Barrows	Little Haldon	(R)	00 00 00·00	+0·089	00 00 00·089
(P)	Furland	(Q)	46 53 47·28	+0·324	46 53 47·604
	Portlemouth	(L)	93 43 23·08	+0·114	93 43 23·194
	Wembury	(K)	164 37 07·84	0·0	164 37 07·840
	Bin Down	(J)	198 12 46·28	−0·520	198 12 45·760
	Ryders Hill	(O)	301 22 20·20	−0·007	301 22 20·193

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(1) Fixed direction from previous Figures.

(2) Mean observed direction plus overlap correction from previous Figures.

4.1 *continued*

From	*To*		*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Trecastle (S$_1$)	Mynydd Rhos-Wen	(W$_1$)	00° 00′ 00″·00	−0″·304	359° 59′ 59″·696
	Talsarn	(Y$_1$)	34 45 26·88	−0·425	34 45 26·455
	Llyn Du	(Z$_1$)	63 46 28·85	−0·378	63 46 28·472
	Drygarn	(A$_2$)	77 06 53·48	−0·008	77 06 53·472
	Radnor Forest	(B$_2$)	111 37 53·21	−0·255	111 37 52·955
	Gwynydd Bach	(T$_1$)	149 30 23·19	+0·570	149 30 23·760
	Mynydd Maen	(O$_1$)	190 14 37·40	−0·040	190 14 37·360
	Llangeinor	(N$_1$)	233 25 42·56	+0·263	233 25 42·823
	Mynydd Margam	(M$_1$)	252 25 57·16	+0·158	252 25 57·318
	Cefn Bryn	(L$_1$)	294 27 59·03	+0·419	294 27 59·449
Tregonning Hill (B)	St. Agnes Beacon	(G)	00 00 00·00	+0·322	00 00 00·322
	Carnmenellis	(F)	27 37 05·25	−0·036	27 37 05·214
	Goonhilly Down	(C)	96 30 08·87	+0·265	96 30 09·135
	Bartinney	(A)	239 13 13·95	+0·060	239 13 14·010
	Carn Galver	(D)	258 50 15·20	−0·432	258 50 14·768
	Trendrine Hill	(E)	277 06 00·79	−0·179	277 06 00·611
Trendrine Hill (E)	St. Agnes Beacon	(G)	00 00 00·00	−0·177	359 59 59·823
	Carnmenellis	(F)	32 27 08·84	−0·348	32 27 08·492
	Tregonning Hill	(B)	62 14 20·77	+0·501	62 14 21·271
	Bartinney	(A)	158 08 23·97	−0·146	158 08 23·824
	Carn Galver	(D)	178 43 37·23	+0·170	178 43 37·400
Trevose Head (M)	St. Agnes Beacon	(G)	00 00 00·00	−0·232	359 59 59·768
	Hendon Moor	(A$_1$)	195 53 01·07	−0·095	195 53 00·975
	Brown Willy	(N)	234 59 30·98	−0·129	234 59 30·851
	Hensbarrow	(I)	294 12 39·42	+0·456	294 12 39·876
Titterstone Clee (D$_2$)	Wrekin	(H$_2$)	359 59 59·790(1)	—	359 59 59·790
	Malvern	(C$_2$)	144 36 07·392(1)	—	144 36 07·392
	Gwynydd Bach	(T$_1$)	211 09 01·308(1)	—	211 09 01·308
	Radnor Forest	(B$_2$)	244 06 35·524(2)	−0·921	244 06 34·603
	Stiperstones	(G$_2$)	305 48 46·364(2)	−0·250	305 48 46·114
Wingreen (Z)	Bulbarrow	(Y)	00 00 00·089(1)	—	00 00 00·089
	Gore Hill	(X)	15 22 46·609(2)	−0·064	15 22 46·545
	Coringdon	(W)	303 43 27·202(1)	—	303 43 27·202
Wrekin (H$_2$)	Stiperstones	(G$_2$)	00 00 00·075(2)	−0·773	359 59 59·302
	Cader Berwyn	(L$_2$)	43 49 25·185(2)	−0·888	43 49 24·297
	Wirswall	(R$_2$)	97 36 41·855(2)	+0·112	97 36 41·967
	Delamere	(X$_2$)	102 04 46·457(1)	—	102 04 46·457
	Hanchurch Wtr Twr	(M$_2$)	143 42 37·957(1)	—	143 42 37·957
	Titterstone Clee	(D$_2$)	296 52 15·771(1)	—	296 52 15·771

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(1) Fixed direction from previous Figures.

(2) Mean observed direction plus overlap correction from previous Figures.

4.1 *continued*

From	*To*		*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Yes Tor (S)	Little Haldon	(R)	00° 00′ 00″00	−0″171	359° 59′ 59″829
	Ryders Hill	(O)	45 31 46·60	+0·132	45 31 46·732
	Bin Down	(J)	109 16 17·17	−0·220	109 16 16·950
	Brown Willy	(N)	142 31 37·62	+1·026	142 31 38·646
	Hendon Moor	(A_1)	197 17 40·72	−0·283	197 17 40·437
	Eastacott Hill	(B_1)	233 57 55·84	−0·195	233 57 55·645
	Parracombe	(C_1)	258 37 38·48	+0·160	258 37 38·640
	Dunkery	(D_1)	277 09 08·98	−0·467	277 09 08·513
	Bagborough	(E_1)	298 26 03·71	+0·018	298 26 03·728
Yr Eifl (N_2)	Rhiw	(I_2)	00 00 00·00	−0·110	359 59 59·890
	Holyhead	(S_2)	117 23 54·07	−0·024	117 23 54·046
	Llaneilian	(T_2)	151 18 13·39	+0·197	151 18 13·587
	Garnedd Ugain	(O_2)	205 25 08·30	+0·322	205 25 08·622
	Arenig	(P_2)	237 57 09·83	+0·010	237 57 09·840
	Cader Idris	(J_2)	270 51 59·90	−0·394	270 51 59·506

4.2 Triangle misclosures and spherical excesses

Triangle	*Spherical Excess* (ε)	*Triangle Misclosure*	*Triangle*	*Spherical Excess* (ε)	*Triangle Misclosure*	*Triangle*	*Spherical Excess* (ε)	*Triangle Misclosure*
GFH	1″057	−0″707	GFI	0″973	+0″597	FHI	1″412	−0″842
IHG	1·327	+0·463	GFE	0·765	+0·155	GFB	0·312	+1·118
GEB	0·860	−1·760	FBA	0·314	−0·054	FBE	0·408	−0·798
FAE	0·567	−0·637	EAB	0·474	+0·106	HIJ	1·281	+1·419
HIN	0·773	+1·637	NJH	2·088	+1·132	NJI	1·580	+1·350
GMI	1·642	+1·148	MIN	1·597	−2·337	MNA_1	2·876	−1·506
NJS	2·690	−2·620	NJO	2·514	+1·026	SOJ	2·281	+1·559
SON	2·457	−2·087	A_1NS	3·828	+3·582	JOP	0·613	−2·113
OPL	0·202	−0·502	PLQ	1·259	−0·799	OLQ	1·470	−1·220
PQO	0·412	−0·922	PQR	1·400	−1·140	ORP	0·412	−0·382
ORQ	1·400	−0·600	ORS	1·496	+0·004	A_1SB_1	3·477	+1·223
A_1SC_1	5·175	+0·495	B_1C_1S	3·115	+0·145	$B_1C_1A_1$	1·416	+0·874
SC_1D_1	2·642	+1·038	SRD_1	5·550	+0·210	D_1E_1S	4·076	−0·276
SRE_1	6·039	+0·771	D_1E_1R	4·564	+0·286	RUQ	2·629	−0·839
RUE_1	5·912	+0·558	UG_1E_1	4·296	+1·204	UG_1H_1	3·100	−2·710
G_1H_1Y	1·824	−1·604	ZYW	1·792	−1·892	$E_1G_1O_1$	6·009	−1·289
$E_1D_1O_1$	4·502	+0·008	$D_1N_1O_1$	4·661	+0·709	$D_1C_1N_1$	2·589	−0·319
$D_1C_1L_1$	2·118	−1·398	$L_1B_1C_1$	2·699	+0·081	$P_1R_1V_1$	2·486	+2·114
$V_1R_1W_1$	2·164	−2·124	$L_1R_1W_1$	2·993	−1·073	$W_1L_1S_1$	3·786	+1·244
$S_1T_1O_1$	3·270	+0·230	$S_1T_1N_1$	2·851	−0·331	$N_1O_1S_1$	2·444	−1·074
$N_1O_1T_1$	2·863	−0·513	$S_1T_1A_2$	3·538	−2·188	$S_1A_2B_2$	2·912	+0·178
$T_1C_2D_2$	5·274	+1·476	$B_2D_2C_2$	4·025	+1·745	$W_1S_1A_2$	3·316	−1·046
$W_1A_2X_1$	2·214	+0·256	$W_1X_1V_1$	1·615	+0·705	$F_2A_2B_2$	2·416	−0·976

4.2 *continued*

Triangle	*Spherical Excess (ε)*	*Triangle Misclosure*	*Triangle*	*Spherical Excess (ε)*	*Triangle Misclosure*	*Triangle*	*Spherical Excess (ε)*	*Triangle Misclosure*
$E_2F_2J_2$	1″374	−0″234	$F_2J_2K_2$	1″188	+0″722	$F_2B_2K_2$	3″949	+0″381
$K_2L_2B_2$	3·940	−1·960	$B_2L_2G_2$	4·199	+2·621	$B_2G_2F_2$	4·536	+2·694
$B_2D_2G_2$	2·942	−2·212	$G_2D_2H_2$	1·902	+1·638	$G_2H_2M_2$	1·582	−1·912
$G_2L_2M_2$	7·150	−1·050	$G_2H_2L_2$	2·955	−1·645	$L_2M_2H_2$	5·777	−1·317
$H_2M_2X_2$	3·980	−1·080	$L_2M_2W_2$	5·661	−0·021	$W_2L_2Q_2$	0·689	+0·091
$W_2Q_2X_2$	1·349	−2·409	$L_2W_2X_2$	2·735	−1·195	$L_2Q_2X_2$	0·697	+1·123
$L_2X_2M_2$	6·337	+0·263	$W_2X_2M_2$	3·412	−0·912	$L_2X_2H_2$	8·134	+0·026
$W_2X_2Y_2$	4·172	−1·042	$L_2P_2Q_2$	1·203	+2·177	$L_2P_2W_2$	1·955	+0·085
$P_2L_2K_2$	0·866	+1·234	$P_2Q_2W_2$	1·441	−2·001	$P_2K_2J_2$	0·644	+0·246
$P_2J_2N_2$	3·027	−0·357	$P_2J_2O_2$	1·844	−1·854	$N_2O_2P_2$	1·704	+0·336
$N_2O_2J_2$	2·886	+1·834	$P_2O_2W_2$	2·951	+3·209	$N_2T_2O_2$	2·639	−0·549

Unclosed Triangles

Triangle	*Spherical Excess (ε)*	*Triangle*	*Spherical Excess (ε)*	*Triangle*	*Spherical Excess (ε)*
ACB	0″486	ACF	1″222	BCF	0″422
FCH	1·211	ADE	0·073	ADF	0·440
ADB	0·326	BDE	0·221	BDF	0·428
EDF	0·201	JKP	0·906	LKP	1·069
RTD_1	4·544	RTE_1	2·124	RTU	1·921
UTE_1	1·867	D_1TE_1	2·144	UVW	1·032
WVY	2·069	UXW	1·542	WXY	0·968
YXZ	0·472	WXZ	3·231	YXG_1	1·629
G_1XE_1	4·795	E_1XU	2·100	G_1XU	2·598
$E_1F_1G_1$	1·134	$G_1F_1O_1$	0·337	$O_1F_1E_1$	4·539
$A_1I_1B_1$	2·063	$A_1I_1C_1$	3·573	$B_1I_1C_1$	0·094
$B_1I_1L_1$	3·792	$C_1I_1L_1$	6·397	$L_1I_1R_1$	5·229
$A_1J_1B_1$	2·171	$A_1J_1C_1$	3·719	$B_1J_1C_1$	0·132
$B_1K_1C_1$	0·358	$B_1K_1L_1$	4·005	$C_1K_1L_1$	6·346
$L_1K_1R_1$	4·978	$C_1M_1D_1$	2·257	$C_1M_1N_1$	0·898
$D_1M_1N_1$	1·229	$C_1M_1L_1$	3·465	$D_1M_1L_1$	3·603
$L_1M_1S_1$	2·512	$S_1M_1N_1$	0·772	$P_1Q_1V_1$	2·405
$P_1Q_1R_1$	0·147	$V_1Q_1R_1$	0·228	$R_1Q_1L_1$	0·289
$P_1U_1V_1$	1·906	$V_1U_1X_1$	1·460	$A_2X_1F_2$	3·704
$X_1Y_1F_2$	0·499	$X_1Y_1W_1$	0·965	$W_1Y_1S_1$	2·504
$E_2Y_1F_2$	1·124	$X_1Z_1A_2$	0·474	$S_1Z_1A_2$	0·841
$S_1Z_1T_1$	3·938	$T_1Z_1A_2$	0·440	$A_2Z_1F_2$	0·600
$E_2Z_1F_2$	1·302	$X_1Z_1F_2$	2·630	$A_2B_2T_1$	2·795
$S_1B_2T_1$	3·421	$T_1B_2C_2$	4·884	$T_1B_2D_2$	3·635
$J_2I_2N_2$	2·442	$J_2I_2O_2$	4·732	$O_2I_2N_2$	0·596
$G_2R_2H_2$	2·546	$H_2R_2X_2$	0·448	$X_2R_2Q_2$	2·186
$N_2S_2O_2$	2·763	$N_2S_2T_2$	2·784	$O_2S_2T_2$	2·660
$P_2U_2W_2$	4·318	$T_2V_2O_2$	3·789	$O_2V_2P_2$	2·568
$P_2V_2W_2$	2·277	$O_2V_2W_2$	1·894		

4.3 Symbolic statement of condition equations

Angle Closure	*Side Closure*	*Remarks*	*Angle Closure*	*Side Closure*	*Remarks*
	$Y(WVH_1)$	Fixed sides	HGF		
		Contains artificial direction H_1-V	HIF		
				$x(IHFG)$	Pole at intersection of diagonals
	$H_1(YVUG_1)$	Eliminator for artificial direction			
			IGH		
	$X(UWYG_1)$			$B(AFC)$	
$UWYG_1$		Polygon closure		$D(ABF)$	
	$X(ZYW)$			$x(BEGF)$	Pole at intersection of diagonals
	$Y(WXG_1)$	Fixed sides			
	$G_1(YXUE_1O_1)$	Fixed sides		$B(AEF)$	
UH_1G_1			GFB		
	$G_1(O_1E_1UH_1)$	Fixed sides		$D(EFB)$	
	$X(UE_1G_1)$		EFA		
	$E_1(O_1G_1URD_1)$		FGE		
UE_1G_1			EBA		
$O_1G_1E_1$			GBE		
	$F_1(E_1O_1G_1)$		IMG		
	$O_1(G_1E_1D_1N_1T_1)$	Fixed sides	MIN		
RUQ			A_1NM		
	$R(OSE_1UQ)$		SNA_1		
UE_1R			C_1SA_1		
	$T(RE_1U)$		SA_1B_1		
PQL				$A_1(B_1C_1S)$	
QOL			B_1C_1S		
	$P(OQLKJ)$			$C_1(D_1SB_1L_1)$	
QOP			D_1C_1S		
RQP				$D_1(E_1SC_1N_1O_1)$	
	$O(RQP)$		E_1D_1S		
ROQ			$O_1D_1E_1$		
	$O(SRPJ)$			$M_1(D_1C_1L_1)$	
ROS				$M_1(N_1D_1C_1)$	
	$S(NA_1C_1D_1RO)$		$D_1C_1L_1$		
D_1SR			$N_1D_1C_1$		
E_1SR			$D_1L_1S_1N_1$		Polygon closure
	$x(D_1E_1RS)$	Pole at intersection of diagonals		$M_1(D_1L_1S_1N_1)$	
				$N_1(S_1O_1D_1M_1)$	
	$T(E_1D_1R)$		$O_1N_1D_1$		
	$Q(LPO)$			$A_1(J_1B_1C_1)$	
POJ				$A_1(I_1B_1C_1)$	
SOJ			$C_1L_1B_1$		
NOJ				$L_1(I_1B_1C_1)$	
	$x(NSOJ)$	Pole at intersection of diagonals		$L_1(K_1B_1C_1)$	
				$L_1(R_1K_1C_1I_1)$	
HJI				$L_1(K_1R_1W_1S_1M_1C_1)$	
NJH			$R_1W_1L_1$		
	$I(MNJHG)$		$L_1W_1S_1$		
	$I(NJH)$			$S_1(N_1M_1L_1W_1A_2T_1)$	
	$N(JIMA_1S)$			$W_1(L_1S_1A_2X_1V_1R_1)$	
JNI				$R_1(V_1W_1L_1Q_1)$	
JSN					
	$F(BGHC)$				

4.3 *continued*

Angle Closure	*Side Closure*	*Remarks*
$O_1S_1N_1$		
$T_1N_1S_1$		
	$x(T_1O_1N_1S_1)$	Pole at intersection of diagonals
$O_1T_1S_1$		
	$T_1(O_1S_1B_2C_2)$	Fixed sides
$A_2T_1S_1$		
	$x(B_2A_2S_1T_1)$	Pole at intersection of diagonals
	$A_2(F_2B_2T_1Z_1)$	
$A_2B_2D_2T_1$		Polygon closure
	$B_2(D_2T_1A_2F_2G_2)$	
	$B_2(D_2C_2T_1)$	
$A_2S_1W_1$		
	$A_2(X_1F_2B_2S_1W_1)$	
	$W_1(X_1Y_1S_1A_2)$	
	$Y_1(A_2S_1W_1)$	Contains artificial direction A_2-Y_1
	$A_2(F_2B_2S_1Y_1)$	Eliminator for artificial direction
	$F_2(E_2Y_1A_2Z_1)$	Contains artificial direction A_2-Y_1
	$Y_1(A_2S_1W_1)$	Eliminator for artificial direction
	$A_2(F_2B_2S_1Z_1)$	
$A_2B_2S_1$		
$V_1R_1P_1$		
	$V_1(X_1W_1R_1P_1U_1)$	
	$V_1(R_1Q_1P_1)$	
$V_1R_1W_1$		
$X_1W_1V_1$		
	$F_2(A_2Z_1X_1)$	
$A_2W_1X_1$		
$A_2B_2F_2$		
	$F_2(A_2B_2K_2J_2E_2Z_1)$	
$K_2B_2F_2$		
$G_2B_2F_2$		
	$K_2(J_2P_2L_2B_2F_2)$	
$L_2K_2B_2$		
	$L_2(K_2P_2W_2X_2H_2G_2B_2)$	
$L_2G_2B_2$		
	$G_2(L_2H_2D_2B_2)$	
$D_2G_2B_2$		
	$D_2(C_2B_2G_2H_2)$	Fixed sides
$C_2D_2B_2$		
$H_2D_2G_2$		
	$H_2(D_2G_2M_2)$	Fixed sides
	$B_2(F_2K_2L_2G_2)$	

Angle Closure	*Side Closure*	*Remarks*
$H_2L_2G_2$		
$M_2G_2L_2$		
	$H_2(G_2L_2M_2)$	
	$x(L_2R_2H_2G_2)$	Contains artificial direction L_2-R_2 with Pole at intersection of diagonals
	$L_2(X_2R_2H_2)$	Eliminator for artificial direction
$J_2K_2F_2$		
$J_2E_2F_2$		
	$I_2(N_2O_2J_2)$	
$J_2N_2O_2$		
$P_2J_2N_2$		
	$O_2(P_2J_2N_2)$	
$J_2O_2P_2$		
	$P_2(O_2W_2L_2K_2J_2)$	
$J_2K_2P_2$		
$K_2L_2P_2$		
$L_2W_2P_2$		
	$x(W_2Q_2L_2P_2)$	Pole at intersection of diagonals
$W_2M_2L_2$		
$L_2W_2Q_2$		
	$W_2(X_2M_2L_2)$	
	$W_2(X_2Q_2L_2)$	
	$R_2(L_2Q_2X_2)$	Contains artificial direction L_2-R_2
	$L_2(X_2R_2H_2)$	Eliminator for artificial direction
	$x(X_2M_2H_2L_2)$	Fixed sides with Pole at intersection of diagonals
$K_2M_2L_2$		
$H_2M_2L_2$		
$M_2X_2W_2$		
	$X_2(Y_2W_2M_2)$	Fixed sides
$Q_2W_2P_2$		
$W_2X_2Q_2$		
	$U_2(W_2P_2O_2)$	Contains artificial direction O_2-U_2
	$O_2(P_2N_2T_2U_2)$	Eliminator for artificial direction
	$O_2(T_2V_2P_2N_2)$	
$O_2P_2W_2$		
	$V_2(W_2P_2O_2)$	
	$x(T_2O_2N_2S_2)$	Pole at intersection of diagonals
$N_2O_2T_2$		
$X_2Y_2W_2$		

APPENDIX 5

FIGURE 5 (see DIAGRAM 9)

5.1 Mean observed directions for whole figure, $(t-T)$ corrections, mean plane observed directions, adjustment corrections, and plane adjusted directions from the whole-figure adjustment

From	*To*		*Mean Observed Direction*	$(t-T)$	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Abberton Wtr Twr (Q_1)	Stoke by Nayland Ch Twr	(U_1)	00° 00′ 00″·00	− 8″·740	359° 59′ 51″·260	+0″·868	359° 59′ 52″·128
	Manningtree	(S_1)	42 55 53·54	− 5·418	42 55 48·122	+0·053	42 55 48·175
	Walton on the Naze Twr	(R_1)	86 07 13·50	− 2·399	86 07 11·101	−0·232	86 07 10·869
	Rumfields Wtr Twr	(T)	149 52 27·11	+27·635	149 52 54·745	−0·394	149 52 54·351
	Shurland	(U)	186 15 35·42	+24·022	186 15 59·442	−0·117	186 15 59·325
	Hockley Wtr Twr	(X)	219 47 40·29	+13·202	219 47 53·492	−0·948	219 47 52·544
	Maplestead	(V_1)	317 37 08·43	− 7·623	317 37 00·807	+0·771	317 37 01·578
Acre (V_3)	Lincoln Minster	(U_3)	00 00 00·00	+ 6·704	00 00 06·704	−0·390	00 00 06·314
	Cold Harbour	(W_3)	286 20 50·49	+ 4·517	286 20 55·007	+1·431	286 20 56·438
	Boston Tower	(N_3)	308 24 32·88	+15·754	308 24 48·634	−1·040	308 24 47·594
Alport Heights (J_3)	Loath Hill	(K_3)	00 00 00·00	− 0·225	359 59 59·775	+0·499	00 00 00·274
	Charnwood	(G_3)	64 42 55·32	+ 3·479	64 42 58·799	−0·436	64 42 58·363
	Bardon Hill	(H_3)	71 48 18·98	+ 3·472	71 48 22·452	−0·062	71 48 22·390
Bardon Hill (H_3)	Cold Ashby	(H_2)	00 00 00·00	+ 4·834	00 00 04·834	−0·518	00 00 04·316
	Alport Heights	(J_3)	184 49 32·32	− 3·972	184 49 28·348	+0·163	184 49 28·511
	Loath Hill	(K_3)	230 05 26·66	− 5·320	230 05 21·340	−0·255	230 05 21·085
	Belvoir Castle	(L_3)	267 02 11·28	− 3·014	267 02 08·266	+0·109	267 02 08·375
	Charnwood	(G_3)	278 38 09·76	− 0·195	278 38 09·565	+0·502	278 38 10·067
Beachy Head (H)	Ditchling	(F)	00 00 00·00	− 6·584	359 59 53·416	−0·155	359 59 53·261
	Firle Beacon	(G)	10 18 16·66	− 3·994	10 18 12·666	+0·336	10 18 13·002
	Fairlight Down	(I)	113 45 07·44	− 6·847	113 45 00·593	−0·179	113 45 00·414
Belvoir Castle (L_3)	Lincoln Minster	(U_3)	00 00 00·00	− 8·419	359 59 51·581	+1·179	359 59 52·760
	Buckminster Wtr Twr	(I_3)	127 33 03·46	+ 2·291	127 33 05·751	+0·161	127 33 05·912
	Bardon Hill	(H_3)	217 45 43·03	+ 3·637	217 45 46·667	−0·049	217 45 46·618
	Loath Hill	(K_3)	294 42 57·57	− 3·842	294 42 53·728	−1·290	294 42 52·438

5.1 *continued*

From	*To*		*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Benfleet Wtr Twr	Warley Wtr Twr	(Y)	00° 00′ 00″·00	− 2″·103	359° 59′ 57″·897	+0″·974	359° 59′ 58″·871
(W)	Hockley Wtr Twr	(X)	108 05 17·43	− 2·509	108 05 14·921	+0·685	108 05 15·606
	Rumfields Wtr Twr	(T)	184 18 48·75	+ 9·534	184 18 58·284	+0·658	184 18 58·942
	Shurland	(U)	201 53 16·47	+ 7·088	201 53 23·558	−0·860	201 53 22·698
	Lenham Wtr Twr	(R)	234 39 42·51	+15·753	234 39 58·263	−1·207	234 39 57·056
	Wrotham	(Q)	292 52 52·91	+11·672	292 53 04·582	−0·596	292 53 03·986
	Severndroog Castle	(V)	330 05 33·17	+ 4·451	330 05 37·621	+0·347	330 05 37·968
Bignor Beacon	Ditchling	(F)	00 00 00·00	+ 0·015	00 00 00·015	+1·258	00 00 01·273
(E)	Selsey	(B)	119 16 10·59	+ 4·109	119 16 14·699	−0·158	119 16 14·541
	Dunnose	(A)	140 17 20·46	+ 6·961	140 17 27·421	−1·396	140 17 26·025
	Butser	(C)	196 02 46·55	− 1·612	196 02 44·938	+0·028	196 02 44·966
	Leith Hill Tower	(O)	299 55 37·11	− 7·796	299 55 29·314	+0·266	299 55 29·580
Bolnhurst	Fayway	(J_2)	00 00 00·00	− 5·033	359 59 54·967	−0·574	359 59 54·393
(X_1)	Wyton Wtr Twr	(K_2)	55 19 50·96	− 4·030	55 19 46·930	+0·704	55 19 47·634
	Ely Cathedral	(L_2)	64 30 10·77	− 6·332	64 30 04·438	+0·364	64 30 04·802
	Therfield	(N_1)	127 05 09·77	+ 6·565	127 05 16·335	−0·389	127 05 15·946
	Dunstable Down	(E_1)	184 36 40·65	+10·656	184 36 51·306	−0·040	184 36 51·266
	Faxton	(I_2)	299 16 45·02	− 3·860	299 16 41·160	−0·065	299 16 41·095
Boston Tower	Coldharbour	(W_3)	00 00 00·00	−12·255	359 59 47·745	+0·601	359 59 48·346
(N_3)	Dexthorpe	(T_3)	24 48 40·51	− 9·885	24 48 30·625	+1·071	24 48 31·696
	Skegness Wtr Twr	(S_3)	58 07 13·60	− 7·192	58 07 06·408	+0·104	58 07 06·512
	Docking Ch Twr	(O_3)	108 37 38·10	+ 2·689	108 37 40·789	+0·707	108 37 41·496
	Walpole St Peters	(B_3)	156 48 18·40	+ 9·668	156 48 28·068	+0·057	156 48 28·125
	Peterborough Cathedral	(C_3)	205 29 27·28	+14·791	205 29 42·071	−0·160	205 29 41·911
	Collyweston	(D_3)	227 46 25·63	+12·643	227 46 38·273	−1·072	227 46 37·201
	Harrowby	(M_3)	266 49 20·16	+ 2·556	266 49 22·716	−1·189	266 49 21·527
	Lincoln Minster	(U_3)	317 41 42·55	− 8·470	317 41 34·080	+0·310	317 41 34·390
	Acre	(V_3)	347 50 56·01	−16·662	347 50 39·348	−0·428	347 50 38·920
Broadway Tower	Cleeve Hill	(J_1)	00 00 00·00	+ 0·220	00 00 00·220	−0·904	359 59 59·316
(K_1)	Charwelton	(M_1)	198 24 05·71	− 1·246	198 24 04·464	−0·600	198 24 03·864
	Rollright	(L_1)	245 56 00·22	+ 0·272	245 56 00·492	−0·041	245 56 00·451
	Icomb Tower	(I_1)	281 25 16·29	+ 0·483	281 25 16·773	−0·251	281 25 16·522
	Wyck Beacon	(H_1)	285 06 26·78	+ 0·559	285 06 27·339	+1·163	285 06 28·502
	White Horse Hill	(B_1)	294 17 38·31	+ 2·223	294 17 40·533	+0·630	294 17 41·163
Buckminster Wtr Twr	Collyweston	(D_3)	00 00 00·00	+ 4·610	00 00 04·610	+0·043	00 00 04·653
(I_3)	Uppingham	(E_3)	38 18 37·13	+ 5·313	38 18 42·443	+0·225	38 18 42·668
	Tilton Pile	(F_3)	64 55 46·19	+ 3·643	64 55 49·833	+0·330	64 55 50·163
	Charnwood	(G_3)	108 45 08·70	+ 1·563	108 45 10·263	−0·640	108 45 09·623
	Belvoir Castle	(L_3)	181 11 15·00	− 2·347	181 11 12·653	+0·041	181 11 12·694
Bunwell Ch Twr	Hingham Ch Twr	(X_2)	00 00 00·00	− 4·956	359 59 55·044	+0·149	359 59 55·193
(U_2)	Framingham	(V_2)	102 11 26·95	− 5·432	102 11 21·518	−0·144	102 11 21·374
	Topcroft Ch Twr	(T_2)	137 28 49·71	− 0·069	137 28 49·641	+0·286	137 28 49·927
	Metfield	(O_2)	172 17 52·81	+ 7·072	172 17 59·882	+0·376	172 18 00·258
	Crown Corner	(C_2)	198 08 21·70	+12·416	198 08 34·116	+0·040	198 08 34·156
	South Lopham Ch Twr	(N_2)	265 55 45·88	+ 5·851	265 55 51·731	−0·025	265 55 51·706
	Frog Hill	(M_2)	314 12 11·85	+ 0·868	314 12 12·718	−0·683	314 12 12·035

5.1 *continued*

From	*To*		*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Burrough Green Wtr Twr (F_2)	Helion Bumpstead	(W_1)	00° 00′ 00″·00	+ 6″·101	00° 00′ 06″·101	−0″·184	00° 00′ 05″·917
	Therfield	(N_1)	54 40 04·01	+ 7·436	54 40 11·446	−0·022	54 40 11·424
	Wyton Wtr Twr	(K_2)	113 37 23·18	− 6·686	113 37 16·494	−0·190	113 37 16·304
	Ely Cathedral	(L_2)	156 12 21·12	− 9·688	156 12 11·432	−0·034	156 12 11·398
	Chedburgh	(E_2)	269 12 56·52	+ 0·232	269 12 56·752	+0·429	269 12 57·181
Butser (C)	Hindhead	(P)	00 00 00·00	− 3·073	359 59 56·927	−0·516	359 59 56·411
	Ditchling	(F)	47 09 17·32	+ 1·695	47 09 19·015	+1·282	47 09 20·297
	Linch Ball	(D)	53 05 22·95	+ 0·568	53 05 23·518	−0·009	53 05 23·509
	Bignor Beacon	(E)	56 33 04·07	+ 1·460	56 33 05·530	+0·654	56 33 06·184
	Selsey	(B)	98 46 41·88	+ 4·779	98 46 46·659	−0·276	98 46 46·383
	Dunnose	(A)	150 46 11·80	+ 6·792	150 46 18·592	−0·384	150 46 18·208
	Inkpen	(S)	270 41 12·95	− 6·305	270 41 06·645	−0·750	270 41 05·895
Caister Wtr Twr (W_2)	North Walsham Wtr Twr	(R_3)	00 00 00·00	− 9·886	359 59 50·114	−0·451	359 59 49·663
	Church Farm Wtr Twr	(R_2)	227 51 39·67	+12·032	227 51 51·702	+0·655	227 51 52·357
	Framingham	(V_2)	303 04 36·82	+ 6·483	303 04 43·303	−1·043	303 04 42·260
	Piggs Grave	(Q_3)	347 54 29·43	−11·808	347 54 17·622	+0·838	347 54 18·460
Charnwood (G_3)	Tilton Pile	(F_3)	00 00 00·00	+ 1·343	00 00 01·343	−0·380	00 00 00·963
	Cold Ashby	(H_2)	51 31 27·05	+ 5·367	51 31 32·417	+1·304	51 31 33·721
	Bardon Hill	(H_3)	142 52 21·65	+ 0·202	142 52 21·852	−0·896	142 52 20·956
	Alport Heights	(J_3)	221 58 19·78	− 4·112	221 58 15·668	−0·295	221 58 15·373
	Loath Hill	(K_3)	268 51 22·53	− 5·433	268 51 17·097	+0·365	268 51 17·462
	Buckminster Wtr Twr	(I_3)	328 37 36·89	− 1·307	328 37 35·583	−0·097	328 37 35·486
Charwelton (M_1)	Broadway Tower	(K_1)	00 00 00·00	+ 1·918	00 00 01·918	+1·031	00 00 02·949
	Cold Ashby	(H_2)	149 03 28·58	− 2·888	149 03 25·692	+0·975	149 03 26·667
	Dunstable Down	(E_1)	243 00 52·35	+ 6·311	243 00 58·661	−0·829	243 00 57·832
	Muswell Hill	(G_1)	279 05 20·14	+ 5·752	279 05 25·892	−1·524	279 05 24·368
	Rollright	(L_1)	338 15 11·33	+ 2·896	338 15 14·226	+0·349	338 15 14·575
Chedburgh (E_2)	Maplestead	(V_1)	00 00 00·00	+ 9·760	00 00 09·760	−0·191	00 00 09·569
	Helion Bumpstead	(W_1)	60 07 45·14	+ 6·249	60 07 51·389	−0·426	60 07 50·963
	Burrough Green Wtr Twr	(F_2)	103 26 56·60	− 0·239	103 26 56·361	−0·149	103 26 56·212
	Ely Cathedral	(L_2)	146 10 49·79	−10·547	146 10 39·243	+0·526	146 10 39·769
	Puttocks Hill	(G_2)	230 28 36·60	− 6·343	230 28 30·257	−0·433	230 28 29·824
	South Lopham Ch Twr	(N_2)	235 44 12·69	−12·277	235 44 00·413	−0·132	235 44 00·281
	Woolpit	(D_2)	264 21 37·65	− 3·027	264 21 34·623	+1·291	264 21 35·914
	Nedging Tye	(Y_1)	296 13 14·01	+ 2·902	296 13 16·912	+0·292	296 13 17·204
	Stoke by Nayland Ch Twr	(U_1)	325 58 10·05	+ 9·195	325 58 19·245	−0·780	325 58 18·465

5.1 *continued*

From	*To*	*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Chipping Barnet Ch Twr (Z)	Epping Wtr Twr (D_1)	00° 00′ 00″·00	− 2″·107	359° 59′ 57″·893	−0″·187	359° 59′ 57″·706
	Warley Wtr Twr (Y)	23 59 45·55	+ 1·702	23 59 47·252	+0·046	23 59 47·298
	Severndroog Castle (V)	63 14 43·91	+ 6·713	63 14 50·623	−0·403	63 14 50·220
	Leith Hill Tower (O)	117 06 02·85	+16·342	117 06 19·192	+0·309	117 06 19·501
	Hindhead (P)	135 34 28·34	+17·340	135 34 45·680	+0·445	135 34 46·125
	Shirburn Hill (A_1)	194 31 38·34	+ 0·332	194 31 38·672	−0·526	194 31 38·146
	Coombe Hill (F_1)	216 45 19·26	− 3·865	216 45 15·395	−0·192	216 45 15·203
	Dunstable Down (E_1)	240 00 23·27	− 6·785	240 00 16·485	+0·529	240 00 17·014
	Sibleys Wtr Twr (O_1)	329 28 52·23	−11·485	329 28 40·745	−0·020	329 28 40·725
Church Farm Wtr Twr (R_2)	Southwold Ch Twr (P_2)	00 00 00·00	+11·508	00 00 11·508	+0·140	00 00 11·648
	Metfield (O_2)	47 25 10·99	+ 8·953	47 25 19·943	+0·438	47 25 20·381
	Ilketshall St Andrews Ch Twr (S_2)	55 48 50·99	+ 4·479	55 48 55·469	−0·797	55 48 54·672
	Framingham (V_2)	96 14 11·40	− 5·144	96 14 06·256	+0·307	96 14 06·563
	Caister Wtr Twr (W_2)	161 41 54·75	−12·074	161 41 42·676	−0·598	161 41 42·078
	Kessingland Ch Twr (Q_2)	358 29 02·09	+ 5·194	358 29 07·284	+0·510	358 29 07·794
Cleeve Hill (J_1)	Broadway Tower (K_1)	00 00 00·00	− 0·106	359 59 59·894	+0·478	00 00 00·372
	Rollright (L_1)	34 17 25·69	− 0·121	34 17 25·569	+0·083	34 17 25·652
	Icomb Tower (I_1)	49 39 10·93	+ 0·028	49 39 10·958	−0·297	49 39 10·661
	Wyck Beacon (H_1)	55 22 47·79	+ 0·063	55 22 47·853	−0·393	55 22 47·460
	White Horse Hill (B_1)	96 23 06·46	+ 0·951	96 23 07·411	+0·154	96 23 07·565
	Liddington Castle (C_1)	109 28 47·15	+ 0·771	109 28 47·921	−0·026	109 28 47·895
Cold Ashby (H_2)	Charwelton (M_1)	14 11 30·52	+ 3·114	14 11 33·634	−0·368	14 11 33·266
	Bardon Hill (H_3)	134 53 17·36	− 5·404	134 53 11·956	−0·300	134 53 11·656
	Charnwood (G_3)	142 10 35·73	− 5·802	142 10 29·928	+0·244	142 10 30·172
	Tilton Pile (F_3)	184 24 13·04	− 5·089	184 24 07·951	+0·039	184 24 07·990
	Uppingham (E_3)	204 28 48·55	− 4·029	204 28 44·521	+0·178	204 28 44·699
	Faxton (I_2)	255 46 08·69	+ 0·208	255 46 08·898	+0·207	255 46 09·105
Cold Harbour (W_3)	Acre (V_3)	00 00 00·00	− 4·704	359 59 55·296	+0·064	359 59 55·360
	Mablethorpe Wtr Twr (X_3)	126 29 42·20	− 1·006	126 29 41·194	+0·388	126 29 41·582
	Dexthorpe (T_3)	163 44 14·87	+ 2·725	163 44 17·595	+0·139	163 44 17·734
	Boston Tower (N_3)	214 12 43·33	+12·064	214 12 55·394	+0·548	214 12 55·942
	Harrowby (M_3)	258 38 08·00	+13·345	258 38 21·345	−0·360	258 38 20·985
	Lincoln Minster (U_3)	295 25 20·91	+ 2·788	295 25 23·698	−0·780	295 25 22·918
Collyweston (D_3)	Tilton Pile (F_3)	00 00 00·00	− 0·632	359 59 59·368	−0·508	359 59 58·860
	Buckminster Wtr Twr (I_3)	52 18 10·47	− 4·809	52 18 05·661	−0·016	52 18 05·645
	Harrowby (M_3)	73 52 33·85	− 8·107	73 52 25·743	+0·840	73 52 26·583
	Boston Tower (N_3)	121 52 26·26	−11·517	121 52 14·743	+0·758	121 52 15·501
	Walpole St Peters (B_3)	158 23 51·94	− 3·971	158 23 47·969	−0·350	158 23 47·619
	Peterborough Cathedral (C_3)	186 37 48·20	+ 1·229	186 37 49·429	+0·283	186 37 49·712
	Ely Cathedral (L_2)	196 23 59·38	+ 6·856	196 24 06·236	−0·160	196 24 06·076
	Wyton Wtr Twr (K_2)	219 41 28·85	+ 8·146	219 41 36·996	−0·331	219 41 36·665
	Fayway (J_2)	248 25 45·56	+ 6·401	248 25 51·961	+0·204	248 25 52·165
	Uppingham (E_3)	337 18 32·12	+ 1·039	337 18 33·159	−0·721	337 18 32·438

5.1 *continued*

From	*To*		*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Coombe Hill (F$_1$)	Dunstable Down	(E$_1$)	00° 00′ 00″·00	− 2″·220	359° 59′ 57″·780	+0″·761	359° 59′ 58″·541
	Chipping Barnet Ch Twr	(Z)	59 27 38·02	+ 3·460	59 27 41·480	+1·050	59 27 42·530
	Hindhead	(P)	127 51 50·95	+16·777	127 52 07·727	−2·545	127 52 05·182
	Shirburn Hill	(A$_1$)	177 09 05·66	+ 3·122	177 09 08·782	+0·320	177 09 09·102
	White Horse Hill	(B$_1$)	196 45 00·60	+ 4·154	196 45 04·754	+0·639	196 45 05·393
	Muswell Hill	(G$_1$)	230 34 15·07	− 1·084	230 34 13·986	−0·225	230 34 13·761
Crimplesham (A$_3$)	Frog Hill	(M$_2$)	00 00 00·00	+ 5·753	00 00 05·753	−0·155	00 00 05·598
	Ely Cathedral	(L$_2$)	83 41 50·20	+ 9·801	83 42 00·001	−0·459	83 41 59·542
	Peterborough Cathedral	(C$_3$)	142 25 25·38	+ 2·133	142 25 27·513	+0·044	142 25 27·557
	Walpole St Peters	(B$_3$)	189 38 39·83	− 5·005	189 38 34·825	+0·440	189 38 35·265
	Massingham	(P$_3$)	282 11 06·42	− 6·823	282 10 59·597	−0·213	282 10 59·384
	Swaffham	(Z$_2$)	314 50 36·18	− 2·161	314 50 34·019	+0·341	314 50 34·360
Crowborough (M)	Ditchling	(F)	00 00 00·00	+ 6·511	00 00 06·511	−0·459	00 00 06·052
	Leith Hill Tower	(O)	62 56 07·13	− 4·361	62 56 02·769	−1·690	62 56 01·079
	East Grinstead Ch Twr	(N)	76 37 06·59	− 2·703	76 37 03·887	+0·578	76 37 04·465
	Wrotham	(Q)	150 05 21·86	−11·404	150 05 10·456	+1·000	150 05 11·456
	Brenchley Air Beacon	(L)	190 10 04·78	− 4·559	190 10 00·221	−0·376	190 09 59·845
	Lenham Wtr Twr	(R)	196 26 18·33	− 9·232	196 26 09·098	+0·258	196 26 09·356
	Bethersden Air Beacon	(K)	211 19 51·60	− 4·111	211 19 47·489	+1·154	211 19 48·643
	Fairlight Down	(I)	254 05 51·67	+ 7·744	254 05 59·414	−0·876	254 05 58·538
	Firle Beacon	(G)	320 30 27·75	+ 9·461	320 30 37·211	+0·412	320 30 37·623
Crown Corner (C$_2$)	South Lopham Ch Twr	(N$_2$)	00 00 00·00	− 6·408	359 59 53·592	−0·647	359 59 52·945
	Bunwell Ch Twr	(U$_2$)	31 53 37·66	−12·664	31 53 24·996	−0·612	31 53 24·384
	Topcroft Ch Twr	(T$_2$)	64 25 00·74	−13·003	64 24 47·737	+0·440	64 24 48·177
	Metfield	(O$_2$)	91 57 51·83	− 5·669	91 57 46·161	+1·088	91 57 47·249
	Salle	(B$_2$)	172 27 38·01	+ 2·270	172 27 40·280	−0·119	172 27 40·161
	Felixstowe Wtr Twr	(T$_1$)	236 21 10·24	+19·393	236 21 29·633	−0·556	236 21 29·077
	Swilland	(Z$_1$)	265 42 55·63	+ 9·244	265 43 04·874	−0·324	265 43 04·550
	Nedging Tye	(Y$_1$)	290 44 57·12	+11·280	290 45 08·400	+0·318	290 45 08·718
	Woolpit	(D$_2$)	314 48 29·06	+ 4·329	314 48 33·389	+0·411	314 48 33·800
Dexthorpe (T$_3$)	Mablethorpe Wtr Twr	(X$_3$)	00 00 00·00	− 4·064	359 59 55·936	−0·480	359 59 55·456
	Skegness Wtr Twr	(S$_3$)	78 03 44·14	+ 3·177	78 03 47·317	+0·553	78 03 47·870
	Boston Tower	(N$_3$)	153 55 09·29	+10·080	153 55 19·370	−0·759	153 55 18·611
	Cold Harbour	(W$_3$)	258 37 59·19	− 2·823	258 37 56·367	+0·686	258 37 57·053

5.1 *continued*

From	*To*		*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Ditchling (F)	Beachy Head	(H)	00° 00′ 00″00	+ 6″207	00° 00′ 06″207	−0″946	00° 00′ 05″261
	Dunnose	(A)	122 57 40·79	+ 8·983	122 57 49·773	−0·002	122 57 49·771
	Bignor Beacon	(E)	146 21 35·86	− 0·016	146 21 35·844	−0·415	146 21 35·429
	Linch Ball	(D)	151 22 07·85	− 1·278	151 22 06·572	−1·414	151 22 05·158
	Butser	(C)	153 00 36·91	− 2·072	153 00 34·838	−1·604	153 00 33·234
	Leith Hill Tower	(O)	203 43 37·20	− 9·669	203 43 27·531	+0·772	203 43 28·303
	East Grinstead Ch Twr	(N)	250 49 09·18	− 8·552	250 49 00·628	+0·295	250 49 00·923
	Crowborough	(M)	281 46 21·51	− 6·242	281 46 15·268	+0·453	281 46 15·721
	Fairlight Down	(I)	327 33 05·79	+ 0·434	327 33 06·224	+1·693	327 33 07·917
	Firle Beacon	(G)	351 09 36·40	+ 2·503	351 09 38·903	+1·168	351 09 40·071
Docking Ch Twr (O_3)	Piggs Grave	(Q_3)	00 00 00·00	+ 1·864	00 00 01·864	+0·043	00 00 01·907
	Massingham	(P_3)	71 20 05·35	+ 7·568	71 20 12·918	−0·026	71 20 12·892
	Walpole St Peters	(B_3)	133 37 55·71	+ 8·647	133 38 04·357	−0·204	133 38 04·153
	Boston Tower	(N_3)	180 41 40·30	− 2·955	180 41 37·345	−0·452	180 41 36·893
	Skegness Wtr Twr	(S_3)	224 17 45·96	−11·787	224 17 34·173	+0·638	224 17 34·811
Dunmow (P_1)	Sibleys Wtr Twr	(O_1)	00 00 00·00	− 3·139	359 59 56·861	+0·603	359 59 57·464
	Helion Bumpstead	(W_1)	40 38 25·51	− 8·013	40 38 17·497	−0·129	40 38 17·368
	Maplestead	(V_1)	103 57 21·61	− 5·249	103 57 16·361	+0·377	103 57 16·738
	Hockley Wtr Twr	(X)	197 29 55·20	+13·039	197 30 08·239	−0·425	197 30 07·814
	Warley Wtr Twr	(Y)	238 20 32·73	+12·726	238 20 45·456	+0·213	238 20 45·669
	Epping Wtr Twr	(D_1)	270 35 12·01	+ 7·881	270 35 19·891	−0·641	270 35 19·250
Dunnose (A)	Butser	(C)	00 00 00·00	− 6·287	359 59 53·713	−0·876	359 59 52·837
	Linch Ball	(D)	16 37 21·92	− 6·238	16 37 15·682	−0·573	16 37 15·109
	Bignor Beacon	(E)	30 01 27·17	− 5·853	30 01 21·317	+0·555	30 01 21·872
	Selsey	(B)	42 12 54·13	− 2·640	42 12 51·490	−0·010	42 12 51·480
	Ditchling	(F)	46 20 17·42	− 6·860	46 20 10·560	+0·902	46 20 11·462
Dunstable Down (E_1)	Muswell Hill	(G_1)	00 00 00·00	+ 0·926	00 00 00·926	+0·032	00 00 00·958
	Charwelton	(M_1)	42 56 38·69	− 7·847	42 56 30·843	−1·967	42 56 28·876
	Faxton	(I_2)	76 29 09·03	−13·352	76 28 55·678	+0·967	76 28 56·645
	Bolnhurst	(X_1)	103 27 55·04	−10·486	103 27 44·554	+0·823	103 27 45·377
	Therfield	(N_1)	157 30 52·42	− 5·042	157 30 47·378	+1·325	157 30 48·703
	Epping Wtr Twr	(D_1)	206 22 19·33	+ 4·901	206 22 24·231	−0·564	206 22 23·667
	Chipping Barnet Ch Twr	(Z)	230 32 06·86	+ 6·326	230 32 13·186	−0·009	230 32 13·177
	Coombe Hill	(F_1)	327 49 25·67	+ 2·314	327 49 27·984	−0·608	327 49 27·376
East Grinstead Ch Twr (N)	Ditchling	(F)	00 00 00·00	+ 8·688	00 00 08·688	−0·817	00 00 07·871
	Leith Hill Tower	(O)	86 49 19·52	− 1·714	86 49 17·806	+0·617	86 49 18·423
	Crowborough	(M)	287 34 18·25	+ 2·633	287 34 20·883	+0·200	287 34 21·083

5.1 *continued*

From	*To*	*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Ely Cathedral (L_2)	Wyton Wtr Twr (K_2)	00° 00′ 00″·00	+ 2″·379	00° 00′ 02″·379	−0″·209	00° 00′ 02″·170
	Fayway (J_2)	11 51 01·43	+ 0·625	11 51 02·055	−0·802	11 51 01·253
	Collyweston (D_3)	37 01 17·42	− 7·901	37 01 09·519	−0·318	37 01 09·201
	Peterborough Cathedral (C_3)	41 57 30·31	− 6·632	41 57 23·678	+0·033	41 57 23·711
	Walpole St Peters (B_3)	97 58 09·64	−14·066	97 57 55·574	+1·225	97 57 56·799
	Crimplesham (A_3)	128 13 26·86	− 9·583	128 13 17·277	+0·988	128 13 18·265
	Swaffham (Z_2)	149 52 21·57	−12·039	149 52 09·531	+0·234	149 52 09·765
	Frog Hill (M_2)	175 56 26·08	− 4·523	175 56 21·557	−0·730	175 56 20·827
	Puttocks Hill (G_2)	210 38 47·98	+ 4·496	210 38 52·476	−0·316	210 38 52·160
	Chedburgh (E_2)	238 44 30·13	+10·038	238 44 40·168	−0·235	238 44 39·933
	Burrough Green Wtr Twr (F_2)	263 00 01·22	+ 9·503	263 00 10·723	−0·130	263 00 10·593
	Therfield (N_1)	309 51 59·44	+16·036	309 52 15·476	+0·094	309 52 15·570
	Bolnhurst (X_1)	350 57 08·94	+ 7·166	350 57 16·106	+0·166	350 57 16·272
Epping Wtr Twr (D_1)	Warley Wtr Twr (Y)	00 00 00·00	+ 4·295	00 00 04·295	−0·055	00 00 04·240
	Wrotham (Q)	31 21 56·18	+16·350	31 22 12·530	+0·201	31 22 12·731
	Severndroog Castle (V)	55 21 13·62	+ 9·796	55 21 23·416	−0·516	55 21 22·900
	Chipping Barnet Ch Twr (Z)	121 56 21·94	+ 2·225	121 56 24·165	+0·460	121 56 24·625
	Dunstable Down (E_1)	157 47 00·78	− 5·546	157 46 55·234	−0·809	157 46 54·425
	Therfield (N_1)	206 23 58·71	−12·422	206 23 46·288	+0·137	206 23 46·425
	Sibleys Wtr Twr (O_1)	247 33 34·14	−10·346	247 33 23·794	−0·015	247 33 23·779
	Dunmow (P_1)	270 40 55·56	− 7·581	270 40 47·979	+0·598	270 40 48·577
Fairlight Down (I)	Beachy Head (H)	00 00 00·00	+ 7·192	00 00 07·192	−0·016	00 00 07·176
	Firle Beacon (G)	23 00 15·21	+ 2·622	23 00 17·832	−1·068	23 00 16·764
	Ditchling (F)	33 48 04·07	− 0·483	33 48 03·587	−0·908	33 48 02·679
	Crowborough (M)	62 07 11·31	− 8·272	62 07 03·038	−0·069	62 07 02·969
	Brenchley Air Beacon (L)	94 09 05·09	−13·742	94 08 51·348	+1·193	94 08 52·541
	Wrotham (Q)	95 02 33·29	−21·446	95 02 11·844	+0·969	95 02 12·813
	Lenham Wtr Twr (R)	133 54 31·37	−19·401	133 54 11·969	−0·718	133 54 11·251
	Bethersden Air Beacon (K)	139 34 07·60	−13·602	139 33 53·998	+0·506	139 33 54·504
	Paddlesworth (J)	174 47 05·81	−13·727	174 46 52·083	+0·110	174 46 52·193
Faxton (I_2)	Cold Ashby (H_2)	42 32 31·07	− 0·224	42 32 30·846	−0·155	42 32 30·691
	Tilton Pile (F_3)	121 11 20·56	− 6·126	121 11 14·434	+0·731	121 11 15·165
	Uppingham (E_3)	139 18 31·28	− 4·882	139 18 26·398	−0·937	139 18 25·461
	Fayway (J_2)	211 39 08·26	− 0·697	211 39 07·563	+0·715	211 39 08·278
	Bolnhurst (X_1)	250 06 28·81	+ 3·526	250 06 32·336	+0·011	250 06 32·347
	Dunstable Down (E_1)	288 27 41·76	+12·392	288 27 54·152	−0·366	288 27 53·786

5.1 *continued*

From	*To*		*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Fayway (J$_2$)	Uppingham	(E$_3$)	00° 00′ 00″00	− 5″140	359° 59′ 54″860	−0″832	359° 59′ 54″028
	Collyweston	(D$_3$)	31 37 59·07	− 6·539	31 37 52·531	+0·526	31 37 53·057
	Ely Cathedral	(L$_2$)	134 25 59·71	− 0·553	134 25 59·157	−0·139	134 25 59·018
	Wyton Wtr Twr	(K$_2$)	148 52 24·70	+ 1·349	148 52 26·049	+0·166	148 52 26·215
	Therfield	(N$_1$)	193 51 45·14	+12·072	193 51 57·212	+0·533	193 51 57·745
	Bolnhurst	(X$_1$)	229 01 58·26	+ 5·045	229 02 03·305	+0·322	229 02 03·627
	Faxton	(I$_2$)	309 51 26·07	+ 0·765	309 51 26·835	−0·574	309 51 26·261
Felixstowe Wtr Twr (T$_1$)	Walton on the Naze Twr	(R$_1$)	00 00 00·00	+ 7·419	00 00 07·419	−0·064	00 00 07·355
	Manningtree	(S$_1$)	61 39 50·19	+ 3·847	61 39 54·037	−0·219	61 39 53·818
	Stoke by Nayland Ch Twr	(U$_1$)	80 01 24·33	+ 0·061	80 01 24·391	−0·008	80 01 24·383
	Nedging Tye	(Y$_1$)	106 44 37·55	− 7·423	106 44 30·127	+0·230	106 44 30·357
	Swilland	(Z$_1$)	139 16 18·42	− 9·945	139 16 08·475	−0·029	139 16 08·446
	Crown Corner	(C$_2$)	164 51 21·91	−19·485	164 51 02·425	−0·033	164 51 02·392
	Salle	(B$_2$)	183 43 12·27	−17·490	183 42 54·780	−0·072	183 42 54·708
	Orford Castle	(A$_2$)	214 42 16·19	− 7·970	214 42 08·220	+0·195	214 42 08·415
Firle Beacon (G)	Beachy Head	(H)	00 00 00·00	+ 3·904	00 00 03·904	+0·150	00 00 04·054
	Ditchling	(F)	160 51 22·27	− 2·596	160 51 19·674	−0·551	160 51 19·123
	Leith Hill Tower	(O)	183 04 20·17	−12·931	183 04 07·239	−0·553	183 04 06·686
	Crowborough	(M)	231 58 35·45	− 9·406	231 58 26·044	+0·300	231 58 26·344
	Fairlight Down	(I)	306 27 02·84	− 2·441	306 27 00·399	+0·654	306 27 01·053
Framingham (V$_2$)	Hingham Ch Twr	(X$_2$)	00 00 00·00	+ 0·287	00 00 00·287	+0·172	00 00 00·459
	Piggs Grave	(Q$_3$)	53 23 36·21	−16·791	53 23 19·419	+0·212	53 23 19·631
	North Walsham Wtr Twr	(R$_3$)	94 42 22·58	−15·255	94 42 07·325	+0·038	94 42 07·363
	Caister Wtr Twr	(W$_2$)	158 32 08·42	− 6·259	158 32 02·161	+0·973	158 32 03·134
	Church Farm Wtr Twr	(R$_2$)	197 51 33·35	+ 4·949	197 51 38·299	−0·582	197 51 37·717
	Ilketshall St Andrews Ch Twr	(S$_2$)	234 06 09·94	+ 8·982	234 06 18·922	−0·579	234 06 18·343
	Metfield	(O$_2$)	258 45 34·03	+13·069	258 45 47·099	+0·351	258 45 47·450
	Topcroft Ch Twr	(T$_2$)	269 15 23·93	+ 5·591	269 15 29·521	+0·272	269 15 29·793
	Bunwell Ch Twr	(U$_2$)	325 25 54·32	+ 5·546	325 25 59·866	−0·856	325 25 59·010
Fransham (Y$_2$)	Piggs Grave	(Q$_3$)	26 47 28·56	−11·205	26 47 17·355	−0·469	26 47 16·886
	Hingham Ch Twr	(X$_2$)	133 16 21·22	+ 4·111	133 16 25·331	−0·420	133 16 24·911
	Frog Hill	(M$_2$)	197 56 46·31	+ 9·339	197 56 55·649	+0·066	197 56 55·715
	Swaffham	(Z$_2$)	264 52 26·98	+ 0·560	264 52 27·540	+0·925	264 52 28·465
	Massingham	(P$_3$)	309 19 51·33	− 4·634	309 19 46·696	−0·100	309 19 46·596

5.1 *continued*

From	*To*		*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Frog Hill (M$_2$)	Swaffham	(Z$_2$)	00° 00′ 00″·00	− 8″·563	359° 59′ 51″·437	+0″·024	359° 59′ 51″·461
	Fransham	(Y$_2$)	25 36 53·24	− 9·253	25 36 43·987	−0·187	25 36 43·800
	Hingham Ch Twr	(X$_2$)	63 49 52·63	− 5·373	63 49 47·257	+0·734	63 49 47·991
	Bunwell Ch Twr	(U$_2$)	96 28 02·06	− 0·832	96 28 01·228	−0·879	96 28 00·349
	South Lopham Ch Twr	(N$_2$)	129 22 23·92	+ 4·559	129 22 28·479	+0·303	129 22 28·782
	Woolpit	(D$_2$)	166 53 56·34	+13·959	166 54 10·299	+0·090	166 54 10·389
	Puttocks Hill	(G$_2$)	183 18 26·35	+10·247	183 18 36·597	+0·483	183 18 37·080
	Ely Cathedral	(L$_2$)	262 11 20·62	+ 4·826	262 11 25·446	−0·274	262 11 25·172
	Crimplesham	(A$_3$)	310 46 34·91	− 6·002	310 46 28·908	−0·243	310 46 28·665
	Massingham	(P$_3$)	355 22 59·37	−13·587	355 22 45·783	−0·050	355 22 45·733
Harrowby (M$_3$)	Loath Hill	(K$_3$)	00 00 00·00	− 3·831	359 59 56·169	−0·742	359 59 55·427
	Lincoln Minster	(U$_3$)	65 03 39·44	− 8·735	65 03 30·705	+1·934	65 03 32·639
	Cold Harbour	(W$_3$)	95 09 14·35	−12·120	95 09 02·230	+0·147	95 09 02·377
	Boston Tower	(N$_3$)	137 33 13·38	− 2·286	137 33 11·094	−0·579	137 33 10·515
	Collyweston	(D$_3$)	230 30 29·61	+ 7·956	230 30 37·566	−0·295	230 30 37·271
	Tilton Pile	(F$_3$)	270 56 05·33	+ 6·707	270 56 12·037	−0·335	270 56 11·702
	Belvoir Castle	(L$_3$)	320 47 45·84	+ 0·470	320 47 46·310	−0·131	320 47 46·179
Helion Bumpstead (W$_1$)	Sibleys Wtr Twr	(O$_1$)	00 00 00·00	+ 4·728	00 00 04·728	+0·205	00 00 04·933
	Therfield	(N$_1$)	54 09 30·55	+ 1·695	54 09 32·245	−0·479	54 09 31·766
	Burrough Green Wtr Twr	(F$_2$)	155 27 24·71	− 6·092	155 27 18·618	+0·038	155 27 18·656
	Chedburgh	(E$_2$)	201 21 10·34	− 6·055	201 21 04·285	+0·387	201 21 04·672
	Maplestead	(V$_1$)	261 52 14·57	+ 3·068	261 52 17·638	+0·572	261 52 18·210
	Dunmow	(P$_1$)	325 34 39·59	+ 7·973	325 34 47·563	−0·723	325 34 46·840
Hindhead (P)	Linch Ball	(D)	00 00 00·00	+ 4·146	00 00 04·146	−0·197	00 00 03·949
	Butser	(C)	33 57 52·80	+ 3·314	33 57 56·114	+0·163	33 57 56·277
	Inkpen	(S)	100 25 38·04	− 4·720	100 25 33·320	+0·644	100 25 33·964
	Shirburn Hill	(A$_1$)	147 50 06·43	−12·645	147 49 53·785	−0·238	147 49 53·547
	Coombe Hill	(F$_1$)	163 41 10·35	−16·837	163 40 53·513	−1·190	163 40 52·323
	Chipping Barnet Ch Twr	(Z)	194 06 15·78	−15·575	194 06 00·205	+0·387	194 06 00·592
	Leith Hill Tower	(O)	237 33 12·87	−1·800	237 33 11·070	+0·431	237 33 11·501
Hingham Ch Twr (X$_2$)	South Lopham Ch Twr	(N$_2$)	00 00 00·00	+10·463	00 00 10·463	−0·742	00 00 09·721
	Puttocks Hill	(G$_2$)	25 49 04·88	+16·327	25 49 21·207	−0·787	25 49 20·420
	Frog Hill	(M$_2$)	58 38 13·17	+ 5·513	58 38 18·683	−0·187	58 38 18·496
	Swaffham	(Z$_2$)	116 24 22·38	− 3·540	116 24 18·840	−0·578	116 24 18·262
	Fransham	(Y$_2$)	135 44 46·94	− 4·179	135 44 42·761	+0·740	135 44 43·501
	Piggs Grave	(Q$_3$)	185 59 41·60	−15·823	185 59 25·777	+0·732	185 59 26·509
	Framingham	(V$_2$)	273 49 41·46	− 0·277	273 49 41·183	+0·457	273 49 41·640
	Bunwell Ch Twr	(U$_2$)	317 04 08·77	+ 4·874	317 04 13·644	+0·366	317 04 14·010

5.1 *continued*

From	*To*		*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Hockley Wtr Twr (X)	Warley Wtr Twr	(Y)	00° 00′ 00″00	+ 0″301	00° 00′ 00″301	−0″541	359° 59′ 59″760
	Dunmow	(P_1)	61 27 37·36	−13·487	61 27 23·873	−0·903	61 27 22·970
	Maplestead	(V_1)	92 27 30·67	−19·557	92 27 11·113	+1·604	92 27 12·717
	Abberton Wtr Twr	(Q_1)	125 30 13·77	−12·796	125 30 00·974	+1·068	125 30 02·042
	Rumfields Wtr Twr	(T)	205 30 31·23	+12·440	205 30 43·670	−0·007	205 30 43·663
	Shurland	(U)	230 52 30·67	+ 9·797	230 52 40·467	+0·734	230 52 41·201
	Lenham Wtr Twr	(R)	257 13 58·42	+18·534	257 14 16·954	−1·337	257 14 15·617
	Benfleet Wtr Twr	(W)	303 19 47·07	+ 2·525	303 19 49·595	−0·618	303 19 48·977
Ilketshall St Andrews Ch Twr (S_2)	Kessingland Ch Twr	(Q_2)	00 00 00·00	+ 0·599	00 00 00·599	−0·351	00 00 00·248
	Southwold Ch Twr	(P_2)	36 28 07·02	+ 6·657	36 28 13·677	−0·714	36 28 12·963
	Metfield	(O_2)	128 53 15·79	+ 4·317	128 53 20·107	−0·925	128 53 19·182
	Topcroft Ch Twr	(T_2)	202 46 38·45	− 3·355	202 46 35·095	+0·842	202 46 35·937
	Framingham	(V_2)	229 07 07·59	− 9·134	229 06 58·456	+0·214	229 06 58·670
	Church Farm Wtr Twr	(R_2)	332 27 09·60	− 4·383	332 27 05·217	+0·935	332 27 06·152
Inkpen (S)	White Horse Hill	(B_1)	00 00 00·00	− 2·190	359 59 57·810	−1·025	359 59 56·785
	Shirburn Hill	(A_1)	62 30 27·37	− 4·175	62 30 23·195	+0·278	62 30 23·473
	Hindhead	(P)	132 23 12·52	+ 3·577	132 23 16·097	+2·867	132 23 18·964
	Butser	(C)	156 36 46·18	+ 5·107	156 36 51·287	−0·525	156 36 50·762
	Liddington Castle	(C_1)	334 16 50·77	− 1·465	334 16 49·305	−1·596	334 16 47·709
Kessingland Ch Twr (Q_2)	Southwold Ch Twr	(P_2)	00 00 00·00	+ 6·307	00 00 06·307	+0·540	00 00 06·847
	Metfield	(O_2)	62 10 06·73	+ 3·892	62 10 10·622	−0·424	62 10 10·198
	Ilketshall St Andrews Ch Twr	(S_2)	82 07 50·08	− 0·611	82 07 49·469	+0·496	82 07 49·965
	Church Farm Wtr Twr	(R_2)	177 15 14·79	− 5·186	177 15 09·604	−0·613	177 15 08·991
Leith Hill Tower (O)	Linch Ball	(D)	00 00 00·00	+ 6·812	00 00 06·812	−0·352	00 00 06·460
	Hindhead	(P)	24 40 12·83	+ 1·947	24 40 14·777	+0·214	24 40 14·991
	Shirburn Hill	(A_1)	92 53 24·97	−13·207	92 53 11·763	+0·549	92 53 12·312
	Chipping Barnet Ch Twr	(Z)	142 44 51·81	−15·867	142 44 35·943	+1·515	142 44 37·458
	Severndroog Castle	(V)	173 01 02·97	−10·356	173 00 52·614	−1·414	173 00 51·200
	Wrotham	(Q)	201 08 34·34	− 5·509	201 08 28·831	+0·218	201 08 29·049
	East Grinstead Ch Twr	(N)	232 51 59·48	+ 1·602	232 52 01·082	+0·811	232 52 01·893
	Crowborough	(M)	239 55 56·96	+ 3·971	239 56 00·931	+0·237	239 56 01·168
	Firle Beacon	(G)	268 36 06·95	+11·842	268 36 18·792	−0·739	268 36 18·053
	Ditchling	(F)	278 57 09·33	+ 9·180	278 57 18·510	+0·211	278 57 18·721
	Bignor Beacon	(E)	341 30 47·17	+ 8·235	341 30 55·405	−1·250	341 30 54·155

5.1 *continued*

From	*To*		*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Lenham Wtr Twr (R)	Shurland	(U)	00° 00′ 00″00	− 9″313	359° 59′ 50″687	+1″326	359° 59′ 52″013
	Rumfields Wtr Twr	(T)	49 47 29·36	− 7·852	49 47 21·508	−1·143	49 47 20·365
	Bethersden Air Beacon	(K)	155 29 58·94	+ 5·988	155 30 04·928	+0·289	155 30 05·217
	Fairlight Down	(I)	169 26 33·86	+19·684	169 26 53·544	−2·467	169 26 51·077
	Crowborough	(M)	219 59 42·32	+10·003	219 59 52·323	+1·290	219 59 53·613
	Brenchley Air Beacon	(L)	224 45 06·18	+ 4·956	224 45 11·136	+0·430	224 45 11·566
	Wrotham	(Q)	260 13 33·30	− 3·294	260 13 30·006	−0·463	260 13 29·543
	Warley Wtr Twr	(Y)	297 12 24·84	−17·784	297 12 07·056	−0·413	297 12 06·643
	Benfleet Wtr Twr	(W)	316 18 35·13	−16·141	316 18 18·989	+0·815	316 18 19·804
	Hockley Wtr Twr	(X)	323 38 23·53	−18·873	323 38 04·657	+0·338	323 38 04·995
Liddington Castle (C$_1$)	Inkpen	(S)	00 00 00·00	+ 1·214	00 00 01·214	+1·110	00 00 02·324
	Cleeve Hill	(J$_1$)	196 40 37·64	− 1·578	196 40 36·062	−1·174	196 40 34·888
	White Horse Hill	(B$_1$)	276 02 02·01	− 0·403	276 02 01·607	+0·063	276 02 01·670
Linch Ball (D)	Hindhead	(P)	00 00 00·00	− 4·065	359 59 55·935	−0·166	359 59 55·769
	Leith Hill Tower	(O)	32 53 00·19	− 6·177	32 52 54·013	+0·777	32 52 54·790
	Selsey	(B)	159 02 29·92	+ 4·685	159 02 34·605	+0·325	159 02 34·930
	Dunnose	(A)	201 21 25·91	+ 7·118	201 21 33·028	−0·862	201 21 32·166
	Butser	(C)	267 03 15·87	− 0·601	267 03 15·269	−0·074	267 03 15·195
Lincoln Minster (U$_3$)	Acre	(V$_3$)	00 00 00·00	− 6·406	359 59 53·594	−0·833	359 59 52·761
	Cold Harbour	(W$_3$)	41 46 12·42	− 2·559	41 46 09·861	+0·582	41 46 10·443
	Boston Tower	(N$_3$)	98 15 19·85	+ 7·657	98 15 27·507	+2·005	98 15 29·512
	Harrowby	(M$_3$)	154 53 29·21	+ 8·831	154 53 38·041	+0·732	154 53 38·773
	Belvoir Castle	(L$_3$)	172 24 08·99	+ 8·927	172 24 17·917	−0·562	172 24 17·355
	Loath Hill	(K$_3$)	212 02 58·40	+ 3·957	212 03 02·357	−1·924	212 03 00·433
Loath Hill (K$_3$)	Lincoln Minster	(U$_3$)	00 00 00·00	− 3·433	359 59 56·567	+0·193	359 59 56·760
	Harrowby	(M$_3$)	57 46 55·47	+ 3·359	57 46 58·829	−0·942	57 46 57·887
	Belvoir Castle	(L$_3$)	75 04 08·44	+ 3·530	75 04 11·970	+1·389	75 04 13·359
	Charnwood	(G$_3$)	135 42 00·02	+ 5·845	135 42 05·865	−0·127	135 42 05·738
	Bardon Hill	(H$_3$)	141 10 14·41	+ 5·919	141 10 20·329	−0·079	141 10 20·250
	Alport Heights	(J$_3$)	204 06 05·71	+ 0·284	204 06 05·994	−0·434	204 06 05·560
Mablethorpe Wtr Twr (X$_3$)	Skegness Wtr Twr	(S$_3$)	00 00 00·00	+ 7·621	00 00 07·621	−0·039	00 00 07·582
	Dexthorpe	(T$_3$)	56 24 55·48	+ 4·157	56 24 59·637	+0·626	56 25 00·263
	Cold Harbour	(W$_3$)	97 48 25·23	+ 1·065	97 48 26·295	−0·587	97 48 25·708
Manningtree (S$_1$)	Abberton Wtr Twr	(Q$_1$)	00 00 00·00	+ 5·488	00 00 05·488	+0·404	00 00 05·892
	Maplestead	(V$_1$)	64 03 49·82	− 2·497	64 03 47·323	−1·102	64 03 46·221
	Stoke by Nayland Ch Twr	(U$_1$)	87 43 19·11	− 3·498	87 43 15·612	+0·157	87 43 15·769
	Nedging Tye	(Y$_1$)	125 33 28·53	−10·539	125 33 17·991	+0·290	125 33 18·281
	Swilland	(Z$_1$)	165 15 32·78	−13·015	165 15 19·765	−0·572	165 15 19·193
	Felixstowe Wtr Twr	(T$_1$)	214 28 25·09	− 3·730	214 28 21·360	+0·331	214 28 21·691
	Walton on the Naze Twr	(R$_1$)	251 19 54·19	+ 3·260	251 19 57·450	+0·493	251 19 57·943

5.1 *continued*

From	*To*	*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Maplestead (V_1)	Abberton Wtr Twr (Q_1)	00° 00′ 00″·00	+ 7″·396	00° 00′ 07″·396	−0″·239	00° 00′ 07″·157
	Hockley Wtr Twr (X)	49 07 50·17	+19·576	49 08 09·746	−0·949	49 08 08·797
	Dunmow (P_1)	104 35 22·63	+ 5·435	104 35 28·065	−0·091	104 35 27·974
	Sibleys Wtr Twr (O_1)	128 46 45·33	+ 1·976	128 46 47·306	−0·452	128 46 46·854
	Helion Bumpstead (W_1)	157 34 03·30	− 3·192	157 34 00·108	−0·135	157 33 59·973
	Chedburgh (E_2)	216 55 14·37	− 9·838	216 55 04·532	+0·509	216 55 05·041
	Nedging Tye (Y_1)	279 33 01·65	− 7·312	279 32 54·338	+0·456	279 32 54·794
	Stoke by Nayland Ch Twr (U_1)	311 45 10·06	− 0·860	311 45 09·200	+0·928	311 45 10·128
	Manningtree (S_1)	329 22 31·72	+ 2·391	329 22 34·111	−0·027	329 22 34·084
Massingham (P_3)	Fransham (Y_2)	00 00 00·00	+ 4·527	00 00 04·527	+0·052	00 00 04·579
	Swaffham (Z_2)	31 07 18·06	+ 4·990	31 07 23·050	−0·215	31 07 22·835
	Frog Hill (M_2)	38 23 01·26	+13·397	38 23 14·657	+0·974	38 23 15·631
	Crimplesham (A_3)	95 57 45·30	+ 7·019	95 57 52·319	+0·031	95 57 52·350
	Walpole St Peters (B_3)	136 24 57·16	+ 1·512	136 24 58·672	−0·447	136 24 58·225
	Docking Ch Twr (O_3)	223 14 40·47	− 7·611	223 14 32·859	+0·305	223 14 33·164
	Piggs Grave (Q_3)	294 14 12·72	− 6·098	294 14 06·622	−0·702	294 14 05·920
Metfield (O_2)	Southwold Ch Twr (P_2)	00 00 00·00	+ 2·180	00 00 02·180	−0·117	00 00 02·063
	Salle (B_2)	60 55 44·41	+ 8·110	60 55 52·520	+0·822	60 55 53·342
	Crown Corner (C_2)	109 41 51·74	+ 5·716	109 41 57·456	−0·898	109 41 56·558
	Bunwell Ch Twr (U_2)	203 47 07·11	− 7·274	203 46 59·836	−0·041	203 46 59·795
	Topcroft Ch Twr (T_2)	239 33 13·78	− 7·498	239 33 06·282	+0·108	239 33 06·390
	Framingham (V_2)	247 00 23·20	−13·166	247 00 10·034	−0·683	247 00 09·351
	Ilketshall St Andrews Ch Twr (S_2)	302 07 04·04	− 4·276	302 06 59·764	+0·992	302 07 00·756
	Church Farm Wtr Twr (R_2)	317 17 22·49	− 8·677	317 17 13·813	−0·377	317 17 13·436
	Kessingland Ch Twr (Q_2)	333 16 05·64	− 3·779	333 16 01·861	+0·194	333 16 02·055
Muswell Hill (G_1)	Dunstable Down (E_1)	00 00 00·00	− 0·798	359 59 59·202	+1·060	00 00 00·262
	Coombe Hill (F_1)	18 23 41·52	+ 0·973	18 23 42·493	−0·593	18 23 41·900
	Shirburn Hill (A_1)	74 07 26·67	+ 3·398	74 07 30·068	−0·549	74 07 29·519
	White Horse Hill (B_1)	146 03 10·65	+ 3·868	146 03 14·518	+1·065	146 03 15·583
	Wyck Beacon (H_1)	193 31 56·26	− 0·689	193 31 55·571	−0·611	193 31 54·960
	Icomb Tower (I_1)	196 11 36·90	− 0·951	196 11 35·949	−0·608	196 11 35·341
	Charwelton (M_1)	259 01 00·67	− 6·193	259 00 54·477	+0·238	259 00 54·715
Nedging Tye (Y_1)	Stoke by Nayland Ch Twr (U_1)	00 00 00·00	+ 6·839	00 00 06·839	−0·591	00 00 06·248
	Maplestead (V_1)	37 05 44·76	+ 7·556	37 05 52·316	−0·431	37 05 51·885
	Chedburgh (E_2)	90 41 13·09	− 3·023	90 41 10·067	−0·303	90 41 09·764
	Puttocks Hill (G_2)	134 27 28·03	− 9·963	134 27 18·067	+0·724	134 27 18·791
	Woolpit (D_2)	155 22 45·73	− 6·411	155 22 39·319	+0·125	155 22 39·444
	Crown Corner (C_2)	214 55 05·83	−10·874	214 54 54·956	−0·096	214 54 54·860
	Swilland (Z_1)	241 45 30·50	− 2·154	241 45 28·346	+0·356	241 45 28·702
	Felixstowe Wtr Twr (T_1)	282 24 36·15	+ 7·122	282 24 43·272	−0·088	282 24 43·184
	Manningtree (S_1)	328 24 52·50	+10·431	328 25 02·931	+0·303	328 25 03·234

5.1 *continued*

From	*To*	*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
North Walsham Wtr Twr (R_3)	Piggs Grave (Q_3)	00° 00′ 00″·00	− 2″·111	359° 59′ 57″·889	−0″·475	359° 59′ 57″·414
	Caister Wtr Twr (W_2)	205 38 19·49	+ 9·567	205 38 29·057	+0·557	205 38 29·614
	Framingham (V_2)	264 53 11·23	+15·290	264 53 26·520	−0·081	264 53 26·439
Orford Castle (A_2)	Felixstowe Wtr Twr (T_1)	00 00 00·00	+ 8·120	00 00 08·120	−0·217	00 00 07·903
	Swilland (Z_1)	54 57 36·18	− 2·334	54 57 33·846	−0·627	54 57 33·219
	Salle (B_2)	115 09 20·94	− 9·956	115 09 10·984	−0·115	115 09 10·869
	Walton on the Naze Twr (R_1)	345 56 09·46	+15·801	345 56 25·261	+0·960	345 56 26·221
Paddlesworth (J)	Rumfields Wtr Twr (T)	00 00 00·00	−16·167	359 59 43·833	+0·392	359 59 44·225
	Fairlight Down (I)	200 05 23·58	+14·558	200 05 38·138	+0·770	200 05 38·908
	Crowborough (M)	230 34 41·78	+ 4·378	230 34 46·158	+0·106	230 34 46·264
	Bethersden Air Beacon (K)	240 05 16·90	− 0·565	240 05 16·335	+0·863	240 05 17·198
	Brenchley Air Beacon (L)	240 49 06·70	− 1·391	240 49 05·309	−2·131	240 49 03·178
Peterborough Cathedral (C_3)	Wyton Wtr Twr (K_2)	00 00 00·00	+ 7·695	00 00 07·695	−0·049	00 00 07·646
	Collyweston (D_3)	122 36 27·63	− 1·303	122 36 26·327	−0·107	122 36 26·220
	Boston Tower (N_3)	215 34 09·91	−14·284	215 33 55·626	+1·094	215 33 56·720
	Walpole St Peters (B_3)	259 04 41·07	− 5·906	259 04 35·164	+0·133	259 04 35·297
	Ely Cathedral (L_2)	317 18 52·07	+ 6·095	317 18 58·165	−1·071	317 18 57·094
Piggs Grave (Q_3)	Docking Ch Twr (O_3)	00 00 00·00	− 1·952	359 59 58·048	+0·424	359 59 58·472
	North Walsham Wtr Twr (R_3)	179 55 52·62	+ 2·030	179 55 54·650	+0·557	179 55 55·207
	Caister Wtr Twr (W_2)	193 28 46·07	+10·991	193 28 57·061	−0·856	193 28 56·205
	Framingham (V_2)	223 30 20·65	+16·186	223 30 36·836	−0·336	223 30 36·500
	Hingham Ch Twr (X_2)	262 16 47·04	+15·835	262 17 02·875	−0·678	262 17 02·197
	Fransham (Y_2)	285 32 58·85	+11·398	285 33 10·248	+0·915	285 33 11·163
	Massingham (P_3)	322 19 35·89	+ 6·350	322 19 42·240	−0·025	322 19 42·215
Puttocks Hill (G_2)	South Lopham Ch Twr (N_2)	00 00 00·00	− 5·999	359 59 54·001	−0·527	359 59 53·474
	Woolpit (D_2)	77 19 57·98	+ 3·567	77 20 01·547	−1·140	77 20 00·407
	Nedging Tye (Y_1)	99 16 21·46	+ 9·759	99 16 31·219	+0·132	99 16 31·351
	Chedburgh (E_2)	169 45 28·00	+ 6·472	169 45 34·472	+0·475	169 45 34·947
	Ely Cathedral (L_2)	237 22 02·31	− 4·819	237 21 57·491	−0·371	237 21 57·120
	Frog Hill (M_2)	303 46 47·53	−10·295	303 46 37·235	+0·461	303 46 37·696
	Hingham Ch Twr (X_2)	331 29 05·55	−15·989	331 28 49·561	+0·970	331 28 50·531
Rollright (L_1)	Wyck Beacon (H_1)	00 00 00·00	+ 0·582	00 00 00·582	+0·346	00 00 00·928
	Icomb Tower (I_1)	07 30 42·39	+ 0·448	07 30 42·838	−0·133	07 30 42·705
	Cleeve Hill (J_1)	39 07 14·73	+ 0·247	39 07 14·977	−0·179	39 07 14·798
	Broadway Tower (K_1)	70 45 51·02	− 0·360	70 45 50·660	−0·007	70 45 50·653
	Charwelton (M_1)	181 29 07·16	− 2·376	181 29 04·784	+0·909	181 29 05·693
	White Horse Hill (B_1)	316 48 08·18	+ 3·152	316 48 11·332	−0·935	316 48 10·397

5.1 *continued*

From	*To*		*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Rumfields Wtr Twr (T)	Lenham Wtr Twr	(R)	00° 00′ 00″00	+ 8″421	00° 00′ 08″421	−0″391	00° 00′ 08″030
	Shurland	(U)	24 13 29·78	− 2·233	24 13 27·547	+0·055	24 13 27·602
	Benfleet Wtr Twr	(W)	36 10 20·36	−10·474	36 10 09·886	−0·531	36 10 09·355
	Hockley Wtr Twr	(X)	42 07 34·49	−13·583	42 07 20·907	−0·201	42 07 20·706
	Abberton Wtr Twr	(Q_1)	72 12 07·69	−29·255	72 11 38·435	+2·457	72 11 40·892
	Walton on the Naze Twr	(R_1)	96 52 15·65	−33·069	96 51 42·581	−1·511	96 51 41·070
	Paddlesworth	(J)	320 26 08·29	+16·589	320 26 24·879	+0·121	320 26 25·000
Salle (B_2)	Southwold Ch Twr	(P_2)	00 00 00·00	− 6·182	359 59 53·818	+0·155	359 59 53·973
	Orford Castle	(A_2)	103 52 37·74	+ 9·871	103 52 47·611	+0·603	103 52 48·214
	Felixstowe Wtr Twr	(T_1)	137 44 14·34	+17·670	137 44 32·010	−0·468	137 44 31·542
	Swilland	(Z_1)	179 01 42·02	+ 7·250	179 01 49·270	+0·191	179 01 49·461
	Crown Corner	(C_2)	234 58 52·45	− 2·304	234 58 50·146	+0·164	234 58 50·310
	Metfield	(O_2)	285 43 02·99	− 8·164	285 42 54·826	−0·644	285 42 54·182
Severndroog Castle (V)	Epping Wtr Twr	(D_1)	00 00 00·00	− 9·717	359 59 50·283	+0·088	359 59 50·371
	Warley Wtr Twr	(Y)	38 31 57·61	− 5·767	38 31 51·843	+0·336	38 31 52·179
	Benfleet Wtr Twr	(W)	66 06 57·88	− 4·132	66 06 53·748	−0·767	66 06 52·981
	Wrotham	(Q)	127 33 08·49	+ 6·097	127 33 14·587	−0·035	127 33 14·552
	Leith Hill Tower	(O)	213 57 16·12	+11·171	213 57 27·291	+0·343	213 57 27·634
	Chipping Barnet Ch Twr	(Z)	309 49 51·61	− 7·033	309 49 44·577	+0·034	309 49 44·611
Shirburn Hill (A_1)	Muswell Hill	(G_1)	00 00 00·00	− 3·537	359 59 56·463	−0·714	359 59 55·749
	Coombe Hill	(F_1)	70 51 05·59	− 2·913	70 51 02·677	+0·794	70 51 03·471
	Chipping Barnet Ch Twr	(Z)	110 55 58·87	− 0·278	110 55 58·592	+1·250	110 55 59·842
	Leith Hill Tower	(O)	163 39 06·91	+11·376	163 39 18·286	−2·235	163 39 16·051
	Hindhead	(P)	185 42 50·16	+11·760	185 43 01·920	−1·144	185 43 00·776
	Inkpen	(S)	248 25 40·07	+ 5·168	248 25 45·238	+0·464	248 25 45·702
	Liddington Castle	(C_1)	275 29 38·53	+ 2·167	275 29 40·697	+0·963	275 29 41·660
	White Horse Hill	(B_1)	280 25 35·98	+ 1·309	280 25 37·289	+0·623	280 25 37·912
Shurland (U)	Benfleet Wtr Twr	(W)	00 00 00·00	− 7·356	359 59 52·644	+1·389	359 59 54·033
	Hockley Wtr Twr	(X)	13 44 49·94	−10·104	13 44 39·836	−0·670	13 44 39·166
	Abberton Wtr Twr	(Q_1)	54 50 30·94	−24·015	54 50 06·925	−0·136	54 50 06·789
	Walton on the Naze Twr	(R_1)	81 27 45·69	−27·455	81 27 18·235	+0·259	81 27 18·494
	Rumfields Wtr Twr	(T)	150 28 46·12	+ 2·109	150 28 48·229	+0·296	150 28 48·525
	Lenham Wtr Twr	(R)	256 27 51·85	+ 9·433	256 28 01·283	−0·681	256 28 00·602
	Wrotham	(Q)	308 34 52·86	+ 5·519	308 34 58·379	−0·457	308 34 57·922
Sibleys Wtr Twr (O_1)	Epping Wtr Twr	(D_1)	00 00 00·00	+10·570	00 00 10·570	−0·079	00 00 10·491
	Chipping Barnet Ch Twr	(Z)	23 51 40·85	+12·389	23 51 53·239	+1·118	23 51 54·357
	Therfield	(N_1)	87 32 20·91	− 2·731	87 32 18·179	−0·359	87 32 17·820
	Helion Bumpstead	(W_1)	187 35 55·99	− 4·669	187 35 51·321	+0·179	187 35 51·500
	Maplestead	(V_1)	240 40 53·77	− 1·875	240 40 51·895	−0·238	240 40 51·657
	Dunmow	(P_1)	292 32 11·04	+ 3·085	292 32 14·125	−0·621	292 32 13·504

5.1 *continued*

From	*To*		*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Skegness Wtr Twr (S_3)	Dexthorpe	(T_3)	00° 00′ 00″·00	− 3″·287	359° 59′ 56″·713	+0″·249	359° 59′ 56″·962
	Mablethorpe Wtr Twr	(X_3)	45 31 19·67	− 7·708	45 31 11·962	−0·095	45 31 11·867
	Docking Ch Twr	(O_3)	203 16 23·96	+11·306	203 16 35·266	+0·154	203 16 35·420
	Walpole St Peters	(B_3)	246 59 53·95	+18·630	247 00 12·580	+0·899	247 00 13·479
	Boston Tower	(N_3)	289 09 56·14	+ 7·587	289 10 03·727	−1·208	289 10 02·519
South Lopham Ch Twr (N_2)	Hingham Ch Twr	(X_2)	00 00 00·00	−10·495	359 59 49·505	+0·212	359 59 49·717
	Bunwell Ch Twr	(U_2)	42 59 56·18	− 5·771	42 59 50·409	+0·111	42 59 50·520
	Crown Corner	(C_2)	123 18 55·31	+ 6·197	123 19 01·507	+0·025	123 19 01·532
	Woolpit	(D_2)	197 35 00·26	+ 9·985	197 35 10·245	+0·145	197 35 10·390
	Chedburgh	(E_2)	229 21 02·87	+12·829	229 21 15·699	−0·410	229 21 15·289
	Puttocks Hill	(G_2)	234 19 57·20	+ 6·144	234 20 03·344	+0·015	234 20 03·359
	Frog Hill	(M_2)	304 10 44·07	− 4·691	304 10 39·379	−0·096	304 10 39·283
Southwold Ch Twr (P_2)	Kessingland Ch Twr	(Q_2)	00 00 00·00	− 6·290	359 59 53·710	−0·633	359 59 53·077
	Salle	(B_2)	224 06 41·64	+ 6·309	224 06 47·949	−0·443	224 06 47·506
	Metfield	(O_2)	268 53 57·97	− 2·240	268 53 55·730	+0·706	268 53 56·436
	Ilketshall St Andrews Ch Twr	(S_2)	298 35 55·33	− 6·774	298 35 48·556	+0·355	298 35 48·911
	Church Farm Wtr Twr	(R_2)	358 46 10·52	−11·459	358 45 59·061	+0·015	358 45 59·076
Stoke by Nayland Ch Twr (U_1)	Abberton Wtr Twr	(Q_1)	00 00 00·00	+ 8·713	00 00 08·713	−1·130	00 00 07·583
	Maplestead	(V_1)	89 22 19·53	+ 0·883	89 22 20·413	−0·409	89 22 20·004
	Chedburgh	(E_2)	140 30 32·75	− 9·524	140 30 23·226	+0·587	140 30 23·813
	Nedging Tye	(Y_1)	200 04 25·03	− 6·801	200 04 18·229	+0·804	200 04 19·033
	Felixstowe Wtr Twr	(T_1)	275 45 50·19	− 0·058	275 45 50·132	−0·137	275 45 49·995
	Manningtree	(S_1)	310 39 09·78	+ 3·443	310 39 13·223	+0·284	310 39 13·507
Swaffham (Z_2)	Fransham	(Y_2)	57 49 19·96	− 0·551	57 49 19·409	−0·659	57 49 18·750
	Hingham Ch Twr	(X_2)	86 52 45·81	+ 3·430	86 52 49·240	+0·718	86 52 49·958
	Frog Hill	(M_2)	145 16 45·45	+ 8·512	145 16 53·962	−0·300	145 16 53·662
	Ely Cathedral	(L_2)	201 24 04·20	+12·769	201 24 16·969	−0·659	201 24 16·310
	Crimplesham	(A_3)	230 53 56·98	+ 2·241	230 53 59·221	+0·407	230 53 59·628
	Walpole St Peters	(B_3)	257 52 22·08	− 3·223	257 52 18·857	+0·487	257 52 19·344
	Massingham	(P_3)	313 24 00·16	− 5·031	313 23 55·129	+0·008	313 23 55·137
Swilland (Z_1)	Felixstowe Wtr Twr	(T_1)	00 00 00·00	+ 9·791	00 00 09·791	−0·023	00 00 09·768
	Manningtree	(S_1)	53 10 39·09	+13·218	53 10 52·308	+0·333	53 10 52·641
	Nedging Tye	(Y_1)	106 49 15·23	+ 2·211	106 49 17·441	−0·244	106 49 17·197
	Woolpit	(D_2)	145 28 03·64	− 4·554	145 27 59·086	−0·218	145 27 58·868
	Crown Corner	(C_2)	234 56 47·88	− 9·144	234 56 38·736	+0·450	234 56 39·186
	Salle	(B_2)	265 44 21·39	− 7·065	265 44 14·325	−0·377	265 44 13·948
	Orford Castle	(A_2)	310 23 32·72	+ 2·255	310 23 34·975	+0·077	310 23 35·052

5.1 *continued*

From	*To*		*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Therfield (N_1)	Sibleys Wtr Twr	(O_1)	00° 00′ 00″·00	+ 2″·588	00° 00′ 02″·588	−0″·361	00° 00′ 02″·227
	Epping Wtr Twr	(D_1)	51 18 05·38	+12·027	51 18 17·407	+0·139	51 18 17·546
	Dunstable Down	(E_1)	133 49 45·66	+ 5·528	133 49 51·188	−0·607	133 49 50·581
	Bolnhurst	(X_1)	202 15 19·72	− 7·085	202 15 12·635	−0·700	202 15 11·935
	Fayway	(J_2)	219 59 57·94	−12·996	219 59 44·944	−0·445	219 59 44·499
	Wyton Wtr Twr	(K_2)	244 53 15·41	−12·186	244 53 03·224	+0·090	244 53 03·314
	Ely Cathedral	(L_2)	278 35 14·44	−15·279	278 34 59·161	+0·929	278 35 00·090
	Burrough Green Wtr Twr	(F_2)	310 11 01·56	− 6·950	310 10 54·610	+0·529	310 10 55·139
	Helion Bumpstead	(W_1)	334 13 03·90	− 1·587	334 13 02·313	+0·427	334 13 02·740
Tilton Pile (F_3)	Buckminster Wtr Twr	(I_3)	00 00 00·00	− 3·478	359 59 56·522	+0·087	359 59 56·609
	Collyweston	(D_3)	62 46 04·01	+ 0·579	62 46 04·589	−0·275	62 46 04·314
	Uppingham	(E_3)	96 05 49·23	+ 1·414	96 05 50·644	−0·488	96 05 50·156
	Faxton	(I_2)	138 57 31·88	+ 6·027	138 57 37·907	−0·197	138 57 37·710
	Cold Ashby	(H_2)	168 56 45·28	+ 5·394	168 56 50·674	+1·447	168 56 52·121
	Charnwood	(G_3)	255 11 44·90	− 1·537	255 11 43·363	−1·818	255 11 41·545
	Harrowby	(M_3)	357 04 11·48	− 6·256	357 04 05·224	+1·244	357 04 06·468
Topcroft Ch Twr (T_2)	Ilketshall St Andrews Ch Twr	(S_2)	00 00 00·00	+ 3·301	00 00 03·301	−0·737	00 00 02·564
	Metfield	(O_2)	43 32 43·61	+ 7·447	43 32 51·057	+0·385	43 32 51·442
	Crown Corner	(C_2)	66 08 29·43	+13·023	66 08 42·453	+0·086	66 08 42·539
	Bunwell Ch Twr	(U_2)	152 57 34·50	+ 0·071	152 57 34·571	−0·054	152 57 34·517
	Framingham	(V_2)	241 29 42·02	− 5·594	241 29 36·426	+0·321	241 29 36·747
Uppingham (E_3)	Tilton Pile	(F_3)	00 00 00·00	− 1·463	359 59 58·537	−0·252	359 59 58·285
	Buckminster Wtr Twr	(I_3)	57 17 02·71	− 5·251	57 16 57·459	−0·217	57 16 57·242
	Collyweston	(D_3)	123 58 46·37	− 0·984	123 58 45·386	+0·634	123 58 46·020
	Fayway	(J_2)	183 28 02·88	+ 4·769	183 28 07·649	−0·930	183 28 06·719
	Faxton	(I_2)	240 58 50·46	+ 4·972	240 58 55·432	+0·703	240 58 56·135
	Cold Ashby	(H_2)	272 55 32·48	+ 4·418	272 55 36·898	+0·061	272 55 36·959
Walpole St Peters (B_3)	Ely Cathedral	(L_2)	00 00 00·00	+13·947	00 00 13·947	−0·209	00 00 13·738
	Peterborough Cathedral	(C_3)	65 45 12·14	+ 6·373	65 45 18·513	+0·340	65 45 18·853
	Collyweston	(D_3)	81 03 03·84	+ 4·539	81 03 08·379	−0·695	81 03 07·684
	Boston Tower	(N_3)	153 33 35·88	−10·077	153 33 25·803	+0·688	153 33 26·491
	Skegness Wtr Twr	(S_3)	192 42 34·09	−18·407	192 42 15·683	+0·155	192 42 15·838
	Docking Ch Twr	(O_3)	238 19 15·08	− 8·195	238 19 06·885	+0·236	238 19 07·121
	Massingham	(P_3)	269 11 42·56	− 1·425	269 11 41·135	−0·213	269 11 40·922
	Swaffham	(Z_2)	288 22 26·80	+ 3·014	288 22 29·814	−0·076	288 22 29·738
	Crimplesham	(A_3)	316 12 06·30	+ 4·852	316 12 11·152	−0·225	316 12 10·927

5.1 *continued*

From	*To*	*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Walton on the Naze Twr (R$_1$)	Abberton Wtr Twr (Q$_1$)	00° 00′ 00″·00	+ 2″·499	00° 00′ 02″·499	−0″·371	00° 00′ 02″·128
	Manningtree (S$_1$)	28 08 35·02	− 3·352	28 08 31·668	−0·183	28 08 31·485
	Felixstowe Wtr Twr (T$_1$)	109 37 16·59	− 7·395	109 37 09·195	−0·425	109 37 08·770
	Orford Castle (A$_2$)	130 15 43·74	−15·458	130 15 28·282	−0·135	130 15 28·147
	Rumfields Wtr Twr (T)	268 25 13·11	+32·535	268 25 45·645	+0·142	268 25 45·787
	Shurland (U)	306 45 32·71	+28·606	306 46 01·316	+0·973	306 46 02·289
Warley Wtr Twr (Y)	Benfleet Wtr Twr (W)	00 00 00·00	+ 2·022	00 00 02·022	−0·546	00 00 01·476
	Lenham Wtr Twr (R)	35 33 29·99	+16·688	35 33 46·678	−0·178	35 33 46·500
	Wrotham (Q)	76 01 40·21	+12·713	76 01 52·923	−0·327	76 01 52·596
	Severndroog Castle (V)	122 30 34·16	+ 5·973	122 30 40·133	−0·362	122 30 39·771
	Chipping Barnet Ch Twr (Z)	174 33 29·76	− 1·846	174 33 27·914	+1·367	174 33 29·281
	Epping Wtr Twr (D$_1$)	208 37 24·09	− 4·413	208 37 19·677	−0·374	208 37 19·303
	Dunmow (P$_1$)	267 03 42·95	−12·576	267 03 30·374	−0·315	267 03 30·059
	Hockley Wtr Twr (X)	344 45 28·55	− 0·288	344 45 28·262	+0·733	344 45 28·995
White Horse Hill (B$_1$)	Liddington Castle (C$_1$)	25 24 35·34	+ 0·454	25 24 35·794	−0·889	25 24 34·905
	Cleeve Hill (J$_1$)	112 57 31·20	− 1·932	112 57 29·268	−1·475	112 57 27·793
	Broadway Tower (K$_1$)	130 52 06·38	− 3·011	130 52 03·369	−0·922	130 52 02·447
	Wyck Beacon (H$_1$)	135 24 40·67	− 2·336	135 24 38·334	+0·739	135 24 39·073
	Rollright (L$_1$)	148 32 44·24	− 3·234	148 32 41·006	+0·473	148 32 41·479
	Muswell Hill (G$_1$)	201 06 09·57	− 3·036	201 06 06·534	+0·901	201 06 07·435
	Coombe Hill (F$_1$)	219 37 27·47	− 2·977	219 37 24·493	+0·890	219 37 25·383
	Shirburn Hill (A$_1$)	229 36 04·13	− 0·992	229 36 03·138	+0·395	229 36 03·533
	Inkpen (S)	315 05 42·71	+ 2·038	315 05 44·748	−0·113	315 05 44·635
Woolpit (D$_2$)	South Lopham Ch Twr (N$_2$)	00 00 00·00	− 9·915	359 59 50·085	+0·366	359 59 50·451
	Crown Corner (C$_2$)	60 32 25·98	− 4·157	60 32 21·823	+0·625	60 32 22·448
	Swilland (Z$_1$)	101 58 08·94	+ 4·421	101 58 13·361	−0·481	101 58 12·880
	Nedging Tye (Y$_1$)	156 56 35·60	+ 6·387	156 56 41·987	−0·037	156 56 41·950
	Chedburgh (E$_2$)	240 23 28·35	+ 3·141	240 23 31·491	−0·508	240 23 30·983
	Puttocks Hill (G$_2$)	294 04 53·01	− 3·627	294 04 49·383	+0·970	294 04 50·353
	Frog Hill (M$_2$)	324 07 16·15	−14·263	324 07 01·887	−0·936	324 07 00·951
Wrotham (Q)	Benfleet Wtr Twr (W)	00 00 00·00	−11·227	359 59 48·773	+1·191	359 59 49·964
	Shurland (U)	37 35 17·72	− 5·116	37 35 12·604	−0·040	37 35 12·564
	Lenham Wtr Twr (R)	65 41 50·22	+ 3·093	65 41 53·313	−0·540	65 41 52·773
	Fairlight Down (I)	116 02 55·02	+20·427	116 03 15·447	+0·420	116 03 15·867
	Brenchley Air Beacon (L)	117 36 08·88	+ 7·303	117 36 16·183	+0·254	117 36 16·437
	Crowborough (M)	159 07 08·78	+11·605	159 07 20·385	−1·443	159 07 18·942
	Leith Hill Tower (O)	213 10 30·68	+ 6·154	213 10 36·834	−0·387	213 10 36·447
	Severndroog Castle (V)	278 38 52·17	− 6·317	278 38 45·853	−0·336	278 38 45·517
	Epping Wtr Twr (D$_1$)	307 06 27·11	−16·807	307 06 10·303	+0·863	307 06 11·166
	Warley Wtr Twr (Y)	323 08 48·67	−12·720	323 08 35·950	+0·018	323 08 35·968

5.1 *continued*

From	*To*		*Mean Observed Direction*	*(t − T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Wyck Beacon (H$_1$)	Broadway Tower	(K$_1$)	00° 00′ 00″00	− 0″674	359° 59′ 59″326	−0″870	359° 59′ 58″456
	Icomb Tower	(I$_1$)	29 28 57·75	− 0·107	29 28 57·643	−0·043	29 28 57·600
	Rollright	(L$_1$)	70 03 40·84	− 0·523	70 03 40·317	+0·363	70 03 40·680
	Muswell Hill	(G$_1$)	126 53 55·96	+ 0·485	126 53 56·445	−0·963	126 53 55·482
	White Horse Hill	(B$_1$)	193 43 44·75	+ 2·048	193 43 46·798	+0·945	193 43 47·743
	Cleeve Hill	(J$_1$)	310 16 15·92	− 0·129	310 16 15·791	+0·567	310 16 16·358
Wyton Wtr Twr (K$_2$)	Ely Cathedral	(L$_2$)	00 00 00·00	− 2·238	359 59 57·762	−0·103	359 59 57·659
	Burrough Green Wtr Twr	(F$_2$)	40 25 04·24	+ 6·170	40 25 10·410	+0·578	40 25 10·988
	Therfield	(N$_1$)	96 10 02·38	+12·030	96 10 14·410	−0·127	96 10 14·283
	Bolnhurst	(X$_1$)	161 46 50·51	+ 4·294	161 46 54·804	−0·212	161 46 54·592
	Fayway	(J$_2$)	206 17 25·81	− 1·433	206 17 24·377	−0·439	206 17 23·938
	Collyweston	(D$_3$)	240 18 44·99	− 8·843	240 18 36·147	−0·867	240 18 35·280
	Peterborough Cathedral	(C$_3$)	264 38 36·46	− 7·878	264 38 28·582	+1·170	264 38 29·752

5.2 Triangle misclosures and spherical excesses

Triangle	*Spherical Excess (ε)*	*Triangle Misclosure*	*Triangle*	*Spherical Excess (ε)*	*Triangle Misclosure*	*Triangle*	*Spherical Excess (ε)*	*Triangle Misclosure*
ACD	1″446	−0″716	ACE	2″807	−1″817	ACF	6″531	+1″489
AEF	3·060	+2·720	CDP	0·655	−0·775	CEF	0·664	+0·586
CPS	3·272	+2·678	DPO	1·031	−0·881	EFO	2·787	−0·717
FGH	0·206	+2·324	FGI	0·881	−0·841	FGM	1·015	−0·695
FGO	0·825	−1·345	FHI	2·165	+3·555	FMI	2·346	−2·496
FMN	0·849	−0·179	FMO	2·236	+1·574	FNO	1·707	−0·357
GHI	1·078	+2·072	GMI	2·212	−2·642	GMO	2·426	+2·224
IMQ	2·848	+2·702	IMR	3·834	−1·974	IQR	3·599	−1·279
MNO	0·320	−2·110	MOQ	3·014	−3·764	MQR	2·613	+3·397
OPZ	3·041	−1·481	OPA$_1$	3·925	−2·095	OVQ	2·550	−2·060
OVZ	3·064	+2·526	OZA$_1$	7·016	+3·354	PSA$_1$	6·765	−3·315
PZA$_1$	7·900	+2·740	PZF$_1$	6·626	+2·654	PA$_1$F$_1$	3·174	+0·026
QRU	1·724	−1·514	QRW	2·608	−0·158	QRY	2·652	+0·658
QUW	2·180	−0·880	QVW	1·902	−3·202	QVY	1·279	+0·051
QVD$_1$	1·230	−0·360	QWY	1·591	−2·961	QYD$_1$	0·984	+0·636
RTU	1·869	+3·001	RTW	4·387	+3·963	RTX	4·889	+2·621

5.2 *continued*

Triangle	*Spherical Excess* (ϵ)	*Triangle Misclosure*	*Triangle*	*Spherical Excess* (ϵ)	*Triangle Misclosure*	*Triangle*	*Spherical Excess* (ϵ)	*Triangle Misclosure*
RUW	1″296	−2″236	RWX	0″479	+1″651	RWY	1″547	−3″777
RXU	1·240	+1·070	RXY	2·346	−0·636	SA_1B_1	2·814	−0·954
SB_1C_1	0·692	−0·842	TUX	1·780	−1·450	TUW	1·222	+3·198
TUQ_1	4·511	−3·111	TUR_1	5·201	+0·699	TWX	0·981	+0·309
TXQ_1	4·869	−1·029	TQ_1R_1	3·816	+4·644	UWX	0·423	+4·957
UXQ_1	2·138	+0·632	UQ_1R_1	3·126	+0·834	VWY	0·968	+0·292
VYZ	1·542	−1·582	VYD_1	0·935	+0·225	VZD_1	1·435	−0·815
WXY	0·320	+1·490	XYP_1	1·813	−1·323	XP_1V_1	1·923	−2·563
XQ_1V_1	1·884	−0·474	YZD_1	0·828	+0·992	YD_1P_1	1·132	+1·448
A_1ZF_1	1·900	−0·060	ZD_1E_1	1·668	+1·432	ZD_1O_1	1·374	−0·554
ZE_1F_1	1·252	−0·412	$A_1B_1F_1$	1·205	+0·005	$A_1B_1G_1$	2·332	+0·228
$A_1F_1G_1$	1·157	−1·007	$B_1C_1J_1$	1·391	−0·471	$B_1F_1G_1$	2·284	−0·784
$B_1G_1H_1$	3·694	−0·394	$B_1H_1J_1$	1·693	−2·383	$B_1H_1L_1$	0·897	−1·597
$B_1J_1K_1$	2·026	+1·304	$B_1J_1L_1$	3·135	−2·775	$B_1K_1L_1$	1·784	−2·994
$D_1E_1N_1$	3·432	+1·688	$D_1N_1O_1$	1·786	−0·066	$D_1O_1P_1$	0·770	−2·400
$E_1F_1G_1$	0·753	+0·027	$E_1G_1M_1$	3·937	+1·873	$E_1N_1X_1$	3·078	−0·758
$E_1X_1I_2$	2·784	+0·546	$H_1J_1K_1$	0·715	+4·375	$H_1J_1L_1$	0·545	+1·205
$H_1K_1L_1$	0·504	−2·084	$J_1K_1L_1$	0·675	+1·085	$K_1L_1M_1$	1·476	−2·156
$N_1O_1W_1$	0·796	+0·934	$N_1X_1J_2$	1·340	−0·230	$N_1X_1K_2$	2·242	+0·388
$N_1X_1L_2$	4·167	−0·947	$N_1W_1F_2$	1·089	−0·579	$N_1F_2K_2$	3·026	+0·434
$N_1F_2L_2$	2·262	+0·188	$N_1J_2K_2$	1·930	−0·590	$N_1J_2L_2$	5·069	−1·149
$N_1K_2L_2$	2·482	−0·512	$O_1P_1V_1$	0·609	+0·971	$O_1P_1W_1$	0·365	+0·605
$O_1V_1W_1$	0·714	+0·466	$P_1V_1W_1$	0·958	+0·832	$Q_1R_1S_1$	0·604	+0·186
$Q_1S_1U_1$	0·394	+2·476	$Q_1S_1V_1$	0·775	+2·435	$Q_1U_1V_1$	0·691	+0·349
$R_1S_1T_1$	0·625	+0·235	$R_1T_1A_2$	0·355	+1·145	$S_1T_1U_1$	0·517	−0·807
$S_1T_1Y_1$	1·152	−0·882	$S_1T_1Z_1$	1·080	−1·450	$S_1U_1V_1$	0·310	+0·390
$S_1U_1Y_1$	0·389	+1·281	$S_1V_1Y_1$	1·214	−0·174	$S_1Y_1Z_1$	0·898	+1·492
$T_1U_1Y_1$	1·024	+1·206	$T_1Y_1Z_1$	0·826	+0·924	$T_1Y_1C_2$	2·179	−0·619
$T_1Z_1A_2$	0·943	+0·287	$T_1Z_1B_2$	1·109	−0·969	$T_1Z_1C_2$	0·754	+0·246
$T_1A_2B_2$	0·757	+0·703	$T_1B_2C_2$	0·856	−0·156	$U_1V_1Y_1$	0·515	−1·845
$U_1V_1E_2$	0·864	−2·004	$U_1Y_1E_2$	0·846	+0·564	$V_1W_1E_2$	1·034	−0·584
$V_1Y_1E_2$	1·195	+0·405	$W_1E_2F_2$	0·582	−0·012	$X_1I_2J_2$	1·230	+2·110
$X_1J_2K_2$	1·028	−1·208	$X_1J_2L_2$	2·242	−0·432	$X_1K_2L_2$	0·557	+0·823
$Y_1Z_1C_2$	0·599	−1·789	$Y_1Z_1D_2$	0·542	−0·702	$Y_1C_2D_2$	0·870	+0·790
$Y_1D_2E_2$	0·705	+1·045	$Y_1D_2G_2$	0·270	−1·680	$Y_1E_2G_2$	0·982	−2·092
$Z_1A_2B_2$	0·923	−0·553	$Z_1B_2C_2$	0·501	+1·059	$Z_1C_2D_2$	0·927	−0·297
$B_2C_2O_2$	0·315	+3·736	$B_2O_2P_2$	0·637	−2·887	$C_2D_2N_2$	1·191	+0·679
$C_2O_2T_2$	0·304	−1·354	$C_2N_2U_2$	0·853	+0·117	$C_2T_2U_2$	0·806	−0·666
$C_2O_2U_2$	0·651	−2·221	$D_2E_2N_2$	0·962	−1·742	$D_2E_2G_2$	0·547	−4·817
$D_2G_2N_2$	0·563	+1·347	$D_2G_2M_2$	0·486	+3·114	$E_2F_2L_2$	0·923	−1·243
$D_2M_2N_2$	0·928	−0·848	$E_2G_2N_2$	0·148	−1·728	$E_2G_2L_2$	1·545	+1·725
$G_2L_2M_2$	1·878	−0·488	$F_2K_2L_2$	1·718	−0·758	$G_2M_2X_2$	0·888	−0·858
$G_2M_2N_2$	0·851	+0·919	$H_2E_3F_3$	0·841	−1·761	$G_2N_2X_2$	0·785	+1·345
$H_2I_2E_3$	0·975	+1·395	$H_2F_3G_3$	2·194	+1·786	$H_2G_3H_3$	0·534	+2·676
$H_2I_2F_3$	1·238	−2·698	$I_2E_3F_3$	0·578	+2·332	$I_2J_2E_3$	1·517	−3·027
$J_2K_2D_3$	1·267	+0·253	$J_2K_2L_2$	0·657	−0·047	$J_2D_3E_3$	1·008	+1·132
$J_2L_2D_3$	2·995	−0·185	$K_2L_2D_3$	2·385	−0·485	$K_2L_2C_3$	1·771	+0·009
$L_2M_2Z_2$	1·616	+1·024	$K_2C_3D_3$	1·116	−1·366	$L_2Z_2A_3$	1·022	+0·488
$L_2M_2A_3$	1·720	+1·990	$L_2Z_2B_3$	3·032	−0·022	$L_2A_3B_3$	1·228	−0·678
$L_2B_3C_3$	3·007	−0·537	$L_2B_3D_3$	4·746	−1·246	$L_2C_3D_3$	0·502	−0·872
$M_2N_2U_2$	0·672	−0·732	$M_2N_2X_2$	0·822	−0·432	$M_2U_2X_2$	0·644	+1·336

5.2 *continued*

Triangle	*Spherical Excess* (ϵ)	*Triangle Misclosure*	*Triangle*	*Spherical Excess* (ϵ)	*Triangle Misclosure*	*Triangle*	*Spherical Excess* (ϵ)	*Triangle Misclosure*
$M_2X_2Y_2$	0″·584	−2″·334	$M_2X_2Z_2$	0″·781	+0″·699	$M_2Y_2Z_2$	0″·406	−1″·006
$M_2Y_2P_3$	0·769	−0·619	$M_2Z_2A_3$	0·918	−0·478	$M_2Z_2P_3$	0·112	−1·572
$M_2A_3P_3$	1·387	+0·693	$N_2U_2X_2$	0·494	+1·036	$O_2P_2Q_2$	0·506	+2·614
$O_2P_2R_2$	0·917	+0·133	$O_2P_2S_2$	0·418	+1·672	$O_2Q_2R_2$	0·421	−0·311
$O_2Q_2S_2$	0·288	+0·452	$O_2R_2S_2$	0·175	+4·465	$O_2R_2V_2$	1·488	−1·108
$O_2S_2T_2$	0·304	−3·774	$O_2S_2V_2$	0·474	−3·744	$O_2T_2U_2$	0·459	+0·201
$O_2T_2V_2$	0·105	+0·805	$O_2U_2V_2$	0·911	+1·329	$P_2Q_2R_2$	0·010	+2·170
$P_2Q_2S_2$	0·376	+1·394	$P_2R_2S_2$	0·674	+2·926	$Q_2R_2S_2$	0·308	+3·702
$R_2S_2V_2$	0·839	−1·829	$R_2V_2W_2$	1·271	+4·159	$S_2T_2V_2$	0·275	+0·835
$T_2U_2V_2$	0·347	+0·323	$U_2V_2X_2$	0·584	−0·644	$V_2W_2Q_3$	2·564	−3·164
$V_2W_2R_3$	1·650	−0·890	$V_2X_2Q_3$	1·883	+0·577	$V_2Q_3R_3$	1·709	+1·461
$W_2Q_3R_3$	0·795	+3·735	$X_2Y_2Z_2$	0·209	−4·039	$X_2Y_2Q_3$	0·765	−1·635
$Y_2Z_2P_3$	0·251	+1·959	$Y_2P_3Q_3$	0·994	+0·556	$Z_2A_3B_3$	0·782	+0·168
$Z_2A_3P_3$	0·581	−0·401	$Z_2B_3P_3$	0·847	+0·573	$A_3B_3P_3$	1·048	+1·142
$B_3C_3D_3$	1·237	+0·163	$B_3C_3N_3$	2·949	+0·831	$B_3D_3N_3$	4·095	+0·855
$B_3N_3O_3$	2·741	+1·349	$B_3N_3S_3$	2·513	+2·687	$B_3O_3P_3$	1·274	−0·124
$B_3O_3S_3$	2·898	−1·668	$C_3D_3N_3$	2·383	+0·187	$D_3E_3F_3$	0·357	−0·887
$D_3E_3I_3$	0·878	−1·738	$D_3F_3I_3$	1·087	−0·417	$D_3F_3M_3$	1·889	+0·211
$D_3M_3N_3$	3·255	−0·085	$E_3F_3I_3$	0·566	+0·434	$F_3G_3I_3$	1·371	−0·651
$G_3H_3J_3$	0·544	−1·314	$G_3H_3K_3$	0·436	−2·066	$G_3J_3K_3$	3·178	+0·582
$H_3J_3K_3$	3·285	+1·335	$H_3K_3L_3$	2·784	+2·346	$K_3L_3U_3$	2·584	−2·304
$K_3M_3U_3$	2·986	+1·114	$M_3N_3U_3$	3·403	+2·287	$M_3N_3W_3$	3·696	−0·156
$M_3U_3W_3$	2·552	+2·058	$N_3O_3S_3$	2·670	−0·330	$N_3S_3T_3$	1·278	+0·822
$N_3T_3W_3$	1·194	−2·324	$N_3U_3V_3$	3·179	−2·749	$N_3U_3W_3$	2·845	−0·385
$N_3V_3W_3$	1·124	+1·926	$O_3P_3Q_3$	1·085	+0·625	$S_3T_3X_3$	0·644	−1·354
$T_3W_3X_3$	0·602	+2·628	$U_3V_3W_3$	1·458	−0·438			

Unclosed Triangles

Triangle	*Spherical Excess* (ϵ)	*Triangle*	*Spherical Excess* (ϵ)	*Triangle*	*Spherical Excess* (ϵ)	*Triangle*	*Spherical Excess* (ϵ)
ABC	2″·469	ABE	0″·936	ABD	1″·726	CBD	0″·703
CBE	1·275	ADF	4·867	FDO	3·480	FDC	0·218
ILR	2·329	ILM	1·765	ILJ	3·881	ILQ	0·076
JLM	1·627	MLR	0·266	MLQ	1·007	RLQ	1·340
IKM	2·826	IKJ	1·974	IKR	0·312	JKM	0·781
MKR	1·316	IMJ	4·019	SC_1A_1	3·001	$A_1C_1B_1$	0·505
$G_1I_1H_1$	0·232	$H_1I_1J_1$	0·108	$H_1I_1K_1$	0·047	$H_1I_1L_1$	0·041
$J_1I_1K_1$	0·654	$K_1I_1L_1$	0·416	$J_1I_1L_1$	0·396	$L_2C_3A_3$	2·607
$A_3C_3B_3$	1·628	$K_3L_3M_3$	0·736	$M_3L_3U_3$	1·138		

APPENDIX 6

FIGURE 6 (see DIAGRAM 10)

6.1 Mean observed directions, $(t-T)$ corrections, mean plane observed directions, adjustment corrections and plane adjusted directions

From	*To*		*Mean Observed Direction*	$(t-T)$	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Ailsa Craig (E)	Brown Carrick	(F)	00° 00′ 00″00	+ 7″724	00° 00′ 07″724	+0″330	00° 00′ 08″054
	Cairnsmore of Deugh	(C)	33 12 11·49	− 0·837	33 12 10·653	−0·197	33 12 10·456
	Merrick	(B)	50 38 42·72	− 6·673	50 38 36·047	−0·078	50 38 35·969
	Beneraird	(A)	92 45 01·81	−10·480	92 44 51·330	−1·081	92 44 50·249
	Cairn Pat	(A_4)	118 03 58·05	−21·713	118 03 36·337	+0·762	118 03 37·099
	Cnoc Moy	(K)	232 04 01·19	+ 8·259	232 04 09·449	+0·959	232 04 10·408
	Goat Fell	(L)	297 33 42·23	+21·015	297 34 03·245	−0·695	297 34 02·550
An Cuaidh (L_2)	Point of Stoer	(M_2)	00 00 00·00	+24·744	00 00 24·744	+0·038	00 00 24·782
	Anteallach	(D_2)	69 51 27·70	− 2·572	69 51 25·128	−0·199	69 51 24·929
	Beinn Bhan	(C_2)	145 58 03·80	−24·790	145 57 39·010	−0·985	145 57 38·025
	Storr	(K_2)	188 30 54·55	−20·632	188 30 33·918	+0·058	188 30 33·976
	Clisham	(O_2)	257 33 02·51	+11·224	257 33 13·734	+0·789	257 33 14·523
	Muirnag	(Q_2)	305 26 46·66	+35·264	305 27 21·924	+0·300	305 27 22·224
Anteallach (D_2)	Point of Stoer	(M_2)	00 00 00·00	+24·761	00 00 24·761	+0·315	00 00 25·076
	Conival	(E_2)	39 13 53·54	+16·673	39 14 10·213	−0·228	39 14 09·985
	Ben Wyvis	(X_1)	117 57 24·46	− 7·286	117 57 17·174	−0·285	117 57 16·889
	Beinn a' Bha' ach Ard	(W_1)	150 22 14·87	−18·974	150 21 55·896	+0·562	150 21 56·458
	Carn Eige	(L_1)	180 30 44·70	−28·157	180 30 16·543	−0·775	180 30 15·768
	Beinn Bhan	(C_2)	219 52 42·96	−20·095	219 52 22·865	−0·072	219 52 22·793
	Storr	(K_2)	248 00 12·43	−16·276	247 59 56·154	+0·031	247 59 56·185
	An Cuaidh	(L_2)	284 45 09·85	+ 2·449	284 45 12·299	+0·453	284 45 12·752
Askival (A_2)	Meall nan Con	(J_1)	00 00 00·00	−17·618	359 59 42·382	−0·361	359 59 42·021
	Ben Hogh	(I_1)	51 57 40·50	−25·170	51 57 15·330	+0·441	51 57 15·771
	Ben Hynish	(H_1)	59 54 02·22	−38·326	59 53 23·894	−0·021	59 53 23·873
	Heaval	(H_2)	115 35 33·19	+ 3·033	115 35 36·223	+0·130	115 35 36·353
	Beinn Mhor	(I_2)	143 46 06·42	+25·447	143 46 31·867	−0·003	143 46 31·864
	Healaval Beg	(J_2)	182 32 28·32	+31·671	182 32 59·991	−0·008	182 32 59·983
	Beinn na Caillich	(B_2)	238 48 25·32	+18·030	238 48 43·350	+0·237	238 48 43·587
	Sgurr na Ciche	(K_1)	290 35 42·45	+ 0·893	290 35 43·343	−0·415	290 35 42·928

6.1 *continued*

From	*To*		*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Bad Mor (T_2)	Scaraben	(G_2)	00° 00′ 00″00	− 6″989	359° 59′ 53″011	+0″093	359° 59′ 53″104
	Ben Klibreck	(F_2)	72 09 00·62	− 7·246	72 08 53·374	+0·835	72 08 54·209
	Ben Hutig	(S_2)	116 00 18·75	+ 2·990	116 00 21·740	+0·174	116 00 21·914
	Ward Hill	(Z_2)	219 28 13·14	+11·040	219 28 24·180	+0·045	219 28 24·225
	Dunnet Head	(W_2)	237 24 12·00	+ 5·062	237 24 17·062	+0·427	237 24 17·489
	Warth Hill	(Y_2)	261 47 41·08	+ 3·286	261 47 44·366	−0·735	261 47 43·631
	Spital Hill	(U_2)	281 29 03·35	+ 0·140	281 29 03·490	−0·475	281 29 03·015
	Hill of Yarrows	(V_2)	305 51 06·54	− 2·799	305 51 03·741	−0·364	305 51 03·377
Balta (K_3)	Fetlar	(J_3)	00 00 00·00	+ 2·407	00 00 02·407	+0·167	00 00 02·574
	Yell	(I_3)	19 39 52·89	+ 3·553	19 39 56·443	−0·348	19 39 56·095
	Saxavord	(L_3)	144 21 11·98	− 1·391	144 21 10·589	+0·182	144 21 10·771
Beinn Bhan (C_2)	Anteallach	(D_2)	00 00 00·00	+20·976	00 00 20·976	+0·159	00 00 21·135
	Carn Eige	(L_1)	86 29 21·60	− 9·967	86 29 11·633	−0·693	86 29 10·940
	Sgurr na Ciche	(K_1)	134 28 46·96	−26·476	134 28 20·484	−0·042	134 28 20·442
	Beinn na Caillich	(B_2)	188 55 30·68	−12·447	188 55 18·233	−0·349	188 55 17·884
	Healaval Beg	(J_2)	233 12 02·23	− 1·699	233 12 00·531	+0·625	233 12 01·156
	Storr	(K_2)	252 16 58·88	+ 5·245	252 17 04·125	−0·302	252 17 03·823
	An Cuaidh	(L_2)	320 58 58·94	+24·647	320 59 23·587	+0·602	320 59 24·189
Beinn Bheula (Q)	Ben Lomond	(R)	00 00 00·00	+ 2·037	00 00 02·037	−0·351	00 00 01·686
	Hill of Stake	(M)	83 29 42·78	−16·145	83 29 26·635	+0·057	83 29 26·692
	Sliabh Gaoil	(P)	156 25 49·64	−11·936	156 25 37·704	−0·110	156 25 37·594
	Carra Duagh	(U)	216 34 59·42	+ 5·846	216 35 05·266	+0·354	216 35 05·620
	Ben Cruachan	(V)	267 14 12·12	+15·245	267 14 27·365	+0·050	267 14 27·415
Beinn Bhreac Mhor (M_1)	Carn nan-tri-tighearnan	(R_1)	00 00 00·00	+ 6·180	00 00 06·180	+0·032	00 00 06·212
	Corryhabbie	(O_1)	44 23 43·76	+ 2·553	44 23 46·313	−0·157	44 23 46·156
	Ben Macdhui	(G_1)	86 49 31·32	− 6·450	86 49 24·870	+0·771	86 49 25·641
	Carn Gower	(B_1)	110 49 36·18	−14·462	110 49 21·718	−0·844	110 49 20·874
	Carn an Fhreiceadain	(N_1)	122 24 41·05	− 4·207	122 24 36·843	+0·084	122 24 36·927
	Ben Alder	(A_1)	163 39 02·93	−16·795	163 38 46·135	−0·625	163 38 45·510
	Carn Eige	(L_1)	239 24 49·29	+ 2·420	239 24 51·710	−1·260	239 24 50·450
	Beinn a' Bha' ach Ard	(W_1)	269 32 48·95	+ 8·538	269 32 57·488	+1·091	269 32 58·579
	Ben Wyvis	(X_1)	298 58 49·35	+17·112	298 59 06·462	+0·906	298 59 07·368
Beinn Mhor (I_2)	Heaval	(H_2)	00 00 00·00	−25·932	359 59 34·068	−0·224	359 59 33·844
	Marrival	(N_2)	157 36 50·95	+31·444	157 37 22·394	+0·412	157 37 22·806
	Healaval Beg	(J_2)	232 39 29·94	+ 8·580	232 39 38·520	+0·252	232 39 38·772
	Askival	(A_2)	279 09 26·26	−27·217	279 08 59·043	−0·108	279 08 58·935
	Ben Hogh	(I_1)	310 36 02·68	−56·690	310 35 05·990	−0·333	310 35 05·657

6.1 *continued*

From	*To*		*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Beinn na Caillich (B$_2$)	Healaval Beg	(J$_2$)	00° 00′ 00″·00	+12″·083	00° 00′ 12″·083	−0″·121	00° 00′ 11″·962
	Storr	(K$_2$)	44 17 38·81	+18·934	44 17 57·744	+0·064	44 17 57·808
	Beinn Bhan	(C$_2$)	106 15 44·31	+12·817	106 15 57·127	+0·570	106 15 57·697
	Carn Eige	(L$_1$)	150 10 07·76	+ 1·621	150 10 09·381	−0·615	150 10 08·766
	Sgurr na Ciche	(K$_1$)	194 51 44·90	−15·489	194 51 29·411	+0·156	194 51 29·567
	Meall nan Con	(J$_1$)	253 22 45·73	−33·940	253 22 11·790	+0·398	253 22 12·188
	Askival	(A$_2$)	279 54 59·32	−17·537	279 54 41·783	−0·452	279 54 41·331
Beinn Tart a' Mhill (S)	Sliabh Gaoil	(P)	00 00 00·00	+11·280	00 00 11·280	−0·695	00 00 10·585
	Cnoc Moy	(K)	62 00 34·58	−28·079	62 00 06·501	−0·062	62 00 06·439
	Ben Hynish	(H$_1$)	269 32 28·54	+60·404	269 33 28·944	−0·436	269 33 28·508
	Ben More (Mull)	(X)	308 18 59·27	+51·698	308 19 50·968	+0·007	308 19 50·975
	Jura	(T)	343 49 48·44	+12·242	343 50 00·682	+0·315	343 50 00·997
Ben Alder (A$_1$)	Carn an Fhreiceadain •	(N$_1$)	00 00 00·00	+12·742	00 00 12·742	+0·362	00 00 13·104
	Ben Macdhui	(G$_1$)	28 09 59·63	+ 9·181	28 10 08·811	+0·150	28 10 08·961
	Carn Gower	(B$_1$)	55 20 45·98	+ 0·454	55 20 46·434	−0·860	55 20 45·574
	Ben Lawers	(W)	122 21 51·47	−11·226	122 21 40·244	+0·132	122 21 40·376
	Ben Cruachan	(V)	192 49 52·51	−17·238	192 49 35·272	−0·379	192 49 34·893
	Ben Nevis	(Z)	235 58 05·22	− 0·232	235 58 04·988	+0·078	235 58 05·066
	Carn Eige	(L$_1$)	292 31 11·28	+22·392	292 31 33·672	−0·262	292 31 33·410
	Beinn Bhreac Mhor	(M$_1$)	347 42 37·76	+17·532	347 42 55·292	+0·779	347 42 56·071
Ben Cruachan (V)	Carra Duagh	(U)	00 00 00·00	−10·168	359 59 49·832	−0·134	359 59 49·698
	Ben More (Mull)	(X)	51 31 51·21	+ 1·400	51 31 52·610	+1·029	51 31 53·639
	Creach Bheinn	(Y)	102 33 51·02	+13·737	102 34 04·757	−0·583	102 34 04·174
	Ben Nevis	(Z)	152 09 47·65	+19·605	152 10 07·255	+0·580	152 10 07·835
	Ben Alder	(A$_1$)	184 38 39·20	+18·727	184 38 57·927	+0·039	184 38 57·966
	Ben Lawers	(W)	217 49 50·34	+ 4·824	217 49 55·164	−0·258	217 49 54·906
	Ben Lomond	(R)	271 39 37·22	−12·797	271 39 24·423	−0·360	271 39 24·063
	Beinn Bheula	(Q)	303 57 29·12	−15·476	303 57 13·644	−0·313	303 57 13·331
Beneraird (A)	Cairn Pat	(A$_4$)	00 00 00·093(1)	−10·633	359 59 49·460	−0·048	359 59 49·412
	Cnoc Moy	(K)	102 39 06·682(2)	+18·961	102 39 25·643	+0·748	102 39 26·391
	Ailsa Craig	(E)	129 00 42·562(2)	+10·271	129 00 52·833	−0·275	129 00 52·558
	Goat Fell	(L)	144 45 18·612(2)	+30·517	144 45 49·129	+0·377	144 45 49·506
	Brown Carrick	(F)	179 13 22·211(3)	+17·203	179 13 39·414	−0·691	179 13 38·723
	Merrick	(B)	234 06 16·109(1)	+ 3·144	234 06 19·253	−0·059	234 06 19·194
	Cairnsmore of Fleet	(B$_4$)	264 59 44·715(1)	− 5·050	264 59 39·665	−0·053	264 59 39·612
Ben Hogh (I$_1$)	Ben Hynish	(H$_1$)	00 00 00·00	−13·132	359 59 46·868	−0·423	359 59 46·445
	Heaval	(H$_2$)	79 31 18·65	+31·259	79 31 49·909	−0·027	79 31 49·882
	Beinn Mhor	(I$_2$)	103 04 10·00	+54·398	103 05 04·398	+0·225	103 05 04·623
	Askival	(A$_2$)	159 49 16·63	+25·834	159 49 42·464	−0·655	159 49 41·809
	Meall nan Con	(J$_1$)	202 47 39·38	+ 6·900	202 47 46·280	+0·760	202 47 47·040
	Ben More (Mull)	(X)	256 03 51·14	−17·113	256 03 34·027	+0·120	256 03 34·147

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(1) Fixed direction from Figure 3.
(2) Mean observed direction plus overlap correction from Figure 3.
(3) Mean of 1938 and 1951 observations plus adjustment correction from Figure 3.

6.1 *continued*

From	*To*		*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Ben Hutig (S_2)	Bad Mor	(T_2)	00° 00′ 00″00	− 3″387	359° 59′ 56″613	+0″310	359° 59′ 56″923
	Ben Klibreck	(F_2)	69 56 44·49	−12·939	69 56 31·551	−0·169	69 56 31·382
	Conival	(E_2)	104 52 57·88	−17·666	104 52 40·214	−0·189	104 52 40·025
	Creag Riabhach	(R_2)	164 11 05·41	− 0·580	164 11 04·830	+0·176	164 11 05·006
	Ward Hill	(Z_2)	319 17 02·47	+11·512	319 17 13·982	−0·104	319 17 13·878
	Dunnet Head	(W_2)	337 54 00·24	+ 3·517	337 54 03·757	−0·024	337 54 03·733
Ben Hynish (H_1)	Ben Hogh	(I_1)	00 00 00·00	+13·454	00 00 13·454	+0·381	00 00 13·835
	Meall nan Con	(J_1)	12 30 39·84	+20·227	12 31 00·067	−0·730	12 30 59·337
	Ben More (Mull)	(X)	47 18 37·15	− 5·080	47 18 32·070	−0·466	47 18 31·604
	Jura	(T)	91 00 16·71	−47·110	90 59 29·600	−0·741	90 59 28·859
	Beinn Tart a' Mhill	(S)	113 51 49·77	−62·103	113 50 47·667	+0·694	113 50 48·361
	Heaval	(H_2)	284 03 37·65	+46·963	284 04 24·613	+0·484	284 04 25·097
	Askival	(A_2)	347 45 36·62	+40·304	347 46 16·924	+0·377	347 46 17·301
Ben Klibreck (F_2)	Cnoc an t'Sabhail	(Y_1)	00 00 00·00	−16·700	359 59 43·300	−0·251	359 59 43·049
	Ben Wyvis	(X_1)	27 02 16·44	−22·662	27 01 53·778	+0·610	27 01 54·388
	Conival	(E_2)	86 18 12·90	− 3·807	86 18 09·093	−0·264	86 18 08·829
	Ben Hutig	(S_2)	188 16 20·38	+12·800	188 16 33·180	+0·219	188 16 33·399
	Bad Mor	(T_2)	254 28 23·55	+ 8·122	254 28 31·672	−0·436	254 28 31·236
	Scaraben	(G_2)	289 27 10·74	− 0·978	289 27 09·762	+0·297	289 27 10·059
	Col Bheinn	(Z_1)	318 05 28·48	− 6·294	318 05 22·186	−0·173	318 05 22·013
Ben Lawers (W)	Meall Dearg	(C_1)	359 59 59·848(1)	+ 0·022	359 59 59·870	−0·184	359 59 59·686
	Ben Cleugh	(O)	56 56 02·443(1)	−13·167	56 55 49·276	−0·172	56 55 49·104
	Ben Lomond	(R)	125 01 00·271(1)	−14·188	125 00 46·083	−0·163	125 00 45·920
	Ben Cruachan	(V)	169 12 32·420(2)	− 4·302	169 12 28·118	+0·678	169 12 28·796
	Ben Nevis	(Z)	212 39 24·560(2)	+11·496	212 39 36·056	+0·215	212 39 36·271
	Ben Alder	(A_1)	245 33 26·680(2)	+10·868	245 33 37·548	−0·210	245 33 37·338
	Carn Gower	(B_1)	316 40 18·567(1)	+10·073	316 40 28·640	−0·164	316 40 28·476
Ben Lomond (R)	Hill of Stake	(M)	00 00 00·309(1)	−16·790	359 59 43·519	+0·032	359 59 43·551
	Beinn Bheula	(Q)	64 43 23·45	− 1·956	64 43 21·494	−0·206	64 43 21·288
	Ben Cruachan	(V)	119 39 45·11	+12·105	119 39 57·215	+0·535	119 39 57·750
	Ben Nevis	(Z)	150 28 18·01	+29·435	150 28 47·445	−0·388	150 28 47·057
	Ben Lawers	(W)	201 38 30·653(1)	+15·062	201 38 45·715	+0·003	201 38 45·718
	Ben Cleugh	(O)	259 10 34·834(1)	− 0·823	259 10 34·011	+0·006	259 10 34·017
	Earls Seat	(N)	300 01 37·454(1)	− 7·556	300 01 29·898	+0·017	300 01 29·915
Ben Macdhui (G_1)	Glas Maol	(F_1)	359 59 59·956(1)	− 5·387	359 59 54·569	−0·205	359 59 54·364
	Carn Gower	(B_1)	42 37 52·290(1)	− 6·628	42 37 45·662	−0·207	42 37 45·455
	Ben Alder	(A_1)	99 43 16·346(2)	− 8·055	99 43 08·291	+0·152	99 43 08·443
	Beinn Bhreac Mhor	(M_1)	162 26 29·706(2)	+ 5·901	162 26 35·607	+0·077	162 26 35·684
	Carn nan-tri-tighearnan	(R_1)	196 02 04·906(2)	+10·819	196 02 15·725	+0·579	196 02 16·304
	Corryhabbie	(O_1)	262 48 09·954(1)	+ 6·919	262 48 16·873	−0·201	262 48 16·672
	Mount Battock	(P_1)	322 59 23·534(1)	− 3·020	322 59 20·514	−0·194	322 59 20·320

(1) Fixed direction from Figure 3.

(2) Mean observed direction plus overlap correction from Figure 3.

6.1 *continued*

From	*To*		*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Ben More (Mull) (X)	Meall nan Con	(J_1)	00° 00′ 00″·00	+22″·020	00° 00′ 22″·020	−0″·130	00° 00′ 21″·890
	Creach Bheinn	(Y)	58 06 18·34	+14·667	58 06 33·007	−0·375	58 06 32·632
	Ben Cruachan	(V)	96 19 29·32	− 1·521	96 19 27·799	−0·199	96 19 27·600
	Carra Duagh	(U)	125 26 23·50	−13·581	125 26 09·919	−1·177	125 26 08·742
	Jura	(T)	186 19 03·32	−36·542	186 18 26·778	+0·507	186 18 27·285
	Beinn Tart a'Mhill	(S)	206 05 32·30	−49·674	206 04 42·626	+0·244	206 04 42·870
	Ben Hynish	(H_1)	280 45 58·34	+ 4·748	280 46 03·088	+1·432	280 46 04·520
	Ben Hogh	(I_1)	309 31 18·37	+16·386	309 31 34·756	−0·304	309 31 34·452
Ben Nevis (Z)	Creach Bheinn	(Y)	00 00 00·00	− 6·664	359 59 53·336	+0·046	359 59 53·382
	Meall nan Con	(J_1)	22 00 09·69	− 1·628	22 00 08·062	−0·098	22 00 07·964
	Sgurr na Ciche	(K_1)	68 32 48·02	+12·354	68 33 00·374	+0·297	68 33 00·671
	Carn Eige	(L_1)	110 12 53·57	+25·678	110 13 19·248	+0·116	110 13 19·364
	Ben Alder	(A_1)	203 43 26·08	+ 0·247	203 43 26·327	+0·165	203 43 26·492
	Ben Lawers	(W)	237 13 13·28	−12·677	237 13 00·603	+0·132	237 13 00·735
	Ben Lomond	(R)	278 24 43·19	−30·591	278 24 12·599	−0·875	278 24 11·724
	Ben Cruachan	(V)	308 06 25·24	−19·270	308 06 05·970	+0·218	308 06 06·188
Ben Wyvis (X_1)	Beinn a' Bha' ach Ard	(W_1)	00 00 00·00	− 9·896	359 59 50·104	+0·632	359 59 50·736
	Carn Eige	(L_1)	16 28 09·11	−17·611	16 27 51·499	−0·708	16 27 50·791
	Anteallach	(D_2)	89 44 01·05	+ 6·754	89 44 07·804	+0·848	89 44 08·652
	Conival	(E_2)	140 25 55·79	+20·747	140 26 16·537	−0·189	140 26 16·348
	Ben Klibreck	(F_2)	168 52 32·02	+23·297	168 52 55·317	+0·459	168 52 55·776
	Cnoc an t' Sabhail	(Y_1)	220 21 51·50	+ 4·895	220 21 56·395	+0·247	220 21 56·642
	Carn nan-tri-tighearnan	(R_1)	286 48 56·16	−10·523	286 48 45·637	−0·333	286 48 45·304
	Beinn Bhreac Mhor	(M_1)	313 44 18·92	−17·992	313 44 00·928	−0·955	313 43 59·973
Brassa (G_3)	Yell	(I_3)	00 00 00·00	− 5·889	359 59 54·111	+0·436	359 59 54·547
	Fetlar	(J_3)	12 32 26·59	− 7·520	12 32 19·070	+0·246	12 32 19·316
	Fair Isle	(E_3)	204 30 38·61	+ 6·685	204 30 45·295	−0·646	204 30 44·649
	Foula	(F_3)	271 03 20·64	− 0·063	271 03 20·577	+0·145	271 03 20·722
	Ronas Hill	(H_3)	336 25 52·10	− 4·946	336 25 47·154	−0·182	336 25 46·972
Brown Carrick (F)	Beneraird	(A)	00 00 00·00	−16·735	359 59 43·265	−1·877	359 59 41·388
	Ailsa Craig	(E)	37 02 20·28	− 7·364	37 02 12·916	+0·112	37 02 13·028
	Cnoc Moy	(K)	67 47 36·15	− 0·348	67 47 35·802	+0·177	67 47 35·979
	Goat Fell	(L)	109 36 42·59	+11·751	109 36 54·341	+1·021	109 36 55·362
	Hill of Stake	(M)	157 10 58·82	+20·485	157 11 19·305	+0·929	157 11 20·234
	Corse Hill	(J)	204 17 50·26	+12·447	204 18 02·707	+1·144	204 18 03·851
	Cairnsmore of Deugh	(C)	278 25 45·87	− 7·336	278 25 38·534	−1·506	278 25 37·028
Cairn Pat (A_4)	Cnoc Moy	(K)	162 10 28·364(2)	+31·310	162 10 59·674	+0·812	162 11 00·486
	Goat Fell	(L)	194 56 22·064(2)	+42·560	194 57 04·624	−0·654	194 57 03·970
	Ailsa Craig	(E)	195 11 11·054(2)	+21·621	195 11 32·675	+0·400	195 11 33·075
	Beneraird	(A)	220 51 32·459(1)	+10·804	220 51 43·263	−0·184	220 51 43·079
	Merrick	(B)	251 12 13·634(1)	+13·510	251 12 27·144	−0·187	251 12 26·957
	Cairnsmore of Fleet	(B_4)	275 18 25·968(1)	+ 4·892	275 18 30·860	−0·186	275 18 30·674

(1) Fixed direction from Figure 3.

(2) Mean observed direction plus overlap correction from Figure 3.

6.1 *continued*

From	*To*	*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Cairnsmore of Deugh (C)	Cairnsmore of Fleet (B$_4$)	359° 59′ 59″·323(1)	−11″·244	359° 59′ 48″·079	−0″·240	359° 59′ 47″·839
	Merrick (B)	36 34 05·518(3)	− 4·602	36 34 00·916	−0·429	36 34 00·487
	Ailsa Craig (E)	75 06 31·200(2)	+ 0·747	75 06 31·947	+1·991	75 06 33·938
	Brown Carrick (F)	103 17 48·774(3)	+ 6·865	103 17 55·639	−0·103	103 17 55·536
	Corse Hill (J)	163 44 33·724(1)	+17·234	163 44 50·958	−0·248	163 44 50·710
	Cairn Table (G)	189 36 43·462(1)	+ 9·047	189 36 52·509	−0·232	189 36 52·277
	Tinto (H)	207 50 45·410(1)	+11·844	207 50 57·254	−0·249	207 50 57·005
	Hart Fell (D)	236 34 25·831(1)	+ 4·860	236 34 30·691	−0·254	236 34 30·437
	Criffell (C$_4$)	298 09 26·148(1)	−11·750	298 09 14·398	−0·235	298 09 14·163
Carn nan-tri-tighearnan (R$_1$)	Beinn Bhreac Mhor (M$_1$)	00 00 00·00	− 5·945	359 59 54·055	−0·267	359 59 53·788
	Beinn a' Bha' ach Ard (W$_1$)	58 23 06·36	+ 1·501	58 23 07·861	−0·200	58 23 07·661
	Ben Wyvis (X$_1$)	92 03 30·18	+ 9·631	92 03 39·811	+0·465	92 03 40·276
	Cnoc an t' Sabhail (Y$_1$)	129 30 36·43	+13·076	129 30 49·506	+0·659	129 30 50·165
	Findlays Seat (S$_1$)	212 50 12·96	+ 4·145	212 50 17·105	−1·069	212 50 16·036
	Corryhabbie (O$_1$)	245 24 47·89	− 2·637	245 24 45·253	+0·426	245 24 45·679
	Ben Macdhui (G$_1$)	300 25 05·23	−11·380	300 24 53·850	−0·013	300 24 53·837
Carn Eige (L$_1$)	Beinn a' Bha' ach Ard (W$_1$)	00 00 00·00	+ 7·861	00 00 07.861	+0·736	00 00 08·597
	Beinn Bhreac Mhor (M$_1$)	42 37 38·70	− 2·717	42 37 35·983	+0·125	42 37 36·108
	Ben Alder (A$_1$)	91 40 33·79	−24·100	91 40 09·690	−1·182	91 40 08·508
	Ben Nevis (Z)	121 36 59·08	−25·877	121 36 33·203	−0·167	121 36 33·036
	Sgurr na Ciche (K$_1$)	162 59 03·62	−14·564	162 58 49·056	+0·364	162 58 49·420
	Beinn na Caillich (B$_2$)	212 56 16·92	− 1·494	212 56 15·426	+0·003	212 56 15·429
	Beinn Bhan (C$_2$)	246 35 47·82	+ 9·458	246 35 57·278	+0·138	246 35 57·416
	Anteallach (D$_2$)	300 44 32·64	+27·890	300 45 00·530	+0·055	300 45 00·585
	Ben Wyvis (X$_1$)	344 55 25·10	+18·818	344 55 43·918	−0·073	344 55 43·845
Carn Gower (B$_1$)	Kings Seat (D$_1$)	00 00 00·089(1)	− 9·591	359 59 50·498	+0·036	359 59 50·534
	Meall Dearg (C$_1$)	47 43 52·943(1)	− 8·484	47 43 44·459	+0·025	47 43 44·484
	Ben Lawers (W)	79 25 35·488(1)	− 9·176	79 25 26·312	+0·012	79 25 26·324
	Ben Alder (A$_1$)	121 17 40·808(2)	− 0·401	121 17 40·407	−0·022	121 17 40·385
	Carn an Fhreiceadain (N$_1$)	177 05 55·068(2)	+ 9·544	177 06 04·612	+0·053	177 06 04·665
	Beinn Bhreac Mhor (M$_1$)	180 50 13·138(2)	+13·311	180 50 26·449	−0·204	180 50 26·245
	Ben Macdhui (G$_1$)	217 01 34·096(1)	+ 6·668	217 01 40·764	+0·019	217 01 40·783
	Glas Maol (F$_1$)	293 09 05·736(1)	+ 0·824	293 09 06·560	+0·044	293 09 06·604
	Craigowl (E$_1$)	342 09 22·252(1)	− 7·514	342 09 14·738	+0·038	342 09 14·776
Carra Duagh (U)	Beinn Bheula (Q)	00 00 00·00	− 6·111	359 59 53·889	−0·650	359 59 53·239
	Sliabh Gaoil (P)	77 05 40·95	−19·450	77 05 21·500	−0·170	77 05 21·330
	Jura (T)	113 39 55·46	−20·014	113 39 35·446	−0·641	113 39 34·805
	Ben More (Mull) (X)	187 20 22·61	+12·875	187 20 35·485	+1·000	187 20 36·485
	Creach Bheinn (Y)	242 48 04·26	+25·357	242 48 29·617	+0·128	242 48 29·745
	Ben Cruachan (V)	286 41 40·60	+10·469	286 41 51·069	+0·333	286 41 51·402

(1) Fixed direction from Figure 3.

(2) Mean observed direction plus overlap correction from Figure 3.

(3) Mean of 1938 and 1951 observations plus adjustment correction from Figure 3.

6.1 *continued*

From	*To*		*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Clisham (O_2)	Muirnag	(Q_2)	00° 00′ 00″·00	+28″·822	00° 00′ 28″·822	+0″·377	00° 00′ 29″·199
	Point of Stoer	(M_2)	34 29 09·64	+17·650	34 29 27·290	+0·344	34 29 27·634
	An Cuaidh	(L_2)	68 38 35·51	−12·160	68 38 23·350	−0·556	68 38 22·794
	Storr	(K_2)	109 27 55·37	−36·817	109 27 18·553	−0·218	109 27 18·335
	Healaval Beg	(J_2)	135 55 01·85	−46·474	135 54 15·376	+0·217	135 54 15·593
	Marrival	(N_2)	184 56 54·09	−27·912	184 56 26·178	−0·346	184 56 25·832
	Mealisval	(P_2)	288 05 45·98	+14·431	288 06 00·411	+0·181	288 06 00·592
Cnoc an t' Sabhail (Y_1)	Col Bheinn	(Z_1)	00 00 00·00	+ 9·071	00 00 09·071	+0·089	00 00 09·160
	Findlays Seat	(S_1)	87 28 35·39	− 7·458	87 28 27·932	+0·184	87 28 28·116
	Carn nan-tri-tighearnan	(R_1)	137 33 25·62	−13·441	137 33 12·179	−0·271	137 33 11·908
	Ben Wyvis	(X_1)	213 39 18·10	− 4·604	213 39 13·496	−0·139	213 39 13·357
	Conival	(E_2)	283 20 45·50	+13·714	283 20 59·214	+0·360	283 20 59·574
	Ben Klibreck	(F_2)	315 07 45·23	+16·146	315 08 01·376	−0·224	315 08 01·152
Cnoc Moy (K)	Goat Fell	(L)	00 00 00·00	+15·060	00 00 15·060	−0·572	00 00 14·488
	Brown Carrick	(F)	34 05 16·09	+ 0·388	34 05 16·478	−0·124	34 05 16·354
	Ailsa Craig	(E)	55 24 05·20	− 8·789	55 23 56·411	−0·655	55 23 55·756
	Beneraird	(A)	69 43 30·09	−20·585	69 43 09·505	−0·075	69 43 09·430
	Cairn Pat	(A_4)	88 23 22·96	−33·461	88 22 49·499	+0·359	88 22 49·858
	Beinn Tart a' Mhill	(S)	260 51 55·96	+26·667	260 52 22·627	+0·958	260 52 23·585
	Jura	(T)	293 56 41·64	+36·682	293 57 18·322	+0·292	293 57 18·614
	Sliabh Gaoil	(P)	324 03 30·68	+34·639	324 04 05·319	−0·183	324 04 05·136
Col Bheinn (Z_1)	Cnoc an t' Sabhail	(Y_1)	00 00 00·00	− 8·669	359 59 51·331	+0·265	359 59 51·596
	Ben Klibreck	(F_2)	93 13 16·32	+ 5·816	93 13 22·136	+0·415	93 13 22·551
	Scaraben	(G_2)	199 50 12·30	+ 4·227	199 50 16·527	−0·343	199 50 16·184
	Findlays Seat	(S_1)	297 15 27·41	−14·065	297 15 13·345	−0·337	297 15 13·008
Conival (E_2)	Cnoc an t' Sabhail	(Y_1)	00 00 00·00	−15·063	359 59 44·937	+0·926	359 59 45·863
	Ben Wyvis	(X_1)	30 22 41·28	−21·441	30 22 19·839	−0·487	30 22 19·352
	Anteallach	(D_2)	80 57 20·95	−15·970	80 57 04·980	−0·228	80 57 04·752
	Point of Stoer	(M_2)	164 44 02·52	+ 6·650	164 44 09·170	−0·290	164 44 08·880
	Creag Riabhach	(R_2)	224 22 54·17	+18·920	224 23 13·090	+0·388	224 23 13·478
	Ben Hutig	(S_2)	254 59 28·05	+18·566	254 59 46·616	−0·183	254 59 46·433
	Ben Klibreck	(F_2)	298 05 09·30	+ 4·044	298 05 13·344	−0·124	298 05 13·220
Corryhabbie (O_1)	Carn nan-tri-tighearnan	(R_1)	00 00 00·113(2)	+ 2·244	00 00 02·357	−0·283	00 00 02·074
	Findlays Seat	(S_1)	72 28 31·455(1)	+ 4·790	72 28 36·245	+0·105	72 28 36·350
	Ben Aigan	(T_1)	86 01 07·453(2)	+ 3·469	86 01 10·922	+0·208	86 01 11·130
	Bin of Cullen	(U_1)	106 49 46·623(2)	+ 5·846	106 49 52·469	−0·931	106 49 51·538
	Knock	(V_1)	121 44 20·920(1)	+ 4·217	121 44 25·137	+0·091	121 44 25·228
	Bennachie	(Q_1)	176 38 41·526(1)	− 0·960	176 38 40·566	+0·095	176 38 40·661
	Mount Battock	(P_1)	226 17 46·048(1)	− 7·073	226 17 38·975	+0·113	226 17 39·088
	Glas Maol	(F_1)	269 46 18·819(1)	−10·018	269 46 08·801	+0·097	269 46 08·898
	Ben Macdhui	(G_1)	301 46 16·681(1)	− 6·182	301 46 10·499	+0·100	301 46 10·599
	Carn an Fhreiceadain	(N_1)	326 06 14·583(2)	− 4·972	326 06 09·611	+0·393	326 06 10·004
	Beinn Bhreac Mhor	(M_1)	338 58 52·213(2)	− 2·096	338 58 50·117	+0·010	338 58 50·127

(1) Fixed direction from Figure 3.

(2) Mean observed direction plus overlap correction from Figure 3.

6.1 *continued*

From	*To*		*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Corse Hill (J)	Cairn Table	(G)	00° 00′ 00″·229(1)	− 7″·656	359° 59′ 52″·573	+0″·008	359° 59′ 52″·581
	Cairnsmore of Deugh	(C)	29 58 55·753(1)	−17·217	29 58 38·536	+0·010	29 58 38·546
	Brown Carrick	(F)	75 24 21·839(1)	−11·637	75 24 10·202	−0·007	75 24 10·195
	Goat Fell	(L)	114 52 04·545(2)	− 1·998	114 52 02·547	−0·025	114 52 02·522
	Hill of Stake	(M)	146 29 11·228(1)	+ 6·322	146 29 17·550	−0·016	146 29 17·534
	Earls Seat	(N)	205 08 15·073(1)	+13·336	205 08 28·409	+0·008	205 08 28·417
	Ben Cleugh	(O)	238 50 03·793(1)	+17·827	238 50 21·620	+0·002	238 50 21·622
	Black Mount	(I)	300 06 18·492(1)	− 0·159	300 06 18·333	+0·003	300 06 18·336
	Tinto	(H)	318 21 23·227(1)	− 3·932	318 21 19·295	+0·017	318 21 19·312
Creach Bheinn (Y)	Carra Duagh	(U)	00 00 00·00	−25·446	359 59 34·554	−0·034	359 59 34·520
	Ben More (Mull)	(X)	57 12 18·49	−13·953	57 12 04·537	+0·612	57 12 05·149
	Meall nan Con	(J_1)	108 37 49·70	+ 5·982	108 37 55·682	−0·354	108 37 55·328
	Sgurr na Ciche	(K_1)	187 17 22·49	+20·932	187 17 43·422	−0·125	187 17 43·297
	Ben Nevis	(Z)	247 56 54·53	+ 7·004	247 57 01·534	−0·027	247 57 01·507
	Ben Cruachan	(V)	326 27 24·92	−14·194	326 27 10·726	−0·074	326 27 10·652
Creag Riabhach (R_2)	Point of Stoer	(M_2)	00 00 00·00	−13·357	359 59 46·643	−0·045	359 59 46·598
	Muirnag	(Q_2)	37 37 51·16	− 7·468	37 37 43·692	+0·349	37 37 44·041
	Ben Hutig	(S_2)	224 53 36·28	+ 0·613	224 53 36·893	−0·075	224 53 36·818
	Conival	(E_2)	314 58 58·12	−19·010	314 58 39·110	−0·229	314 58 38·881
Deerness (B_3)	South Ronaldsay	(A_3)	00 00 00·00	− 2·223	359 59 57·777	+0·119	359 59 57·896
	Ward Hill	(Z_2)	50 12 16·25	− 0·708	50 12 15·542	+0·691	50 12 16·233
	Fitty Hill	(C_3)	128 25 46·44	+ 4·528	128 25 50·968	+0·329	128 25 51·297
	Stronsay	(D_3)	186 03 39·87	+ 1·559	186 03 41·429	−0·598	186 03 40·831
	Fair Isle	(E_3)	192 53 02·47	+ 3·640	192 53 06·110	−0·116	192 53 05·994
	Warth Hill	(Y_2)	356 35 03·83	− 4·718	356 34 59·112	−0·425	356 34 58·687
Dunnet Head (W_2)	Warth Hill	(Y_2)	00 00 00·00	− 1·244	359 59 58·756	−0·063	359 59 58·693
	Hill of Yarrows	(V_2)	53 05 00·26	− 6·521	53 04 53·739	+0·392	53 04 54·131
	Spital Hill	(U_2)	78 21 09·84	− 4·262	78 21 05·578	−0·706	78 21 04·872
	Bad Mor	(T_2)	112 06 35·92	− 4·688	112 06 31·232	+0·270	112 06 31·502
	Ben Hutig	(S_2)	148 36 46·08	− 2·886	148 36 43·194	−0·457	148 36 42·737
	Ward Hill	(Z_2)	253 23 06·29	+ 5·124	253 23 11·414	+0·192	253 23 11·606
	South Ronaldsay	(A_3)	312 17 06·55	+ 2·184	312 17 08·734	+0·374	312 17 09·108
Fair Isle (E_3)	Brassa	(G_3)	00 00 00·00	− 5·064	359 59 54·936	+0·654	359 59 55·590
	Deerness	(B_3)	199 49 11·23	− 0·080	199 49 11·150	+0·257	199 49 11·407
	Stronsay	(D_3)	201 41 04·44	+ 0·447	201 41 04·887	+0·589	201 41 05·476
	Fitty Hill	(C_3)	225 36 25·03	− 0·370	225 36 24·660	−1·220	225 36 23·440
	Foula	(F_3)	314 13 05·03	− 2·031	314 13 02·999	−0·280	314 13 02·719
Fetlar (J_3)	Yell	(I_3)	00 00 00·00	+ 1·240	00 00 01·240	+0·517	00 00 01·757
	Ronas Hill	(H_3)	17 11 26·43	+ 1·311	17 11 27·741	−0·119	17 11 27·622
	Saxavord	(L_3)	126 58 11·88	− 3·652	126 58 08·228	−0·035	126 58 08·193
	Balta	(K_3)	140 04 37·07	− 2·357	140 04 34·713	−0·152	140 04 34·561
	Brassa	(G_3)	317 02 58·14	+ 8·071	317 03 06·211	−0·210	317 03 06·001

(1) Fixed direction from Figure 3.

(2) Mean observed direction plus overlap correction from Figure 3.

6.1 *continued*

From	*To*		*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Findlays Seat	Cnoc an t' Sabhail	(Y_1)	00° 00′ 00″·00	+ 6″·245	00° 00′ 06″·245	+0″·504	00° 00′ 06″·749
(S_1)	Col Bheinn	(Z_1)	29 46 56·37	+12·298	29 47 08·668	+0·538	29 47 09·206
	Scaraben	(G_2)	48 30 12·51	+14·667	48 30 27·177	−0·421	48 30 26·756
	Corryhabbie	(O_1)	238 27 26·18	− 4·840	238 27 21·340	−1·007	238 27 20·333
	Carn nan-tri-tighearnan	(R_1)	313 24 19·59	− 3·563	313 24 16·027	+0·386	313 24 16·413
Fitty Hill	Stronsay	(D_3)	00 00 00·00	− 2·660	359 59 57·340	+0·276	359 59 57·616
(C_3)	Deerness	(B_3)	29 38 55·61	− 4·968	29 38 50·642	−0·722	29 38 49·920
	Ward Hill	(Z_2)	75 15 47·41	− 6·871	75 15 40·539	−0·408	75 15 40·131
	Foula	(F_3)	258 42 21·60	+ 9·517	258 42 31·117	−0·361	258 42 30·756
	Fair Isle	(E_3)	299 53 13·19	+ 2·243	299 53 15·433	+1·216	299 53 16·649
Foula	Brassa	(G_3)	00 00 00·00	+ 0·026	00 00 00·026	+0·038	00 00 00·064
(F_3)	Fair Isle	(E_3)	67 40 30·26	+ 0·578	67 40 30·838	+0·282	67 40 31·120
	Fitty Hill	(C_3)	117 53 10·97	− 5·384	117 53 05·586	+0·361	117 53 05·947
	Ronas Hill	(H_3)	308 17 47·49	− 0·744	308 17 46·746	+0·019	308 17 46·765
	Yell	(I_3)	319 41 30·23	− 1·523	319 41 28·707	−0·699	319 41 28·008
Goat Fell	Sliabh Gaoil	(P)	00 00 00·00	+17·100	00 00 17·100	+0·445	00 00 17·545
(L)	Hill of Stake	(M)	80 34 31·19	+10·401	80 34 41·591	+0·040	80 34 41·631
	Corse Hill	(J)	113 11 40·35	+ 2·249	113 11 42·599	−0·837	113 11 41·762
	Brown Carrick	(F)	159 02 53·04	−12·382	159 02 40·658	+0·289	159 02 40·947
	Ailsa Craig	(E)	204 02 15·04	−21·112	204 01 53·928	−0·821	204 01 53·107
	Cnoc Moy	(K)	263 08 33·81	−14·217	263 08 19·593	+0·104	263 08 19·697
	Jura	(T)	331 56 24·38	+18·384	331 56 42·764	+0·781	331 56 43·545
Healaval Beg	Beinn na Caillich	(B_2)	00 00 00·00	−12·684	359 59 47·316	+0·319	359 59 47·635
(J_2)	Askival	(A_2)	43 39 05·77	−32·337	43 38 33·433	−0·033	43 38 33·400
	Heaval	(H_2)	115 17 23·38	−32·019	115 16 51·361	+0·427	115 16 51·788
	Beinn Mhor	(I_2)	138 22 53·25	− 8·190	138 22 45·060	+0·058	138 22 45·118
	Marrival	(N_2)	187 04 27·03	+20·519	187 04 47·549	+0·611	187 04 48·160
	Clisham	(O_2)	237 10 23·28	+46·091	237 11 09·371	−0·397	237 11 08·974
	Storr	(K_2)	309 42 05·74	+ 8·033	309 42 13·773	−0·292	309 42 13·481
	Beinn Bhan	(C_2)	330 32 15·50	+ 1·835	330 32 17·335	−0·693	330 32 16·642
Heaval	Beinn Mhor	(I_2)	00 00 00·00	+26·281	00 00 26·281	+0·673	00 00 26·954
(H_2)	Healaval Beg	(J_2)	29 34 05·28	+33·992	29 34 39·272	−0·721	29 34 38·551
	Askival	(A_2)	70 59 00·61	− 3·288	70 58 57·322	−0·788	70 58 56·534
	Ben Hogh	(I_1)	107 03 16·96	−33·014	107 02 43·946	+0·079	107 02 44·025
	Ben Hynish	(H_1)	131 35 39·80	−48·412	131 34 51·388	+0·462	131 34 51·850
	Marrival	(N_2)	348 04 58·27	+58·582	348 05 56·852	+0·295	348 05 57·147
Hill of Stake	Brown Carrick	(F)	00 00 00·188(1)	−20·524	359 59 39·664	+0·287	359 59 39·951
(M)	Goat Fell	(L)	53 57 26·616(2)	− 9·890	53 57 16·726	−0·962	53 57 15·764
	Sliabh Gaoil	(P)	105 04 35·056(2)	+ 5·340	105 04 40·396	+0·289	105 04 40·685
	Beinn Bheula	(Q)	162 37 35·666(2)	+15·792	162 37 51·458	−0·294	162 37 51·164
	Ben Lomond	(R)	194 24 31·182(3)	+17·105	194 24 48·287	+0·132	194 24 48·419
	Earls Seat	(N)	236 08 54·875(1)	+ 8·567	236 09 03·442	+0·281	236 09 03·723
	Corse Hill	(J)	298 11 37·418(1)	− 6·776	298 11 30·642	+0·265	298 11 30·907

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(1) Fixed direction from Figure 3.

(2) Mean observed direction plus overlap correction from Figure 3.

(3) Mean of 1938 and 1951 observations plus adjustment correction from Figure 3.

6.1 *continued*

From	*To*		*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Hill of Yarrows	Spital Hill	(U_2)	00° 00′ 00″00	+ 2″427	00° 00′ 02″427	+0″131	00° 00′ 02″558
(V_2)	Dunnet Head	(W_2)	29 53 41·30	+ 6·262	29 53 47·562	+0·108	29 53 47·670
	Warth Hill	(Y_2)	60 28 41·67	+ 4·649	60 28 46·319	−0·100	60 28 46·219
	Scaraben	(G_2)	280 14 13·80	− 3·151	280 14 10·649	−0·636	280 14 10·013
	Bad Mor	(T_2)	337 22 07·94	+ 2·491	337 22 10·431	+0·498	337 22 10·929
Jura	Sliabh Gaoil	(P)	00 00 00·00	− 0·441	359 59 59·559	−0·053	359 59 59·506
(T)	Goat Fell	(L)	32 49 42·07	−19·775	32 49 22·295	−0·708	32 49 21·587
	Cnoc Moy	(K)	77 58 38·79	−37·252	77 58 01·538	+0·326	77 58 01·864
	Beinn Tart a' Mhill	(S)	146 43 13·73	−11·807	146 43 01·923	−0·529	146 43 01·394
	Ben Hynish	(H_1)	229 34 25·89	+44·195	229 35 10·085	+0·191	229 35 10·276
	Ben More (Mull)	(X)	271 25 58·71	+36·678	271 26 35·388	+0·399	271 26 35·787
	Carra Duagh	(U)	316 52 54·00	+21·190	316 53 15·190	+0·374	316 53 15·564
Marrival	Healaval Beg	(J_2)	00 00 00·00	−21·496	359 59 38·504	−0·441	359 59 38·063
(N_2)	Beinn Mhor	(I_2)	56 15 50·59	−31·443	56 15 19·147	−0·092	56 15 19·055
	Heaval	(H_2)	66 43 58·14	−57·802	66 43 00·338	−0·052	66 43 00·286
	Mealisval	(P_2)	256 45 08·24	+45·003	256 45 53·243	−0·057	256 45 53·186
	Clisham	(O_2)	279 07 39·11	+29·000	279 08 08·110	+1·005	279 08 09·115
	Storr	(K_2)	339 21 06·17	−11·998	339 20 54·172	−0·364	339 20 53·808
Mealisval	Marrival	(N_2)	00 00 00·00	−43·976	359 59 16·024	+0·140	359 59 16·164
(P_2)	Muirnag	(Q_2)	223 53 46·62	+15·643	223 54 02·263	+0·064	223 54 02·327
	Clisham	(O_2)	305 31 21·71	−14·653	305 31 07·057	−0·204	305 31 06·853
Meall nan Con	Ben Hogh	(I_1)	00 00 00·00	− 6·626	359 59 53·374	−0·587	359 59 52·787
(J_1)	Askival	(A_2)	85 03 56·30	+17·365	85 04 13·665	+0·141	85 04 13·806
	Beinn na Caillich	(B_2)	117 20 11·84	+34·395	117 20 46·235	−0·006	117 20 46·229
	Sgurr na Ciche	(K_1)	161 40 51·90	+17·068	161 41 08·968	+0·323	161 41 09·291
	Ben Nevis	(Z)	194 35 47·32	+ 1·802	194 35 49·122	−0·108	194 35 49·014
	Creach Bheinn	(Y)	213 16 34·84	− 6·307	213 16 28·533	−0·281	213 16 28·252
	Ben More (Mull)	(X)	283 44 48·79	−22·085	283 44 26·705	+0·626	283 44 27·331
	Ben Hynish	(H_1)	349 42 56·76	−18·960	349 42 37·800	−0·107	349 42 37·693
Merrick	Cairnsmore of						
(B)	Deugh	(C)	359 59 59·506(1)	+ 4·777	00 00 04·283	+0·232	00 00 04·515
	Criffell	(C_4)	60 48 19·904(1)	− 8·372	60 48 11·532	+0·250	60 48 11·782
	Cairnsmore of Fleet	(B_4)	104 53 16·934(1)	− 7·240	104 53 09·694	+0·271	104 53 09·965
	Cairn Pat	(A_4)	179 25 11·490(3)	−12·566	179 24 58·924	−0·433	179 24 58·491
	Beneraird	(A)	203 10 47·127(1)	− 2·970	203 10 44·157	+0·239	203 10 44·396
	Ailsa Craig	(E)	235 58 57·864(2)	+ 6·180	235 59 04·044	−0·564	235 59 03·480
	Cnoc Moy	(K)	236 41 46·584(2)	+13·868	236 42 00·452	−0·541	236 41 59·911
	Goat Fell	(L)	268 47 09·304(2)	+24·353	268 47 33·657	+0·288	268 47 33·945
	Cairn Table	(G)	344 12 12·041(1)	+14·427	344 12 26·468	+0·256	344 12 26·724

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(1) Fixed direction from Figure 3.

(2) Mean observed direction plus overlap correction from Figure 3.

(3) Mean of 1938 and 1951 observations plus adjustment correction from Figure 3.

6.1 *continued*

From	*To*		*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Muirnag	Point of Stoer	(M_2)	00° 00′ 00″·00	− 8″·497	359° 59′ 51″·503	+0″·201	359° 59′ 51″·704
(Q_2)	An Cuaidh	(L_2)	49 34 40·31	−36·703	49 34 03·607	−0·436	49 34 03·171
	Clisham	(O_2)	113 02 29·67	−27·681	113 02 01·989	−0·113	113 02 01·876
	Mealisval	(P_2)	139 30 43·55	−14·798	139 30 28·752	−0·009	139 30 28·743
	Creag Riabhach	(R_2)	334 32 08·17	+ 8·468	334 32 16·638	+0·357	334 32 16·995
Point of Stoer	An Cuaidh	(L_2)	00 00 00·00	−23·775	359 59 36·225	−0·104	359 59 36·121
(M_2)	Clisham	(O_2)	43 23 46·32	−15·666	43 23 30·654	+0·049	43 23 30·703
	Muirnag	(Q_2)	75 52 14·66	+ 7·846	75 52 22·506	−0·410	75 52 22·096
	Creag Riabhach	(R_2)	192 46 36·16	+14·001	192 46 50·161	−0·217	192 46 49·944
	Conival	(E_2)	268 06 44·23	− 7·004	268 06 37·226	+0·403	268 06 37·629
	Anteallach	(D_2)	325 06 13·29	−24·978	325 05 48·312	+0·280	325 05 48·592
Ronas Hill	Fetlar	(J_3)	00 00 00·00	− 1·043	359 59 58·957	+0·168	359 59 59·125
(H_3)	Yell	(I_3)	12 51 35·08	− 0·151	12 51 34·929	−0·058	12 51 34·871
	Brassa	(G_3)	83 45 01·17	+ 4·201	83 45 05·371	−0·211	83 45 05·160
	Foula	(F_3)	146 40 22·94	+ 2·069	146 40 25·009	+0·601	146 40 25·610
	Saxavord	(L_3)	332 05 30·29	− 3·468	332 05 26·822	−0·498	332 05 26·324
Saxavord	Yell	(I_3)	00 00 00·00	+ 4·685	00 00 04·685	−0·330	00 00 04·355
(L_3)	Ronas Hill	(H_3)	22 03 48·68	+ 4·379	22 03 53·059	+0·494	22 03 53·553
	Balta	(K_3)	317 12 40·30	+ 1·368	317 12 41·668	−0·178	317 12 41·490
	Fetlar	(J_3)	339 45 03·24	+ 3·670	339 45 06·910	+0·015	339 45 06·925
Scaraben	Col Bheinn	(Z_1)	00 00 00·00	− 3·984	359 59 56·016	+0·639	359 59 56·655
(G_2)	Ben Klibreck	(F_2)	44 44 51·43	+ 0·853	44 44 52·283	−1·214	44 44 51·069
	Bad Mor	(T_2)	117 37 04·22	+ 6·829	117 37 11·049	+0·092	117 37 11·141
	Spital Hill	(U_2)	150 31 51·92	+ 6·559	150 31 58·479	+0·265	150 31 58·744
	Hill of Yarrows	(V_2)	186 20 16·58	+ 3·461	186 20 20·041	+0·456	186 20 20·497
	Findlays Seat	(S_1)	296 08 27·10	−15·832	296 08 11·268	−0·239	296 08 11·029
Sgurr na Ciche	Meall nan Con	(J_1)	00 00 00·00	−16·109	359 59 43·891	+0·083	359 59 43·974
(K_1)	Askival	(A_2)	33 58 50·00	− 0·832	33 58 49·168	+0·228	33 58 49·396
	Beinn na Caillich	(B_2)	77 08 23·12	+14·813	77 08 37·933	+0·359	77 08 38·292
	Beinn Bhan	(C_2)	114 05 43·36	+26·074	114 06 09·434	−0·455	114 06 08·979
	Carn Eige	(L_1)	162 29 36·47	+15·115	162 29 51·585	−0·104	162 29 51·481
	Ben Nevis	(Z)	259 27 29·32	−12·921	259 27 16·399	+0·006	259 27 16·405
	Creach Bheinn	(Y)	310 15 11·85	−20·828	310 14 51·022	−0·117	310 14 50·905
Sliabh Gaoil	Carra Duagh	(U)	00 00 00·00	+19·675	00 00 19·675	+0·660	00 00 20·335
(P)	Beinn Bheula	(Q)	42 45 12·29	+12·618	42 45 24·908	−0·688	42 45 24·220
	Hill of Stake	(M)	92 16 08·74	− 5·772	92 16 02·968	−0·128	92 16 02·840
	Goat Fell	(L)	140 34 31·95	−17·576	140 34 14·374	−0·542	140 34 13·832
	Cnoc Moy	(K)	187 46 39·80	−33·606	187 46 06·194	+0·439	187 46 06·633
	Beinn Tart a' Mhill	(S)	242 34 39·12	−10·397	242 34 28·723	+0·506	242 34 29·229
	Jura	(T)	259 41 17·58	+ 0·421	259 41 18·001	−0·248	259 41 17·753

6.1 *continued*

From	*To*		*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
South Ronaldsay	Deerness	(B_3)	00° 00′ 00″·00	+ 2″·402	00° 00′ 02″·402	+0″·193	00° 00′ 02″·595
(A_3)	Warth Hill	(Y_2)	172 56 56·06	− 2·722	172 56 53·338	+0·444	172 56 53·782
	Dunnet Head	(W_2)	212 54 56·03	− 1·928	212 54 54·102	−0·279	212 54 53·823
	Ward Hill	(Z_2)	269 45 39·97	+ 2·134	269 45 42·104	−0·359	269 45 41·745
Spital Hill	Hill of Yarrows	(V_2)	00 00 00·00	− 2·566	359 59 57·434	−0·283	359 59 57·151
(U_2)	Scaraben	(G_2)	64 25 49·49	− 6·312	64 25 43·178	−0·325	64 25 42·853
	Bad Mor	(T_2)	133 00 04·85	− 0·131	133 00 04·719	+0·441	133 00 05·160
	Dunnet Head	(W_2)	235 09 48·22	+ 4·328	235 09 52·548	+0·456	235 09 53·004
	Warth Hill	(Y_2)	280 01 14·65	+ 2·750	280 01 17·400	−0·290	280 01 17·110
Storr	Beinn na Caillich	(B_2)	00 00 00·00	−19·209	359 59 40·791	+0·057	359 59 40·848
(K_2)	Healaval Beg	(J_2)	85 24 28·21	− 7·763	85 24 20·447	+0·401	85 24 20·848
	Marrival	(N_2)	122 08 00·67	+11·070	122 08 11·740	−0·468	122 08 11·272
	Clisham	(O_2)	166 25 43·83	+35·287	166 26 19·117	−0·035	166 26 19·082
	An Cuaidh	(L_2)	236 34 21·32	+21·430	236 34 42·750	+0·244	236 34 42·994
	Anteallach	(D_2)	261 09 59·57	+17·743	261 10 17·313	+0·066	261 10 17·379
	Beinn Bhan	(C_2)	305 19 32·42	− 5·479	305 19 26·941	−0·265	305 19 26·676
Stronsay	Deerness	(B_3)	00 00 00·00	− 1·400	359 59 58·600	+0·854	359 59 59·454
(D_3)	Ward Hill	(Z_2)	28 18 20·47	− 2·456	28 18 18·014	−0·137	28 18 17·877
	Fitty Hill	(C_3)	92 43 15·46	+ 2·185	92 43 17·645	−0·029	92 43 17·616
	Fair Isle	(E_3)	188 41 17·62	+ 1·755	188 41 19·375	−0·689	188 41 18·686
Ward Hill	South Ronaldsay	(A_3)	00 00 00·00	− 2·394	359 59 57·606	+0·347	359 59 57·953
(Z_2)	Warth Hill	(Y_2)	35 17 49·53	− 5·930	35 17 43·600	+0·371	35 17 43·971
	Dunnet Head	(W_2)	64 15 17·38	− 5·073	64 15 12·307	+0·222	64 15 12·529
	Bad Mor	(T_2)	85 02 49·76	−10·124	85 02 39·636	−0·476	85 02 39·160
	Ben Hutig	(S_2)	120 52 03·30	− 9·362	120 51 53·938	−0·134	120 51 53·804
	Fitty Hill	(C_3)	264 16 54·18	+ 7·593	264 17 01·773	+0·642	264 17 02·415
	Stronsay	(D_3)	304 36 17·56	+ 3·266	304 36 20·826	−0·665	304 36 20·161
	Deerness	(B_3)	320 26 36·59	+ 0·855	320 26 37·445	−0·305	320 26 37·140
Warth Hill	Dunnet Head	(W_2)	00 00 00·00	+ 1·151	00 00 01·151	+0·041	00 00 01·192
(Y_2)	Ward Hill	(Z_2)	44 25 40·66	+ 5·541	44 25 46·201	−0·653	44 25 45·548
	South Ronaldsay	(A_3)	92 19 08·92	+ 2·855	92 19 11·775	−0·208	92 19 11·567
	Deerness	(B_3)	95 57 15·96	+ 5·344	95 57 21·304	−0·133	95 57 21·171
	Hill of Yarrows	(V_2)	263 39 59·94	− 4·477	263 39 55·463	−0·283	263 39 55·180
	Spital Hill	(U_2)	303 12 33·53	− 2·506	303 12 31·024	+0·453	303 12 31·477
	Bad Mor	(T_2)	316 30 02·18	− 2·821	316 29 59·359	+0·784	316 30 00·143
Yell	Fetlar	(J_3)	00 00 00·00	− 1·153	359 59 58·847	−0·541	359 59 58·306
(I_3)	Brassa	(G_3)	124 30 32·02	+ 5·881	124 30 37·901	−0·121	124 30 37·780
	Foula	(F_3)	175 15 28·50	+ 3·648	175 15 32·148	−0·249	175 15 31·899
	Ronas Hill	(H_3)	210 02 59·51	+ 0·177	210 02 59·687	+0·230	210 02 59·917
	Saxavord	(L_3)	327 13 06·16	− 4·339	327 13 01·821	+0·351	327 13 02·172
	Balta	(K_3)	339 44 27·54	− 3·239	339 44 24·301	+0·330	339 44 24·631

6.2 Triangle misclosures and spherical excesses

Triangle	*Spherical Excess* (ϵ)	*Triangle Misclosure*	*Triangle*	*Spherical Excess* (ϵ)	*Triangle Misclosure*	*Triangle*	*Spherical Excess* (ϵ)	*Triangle Misclosure*
ABE	1″·784	+1″·836	ABA_4	1″·475	−1″·825	AEF	1″·901	−0″·141
AEK	1·747	−1·597	AEA_4	1·146	−0·906	AFK	6·344	−0·644
AKA_4	3·788	−0·108	BCE	1·890	−2·260	BEA_4	4·405	−0·895
CEF	2·471	+0·829	EFK	2·696	+1·094	EFL	2·906	−0·826
EKL	4·198	+0·812	EKA_4	4·389	−0·799	FJL	4·246	−1·346
FJM	3·827	−0·107	FKL	4·408	−1·108	FLM	3·415	+1·215
JLM	2·946	+2·504	KLP	4·292	−0·932	KLT	6·498	−0·848
KPS	8·178	+0·442	KPT	4·827	+0·783	KST	4·861	+1·899
LMP	3·273	−0·433	LPT	2·621	+0·699	MPQ	3·729	+0·191
MQR	2·035	−0·065	PQU	2·614	+0·406	PST	1·510	+2·240
PTU	2·941	−0·011	QRV	1·826	−0·386	QUV	1·873	+1·107
RWV	4·778	−0·328	RWZ	6·603	+0·117	RVZ	3·751	+0·769
STX	4·103	−0·973	STH_1	7·153	−2·033	SXH_1	11·307	−1·917
TUX	5·560	−3·300	TXH_1	8·257	−0·857	UVX	2·898	+0·482
UVY	2·235	+0·205	UXY	4·275	+1·025	VWZ	5·576	+1·214
VWA_1	4·745	+1·695	VXY	3·610	+0·750	VYZ	2·724	−0·944
VZA_1	3·389	+0·031	WZA_1	2·558	+0·512	WA_1B_1	3·700	−0·600
XYJ_1	3·196	+0·304	XH_1I_1	2·915	+3·125	XH_1J_1	4·910	+2·030
XI_1J_1	2·920	+1·680	YZJ_1	2·051	−0·011	YZK_1	2·817	−0·227
YJ_1K_1	3·706	+0·174	ZA_1L_1	4·584	−0·724	ZJ_1K_1	4·472	−0·042
ZK_1L_1	3·399	−0·459	$A_1B_1G_1$	3·085	+1·285	$A_1B_1M_1$	5·699	+1·601
$A_1G_1M_1$	4·740	+2·100	$A_1L_1M_1$	7·028	+0·902	$B_1G_1M_1$	2·126	+1·784
$G_1M_1O_1$	3·899	+0·411	$G_1M_1R_1$	2·277	−0·987	$G_1O_1R_1$	4·217	+2·573
$H_1I_1J_1$	0·925	+2·775	$H_1I_1A_2$	1·040	+0·690	$H_1I_1H_2$	4·516	−0·676
$H_1J_1A_2$	4·460	+0·520	$H_1A_2H_2$	10·424	−1·294	$I_1J_1A_2$	2·495	−2·945
$I_1A_2H_2$	6·948	+0·072	$I_1A_2I_2$	7·420	+1·550	$I_1H_2I_2$	5·398	+0·232
$J_1K_1A_2$	3·531	−0·381	$J_1K_1B_2$	4·856	−0·846	$J_1A_2B_2$	2·215	+1·595
$K_1L_1B_2$	3·738	+0·052	$K_1L_1C_2$	3·446	−0·776	$K_1A_2B_2$	3·540	+1·130
$K_1B_2C_2$	3·016	+1·534	$L_1M_1X_1$	6·462	−2·612	$L_1X_1D_2$	5·578	−0·938
$L_1B_2C_2$	2·724	+0·706	$L_1C_2D_2$	4·449	+0·231	$M_1R_1O_1$	2·595	+1·175
$M_1R_1X_1$	2·825	+0·765	$O_1R_1S_1$	2·959	−2·909	$R_1S_1Y_1$	5·106	+2·064
$R_1X_1Y_1$	3·136	+0·254	$S_1Y_1Z_1$	5·080	−0·730	$S_1Z_1G_2$	4·075	+0·075
$X_1Y_1E_2$	3·912	+0·478	$X_1Y_1F_2$	3·614	−0·564	$X_1D_2E_2$	4·495	+0·835
$X_1E_2F_2$	4·080	+0·590	$Y_1Z_1F_2$	2·996	−0·386	$Y_1E_2F_2$	3·782	−0·452
$Z_1F_2G_2$	2·068	+3·082	$A_2B_2J_2$	3·674	−0·224	$A_2H_2I_2$	5·870	+1·710
$A_2H_2J_2$	8·324	−0·254	$A_2I_2J_2$	5·426	+0·274	$B_2C_2J_2$	3·037	−2·677
$B_2C_2K_2$	2·155	−0·875	$B_2J_2K_2$	2·420	−1·140	$C_2D_2K_2$	3·672	−0·232
$C_2D_2L_2$	3·345	+0·705	$C_2J_2K_2$	1·538	+0·662	$C_2K_2L_2$	3·349	−1·439
$D_2E_2M_2$	3·442	+0·728	$D_2K_2L_2$	3·022	−0·502	$D_2L_2M_2$	3·801	+0·759
$E_2F_2S_2$	2·642	−0·522	$E_2M_2R_2$	3·082	−1·482	$E_2R_2S_2$	2·891	+0·359
$F_2G_2T_2$	3·381	−2·781	$F_2S_2T_2$	3·994	+1·796	$G_2T_2U_2$	1·218	−1·508
$G_2T_2V_2$	1·916	−1·956	$G_2U_2V_2$	1·266	−0·916	$H_2I_2J_2$	2·972	+2·238
$H_2I_2N_2$	1·284	−1·054	$H_2J_2N_2$	8·358	+0·442	$I_2J_2N_2$	4·102	−0·742
$J_2K_2N_2$	3·151	+1·849	$J_2K_2O_2$	4·665	−0·105	$J_2N_2O_2$	6·362	+3·018
$K_2L_2O_2$	6·658	−1·348	$K_2N_2O_2$	7·876	+1·064	$L_2M_2O_2$	8·181	+1·499
$L_2M_2Q_2$	7·105	+1·205	$L_2O_2Q_2$	7·920	+1·100	$M_2O_2Q_2$	6·844	+0·806
$M_2Q_2R_2$	4·921	−0·431	$N_2O_2P_2$	2·983	−1·933	$O_2P_2Q_2$	3·022	−0·032
$S_2T_2W_2$	3·030	+0·140	$S_2T_2Z_2$	6·087	−0·627	$S_2W_2Z_2$	4·274	−0·374
$T_2U_2V_2$	0·568	−0·468	$T_2U_2W_2$	0·889	−0·089	$T_2U_2Y_2$	0·580	+0·140
$T_2V_2W_2$	2·257	+1·303	$T_2V_2Y_2$	2·271	−0·841	$T_2W_2Y_2$	1·248	+1·572
$T_2W_2Z_2$	1·217	+0·393	$T_2Y_2Z_2$	3·586	+3·064	$U_2V_2W_2$	0.800	+1·860

6.2 *continued*

Triangle	*Spherical Excess (ε)*	*Triangle Misclosure*	*Triangle*	*Spherical Excess (ε)*	*Triangle Misclosure*	*Triangle*	*Spherical Excess (ε)*	*Triangle Misclosure*
$U_2V_2Y_2$	1″123	−0″513	$U_2W_2Y_2$	0″939	+1″801	$V_2W_2Y_2$	1″262	−0″572
$W_2Y_2Z_2$	1·121	+1·099	$W_2Y_2A_3$	0·930	+1·410	$W_2Z_2A_3$	1·557	+0·023
$Y_2Z_2A_3$	1·366	+0·334	$Y_2Z_2B_3$	2·972	−2·312	$Y_2A_3B_3$	0·140	−0·870
$Z_2A_3B_3$	1·466	−1·776	$Z_2B_3C_3$	3·405	+0·995	$Z_2B_3D_3$	1·200	+1·920
$Z_2C_3D_3$	3·897	+1·883	$B_3C_3D_3$	1·692	+2·808	$B_3C_3E_3$	8·389	+3·861
$B_3D_3E_3$	0·547	−2·357	$C_3D_3E_3$	6·150	+3·410	$C_3E_3F_3$	14·897	−2·597
$E_3F_3G_3$	9·229	−1·969	$F_3G_3H_3$	6·245	−0·505	$F_3G_3I_3$	6·510	−0·900
$F_3H_3I_3$	2·030	−0·420	$G_3H_3I_3$	2·295	−0·815	$G_3H_3J_3$	4·090	−0·140
$G_3I_3J_3$	1·428	−0·958	$H_3I_3J_3$	0·367	+1·633	$H_3I_3L_3$	1·505	−1·385
$H_3J_3L_3$	1·829	−1·229	$I_3J_3K_3$	0·365	+2·055	$I_3J_3L_3$	0·691	+1·789
$I_3K_3L_3$	0·527	−0·357	$J_3K_3L_3$	0·201	−0·091			

Unclosed Triangles

Triangle	*Spherical Excess (ε)*	*Triangle*	*Spherical Excess (ε)*
ABK	3″652	BEK	0″121
BA_4K	8·915	AKL	7·023
BKL	8·290	ABL	4·919
AEL	1·078	AFL	3·729
AA_4L	2·264	BA_4L	8·658
BEL	4·213	EA_4L	0·040
KA_4L	8·547	$A_1B_1N_1$	4·157
$A_1M_1N_1$	1·163	$B_1M_1N_1$	0·379
$M_1O_1N_1$	2·049	$L_1M_1W_1$	2·809
$L_1X_1W_1$	1·045	$L_1D_2W_1$	3·728
$M_1R_1W_1$	2·405	$M_1X_1W_1$	2·608
$R_1X_1W_1$	3·028	$X_1D_2W_1$	2·895

APPENDIX 7

FIGURE 7 (see DIAGRAM 11)

7.1 Mean observed directions, $(t-T)$ corrections, mean plane observed directions, adjustment corrections, and plane adjusted directions

From	*To*	*Mean Observed Direction*	$(t-T)$	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Aberystwyth (G$_1$)	Talsarn (L$_1$)	00° 00′ 00″00	− 8″452	359° 59′ 51″548	−0″243	359° 59′ 51″305
	Rhiw (Y)	129 33 09·26	+17·836	129 33 27·096	+0·417	129 33 27·513
	Cader Idris (F$_1$)	189 39 13·46	+10·290	189 39 23·750	−0·174	189 39 23·576
Beneraird (C)	Cairn Pat (F)	00 00 00·093(1)	−10·633	359 59 49·460	+0·295	359 59 49·755
	Brown Carrick (A)	179 13 22·211(3)	+17·203	179 13 39·414	−0·348	179 13 39·066
	Merrick (D)	234 06 16·109(1)	+ 3·144	234 06 19·253	+0·284	234 06 19·537
	Cairnsmore of Fleet (G)	264 59 44·715(1)	− 5·050	264 59 39·665	+0·290	264 59 39·955
	Carleton Fell (I)	304 20 22·022(2)	−18·263	304 20 03·759	−0·021	304 20 03·738
	Inshanks (H)	340 30 04·491(2)	−20·374	340 29 44·117	−0·501	340 29 43·616
Black Combe (V)	Rottington (U)	00 00 00·064(2)	+ 6·536	00 00 06·600	+0·907	00 00 07·507
	Skiddaw (P)	49 18 40·298(1)	+ 9·084	49 18 49·382	+1·028	49 18 50·410
	Llaneilian (X)	248 34 55·474(2)	−25·773	248 34 29·701	+0·084	248 34 29·785
	South Barrule (S)	297 06 23·154(2)	− 2·801	297 06 20·353	−1·014	297 06 19·339
	Snaefell (T)	305 20 34·484(2)	+ 0·733	305 20 35·217	−1·006	305 20 34·211
Cader Idris (F$_1$)	Plynlimon (K$_1$)	00 00 00·206(1)	− 8·347	359 59 51·859	−0·120	359 59 51·739
	Aberystwyth (G$_1$)	38 31 30·708(1)	− 9·992	38 31 20·716	−0·144	38 31 20·572
	Rhiw (Y)	125 28 16·234(2)	+ 6·008	125 28 22·242	+0·843	125 28 23·085
	Yr Eifl (Z)	149 14 35·849(1)	+11·282	149 14 47·131	−0·142	149 14 46·989
	Garnedd Ugain (A$_1$)	183 21 15·404(1)	+14·110	183 21 29·514	−0·140	183 21 29·374
	Arenig (E$_1$)	222 36 58·021(1)	+ 7·573	222 37 05·594	−0·146	222 37 05·448
	Aran Fawddwy (J$_1$)	255 05 37·807(1)	+ 2·932	255 05 40·739	−0·150	255 05 40·589
Cairn Pat (F)	Inshanks (H)	00 00 00·214(2)	−10·191	359 59 50·023	−1·021	359 59 49·002
	South Barrule (S)	03 39 28·40	−38·383	03 38 50·017	−1·082	03 38 48·935
	Beneraird (C)	220 51 32·459(1)	+10·804	220 51 43·263	+0·882	220 51 44·145
	Merrick (D)	251 12 13·634(1)	+13·510	251 12 27·144	+0·879	251 12 28·023
	Cairnsmore of Fleet (G)	275 18 25·968(1)	+ 4·892	275 18 30·860	+0·880	275 18 31·740
	Carleton Fell (I)	315 46 33·454(2)	− 8·584	315 46 24·870	−0·540	315 46 24·330

(1) Fixed direction from previous Figures.

(2) Mean observed direction plus overlap correction from previous Figures.

(3) Mean of 1938 and 1951 observations plus adjustment correction from Figure 3.

7.1 *continued*

From	*To*		*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Cairnsmore of Fleet (G)	Carleton Fell	(I)	00° 00′ 00″·039(2)	−11″·314	359° 59′ 48″·725	+0″·673	359° 59′ 49″·398
	Cairn Pat	(F)	58 00 51·678(1)	− 4·479	58 00 47·199	−0·225	58 00 46·974
	Beneraird	(C)	88 33 45·108(1)	+ 4·697	88 33 49·805	−0·226	88 33 49·579
	Merrick	(D)	139 22 47·806(1)	+ 7·124	139 22 54·930	−0·200	139 22 54·730
	Cairnsmore of Deugh	(E)	177 55 25·356(1)	+11·486	177 55 36·842	−0·209	177 55 36·633
	Criffell	(J)	257 43 47·858(1)	− 1·777	257 43 46·081	−0·229	257 43 45·852
	Rottington	(U)	301 13 02·619(2)	−18·335	301 12 44·284	+0·415	301 12 44·699
Capel Cynon (H$_1$)	Talsarn	(L$_1$)	00 00 00·00	+ 4·186	00 00 04·186	−0·586	00 00 03·600
	Prescelly	(I$_1$)	178 41 53·57	− 7·944	178 41 45·626	+0·544	178 41 46·170
	Rhiw	(Y)	291 37 03·54	+33·950	291 37 37·490	+0·042	291 37 37·532
Carleton Fell (I)	Inshanks	(H)	00 00 00·00	− 1·016	359 59 58·984	−0·565	359 59 58·419
	Cairn Pat	(F)	31 58 08·89	+ 8·026	31 58 16·916	+0·191	31 58 17·107
	Cairnsmore of Fleet	(G)	113 29 15·06	+11·558	113 29 26·618	+0·323	113 29 26·941
	Rottington	(U)	208 44 47·53	− 8·787	208 44 38·743	−0·520	208 44 38·223
	Snaefell	(T)	275 14 27·64	−20·171	275 14 07·469	−0·102	275 14 07·367
	South Barrule	(S)	287 50 50·83	−25·829	287 50 25·001	+0·674	287 50 25·675
Criffell (J)	Cairnsmore of Deugh	(E)	359 59 59·860(1)	+10·644	00 00 10·504	+0·009	00 00 10·513
	Cairn Table	(B)	24 38 17·384(1)	+17·689	24 38 35·073	+0·015	24 38 35·088
	Hart Fell	(K)	61 57 14·104(1)	+12·968	61 57 27·072	+0·020	61 57 27·092
	Wisp Hill	(L)	93 59 51·149(1)	+ 8·536	93 59 59·685	−0·013	93 59 59·672
	Whitelyne Common	(M)	118 38 53·911(1)	+ 3·994	118 38 57·905	−0·016	118 38 57·889
	Cold Fell Pike	(N)	140 37 12·962(1)	− 1·306	140 37 11·656	−0·002	140 37 11·654
	Cross Fell	(O)	155 48 07·630(1)	− 5·576	155 48 02·054	−0·002	155 48 02·052
	Skiddaw	(P)	182 22 11·894(1)	− 7·817	182 22 04·077	+0·003	182 22 04·080
	Sca Fell	(R)	199 50 56·372(1)	−13·239	199 50 43·133	+0·003	199 50 43·136
	Rottington	(U)	225 44 00·909(2)	−12·825	225 43 48·084	+0·008	225 43 48·092
	Cairnsmore of Fleet	(G)	321 38 51·849(1)	+ 1·577	321 38 53·426	−0·018	321 38 53·408
	Merrick	(D)	339 12 56·800(1)	+ 7·313	339 13 04·113	−0·010	339 13 04·103
Garnedd Ugain (A$_1$)	Yr Eifl	(Z)	359 59 59·325(1)	− 3·878	359 59 55·447	+0·099	359 59 55·546
	Holyhead	(W)	58 15 44·015(2)	+10·700	58 15 54·715	−0·620	58 15 54·095
	Llaneilian	(X)	92 16 33·244(1)	+13·292	92 16 46·536	+0·099	92 16 46·635
	Moelfre Isaf	(C$_1$)	174 48 50·856(1)	+ 5·884	174 48 56·740	+0·100	174 48 56·840
	Moel Fammau	(D$_1$)	195 12 04·424(1)	+ 2·287	195 12 06·711	+0·105	195 12 06·816
	Arenig	(E$_1$)	243 03 11·363(1)	− 6·076	243 03 05·287	+0·095	243 03 05·382
	Cader Idris	(F$_1$)	279 33 26·878(1)	−14·466	279 33 12·412	+0·099	279 33 12·511
	Rhiw	(Y)	348 58 22·965(2)	− 9·897	348 58 13·068	+0·023	348 58 13·091

(1) Fixed direction from previous Figures.

(2) Mean observed direction plus overlap correction from previous Figures.

7.1 *continued*

From	*To*	*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Holyhead (W)	Rhiw (Y)	00° 00′ 00″·00	−24″·122	359° 59′ 35″·878	+0″·030	359° 59′ 35″·908
	South Barrule (S)	183 27 12·70	+41·639	183 27 54·339	−0·556	183 27 53·783
	Snaefell (T)	190 42 43·06	+45·844	190 43 28·904	+0·244	190 43 29·148
	Llaneilian (X)	252 00 55·06	+ 3·769	252 00 58·829	+0·839	252 00 59·668
	Garnedd Ugain (A_1)	306 22 19·90	−11·621	306 22 08·279	−0·675	306 22 07·604
	Yr Eifl (Z)	340 05 22·97	−16·765	340 05 06·205	+0·119	340 05 06·324
Inshanks (H)	Cairn Pat (F)	00 00 00·00	+10·069	00 00 10·069	−0·384	00 00 09·685
	Beneraird (C)	21 21 38·03	+20·451	21 21 58·481	+0·209	21 21 58·690
	Merrick (D)	50 35 11·55	+22·561	50 35 34·111	+1·067	50 35 35·178
	Carleton Fell (I)	103 48 24·90	+ 1·073	103 48 25·973	+0·352	103 48 26·325
	Snaefell (T)	167 38 14·23	−21·521	167 37 52·709	−0·815	167 37 51·894
	South Barrule (S)	184 58 01·89	−27·744	184 57 34·146	−0·429	184 57 33·717
Llaneilian (X)	Holyhead (W)	00 00 00·003(2)	− 3·581	359 59 56·422	−0·207	359 59 56·215
	South Barrule (S)	94 41 35·353(2)	+34·096	94 42 09·449	−0·773	94 42 08·676
	Snaefell (T)	104 34 04·993(2)	+37·882	104 34 42·875	−0·854	104 34 42·021
	Black Combe (V)	144 16 36·913(2)	+31·017	144 17 07·930	−0·538	144 17 07·392
	Great Ormes Head (B_1)	214 55 24·453(1)	− 3·037	214 55 21·416	−0·523	214 55 20·893
	Garnedd Ugain (A_1)	268 22 08·966(1)	−13·717	268 21 55·249	+1·442	268 21 56·691
	Yr Eifl (Z)	301 58 42·651(1)	−18·597	301 58 24·054	+1·452	301 58 25·506
Merrick (D)	Cairnsmore of Deugh (E)	359 59 59·506(1)	+ 4·777	00 00 04·283	+0·004	00 00 04·287
	Criffell (J)	60 48 19·904(1)	− 8·372	60 48 11·532	+0·022	60 48 11·554
	Cairnsmore of Fleet (G)	104 53 16·934(1)	− 7·240	104 53 09·694	+0·042	104 53 09·736
	Carleton Fell (I)	129 45 12·534(2)	−19·073	129 44 53·461	+0·447	129 44 53·908
	Inshanks (H)	158 48 05·864(2)	−21·237	158 47 44·627	+0·109	158 47 44·736
	Cairn Pat (F)	179 25 11·490(3)	−12·566	179 24 58·924	−0·662	179 24 58·262
	Beneraird (C)	203 10 47·127(1)	− 2·970	203 10 44·157	+0·010	203 10 44·167
	Cairn Table (B)	344 12 12·041(1)	+14·427	344 12 26·468	+0·028	344 12 26·496
Prescelly (I_1)	Capel Cynon (H_1)	00 00 00·00	+ 8·374	00 00 08·374	+0·685	00 00 09·059
	Garn Fawr (M_1)	234 26 30·56	+ 3·856	234 26 34·416	−0·874	234 26 33·542
	Rhiw (Y)	310 57 14·79	+46·316	310 58 01·106	+0·189	310 58 01·295
Rhiw (Y)	Holyhead (W)	00 00 00·00	+24·077	00 00 24·077	+0·780	00 00 24·857
	Yr Eifl (Z)	42 41 30·48	+ 6·714	42 41 37·194	+0·407	42 41 37·601
	Garnedd Ugain (A_1)	57 05 04·66	+10·729	57 05 15·389	+0·159	57 05 15·548
	Cader Idris (F_1)	109 47 16·07	− 6·676	109 47 09·394	−0·714	109 47 08·680
	Aberystwyth (G_1)	142 44 29·11	−19·251	142 44 09·859	+0·246	142 44 10·105
	Capel Cynon (H_1)	170 50 03·92	−34·925	170 49 28·995	−0·177	170 49 28·818
	Prescelly (I_1)	188 52 15·59	−45·198	188 51 30·392	−0·701	188 51 29·691

(1) Fixed direction from previous Figures.

(2) Mean observed direction plus overlap correction from previous Figures.

(3) Mean of 1938 and 1951 observations plus adjustment correction from Figure 3.

7.1 *continued*

From	*To*		*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Rottington (U)	Criffell	(J)	00° 00′ 00″00	+12″846	00° 00′ 12″846	+1″236	00° 00′ 14″082
	Skiddaw	(P)	62 23 16·61	+ 3·760	62 23 20·370	−0·111	62 23 20·259
	Black Combe	(V)	146 04 51·96	− 6·967	146 04 44·993	−0·335	146 04 44·658
	Snaefell	(T)	244 53 29·50	− 7·890	244 53 21·610	−0·382	244 53 21·228
	Carleton Fell	(I)	293 26 27·36	+ 7·648	293 26 35·008	−0·782	293 26 34·226
	Cairnsmore of Fleet	(G)	319 24 01·58	+16·292	319 24 17·872	+0·374	319 24 18·246
Skiddaw (P)	Criffell	(J)	359 59 59·793(1)	+ 6·978	00 00 06·771	+0·086	00 00 06·857
	Wisp Hill	(L)	52 55 48·587(1)	+12·409	52 56 00·996	+0·080	52 56 01·076
	Whitelyne Common	(M)	76 05 53·520(1)	+ 8·215	76 06 01·735	+0·069	76 06 01·804
	Cold Fell Pike	(N)	95 12 30·570(1)	+ 4·198	95 12 34·768	+0·082	95 12 34·850
	Cross Fell	(O)	125 46 18·051(1)	+ 0·791	125 46 18·842	+0·079	125 46 18·921
	High Street	(Q)	177 46 08·472(1)	− 3·104	177 46 05·368	+0·081	177 46 05·449
	Sca Fell	(R)	234 23 35·831(1)	− 4·178	234 23 31·653	+0·061	234 23 31·714
	Black Combe	(V)	238 45 12·890(1)	− 8·624	238 45 04·266	+0·081	238 45 04·347
	Rottington	(U)	285 45 01·014(2)	− 3·351	285 44 57·663	−0·618	285 44 57·045
Snaefell (T)	Inshanks	(H)	00 00 00·00	+20·385	00 00 20·385	−1·146	00 00 19·239
	Carleton Fell	(I)	31 24 41·42	+20·190	31 25 01·610	+1·007	31 25 02·617
	Rottington	(U)	96 22 10·79	+ 9·073	96 22 19·863	+0·613	96 22 20·476
	Black Combe	(V)	122 54 12·47	− 0·896	122 54 11·574	−0·964	122 54 10·610
	Llaneilian	(X)	206 26 19·02	−38·492	206 25 40·528	+0·283	206 25 40·811
	Holyhead	(W)	220 34 08·89	−44·252	220 33 24·638	−0·152	220 33 24·486
	South Barrule	(S)	259 54 02·84	− 5·079	259 53 57·761	+0·359	259 53 58·120
South Barrule (S)	Inshanks	(H)	00 00 00·00	+27·021	00 00 27·021	−0·877	00 00 26·144
	Carleton Fell	(I)	26 41 18·34	+26·586	26 41 44·926	+1·082	26 41 46·008
	Snaefell	(T)	62 34 18·47	+ 5·223	62 34 23·693	−0·492	62 34 23·201
	Black Combe	(V)	97 20 17·09	+ 3·508	97 20 20·598	+0·221	97 20 20·819
	Llaneilian	(X)	179 14 07·70	−35·625	179 13 32·075	+0·473	179 13 32·548
	Holyhead	(W)	195 58 55·87	−41·332	195 58 14·538	−0·336	195 58 14·202
	Cairn Pat	(F)	358 41 25·18	+36·936	358 42 02·116	−0·071	358 42 02·045
Yr Eifl (Z)	Rhiw	(Y)	359 59 59·890(2)	− 6·537	359 59 53·353	−0·803	359 59 52·550
	Holyhead	(W)	117 23 54·075(2)	+16·293	117 24 10·368	−0·146	117 24 10·222
	Llaneilian	(X)	151 18 13·587(1)	+19·025	151 18 32·612	+0·245	151 18 32·857
	Garnedd Ugain	(A$_1$)	205 25 08·622(1)	+ 4·094	205 25 12·716	+0·237	205 25 12·953
	Arenig	(E$_1$)	237 57 09·840(1)	− 2·928	237 57 06·912	+0·233	237 57 07·145
	Cader Idris	(F$_1$)	270 51 59·506(1)	−12·209	270 51 47·297	+0·236	270 51 47·533

(1) Fixed direction from previous Figures.

(2) Mean observed direction plus overlap correction from previous Figures.

7.2 Triangle misclosures and spherical excesses

Triangle	*Spherical Excess* (ϵ)	*Triangle Misclosure*	*Triangle*	*Spherical Excess* (ϵ)	*Triangle Misclosure*
CDF	1″·475	−1″·825	CDG	1″·497	+0″·633
CDH	3·141	−0·221	CFG	2·319	−0·509
CFH	0·872	+0·388	DFG	2·341	+1·949
DFH	2·538	+1·992	DGJ	2·033	−1·863
FGI	3·109	+2·191	FHI	1·561	−1·011
FHS	0·294	+1·036	FIS	6·294	+0·086
GIU	4·680	−0·570	GJU	5·602	−0·972
HIS	4·439	+0·061	HIT	3·634	−0·524
HST	2·557	+0·733	IST	1·752	+0·148
ITU	6·963	+0·377	JPU	3·765	+0·565
PUV	2·906	+1·014	STV	2·364	−2·044
STW	3·177	−1·467	STX	3·649	−0·959
SVX	19·240	+0·610	SWX	5·900	−0·020
TUV	5·089	−0·289	TVX	17·955	−0·475
TWX	6·372	+0·488	WXZ	2·786	+1·754
WXA$_1$	2·662	+2·648	WYZ	1·890	−0·310
WYA$_1$	5·252	+0·848	WZA$_1$	2·765	−1·445
XZA$_1$	2·641	−0·551	YZA$_1$	0·597	+2·603
YZF$_1$	2·444	+2·466	YA$_1$F$_1$	4·734	+1·696
YF$_1$G$_1$	4·121	−2·461	YH$_1$I$_1$	6·321	+0·529
ZA$_1$F$_1$	2·887	+1·833			

Unclosed Triangles

Triangle	*Spherical Excess* (ϵ)	*Triangle*	*Spherical Excess* (ϵ)
BDJ	6″·965	CDI	3″·478
CFI	2·436	CGI	2·992
CHI	3·125	DEG	1·013
DEJ	2·670	DFI	4·439
DGI	1·011	DHI	3·462
EGJ	3·690	JLP	6·439
JMP	6·809	JNP	4·903
JOP	3·946	JPR	2·053
ZA$_1$E$_1$	1·704	ZE$_1$F$_1$	3·027
A$_1$E$_1$F$_1$	1·844		

APPENDIX 8

ADDITIONAL PRIMARY WORK

8.1 Liddington Castle reco-ordination

(see Diagram 12)

From	*To*		*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Cleeve Hill (F)	White Horse Hill	(D)	96° 23′ 06″857(1)	—	96° 23′ 06″857
	Liddington Castle	(C)	109 28 47·156(2)	+0″210	109 28 47·366
	Peglers Tump	(E)	175 02 42·039(1)	—	175 02 42·039
Inkpen (B)	White Horse Hill	(D)	359 59 59·447(1)	—	359 59 59·447
	Martinsell	(A)	292 54 35·723(1)	—	292 54 35·723
	Liddington Castle	(C)	334 16 50·768(2)	−1·128	334 16 49·640
Liddington Castle (C)	Inkpen	(B)	00 00 00·00	+0·838	00 00 00·838
	Martinsell	(A)	53 16 18·18	+0·101	53 16 18·281
	Peglers Tump	(E)	157 49 50·23	+0·664	157 49 50·894
	Cleeve Hill	(F)	196 40 37·64	−1·434	196 40 36·206
	White Horse Hill	(D)	276 02 02·01	−0·169	276 02 01·841
Martinsell (A)	Inkpen	(B)	359 59 59·551(1)	—	359 59 59·551
	Peglers Tump	(E)	216 22 11·925(1)	—	216 22 11·925
	Liddington Castle	(C)	274 38 30·580(2)	−0·472	274 38 30·108
Peglers Tump (E)	Cleeve Hill	(F)	00 00 00·477(1)	—	00 00 00·477
	Liddington Castle	(C)	75 35 25·001(2)	−0·828	75 35 24·173
	Martinsell	(A)	92 45 35·230(1)	—	92 45 35·230
White Horse Hill (D)	Liddington Castle	(C)	25 24 35·348(2)	+0·026	25 24 35·374
	Cleeve Hill	(F)	112 57 30·621(1)	—	112 57 30·621
	Inkpen	(B)	315 05 43·488(1)	—	315 05 43·488

(1) Fixed direction from Figure 1. (2) Mean observed direction plus overlap correction from Figure 1.

Triangle misclosures and spherical excesses

Triangle	*Spherical Excess* (ϵ)	*Triangle Misclosure*
ABC	0″803	+2″347
ACE	1·853	−1·173
BCD	0·693	−0·713
CDF	1·391	−0·471
CEF	3·681	+3·999

Symbolic statement of condition equations

Angle Closure	*Side Closure*	*Remarks*
ABC		
AEC	A(ECB)	Fixed sides
EFC	E(FCA)	Fixed sides
FDC	F(DCE)	Fixed sides

8.2 Spurn Head Extension (see Diagram 12)

From	*To*		*Mean Observed Direction*	*Adjustment Correction*	*Adjusted Direction*
Acre	Cave Wold	(A)	124° 08′ 37″·713(1)	—	124° 08′ 37″·713
(C)	Tunstall	(B)	175 16 12·724(2)	−0″·251	175 16 12·473
	Stone Creek	(E)	179 31 18·824(2)	−0·100	179 31 18·724
	Dimlington	(D)	198 29 17·364(2)	−0·966	198 29 16·398
Cave Wold	Acre	(C)	352 21 37·857(1)	—	352 21 37·857
(A)	Tunstall	(B)	285 19 19·717(2)	+0·194	285 19 19·911
	Dimlington	(D)	302 24 38·387(2)	−0·379	302 24 38·008
	Stone Creek	(E)	312 00 06·477(2)	+1·594	312 00 08·071
Dimlington	Cave Wold	(A)	01 53 59·98	−0·637	01 53 59·343
(D)	Tunstall	(B)	40 48 57·30	+0·119	40 48 57·419
	Acre	(C)	306 11 33·68	+0·668	306 11 34·348
	Stone Creek	(E)	340 24 32·07	−0·149	340 24 31·921
Stone Creek	Dimlington	(D)	00 00 00·00	+0·526	00 00 00·526
(E)	Acre	(C)	126 49 05·48	+0·574	126 49 06·054
	Cave Wold	(A)	211 04 58·34	−0·964	211 04 57·376
	Tunstall	(B)	295 36 32·94	−0·136	295 36 32·804
Tunstall	Dimlington	(D)	00 00 00·00	−0·053	359 59 59·947
(B)	Stone Creek	(E)	55 12 07·04	+0·221	55 12 07·261
	Acre	(C)	62 09 34·60	−0·140	62 09 34·460
	Cave Wold	(A)	123 59 45·00	−0·028	123 59 44·972

(1) Fixed direction from Figure 2.
(2) Mean observed direction plus overlap correction from Figure 2.

Triangle misclosures and spherical excesses

Triangle	*Spherical Excess (ε)*	*Triangle Misclosure*
ABC	3″·217	+0″·543
ABD	1·197	−0·207
ABE	1·298	−1·978
ACD	3·530	+2·100
ACE	2·119	+3·441
ADE	0·636	−2·976
BCD	1·510	+1·350
BCE	0·199	+0·921
BDE	0·535	−1·205
CDE	0·776	+1·634

Symbolic statement of condition equations

Angle Closure	*Side Closure*	*Remarks*
ACE		
ABE		
BDE		
CDE		
	E(ABDC)	
ABD		
	E(ABD)	
BAC		
	x(ABDC)	Pole at intersection of diagonals

8.3 Fixation of Frittenfield and Paddlesworth (see Diagram 12)

From	*To*		*Mean Observed Direction*	*(t—T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Crowborough (B)	Firle Beacon	(C)	00° 00′ 00″00	+ 9″461	00° 00′ 09″461	−0″587	00° 00′ 08″874
	Wrotham	(A)	189 34 53·51	−11·405	189 34 42·105	+0·712	189 34 42·817
	Frittenfield	(F)	242 48 04·97	− 7·701	242 47 57·269	+0·023	242 47 57·292
	Paddlesworth	(I)	256 44 32·23	− 3·873	256 44 28·357	+0·643	256 44 29·000
	Fairlight Down	(E)	293 35 22·78	+ 7·742	293 35 30·522	−0·792	293 35 29·730
Fairlight Down (E)	Beachy Head	(D)	00 00 00·00	+ 7·194	00 00 07·194	+1·084	00 00 08·278
	Firle Beacon	(C)	23 00 14·49	+ 2·625	23 00 17·115	+0·647	23 00 17·762
	Crowborough	(B)	62 07 13·11	− 8·269	62 07 04·841	−1·891	62 07 02·950
	Wrotham	(A)	95 02 33·67	−21·442	95 02 12·228	−0·050	95 02 12·178
	Frittenfield	(F)	142 55 39·83	−17·738	142 55 22·092	−1·324	142 55 20·768
	Paddlesworth	(I)	174 47 05·80	−13·730	174 46 52·070	+1·536	174 46 53·606
Frittenfield (F)	Rumfields Wtr Twr	(H)	00 00 00·00	−10·071	359 59 49·929	+1·315	359 59 51·244
	Paddlesworth	(I)	48 42 31·77	+ 4·914	48 42 36·684	−0·082	48 42 36·602
	Fairlight Down	(E)	135 45 37·65	+18·169	135 45 55·819	−1·461	135 45 54·358
	Crowborough	(B)	184 09 55·44	+ 8·424	184 10 03·864	+0·238	184 10 04·102
	Wrotham	(A)	221 14 47·98	− 5·175	221 14 42·805	+1·474	221 14 44·279
	Shurland	(G)	300 30 34·44	−11·440	300 30 23·000	−1·482	300 30 21·518
Paddlesworth (I)	Frittenfield	(F)	00 00 00·00	− 5·087	359 59 54·913	−0·275	359 59 54·638
	Rumfields Wtr Twr	(H)	98 49 07·57	−16·169	98 48 51·401	+0·514	98 48 51·915
	Fairlight Down	(E)	298 54 30·27	+14·562	298 54 44·832	+0·400	298 54 45·232
	Crowborough	(B)	329 23 50·10	+ 4·384	329 23 54·484	−0·638	329 23 53·846
Rumfields Wtr Twr (H)	Shurland	(G)	00 00 00·00	− 2·228	359 59 57·772	−1·638	359 59 56·134
	Paddlesworth	(I)	296 12 38·43	+16·592	296 12 55·022	+0·756	296 12 55·778
	Frittenfield	(F)	328 41 01·56	+10·700	328 41 12·260	+0·883	328 41 13·143
Shurland (G)	Rumfields Wtr Twr	(H)	00 00 00·00	+ 2·105	00 00 02·105	−0·157	00 00 01·948
	Frittenfield	(F)	89 11 38·64	+11·479	89 11 50·119	−0·888	89 11 49·231
	Wrotham	(A)	158 06 05·87	+ 5·523	158 06 11·393	+1·045	158 06 12·438
Wrotham (A)	Frittenfield	(F)	00 00 00·00	+ 4·813	00 00 04·813	+0·285	00 00 05·098
	Fairlight Down	(E)	46 37 47·90	+20·426	46 38 08·326	−1·739	46 38 06·587
	Crowborough	(B)	89 41 59·13	+11·606	89 42 10·736	−0·290	89 42 10·446
	Shurland	(G)	328 10 08·92	− 5·119	328 10 03·801	+1·743	328 10 05·544

Triangle misclosures and spherical excesses

Triangle	*Spherical Excess (ε)*	*Triangle Misclosure*	*Triangle*	*Spherical Excess (ε)*	*Triangle Misclosure*
ABE	2″846	−1″786	ABF	3″102	+0″028
AEF	4·027	+0·363	AFG	2·289	+2·481
BEF	3·771	−1·451	BEI	4·024	−0·954
BFI	2·133	−1·303	EFI	2·386	−0·806
FGH	2·185	+0·455	FHI	1·989	+0·481

Unclosed Triangle

Triangle	*Spherical Excess (ε)*
BCE	2″210

8.4 Hillhead Farm co-ordination (see Diagram 13)

From	*To*		*Mean Observed Direction*	*(t − T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Bad Mor	Scaraben	(G)	00° 00′ 00″00	− 6″989	359° 59′ 53″011	+0″295	359° 59′ 53″306
(E)	Dunnet Head	(A)	237 24 12·00	+ 5·062	237 24 17·062	+0·627	237 24 17·689
	Warth Hill	(B)	261 47 41·08	+ 3·286	261 47 44·366	−0·535	261 47 43·831
	Hillhead Farm	(C)	266 56 13·59	+ 1·906	266 56 15·496	+0·046	266 56 15·542
	Spital Hill	(D)	281 29 03·35	+ 0·140	281 29 03·490	−0·270	281 29 03·220
	Hill of Yarrows	(F)	305 51 06·54	− 2·799	305 51 03·741	−0·162	305 51 03·579
Dunnet Head	Warth Hill	(B)	00 00 00·00	− 1·244	359 59 58·756	+0·058	359 59 58·814
(A)	Hillhead Farm	(C)	39 13 58·72	− 2·565	39 13 56·155	−0·380	39 13 55·775
	Hill of Yarrows	(F)	53 05 00·26	− 6·521	53 04 53·739	+0·515	53 04 54·254
	Spital Hill	(D)	78 21 09·84	− 4·262	78 21 05·578	−0·583	78 21 04·995
	Bad Mor	(E)	112 06 35·92	− 4·688	112 06 31·232	+0·392	112 06 31·624
Hillhead Farm	Hill of Yarrows	(F)	00 00 00·00	− 3·723	359 59 56·277	+0·809	359 59 57·086
(C)	Scaraben	(G)	35 10 29·65	− 7·322	35 10 22·328	−1·199	35 10 21·129
	Spital Hill	(D)	60 04 37·19	− 1·480	60 04 35·710	−0·303	60 04 35·407
	Bad Mor	(E)	78 30 43·22	− 1·710	78 30 41·510	−0·156	78 30 41·354
	Dunnet Head	(A)	156 06 04·16	+ 2·485	156 06 06·645	+1·006	156 06 07·651
	Warth Hill	(B)	240 04 46·59	+ 1·139	240 04 47·729	−0·157	240 04 47·572
Hill of Yarrows	Spital Hill	(D)	00 00 00·00	+ 2·427	00 00 02·427	+0·034	00 00 02·461
(F)	Dunnet Head	(A)	29 53 41·30	+ 6·262	29 53 47·562	+0·014	29 53 47·576
	Hillhead Farm	(C)	39 56 34·37	+ 3·692	39 56 38·062	+0·470	39 56 38·532
	Warth Hill	(B)	60 28 41·67	+ 4·649	60 28 46·319	−0·191	60 28 46·128
	Scaraben	(G)	280 14 13·80	− 3·151	280 14 10·649	−0·731	280 14 09·918
	Bad Mor	(E)	337 22 07·94	+ 2·491	337 22 10·431	+0·405	337 22 10·836
Scaraben	Bad Mor	(E)	117 37 04·22	+ 6·829	117 37 11·049	−0·125	117 37 10·924
(G)	Spital Hill	(D)	150 31 51·92	+ 6·559	150 31 58·479	+0·046	150 31 58·525
	Hillhead Farm	(C)	161 13 05·12	+ 7·974	161 13 13·094	−0·158	161 13 12·936
	Hill of Yarrows	(F)	186 20 16·58	+ 3·461	186 20 20·041	+0·238	186 20 20·279
Spital Hill	Hill of Yarrows	(F)	00 00 00·00	− 2·566	359 59 57·434	−0·437	359 59 56·997
(D)	Scaraben	(G)	64 25 49·49	− 6·312	64 25 43·178	−0·478	64 25 42·700
	Bad Mor	(E)	133 00 04·85	− 0·131	133 00 04·719	+0·294	133 00 05·013
	Dunnet Head	(A)	235 09 48·22	+ 4·328	235 09 52·548	+0·305	235 09 52·853
	Hillhead Farm	(C)	280 01 09·08	+ 1·551	280 01 10·631	+0·758	280 01 11·389
	Warth Hill	(B)	280 01 14·65	+ 2·750	280 01 17·400	−0·440	280 01 16·960
Warth Hill	Dunnet Head	(A)	00 00 00·00	+ 1·151	00 00 01·151	−0·328	00 00 00·823
(B)	Hill of Yarrows	(F)	263 39 59·94	− 4·477	263 39 55·463	−0·647	263 39 54·816
	Spital Hill	(D)	303 12 33·53	− 2·506	303 12 31·024	+0·087	303 12 31·111
	Hillhead Farm	(C)	303 12 38·32	− 1·087	303 12 37·233	+0·472	303 12 37·705
	Bad Mor	(E)	316 30 02·18	− 2·821	316 29 59·359	+0·416	316 29 59·775

8.4 *continued*

Triangle misclosures and spherical excesses

Triangle	*Spherical Excess* (ε)	*Triangle Misclosure*	*Triangle*	*Spherical Excess* (ε)	*Triangle Misclosure*
ABC	0″·429	+2″·401	BDE	0″·580	+0″·140
ABD	0·939	+1·801	BDF	1·123	−0·513
ABE	1·248	+1·572	BEF	2·271	−0·841
ABF	1·262	−0·572	CDE	0·314	−0·294
ACD	0·509	−1·559	CDF	0·609	+1·871
ACE	1·084	−1·354	CDG	0·606	+0·544
ACF	0·318	−1·548	CEF	1·491	+1·109
ADE	0·889	−0·089	CEG	2·138	−1·258
ADF	0·800	+1·860	CFG	1·269	+0·411
AEF	2·257	+1·303	DEF	0·568	−0·468
BCD	0·001	+0·959	DEG	1·218	−1·508
BCE	0·265	−0·525	DFG	1·266	−0·916
BCF	0·515	−1·425	EFG	1·916	−1·956

8.5 Connection with N. Ireland and Eire (see Diagram 14)

From	*To*	*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Ballycreen (B$_1$)	Tara (C$_1$)	00° 00′ 00″·00	−18″·391	359° 59′ 41″·609	−0″·303	359° 59′ 41″·306
	Forth Mountain (D$_1$)	33 40 32·35	−50·116	33 39 42·234	−0·330	33 39 41·904
	Kippure (A$_1$)	206 57 52·67	+21·257	206 58 13·927	−1·274	206 58 12·653
	Holyhead (X)	270 30 37·28	+24·750	270 31 02·030	+1·206	270 31 03·236
	Rhiw (Y)	296 22 18·30	− 9·830	296 22 08·470	+0·396	296 22 08·866
	Prescelly (F$_1$)	336 44 45·60	−74·489	336 43 31·111	+0·306	336 43 31·417
Beinn Tart a' Mhill (E)	Sliabh Gaoil (C)	00 00 00·00	+11·280	00 00 11·280	−0·474	00 00 10·806
	Cnoc Moy (I)	62 00 34·58	−28·079	62 00 06·501	+0·156	62 00 06·657
	Knocklayd (G)	98 24 20·33	−44·345	98 23 35·985	−0·859	98 23 35·126
	Slieve Snaght (F)	153 34 18·91	−41·524	153 33 37·386	−0·234	153 33 37·152
	Ben Hynish (A)	269 32 28·54	+60·403	269 33 28·943	+0·656	269 33 29·599
	Ben More (Mull) (B)	308 18 59·27	+51·699	308 19 50·969	+0·225	308 19 51·194
	Jura (D)	343 49 48·44	+12·242	343 50 00·682	+0·531	343 50 01·213
Cairn Pat (J)	Inshanks (Q)	00 00 00·00	−10·191	359 59 49·809	−1·447	359 59 48·362
	South Barrule (T)	03 39 28·40	−38·383	03 38 50·017	−1·721	03 38 48·296
	Slieve Donard (New) (S)	57 56 33·95	−39·961	57 55 53·989	−1·250	57 55 52·739
	Divis (R)	87 10 41·93	−13·672	87 10 28·258	−1·464	87 10 26·794
	Trostan (H)	127 24 41·85	+13·251	127 24 55·101	+1·337	127 24 56·438
	Knocklayd (G)	134 51 37·13	+20·792	134 51 57·922	+1·658	134 51 59·580
	Cnoc Moy (I)	162 10 28·15	+31·310	162 10 59·460	+1·455	162 11 00·915
	Goat Fell (K)	194 56 21·85	+42·561	194 57 04·411	−0·012	194 57 04·399
	Ailsa Craig (L)	195 11 10·84	+21·621	195 11 32·461	+1·043	195 11 33·504
	Beneraird (M)	220 51 32·21	+10·804	220 51 43·014	+0·493	220 51 43·507
	Merrick (N)	251 12 13·22	+13·510	251 12 26·730	+0·655	251 12 27·385
	Cairnsmore of Fleet (O)	275 18 25·99	+ 4·892	275 18 30·882	+0·220	275 18 31·102
	Carleton Fell (P)	315 46 33·24	− 8·583	315 46 24·657	−0·964	315 46 23·693
Cnoc Moy (I)	Knocklayd (G)	00 00 00·00	−13·680	359 59 46·320	−0·198	359 59 46·122
	Slieve Snaght (F)	26 49 52·64	− 9·022	26 49 43·618	−0·547	26 49 43·071
	Beinn Tart a' Mhill (E)	80 25 44·32	+26·667	80 26 10·987	+1·184	80 26 12·171
	Cairn Pat (J)	267 57 10·38	−33·461	267 56 36·919	+1·528	267 56 38·447
	Trostan (H)	341 24 15·00	−21·930	341 23 53·070	−1·966	341 23 51·104
Divis (R)	Knocklayd (G)	123 47 24·15	+41·393	123 48 05·543	+0·698	123 48 06·241
	Trostan (H)	127 07 06·81	+32·306	127 07 39·116	−0·452	127 07 38·664
	Cairn Pat (J)	202 45 50·12	+15·012	202 46 05·132	+0·339	202 46 05·471
	Inshanks (Q)	220 47 32·23	+ 2·431	220 47 34·661	+1·288	220 47 35·949
	South Barrule (T)	257 12 56·27	−32·483	257 12 23·787	−0·822	257 12 22·965
	Slieve Donard (New) (S)	309 49 29·89	−31·566	309 48 58·324	−1·051	309 48 57·273
Slieve Donard (New) (S)	Trostan (H)	73 56 40·91	+63·672	73 57 44·582	−0·822	73 57 43·760
	Divis (R)	75 18 29·58	+31·420	75 19 01·000	−0·316	75 19 00·684
	Cairn Pat (J)	119 00 50·65	+43·680	119 01 34·330	+0·497	119 01 34·827
	Inshanks (Q)	131 38 26·51	+30·942	131 38 57·452	+1·584	131 38 59·036
	South Barrule (T)	174 40 11·82	− 4·195	174 40 07·625	−1·112	174 40 06·513
	Holyhead (X)	221 51 31·72	−58·346	221 50 33·374	+0·596	221 50 33·970
	Howth (Z)	269 03 23·89	−58·858	269 02 25·032	+0·160	269 02 25·192
	Kippure (A$_1$)	276 38 13·59	−74·237	276 36 59·353	−0·587	276 36 58·766

8.5 *continued*

From	*To*	*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Forth Mountain	Tara (C$_1$)	00° 00′ 00″·00	+31″·485	00° 00′ 31″·485	+0″·341	00° 00′ 31″·826
(D$_1$)	Prescelly (F$_1$)	79 14 23·80	−32·479	79 13 51·321	+0·358	79 13 51·679
	Ballycreen (B$_1$)	342 28 45·61	+51·090	342 29 36·700	−0·699	342 29 36·001
Holyhead	Rhiw (Y)	00 00 00·00	−24·121	359 59 35·879	+0·655	359 59 36·534
(X)	Ballycreen (B$_1$)	72 39 38·20	−21·022	72 39 17·178	+0·245	72 39 17·423
	Kippure (A$_1$)	86 06 57·21	− 5·227	86 06 51·983	−1·147	86 06 50·836
	Howth (Z)	97 52 07·97	+ 5·857	97 52 13·827	−1·576	97 52 12·251
	Slieve Donard (New)(S)	143 16 55·32	+51·782	143 17 47·102	−1·266	143 17 45·836
	South Barrule (T)	183 27 12·70	+41·641	183 27 54·341	+0·067	183 27 54·408
	Snaefell (U)	190 42 43·06	+45·845	190 43 28·905	+0·869	190 43 29·774
	Llaneilian (W)	252 00 55·06	+ 3·769	252 00 58·829	+1·461	252 01 00·290
	Garnedd Ugain (H$_1$)	306 22 19·90	−11·621	306 22 08·279	−0·052	306 22 08·227
	Yr Eifl (G$_1$)	340 05 22·97	−16·765	340 05 06·205	+0·743	340 05 06·948
Howth	Slieve Donard (New)(S)	48 35 55·79	+59·977	48 36 55·767	−1·602	48 36 54·165
(Z)	South Barrule (T)	88 47 08·87	+49·465	88 47 58·335	+1·040	88 47 59·375
	Holyhead (X)	135 59 35·34	− 6·722	135 59 28·618	+0·739	135 59 29·357
	Rhiw (Y)	163 51 04·19	−39·193	163 50 24·997	+0·204	163 50 25·201
	Kippure (A$_1$)	261 27 20·60	−14·483	261 27 06·117	−0·381	261 27 05·736
Inshanks	Cairn Pat (J)	00 00 00·00	+10·069	00 00 10·069	−0·624	00 00 09·445
(Q)	Beneraird (M)	21 21 38·03	+20·452	21 21 58·482	−0·030	21 21 58·452
	Merrick (N)	50 35 11·55	+22·562	50 35 34·112	+0·828	50 35 34·940
	Carleton Fell (P)	103 48 24·90	+ 1·073	103 48 25·973	+0·110	103 48 26·083
	Snaefell (U)	167 38 14·23	−21·521	167 37 52·709	−1·056	167 37 51·653
	South Barrule (T)	184 58 01·89	−27·743	184 57 34·147	−0·668	184 57 33·479
	Slieve Donard (New)(S)	250 34 05·11	−27·974	250 33 37·136	+0·896	250 33 38·032
	Divis (R)	285 12 20·00	− 2·189	285 12 17·811	+0·545	285 12 18·356
Kippure	Howth (Z)	00 00 00·00	+14·811	00 00 14·811	+0·302	00 00 15·113
(A$_1$)	Holyhead (X)	42 47 10·58	+ 6·125	42 47 16·705	+0·614	42 47 17·319
	Rhiw (Y)	69 07 21·91	−28·062	69 06 53·848	+0·004	69 06 53·852
	Tara (C$_1$)	132 57 05·07	−39·169	132 56 25·901	+0·432	132 56 26·333
	Ballycreen (B$_1$)	145 47 14·74	−21·158	145 46 53·582	−0·260	145 46 53·322
	Slieve Donard (New)(S)	334 43 20·85	+77·358	334 44 38·208	−1·093	334 44 37·115
Knocklayd	Trostan (H)	00 00 00·00	− 9·061	359 59 50·939	−1·176	359 59 49·763
(G)	Divis (R)	11 29 29·10	−41·983	11 28 47·117	−0·959	11 28 46·158
	Slieve Snaght (F)	118 52 56·15	+ 6·375	118 53 02·525	+0·612	118 53 03·137
	Beinn Tart a' Mhill (E)	194 19 36·37	+43·904	194 20 20·274	+1·426	194 20 21·700
	Cnoc Moy (I)	257 30 13·32	+14·262	257 30 27·582	−0·399	257 30 27·183
	Cairn Pat (J)	318 08 40·83	−23·152	318 08 17·678	+0·495	318 08 18·173
Prescelly	Capel Cynon (K$_1$)	00 00 00·00	+ 8·374	00 00 08·374	+1·024	00 00 09·398
(F$_1$)	Garn Fawr (E$_1$)	234 26 30·56	+ 3·856	234 26 34·416	−0·535	234 26 33·881
	Forth Mountain (D$_1$)	234 39 05·32	+27·670	234 39 32·990	−0·954	234 39 32·036
	Tara (C$_1$)	256 20 30·10	+49·766	256 21 19·866	−0·444	256 21 19·422
	Ballycreen (B$_1$)	260 58 00·85	+64·639	260 59 05·489	+0·381	260 59 05·870
	Rhiw (Y)	310 57 14·79	+46·316	310 58 01·106	+0·527	310 58 01·633

8.5 *continued*

From	*To*		*Mean Observed Direction*	$(t-T)$	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Rhiw	Holyhead	(X)	00° 00′ 00″·00	+24″·077	00° 00′ 24″·077	+0″·820	00° 00′ 24″·897
(Y)	Yr Eifl	(G_1)	42 41 30·48	+ 6·714	42 41 37·194	+0·450	42 41 37·644
	Garnedd Ugain	(H_1)	57 05 04·66	+10·729	57 05 15·389	+0·199	57 05 15·588
	Cader Idris	(I_1)	109 47 16·07	− 6·676	109 47 09·394	−0·673	109 47 08·721
	Aberystwyth	(J_1)	142 44 29·11	−19·251	142 44 09·859	+0·284	142 44 10·143
	Capel Cynon	(K_1)	170 50 03·92	−34·925	170 49 28·995	−0·140	170 49 28·855
	Prescelly	(F_1)	188 52 15·59	−45·199	188 51 30·391	−0·662	188 51 29·729
	Tara	(C_1)	265 52 59·46	− 5·264	265 52 54·196	+0·393	265 52 54·589
	Ballycreen	(B_1)	278 31 03·23	+ 8·331	278 31 11·561	−0·145	278 31 11·416
	Kippure	(A_1)	292 26 51·77	+23·895	292 27 15·665	+0·068	292 27 15·733
	Howth	(Z)	305 43 22·96	+34·095	305 43 57·055	−0·597	305 43 56·458
Slieve Snaght	Knocklayd	(G)	00 00 00·00	− 6·873	359 59 53·127	−0·217	359 59 52·910
(F)	Beinn Tart a' Mhill	(E)	310 36 28·74	+44·319	310 37 13·059	+0·440	310 37 13·499
	Cnoc Moy	(I)	345 27 04·00	+10·128	345 27 14·128	−0·223	345 27 13·905
South Barrule	Inshanks	(Q)	00 00 00·00	+27·021	00 00 27·021	−0·781	00 00 26·240
(T)	Carleton Fell	(P)	26 41 18·34	+26·586	26 41 44·926	+1·174	26 41 46·100
	Snaefell	(U)	62 34 18·47	+ 5·223	62 34 23·693	−0·392	62 34 23·301
	Black Combe	(V)	97 20 17·09	+ 3·508	97 20 20·598	+0·317	97 20 20·915
	Llaneilian	(W)	179 14 07·70	−35·624	179 13 32·076	+0·566	179 13 32·642
	Holyhead	(X)	195 58 55·87	−41·331	195 58 14·539	−0·241	195 58 14·298
	Howth	(Z)	243 11 43·93	−42·802	243 11 01·128	+1·030	243 11 02·158
	Slieve Donard (New)	(S)	288 37 35·57	+ 3·695	288 37 39·265	−0·996	288 37 38·269
	Divis	(R)	316 39 30·34	+28·492	316 39 58·832	−0·700	316 39 58·132
	Cairn Pat	(J)	358 41 25·18	+36·936	358 42 02·116	+0·023	358 42 02·139
Tara	Forth Mountain	(D_1)	00 00 00·00	−30·583	359 59 29·417	+0·400	359 59 29·817
(C_1)	Ballycreen	(B_1)	128 48 15·93	+18·211	128 48 34·141	−0·747	128 48 33·394
	Kippure	(A_1)	142 55 57·29	+38·967	142 56 36·257	+1·494	142 56 37·751
	Rhiw	(Y)	232 32 36·98	+ 6·148	232 32 43·128	+0·999	232 32 44·127
	Prescelly	(F_1)	280 55 36·01	−56·808	280 54 39·202	−2·146	280 54 37·056
Trostan	Knocklayd	(G)	00 00 00·00	+ 9·001	00 00 09·001	+1·074	00 00 10·075
(H)	Cnoc Moy	(I)	58 54 29·55	+22·712	58 54 52·262	+0·215	58 54 52·477
	Cairn Pat	(J)	130 41 49·25	−14·660	130 41 34·590	+0·754	130 41 35·344
	Divis	(R)	194 49 12·37	−32·552	194 48 39·818	−0·925	194 48 38·893
	Slieve Donard (New)	(S)	196 09 46·15	−64·454	196 08 41·696	−1·118	196 08 40·578

For the British National Grid co-ordinates of the Irish stations see Table 3.1, page 83.

8.5 *continued*

Triangle misclosures and spherical excesses

Triangle	*Spherical Excess (ε)*	*Triangle Misclosure*	*Triangle*	*Spherical Excess (ε)*	*Triangle Misclosure*
EFG	10″·842	−0″·782	STX	19″·266	−2″·286
EFI	11·947	−0·677	STZ	18·678	−1·888
EGI	5·561	+1·459	SXZ	21·286	−2·216
FGI	4·456	+1·354	SXA$_1$	30·115	−0·405
GHI	1·362	−0·132	SZA$_1$	3·466	+0·574
GHJ	2·029	+1·671	TXZ	21·874	−2·614
GHR	0·456	−1·066	XYZ	12·511	+1·349
GIJ	7·115	+1·035	XYA$_1$	15·111	+1·659
GJR	10·748	−1·308	XYB$_1$	15·735	+0·255
HIJ	7·782	+2·838	XZA$_1$	5·363	+1·237
HJR	8·263	−1·913	XA$_1$B$_1$	7·995	−0·215
HJS	16·574	−2·034	YZA$_1$	7·963	+1·547
HRS	0·282	−0·912	YA$_1$B$_1$	8·619	−1·619
JQR	3·804	+0·236	YA$_1$C$_1$	14·767	+0·393
JQS	4·465	+0·235	YB$_1$C$_1$	7·029	−0·509
JQT	0·295	+1·035	YB$_1$F$_1$	29·452	−0·572
JRS	8·029	+0·791	YC$_1$F$_1$	26·471	+1·119
JRT	14·340	+0·180	A$_1$B$_1$C$_1$	0·881	−2·521
JST	16·212	+0·118	B$_1$C$_1$D$_1$	2·536	+0·134
QRS	8·690	+0·790	B$_1$C$_1$F$_1$	4·048	+1·182
QRT	10·831	+0·979	B$_1$D$_1$F$_1$	22·227	−1·757
QST	12·042	+0·918	C$_1$D$_1$F$_1$	15·643	−3·073
RST	9·901	+0·729			

Unclosed Triangles

Triangle	*Spherical Excess (ε)*	*Triangle*	*Spherical Excess (ε)*
JMQ	0″·872	TUX	3″·177
JNQ	2·538	TWX	5·900
JPQ	1·561	XYG$_1$	1·890
JPT	6·294	XYH$_1$	5·252
PQT	4·439	YF$_1$K$_1$	6·321
QTU	2·557		

8.6 Herstmonceux co-ordination (see Diagram 12)

From	*To*		*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Beachy Head (A)	Ditchling	(C)	00° 00′ 00″00	−6″584	359° 59′ 53″416	−0″144	359° 59′ 53″272
	Firle Beacon	(B)	10 18 16·48	−3·994	10 18 12·486	+0·176	10 18 12·662
	Herstmonceux	(E)	79 17 17·66	−5·800	79 17 11·860	−0·116	79 17 11·744
	Fairlight Down	(D)	113 45 08·12	−6·847	113 45 01·273	+0·083	113 45 01·356
Ditchling (C)	Firle Beacon	(B)	00 00 00·00	+2·503	00 00 02·503	−0·876	00 00 01·627
	Beachy Head	(A)	08 50 21·01	+6·207	08 50 27·217	−0·733	08 50 26·484
	Fairlight Down	(D)	336 23 28·93	+0·434	336 23 29·364	+0·492	336 23 29·856
	Herstmonceux	(E)	340 35 50·46	+1·116	340 35 51·576	+1·118	340 35 52·694
Fairlight Down (D)	Beachy Head	(A)	00 00 00·00	+7·192	00 00 07·192	−0·660	00 00 06·532
	Herstmonceux	(E)	26 49 28·37	+0·867	26 49 29·237	+0·557	26 49 29·794
	Ditchling	(C)	33 48 02·20	−0·483	33 48 01·717	+0·103	33 48 01·820
Firle Beacon (B)	Ditchling	(C)	00 00 00·00	−2·595	359 59 57·405	+0·247	359 59 57·652
	Herstmonceux	(E)	141 14 45·89	−1·592	141 14 44·298	−0·212	141 14 44·086
	Beachy Head	(A)	199 08 38·03	+3·904	199 08 41·934	−0·035	199 08 41·899
Herstmonceux (E)	Fairlight Down	(D)	00 00 00·00	−0·836	359 59 59·164	−0·311	359 59 58·853
	Beachy Head	(A)	118 42 39·71	+5·872	118 42 45·582	+0·398	118 42 45·980
	Firle Beacon	(B)	171 49 47·99	+1·649	171 49 49·639	−0·554	171 49 49·085
	Ditchling	(C)	191 10 54·45	−1·199	191 10 53·251	+0·467	191 10 53·718

Triangle misclosures and spherical excesses

Triangle	*Spherical Excess (ε)*	*Triangle Misclosure*
ABC	0″205	−0″745
ABE	0·533	+1·067
ACD	2·165	+0·235
ACE	1·196	+1·754
ADE	0·664	−2·124
BCE	0·458	+1·432
CDE	0·305	+0·605

8.7 Greenwich Observatory co-ordination (see Diagram 12)

From	*To*	*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Chipping Barnet Ch Twr (A)	Epping (B)	00° 00′ 00″·00	−2″·112	359° 59′ 57″·888	+1″·543	359° 59′ 59″·431
	Warley Wtr Twr (C)	24 02 16·25	+1·702	24 02 17·952	+0·745	24 02 18·697
	Severndroog Castle (D)	63 17 14·77	+6·713	63 17 21·483	−0·854	63 17 20·629
	Greenwich Observatory (E)	69 03 08·67	+6·270	69 03 14·940	−1·433	69 03 13·507
Epping (B)	Warley Wtr Twr (C)	00 00 00·00	+4·301	00 00 04·301	−1·694	00 00 02·607
	Severndroog Castle (D)	55 18 33·50	+9·802	55 18 43·302	+0·916	55 18 44·218
	Greenwich Observatory (E)	64 51 11·80	+9·293	64 51 21·093	−0·545	64 51 20·548
	Chipping Barnet Ch Twr (A)	121 52 08·86	+2·230	121 52 11·090	+1·323	121 52 12·413
Greenwich Observatory (E)	*Pole Hill Obelisk (Ref. Mark)	00 00 00·00	Not included in adjustment			
	Epping (B)	18 39 34·48	−9·125	18 39 25·355	+0·662	18 39 26·017
	Warley Wtr Twr (C)	56 30 30·62	−5·239	56 30 25·381	+1·375	56 30 26·756
	Severndroog Castle (D)	106 17 09·62	+0·401	106 17 10·021	−0·757	106 17 09·264
	Chipping Barnet Ch Twr (A)	324 43 39·74	−6·502	324 43 33·238	−1·279	324 43 31·959
Severndroog Castle (D)	Epping (B)	00 00 00·00	−9·723	359 59 50·277	−0·477	359 59 49·800
	Warley Wtr Twr (C)	38 32 53·86	−5·767	38 32 48·093	+1·308	38 32 49·401
	Greenwich Observatory (E)	277 10 09·67	−0·406	277 10 09·264	+0·113	277 10 09·377
	Chipping Barnet Ch Twr (A)	309 50 47·17	−7·033	309 50 40·137	−0·944	309 50 39·193
Warley Wtr Twr (C)	Severndroog Castle (D)	00 00 00·00	+5·973	00 00 05·973	+1·105	00 00 07·078
	Greenwich Observatory (E)	08 50 38·30	+5·481	08 50 43·781	+0·765	08 50 44·546
	Chipping Barnet Ch Twr (A)	52 02 56·82	−1·846	52 02 54·974	−0·036	52 02 54·938
	Epping (B)	86 08 32·12	−4·419	86 08 27·701	−1·835	86 08 25·866

* Reference Mark at Pole Hill Obelisk is 0·122 metres east of the vane.

Triangle misclosures and spherical excesses

Triangle	*Spherical Excess (ε)*	*Triangle Misclosure*	*Triangle*	*Spherical Excess (ε)*	*Triangle Misclosure*
ABC	0″·830	−0″·420	ADE	0″·167	+1″·113
ABD	1·437	+1·523	BCD	0·935	−1·455
ABE	1·304	−0·834	BCE	1·022	+0·738
ACD	1·542	+0·488	BDE	0·300	+3·470
ACE	1·496	+0·324	CDE	0·213	+1·277

8.7 *continued*

Data for the co-ordination of Epping

From	*To*	*Mean Observed Direction*
Epping Wtr Twr	Warley Wtr Twr	00° 00′ 00″
	Chipping Barnet Ch Twr	121 56 23
	Epping	209 24 26

Slope Distance Epping Wtr Twr to Epping = 20·421 metres.
Vertical Angle = 34° 27′ 52″

Data for the co-ordination of the Airy Transit Instrument

From	*To*	*Mean Observed Direction*
Greenwich Observatory	*Pole Hill Obelisk (Ref. Mark)	00° 00′ 00″
	Airy Transit (Centre)	179 55 46

Slope Distance Greenwich Observatory to Airy Transit (Centre) = 9·800 metres.
Vertical Angle = 41° 02′ 21″

* Reference Mark at Pole Hill Obelisk is 0·122 metres east of the vane.

8.8 North Tolsta co-ordination (see Diagram 13)

From	*To*		*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
An Cuaidh (D)	Clisham	(C)	247° 22′ 08″·03	+11″·224	247° 22′ 19″·254	+0″·100	247° 22′ 19″·354
	North Tolsta	(F)	298 40 53·96	+34·090	298 41 28·050	−0·087	298 41 27·963
	Point of Stoer	(E)	349 49 04·88	+24·745	349 49 29·625	−0·013	349 49 29·612
Clisham (C)	Mealisval	(B)	00 00 00·00	+14·432	00 00 14·432	+0·093	00 00 14·525
	North Tolsta	(F)	76 58 22·98	+27·560	76 58 50·540	+0·089	76 58 50·629
	An Cuaidh	(D)	140 32 49·07	−12·161	140 32 36·909	−0·182	140 32 36·727
Mealisval (B)	Muirnag	(A)	00 00 00·00	+15·643	00 00 15·643	−0·278	00 00 15·365
	North Tolsta	(F)	03 43 20·39	+14·427	03 43 34·817	+0·566	03 43 35·383
	Clisham	(C)	81 37 34·83	−14·653	81 37 20·177	−0·288	81 37 19·889
Muirnag (A)	Mealisval	(B)	73 04 29·95	−14·799	73 04 15·151	−0·246	73 04 14·905
	Point of Stoer	(E)	293 33 45·88	− 8·497	293 33 37·383	+0·480	293 33 37·863
	North Tolsta	(F)	296 30 58·51	− 1·003	296 30 57·507	−0·233	296 30 57·274
North Tolsta (F)	Clisham	(C)	35 53 15·54	−26·298	35 52 49·242	+0·215	35 52 49·457
	Mealisval	(B)	61 00 43·13	−13·560	61 00 29·570	−0·722	61 00 28·848
	Muirnag	(A)	100 43 49·88	+ 0·997	100 43 50·877	+0·321	100 43 51·198
	Point of Stoer	(E)	277 28 34·12	− 7·454	277 28 26·666	+0·272	277 28 26·938
	An Cuaidh	(D)	330 46 19·50	−35·250	330 45 44·250	−0·086	330 45 44·164
Point of Stoer (E)	An Cuaidh	(D)	19 50 26·64	−23·776	19 50 02·864	−1·261	19 50 01·603
	North Tolsta	(F)	95 24 35·11	+ 6·927	95 24 42·037	+0·691	95 24 42·728
	Muirnag	(A)	95 42 39·16	+ 7·846	95 42 47·006	+0·571	95 42 47·577

Triangle misclosures and spherical excesses

Triangle	*Spherical Excess* (ε)	*Triangle Misclosure*
ABF	0″·455	−1″·875
AEF	0·038	+0·882
BCF	3·214	+1·796
CDF	7·903	+0·157
DEF	6·438	−1·668

8.9 St Kilda co-ordination (see Diagram 13)

From	*To*		*Mean Observed Direction*	*(t−T)*	*Mean Plane Observed Direction*	*Adjustment Correction*	*Plane Adjusted Direction*
Beinn Mhor	Marrival	(C)	00° 00′ 00″00	+31″444	00° 00′ 31″444	−0″203	00° 00′ 31″241
(D)	Clisham	(B)	24 25 13·67	+59·311	24 26 12·981	−0·699	24 26 12·282
	Heaval	(E)	202 23 07·49	−25·932	202 22 41·558	+0·726	202 22 42·284
	St Kilda	(F)	314 11 05·07	+59·816	314 12 04·886	+0·177	314 12 05·063
Clisham	Marrival	(C)	00 00 00·00	−27·912	359 59 32·088	−1·259	359 59 30·829
(B)	St Kilda	(F)	43 09 41·03	− 5·855	43 09 35·175	−0·051	43 09 35·124
	Mealisval	(A)	103 08 50·07	+14·431	103 09 04·501	+1·088	103 09 05·589
	Beinn Mhor	(D)	341 33 18·67	−57·082	341 32 21·588	+0·222	341 32 21·810
Heaval	Marrival	(C)	00 00 00·00	+58·582	00 00 58·582	+0·159	00 00 58·741
(E)	Beinn Mhor	(D)	11 55 02·46	+26·281	11 55 28·741	−0·188	11 55 28·553
	St Kilda	(F)	319 37 42·70	+89·510	319 39 12·210	+0·029	319 39 12·239
Marrival	Beinn Mhor	(D)	00 00 00·00	−31·443	359 59 28·557	−0·165	359 59 28·392
(C)	Heaval	(E)	10 28 07·08	−57·802	10 27 09·278	+0·345	10 27 09·623
	St Kilda	(F)	112 54 45·95	+26·042	112 55 11·992	−0·199	112 55 11·793
	Mealisval	(A)	200 29 17·51	+45·003	200 30 02·513	+0·007	200 30 02·520
	Clisham	(B)	222 51 49·44	+29·000	222 52 18·440	+0·012	222 52 18·452
Mealisval	Marrival	(C)	00 00 00·00	−43·976	359 59 16·024	+0·035	359 59 16·059
(A)	St Kilda	(F)	53 09 11·25	−22·426	53 08 48·824	+0·041	53 08 48·865
	Clisham	(B)	305 31 21·48	−14·653	305 31 06·827	−0·076	305 31 06·751
St Kilda	Marrival	(C)	00 00 00·00	−27·838	359 59 32·162	+0·319	359 59 32·481
(F)	Beinn Mhor	(D)	21 16 27·25	−63·938	21 15 23·312	−0·410	21 15 22·902
	Heaval	(E)	37 11 18·09	−94·415	37 09 43·675	+0·133	37 09 43·808
	Mealisval	(A)	320 43 31·44	+24·522	320 43 55·962	+0·051	320 43 56·013
	Clisham	(B)	333 06 37·03	+ 6·496	333 06 43·526	−0·092	333 06 43·434

Triangle misclosures and spherical excesses

Triangle	*Spherical Excess (ε)*	*Triangle Misclosure*	*Triangle*	*Spherical Excess (ε)*	*Triangle Misclosure*
ABC	2″983	−2″463	BDF	19″712	+1″468
ABF	5·513	−1·113	CDE	1·284	+0·766
ACF	11·849	−0·479	CDF	6·987	+1·143
BCD	3·406	+2·154	CEF	13·661	+0·599
BCF	9·319	−1·829	DEF	7·958	+0·222

8.10 Connection with France (see Diagram 12)

From	*To*		*Mean Observed Direction*			*Zeros*	$(t-T)$	*Mean Plane Observed Direction*			*Adjustment Correction*	*Plane Adjusted Direction*		
Rumfields Wtr Twr (G)	St. Inglevert	(F)	240°	46′	41″·37	76	+33″·169	240°	47′	14″·539	−0″·055	240°	47′	14″·484
	Mont Lambert	(D)	248	10	54·00	80	+43·730	248	11	37·730	−0·383	248	11	37·347
	Paddlesworth	(E)	296	12	38·43	46	+16·592	296	12	55·022	+0·439	296	12	55·461
Paddlesworth (E)	Rumfields Wtr Twr	(G)	98	49	07·57	51	−16·169	98	48	51·401	+0·791	98	48	52·192
	Gravelines Wtr Twr	(H)	165	00	56·30	59	+ 6·344	165	01	02·644	−0·247	165	01	02·397
	St. Inglevert	(F)	188	19	23·18	58	+14·853	188	19	38·033	−0·508	188	19	37·525
	Mont Lambert	(D)	204	53	31·35	56	+24·964	204	53	56·314	−0·404	204	53	55·910
	La Canche	(C)	213	39	03·90	53	+34·192	213	39	38·092	−0·311	213	39	37·781
	Fairlight Down	(B)	298	54	30·27	64	+14·562	298	54	44·832	+0·680	298	54	45·512
Fairlight Down (B)	Beachy Head	(A)	00	00	00·00	88	+ 7·194	00	00	07·194	−0·765	00	00	06·429
	Paddlesworth	(E)	174	47	05·80	88	−13·730	174	46	52·070	−0·312	174	46	51·758
	St. Inglevert	(F)	210	39	03·48	55	− 1·337	210	39	02·143	+0·569	210	39	02·712
	Mont Lambert	(D)	223	52	38·87	54	+ 7·801	223	52	46·671	+0·732	223	52	47·403
	La Canche	(C)	235	02	59·37	58	+16·088	235	03	15·458	−0·225	235	03	15·233
Beachy Head (A)	Fairlight Down	(B)	103	26	50·15	68	− 6·849	103	26	43·301	−0·795	103	26	42·506
	Mont Lambert	(D)	135	10	07·96	35	− 0·680	135	10	07·280	+0·776	135	10	08·056
	La Canche	(C)	144	12	28·26	49	+ 6·938	144	12	35·198	+0·019	144	12	35·217
						(1)								
Gravelines Wtr Twr (H)	St. Inglevert	(F)	00	00	00·03	15	+10·520	00	00	10·550	−0·247	00	00	10·303
	Paddlesworth	(E)	35	04	52·09	15	− 6·952	35	04	45·138	+0·247	35	04	45·385
Mont Lambert (D)	St. Inglevert	(F)	00	00	00·00	13	−11·316	359	59	48·684	+0·510	359	59	49·194
	La Canche	(C)	171	02	22·66	10	+10·233	171	02	32·893	+0·282	171	02	33·175
	Beachy Head	(A)	259	43	30·15	13	+ 0·798	259	43	30·948	−0·727	259	43	30·221
	Fairlight Down	(B)	271	52	54·61	10	− 8·715	271	52	45·895	−0·250	271	52	45·645
	Paddlesworth	(E)	308	46	26·24	13	−26·315	308	45	59·925	+0·474	308	46	00·399
	Rumfields Wtr Twr	(G)	334	40	23·78	8	−44·924	334	39	38·856	−0·289	334	39	38·567
St. Inglevert (F)	Mont Lambert	(D)	359	59	59·65	11	+11·358	00	00	11·008	+0·058	00	00	11·066
	Fairlight Down	(B)	78	39	21·65	11	+ 1·500	78	39	23·150	−0·325	78	39	22·825
	Paddlesworth	(E)	112	12	19·61	10	−15·716	112	12	03·894	−0·009	112	12	03·885
	Rumfields Wtr Twr	(G)	147	16	11·22	10	−34·202	147	15	37·018	+0·557	147	15	37·575
	Gravelines Wtr Twr	(H)	233	49	03·58	9	−10·152	233	48	53·428	+0·247	233	48	53·675
	La Canche	(C)	355	46	01·70	11	+21·665	355	46	23·365	−0·528	355	46	22·837
La Canche (C)	Mont Lambert	(D)	00	00	00·00	12	−10·231	359	59	49·769	+0·116	359	59	49·885
	St. Inglevert	(F)	04	43	39·72	10	−21·581	04	43	18·139	−0·464	04	43	17·675
	Beachy Head	(A)	277	43	22·87	12	− 8·126	277	43	14·744	−0·652	277	43	14·092
	Fairlight Down	(B)	292	00	47·90	11	−17·970	292	00	29·930	+0·255	292	00	30·185
	Paddlesworth	(E)	326	29	34·27	12	−36·036	326	28	58·234	+0·745	326	28	58·979

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(1) Number of series of six repetitions.

8.10 *continued*

Triangle misclosures and spherical excesses

Triangle	*Spherical Excess* (ϵ)	*Triangle Misclosure*	*Triangle*	*Spherical Excess* (ϵ)	*Triangle Misclosure*
GEF	4″041	+0″239	FDE	1″964	−0″074
GED	4·614	+1·136	FDB	3·321	−0·541
EBF	5·114	−2·384	DCE	1·515	+0·345
EBD	6·471	−2·851	FCE	3·587	+0·493
EBC	7·878	−1·568	FCB	6·351	+1·309
BAD	3·951	−0·551	DCB	2·922	+1·628
BAC	4·951	−1·181	DCA	3·922	+0·998
HFE	3·399	−0·489	FCD	0·108	+0·222
FDG	1·391	−0·971			

APPENDIX 9

ASTRONOMICAL WORK

9.1 Data for Azimuths by Black's Method

Observing Station = Herstmonceux Referring Object (R) = Fairlight Down

Date 1953 (night of)	*Star*	*Approximate*		A_R
		A_S	h_S	
3/4 May	α Aurigae	337° 58′ 14″	10° 41′ 44″	86° 07′ 15″·57
3/4 May	α Leonis	271 22 58	15 01 21	10·22
3/4 May	α Geminorum	315 17 07	06 12 24	15·24
3/4 May	α Virginis	224 05 58	18 07 20	16·15
3/4 May	α Scorpii	180 22 59	12 43 42	14·88
3/4 May	β Andromedae	48 17 29	12 35 00	13·99
3/4 May	α Andromedae	65 30 23	17 26 48	08·01
3/4 May	α Virginis	245 21 49	05 22 04	14·69
5/6 May	α Aurigae	356 33 55	06 55 04	15·86
5/6 May	γ Andromedae	34 27 39	12 01 27	14·20
5/6 May	δ Leonis	288 44 03	11 37 38	13·75
6/7 May	β Corvi	191 24 21	15 15 00	16·56
6/7 May	δ Scorpii	154 01 38	12 42 53	13·71
6/7 May	ε Aquarii	120 06 53	10 39 22	11·94
6/7 May	β Capricorni	134 25 42	12 54 08	11·33
7/8 May	ζ Cassiopeiae	06 30 32	14 53 00	12·14
7/8 May	γ Hydrae	187 22 47	15 54 52	14·88
7/8 May	γ Aquilae	87 10 39	11 16 04	11·76
7/8 May	ζ Hydrae	268 26 29	08 50 40	12·54
7/8 May	τ Scorpii	164 08 23	09 27 03	14·39
7/8 May	ε Aurigae	345 44 00	06 11 48	16·32
8/9 May	ε Cygni	50 51 53	12 27 21	12·68
8/9 May	ε Leonis	288 43 42	15 49 36	11·92
10/11 May	β Cygni	66 03 12	16 36 54	14·27
10/11 May	α Hydrae	238 55 04	12 42 50	10·64
10/11 May	ε Corvi	201 32 15	14 05 51	13·79
10/11 May	ο Andromedae	32 53 57	11 10 06	15·59
10/11 May	α Geminorum	304 31 42	13 35 38	14·34
10/11 May	γ Hydrae	214 21 31	09 14 06	18·18

Mean = 86° 07′ 13″·78

Least squares value = 86 07 13·74
Correction to Mean Pole = −0·19

A_G = 86 07 13·55

9.1 *continued*

Observing Station = Fairlight Down   Referring Object (R) = Herstmonceux

Date 1953 (night of)	*Star*	*Approximate*		A_R
		A_S	h_S	
12/13 May	γ Cygni	47° 04′ 46″	17° 36′ 15″	266° 19′ 58″·75
12/13 May	α Canis Minoris	266 56 57	09 22 13	57·67
12/13 May	α Cassiopeiae	08 02 04	17 46 22	58·24
12/13 May	ζ Aquilae	89 53 27	17 47 17	58·42
12/13 May	δ Scorpii	155 12 37	13 06 40	56·25
12/13 May	β Aurigae	324 36 23	15 49 50	58·08
12/13 May	β Corvi	213 01 48	09 31 44	20′ 01·53
12/13 May	λ Aquilae	133 18 37	23 34 46	19 57·82
12/13 May	κ Ursae Majoris	322 23 04	20 04 02	56·65
12/13 May	α Aquarii	114 31 39	17 58 56	53·39
13/14 May	α Cassiopeiae	02 32 38	17 13 02	57·22
13/14 May	β Cygni	68 49 21	18 55 23	58·24
16/17 May	ε Cygni	58 17 58	18 05 59	20′ 00·46
16/17 May	ζ Aquilae	95 01 12	21 54 28	19 57·16
17/18 May	α Hydrae	234 56 43	15 07 07	20′ 01·01
18/19 May	δ Leonis	287 34 03	12 34 18	19′ 57·38
18/19 May	α Persei	27 55 11	17 15 38	57·53
19/20 May	ζ Ophiuchi	128 05 27	14 32 51	20′ 00·04
19/20 May	β Ophiuchi	108 13 33	19 59 30	19 59·54
19/20 May	α Aurigae	324 54 52	16 49 12	58·37
19/20 May	ε Corvi	209 31 12	11 41 31	20′ 00·14
19/20 May	π Hydrae	190 40 00	11 58 40	00·06
19/20 May	α Leonis	269 16 58	16 22 32	00·14
19/20 May	γ Corvi	228 31 29	08 35 33	00·14
19/20 May	β Virginis	253 07 24	15 49 50	19′ 55·69
19/20 May	η Virginis	249 10 23	15 37 02	58·67
19/20 May	ι Ursae Majoris	329 27 52	16 56 22	58·44

Mean = 266° 19′ 58″·41

Least squares value = 266 19 58·47
Correction to Mean Pole = −0·25

A_G = 266 19 58·22

9.1 *continued*

Observing Station = Herstmonceux Referring Object (R) = Fairlight Down

Date 1953 (night of)	*Star*	*Approximate*		A_R
		A_S	h_S	
30/31 Oct.	β Capricorni	226° 12′ 11″	12° 35′ 30″	86° 07′ 15″·69
30/31 Oct.	κ Aurigae	67 06 15	19 41 02	14·05
30/31 Oct.	η Eridani	136 48 09	20 36 16	13·63
30/31 Oct.	σ Herculis	317 38 54	17 18 58	09·03
30/31 Oct.	ο Herculis	296 02 12	16 04 55	10·90
30/31 Oct.	β Ceti	183 35 03	20 49 44	16·95
30/31 Oct.	γ Aquilae	269 01 30	14 24 06	11·60
30/31 Oct.	τ^3 Eridani	161 24 47	13 19 00	10·78
30/31 Oct.	ζ Ursae Majoris	04 53 39	16 14 56	09·30
30/31 Oct.	β Canis Minoris	93 05 41	13 18 45	15·36
30/31 Oct.	γ Ursae Majoris	21 57 27	19 14 51	10·98
30/31 Oct.	β Draconis	338 30 27	17 19 26	15·54
30/31 Oct.	β Piscium	252 19 51	18 20 49	12·58
30/31 Oct.	υ Ceti	206 03 48	13 56 40	13·59
30/31 Oct.	τ^5 Eridani	186 50 08	17 05 30	14·09
30/31 Oct.	η Ursae Majoris	23 04 00	14 54 34	14·21
1/2 Nov.	η Eridani	126 55 36	15 31 12	10·42
2/3 Nov.	ξ Herculis	296 46 43	16 18 39	09·83
2/3 Nov.	98 Aquarii	203 39 04	15 37 34	14·29
2/3 Nov.	γ Eridani	137 23 30	15 49 32	12·22
2/3 Nov.	ε Orionis	116 34 31	18 30 56	13·93
2/3 Nov.	η Ursae Majoris	01 08 04	10 25 04	13·06
2/3 Nov.	τ^5 Eridani	159 20 50	14 54 48	12·70
2/3 Nov.	ι Cancri	66 28 40	18 22 41	14·84
2/3 Nov.	μ Ursae Majoris	44 05 37	17 30 46	12·63
2/3 Nov.	η Aquarii	248 05 33	16 27 07	12·09
2/3 Nov.	ι Ceti	229 42 23	17 22 22	11·74
2/3 Nov.	α Cancri	91 58 02	17 10 36	08·31
2/3 Nov.	η Ursae Majoris	21 55 43	14 28 31	10·60
2/3 Nov.	γ Cygni	315 40 08	15 41 13	11·19
2/3 Nov.	α Pegasi	272 23 35	17 29 57	10·63
2/3 Nov.	ε Leporis	183 21 28	16 38 54	11·18
2/3 Nov.	θ Cygni	340 09 24	14 21 26	10·82
2/3 Nov.	ν^1 Draconis	02 20 55	16 07 44	13·05

Mean = 86° 07′ 12″·41

Least squares value = 86 07 12·41
Correction to Mean Pole = −0·32

A_G = 86 07 12·09

9.1 *continued*

Observing Station = Fairlight Down Referring Object (R) = Herstmonceux

Date 1953 (night of)	*Star*	*Approximate*		A_R
		A_S	h_S	
4/5 Nov.	ε Geminorum	70° 19′ 46″	16° 33′ 22″	266° 19′ 58″·68
4/5 Nov.	θ Aquilae	251 50 20	13 00 52	57·61
4/5 Nov.	γ Eridani	137 09 12	15 42 28	55·46
4/5 Nov.	δ Orionis	113 31 49	17 34 46	56·39
4/5 Nov.	γ Delphini	272 56 14	18 21 56	57·80
4/5 Nov.	γ Draconis	340 37 00	15 39 32	55·85
4/5 Nov.	η Ursae Majoris	25 29 35	15 56 28	54·54
6/7 Nov.	β Leporis	183 16 24	18 16 31	56·01
6/7 Nov.	β Leonis	90 14 05	19 26 24	58·72
9/10 Nov.	β Cygni	295 54 57	15 01 33	55·52
9/10 Nov.	β Ceti	207 14 53	16 47 16	54·71
9/10 Nov.	μ Ursae Majoris	46 04 33	18 54 48	57·87
9/10 Nov.	41 Cygni	295 50 16	18 11 44	53·93
9/10 Nov.	ι Ceti	227 22 32	18 33 01	59·17
9/10 Nov.	α Leporis	157 36 05	18 32 02	55·94
9/10 Nov.	θ Draconis	01 32 11	19 34 54	59·51
9/10 Nov.	γ Cygni	318 43 05	13 41 24	55·98
9/10 Nov.	β Canis Majoris	156 32 51	18 11 09	58·28
9/10 Nov.	ν Cygni	317 57 40	15 13 42	53·91
9/10 Nov.	ι Cygni	338 57 33	16 25 09	55·37
9/10 Nov.	ξ Draconis	359 19 59	17 46 09	59·14
9/10 Nov.	δ Leporis	187 56 25	17 53 52	57·15
10/11 Nov.	53 Eridani	210 11 31	19 55 14	58·65
11/12 Nov.	β Canum Venaticorum	44 12 10	17 26 00	54·30
11/12 Nov.	β Virginis	107 12 06	16 03 16	59·71
12/13 Nov.	ν Geminorum	74 04 20	13 13 06	56·82
12/13 Nov.	ε Aquilae	273 52 28	16 19 20	57·06
12/13 Nov.	θ Ursae Majoris	29 35 08	20 36 30	20′ 00·55
12/13 Nov.	α Orionis	97 28 33	15 32 40	19 57·73
12/13 Nov.	γ Eridani	136 46 32	15 31 28	57·63

Mean = 266° 19′ 57″·00

Least squares value = 266 19 56·96
Correction to Mean Pole = −0·23

A_G = 266 19 56·73

9.1 *continued*

Observing Station = White Horse Hill Referring Object (R) = Liddington Castle

Date 1953 (night of)	*Star*	*Approximate*		*A_R*
		A_S	*h_S*	
23/24 May	α Cassiopeiae	07° 24′ 19″	18° 21′ 27″	234° 18′ 04″80
24/25 May	γ Aquilae	89 51 22	13 20 52	03·88
24/25 May	α Geminorum	312 18 42	08 30 59	06·91
24/25 May	β Aurigae	337 05 59	10 32 46	05·96
24/25 May	α Leonis	276 19 33	10 35 24	04·96
24/25 May	γ Andromedae	26 23 39	08 50 40	05·78
24/25 May	σ Sagittarii	162 42 08	10 17 08	04·52
24/25 May	γ Aquarii	110 13 22	13 20 27	04·36
24/25 May	β Capricorni	147 34 05	17 57 23	06·26
24/25 May	α Scorpii	206 46 34	07 45 30	05·48
26/27 May	β Geminorum	291 06 16	19 25 22	05·52
27/28 May	α Virginis	220 02 06	19 20 14	06·38
30/31 May	ζ Cygni	65 48 22	19 23 18	08·70
30/31 May	β Virginis	246 56 55	19 43 14	04·70

Mean = 234° 18′ 05″59

Least squares value = 234 18 05·60
Correction to Mean Pole = −0·38

A_G = 234 18 05·22

9.1 *continued*

Observing Station = Liddington Castle Referring Object (R) = White Horse Hill

Date 1953 (night of)	*Star*	*Approximate*		A_R
		A_S	h_S	
1/2 June	τ Scorpii	191° 01′ 09″	09° 36′ 58″	54° 11′ 55″32
1/2 June	δ Persei	21 40 16	13 00 30	55·40
1/2 June	γ Andromedae	42 55 54	17 34 06	53·44
1/2 June	δ Scorpii	213 00 34	09 40 30	55·42
1/2 June	α^2 Librae	234 09 02	06 29 49	53·87
1/2 June	ε Virginis	275 18 41	10 06 23	51·61
6/7 June	α Virginis	221 50 01	18 35 18	53·90
6/7 June	μ Pegasi	68 50 12	14 24 14	55·00
6/7 June	ε Pegasi	93 54 41	15 28 19	52·02
6/7 June	δ Persei	11 29 25	10 11 50	53·38
6/7 June	β Aquarii	115 11 57	11 41 42	54·66
6/7 June	δ Leonis	287 01 19	13 05 46	53·73
8/9 June	α Geminorum	312 16 47	08 29 34	57·69
8/9 June	θ Ophiuchi	159 20 53	11 02 50	53·27
8/9 June	β Aurigae	340 10 47	09 28 58	57·39
8/9 June	σ Librae	203 23 26	10 07 22	55·78
8/9 June	β Capricorni	131 16 24	10 43 14	54·40
8/9 June	μ Virginis	242 40 56	13 29 22	54·54

Mean = 54° 11′ 54″49

Least squares value = 54 11 54·51
Correction to Mean Pole = −0·44

A_G = 54 11 54·07

9.1 *continued*

Observing Station = St. Agnes Beacon   Referring Object (R) = Tregonning Hill

Date 1953 (night of)	*Star*	*Approximate*		A_R
		A_S	h_S	
13/14 June	δ Scorpii	180° 31′ 42″	17° 11′ 43″	206° 19′ 12″·39
13/14 June	π Scorpii	184 57 27	13 33 33	11·13
13/14 June	α Persei	11 29 41	11 07 46	11·84
22/23 June	π Scorpii	191 38 47	12 52 58	13·85
22/23 June	δ Aurigae	355 23 46	14 47 32	12·48
22/23 June	δ Persei	20 26 58	11 25 13	12·39
22/23 June	γ Andromedae	41 46 48	16 03 40	10·53
22/23 June	ω Piscium	104 10 15	19 55 57	11·46
22/23 June	α Arietis	72 34 35	15 52 59	13·03
22/23 June	δ Aquarii	139 50 54	14 42 44	12·91
23/24 June	ε Pegasi	94 48 40	16 32 36	10·52
23/24 June	θ Leonis	276 22 44	15 12 50	09·91
23/24 June	β Librae	229 54 01	17 33 24	12·08
23/24 June	β Comae Berenices	290 13 50	20 03 17	10·13
23/24 June	θ Capricorni	157 01 51	19 23 32	10·23

Mean = 206° 19′ 11″·66

Least squares value = 206 19 11·60
Correction to Mean Pole = −0·53

A_G = 206 19 11·07

9.1 *continued*

Observing Station = Tregonning Hill   Referring Object (R) = St Agnes Beacon

Date 1953 (night of)	*Star*	*Approximate*		A_R
		A_S	h_S	
3/4 July	δ Aurigae	358° 35′ 42″	14° 25′ 28″	26° 12′ 36″·11
3/4 July	45 Ophiuchi	188 20 45	09 36 20	35·70
3/4 July	δ Persei	26 17 53	13 34 10	35·56
3/4 July	τ Scorpii	210 13 10	05 51 28	36·62
3/4 July	β Librae	238 41 49	12 25 31	35·59
3/4 July	ρ Persei	50 15 42	17 47 08	34·89
3/4 July	ζ Ophiuchi	231 27 52	15 20 30	37·53
3/4 July	η Piscium	87 57 59	18 08 23	32·50
3/4 July	α Serpentis	261 28 05	15 34 21	35·90
3/4 July	δ Aquarii	148 46 10	18 29 38	34·11
3/4 July	χ Ursae Majoris	334 53 38	13 29 37	32·47
4/5 July	δ Capricorni	141 17 38	15 13 18	33·29
4/5 July	μ Sagittarii	201 46 35	16 04 34	37·00
4/5 July	ι Ceti	116 18 44	09 10 38	34·75
4/5 July	β Canum Venaticorum	316 14 05	16 42 26	34·39
7/8 July	τ Scorpii	181 50 39	11 43 56	35·74
7/8 July	δ Leonis	282 08 26	17 06 46	35·25
7/8 July	π Sagittarii	156 52 39	15 42 02	33·44
7/8 July	δ Andromedae	61 02 11	15 55 34	34·76
7/8 July	ν Ursae Majoris	300 38 56	18 09 30	36·74

Mean = 26° 12′ 35″·12

Least squares value = 26 12 35·05
Correction to Mean Pole = −0·59

A_G = 26 12 34·46

9.1 *continued*

Observing Station = Cairn Pat   Referring Object (R) = Inshanks

Date 1953 (night of)	*Star*	*Approximate*		A_R
		A_S	h_S	
15/16 July	ε Virginis	265° 07′ 50″	17° 08′ 09″	159° 00′ 02″90
15/16 July	λ Sagittarii	180 24 11	09 41 02	03·55
15/16 July	ε Aurigae	27 08 27	13 42 54	04·26
15/16 July	λ Ursae Majoris	332 39 29	13 07 54	04·35
15/16 July	ε Ophiuchi	238 37 08	14 50 58	03·97
15/16 July	δ Aquarii	152 16 39	15 13 42	02·98
15/16 July	η Tauri	70 28 22	15 40 32	03·39
21/22 July	η Bootis	286 34 04	11 10 17	02·51
21/22 July	ι Ursae Majoris	356 27 04	13 10 56	03·41
21/22 July	γ Bootis	314 44 44	16 40 14	02·68
27/28 July	α Serpentis	263 32 18	12 32 36	00·76
27/28 July	ν Ceti	103 43 22	15 58 50	01·39
28/29 July	33 Piscium	130 10 39	17 47 14	00·77
29/30 July	ζ Ophiuchi	212 05 52	19 46 49	03·79

Mean = 159° 00′ 02″91

Least squares value = 159 00 02·89
Correction to Mean Pole = −0·69

A_G = 159 00 02·20

9.1 *continued*

Observing Station = Inshanks Referring Object (R) = Cairn Pat

Date 1953 (night of)	*Star*	*Approximate*		
		A_S	h_S	A_R
31 July/1 Aug.	ζ Ophiuchi	213° 23′ 37″	19° 33′ 00″	339° 06′ 03″·15
31 July/1 Aug.	δ Persei	33 24 35	20 36 22	01·82
31 July/1 Aug.	ε Aurigae	25 47 12	13 03 58	01·97
31 July/1 Aug.	μ Serpentis	244 18 52	13 13 13	00·02
31 July/1 Aug.	41 Arietis	66 14 38	16 30 00	01·70
31 July/1 Aug.	ι Ursae Majoris	355 05 22	13 05 06	05′ 59·75
31 July/1 Aug.	ζ Capricorni	170 46 47	12 09 22	06 02·37
31 July/1 Aug.	δ Aquarii	152 01 23	15 19 22	01·17
31 July/1 Aug.	χ Ursae Majoris	334 38 45	17 30 30	00·29
31 July/1 Aug.	α Coronae Borealis	294 23 47	15 48 10	01·69
31 July/1 Aug.	κ Ophiuchi	270 29 17	11 15 30	02·83
1/2 Aug.	γ Piscium	104 52 11	13 57 54	01·49
1/2 Aug.	ξ Aquarii	140 37 10	20 02 04	04·23
1/2 Aug.	γ Bootis	310 04 50	19 27 40	05′ 58·27
1/2 Aug.	δ Ceti	117 11 05	18 05 32	59·52

Mean = 339° 06′ 01″·35

Least squares value = 339 06 01·37
Correction to Mean Pole = −0·71

A_G = 339 06 00·66

9.1 *continued*

Observing Station = Spital Hill   Referring Object (R) = Warth Hill

Date 1953 (night of)	*Star*	*Approximate*		A_R
		A_S	h_S	
7/8 Sept.	β Aurigae	28° 13′ 36″	18° 25′ 04″	53° 49′ 21″·11
7/8 Sept.	ι Ursae Majoris	358 49 04	16 42 48	20·35
7/8 Sept.	ι Aquarii	184 16 12	17 20 56	17·72
7/8 Sept.	ε Aquarii	208 57 06	18 13 40	16·46
7/8 Sept.	θ Aquilae	231 55 04	19 38 46	17·96
7/8 Sept.	δ Aquilae	249 06 55	15 46 39	18·28
7/8 Sept.	ε Geminorum	66 47 01	15 25 32	21·28
7/8 Sept.	η Ursae Majoris	344 40 07	19 34 44	20·26
7/8 Sept.	ζ Aquilae	269 53 42	16 18 28	15·88
7/8 Sept.	ν Orionis	90 33 44	17 45 20	19·70
7/8 Sept.	η Eridani	160 05 23	20 44 54	15·60
7/8 Sept.	ν Eridani	137 20 40	20 36 28	14·44
7/8 Sept.	α Orionis	116 40 14	23 44 16	17·29
9/10 Sept.	χ Herculis	322 56 50	19 23 18	17·83
9/10 Sept.	θ Lyrae	305 25 16	23 22 14	17·80

Mean = 53° 49′ 18″·13

Least squares value = 53 49 18·23
Correction to Mean Pole = −0·73

A_G = 53 49 17·50

9.1 *continued*

Observing Station = Warth Hill Referring Object (R) = Spital Hill

Date 1953 (night of)	*Star*	*Approximate*		A_R
		A_S	h_S	
18/19 Oct.	λ Ursae Majoris	18° 52′ 59″	13° 55′ 45″	234° 06′ 58″·95
18/19 Oct.	β Ceti	177 49 20	13 07 39	54·62
18/19 Oct.	α Canum Venaticorum	358 26 36	07 11 48	58·35
19/20 Oct.	γ Tauri	81 22 28	12 58 51	55·42
19/20 Oct.	ζ Tauri	69 20 41	12 13 22	07′ 00·49
19/20 Oct.	α Geminorum	44 31 34	11 11 58	06 59·17
23/24 Oct.	δ Aquilae	248 29 14	16 03 14	55·78
23/24 Oct.	β Aquilae	259 46 48	13 30 02	56·26
23/24 Oct.	ε Herculis	312 13 06	11 37 46	57·68
23/24 Oct.	β Bootis	340 52 34	11 20 10	58·12
23/24 Oct.	δ Orionis	117 59 33	15 36 42	55·92
23/24 Oct.	λ Aquarii	226 50 49	14 10 18	54·68
23/24 Oct.	γ Eridani	158 55 31	15 46 10	56·92
23/24 Oct.	τ Ceti	197 05 08	13 51 35	55·20
23/24 Oct.	41 Cygni	299 56 02	17 24 24	53·94
23/24 Oct.	τ Herculis	354 25 29	15 14 26	59·07
23/24 Oct.	26 Ceti	252 02 24	11 57 07	54·12

Mean = 234° 06′ 56″·75

Least squares value = 234 06 56·84
Correction to Mean Pole = −0·44

A_G = 234 06 56·40

9.1 *continued*

Observing Station = Fetlar Referring Object (R) = Saxavord

Date 1953 (night of)	*Star*	*Approximate*		A_R
		A_S	h_S	
1/2 Oct.	α Canum Venaticorum	06° 06′ 19″	09° 22′ 52″	03° 12′ 09″98
1/2 Oct.	τ Ceti	182 24 58	13 10 58	07·06
1/2 Oct.	β Canis Minoris	95 34 44	12 45 12	09·93
1/2 Oct.	γ Delphini	273 44 40	16 16 22	10·02
1/2 Oct.	κ Orionis	140 56 42	13 26 20	08·48
1/2 Oct.	β Lyrae	319 37 40	12 10 14	10·18
1/2 Oct.	ν Ursae Majoris	55 05 23	19 01 50	09·00
1/2 Oct.	γ Bootis	22 35 56	11 53 18	10·20
2/3 Oct.	γ Piscium	250 54 41	13 50 28	08·03
2/3 Oct.	1 Pegasi	288 58 10	11 53 21	08·72
3/4 Oct.	ε Geminorum	65 35 30	15 18 42	10·28
3/4 Oct.	ν Eridani	110 49 57	07 27 55	07·40
3/4 Oct.	γ Bootis	334 40 43	12 37 24	09·96
3/4 Oct.	γ Eridani	200 48 19	13 54 19	10·00
5/6 Oct.	o^1 Eridani	206 46 15	19 33 22	10·11

Mean = 03° 12′ 09″29

Least squares value = 03 12 09·28
Correction to Mean Pole = −0·64

A_G = 03 12 08·64

9.1 *continued*

Observing Station = Saxavord Referring Object (R) = Fetlar

Date 1953 (night of)	*Star*	*Approximate*		A_R
		A_S	h_S	
13/14 Oct.	λ Ursae Majoris	01° 44′ 58″	13° 59′ 12″	183° 13′ 22″84
13/14 Oct.	α Coronae Borealis	297 17 37	15 43 45	23·09
13/14 Oct.	δ Aquarii	179 56 40	13 06 24	25·49
13/14 Oct.	α Ophiuchi	272 06 27	13 16 52	23.30
13/14 Oct.	δ Aquilae	250 52 38	13 50 23	23·47
13/14 Oct.	μ Ursae Majoris	25 06 57	16 03 20	22·39
13/14 Oct.	ν Eridani	120 36 04	12 03 04	23·48
13/14 Oct.	δ Eridani	138 55 58	12 42 00	23·43
13/14 Oct.	ω^2 Aquarii	202 58 31	12 05 47	21·49
13/14 Oct.	γ Cancri	71 29 31	14 30 36	22·86
13/14 Oct.	β Cancri	86 40 30	08 50 50	24·90
13/14 Oct.	γ Eridani	164 46 10	14 35 20	23·92
13/14 Oct.	ι Ceti	223 42 23	12 23 41	24·21
13/14 Oct.	ι Herculis	346 09 03	17 58 46	22·82
13/14 Oct.	θ Lyrae	343 41 03	10 17 58	23·54
13/14 Oct.	α Monocerotis	163 52 11	18 40 52	22·90

Mean = 183° 13′ 23″38

Least squares value = 183 13 23·36
Correction to Mean Pole = −0·58

A_G = 183 13 22·78

9.2 Data for Azimuths from Polaris

Observing Station = Herstmonceux Referring Object (R) = Fairlight Down

Date 1953 (night of)	*Astronomic Azimuth of R*
9/10 May	86° 07′ 11″268
9/10 May	06·577
9/10 May	10·845
9/10 May	10·367
9/10 May	10·376
9/10 May	09·486
9/10 May	08·501
9/10 May	06·738
9/10 May	12·031
9/10 May	10·550
9/10 May	11·210
9/10 May	07·039
9/10 May	08·701
9/10 May	10·174
10/11 May	08·366
10/11 May	10·120

Mean = 86° 07′ 09″522
Correction to Mean Pole = −0·195

86 07 09·327

Date 1953 (night of)	*Astronomic Azimuth of R*
1/2 Nov.	86° 07′ 09″883
1/2 Nov.	07·528
1/2 Nov.	09·612
1/2 Nov.	11·641
1/2 Nov.	10·234
1/2 Nov.	09·111
1/2 Nov.	07·419
1/2 Nov.	07·274
1/2 Nov.	13·271
2/3 Nov.	06·644
2/3 Nov.	11·494
2/3 Nov.	08·395
2/3 Nov.	08·070
2/3 Nov.	07·601
2/3 Nov.	09·562
2/3 Nov.	08·812
2/3 Nov.	10·948
2/3 Nov.	11·253
2/3 Nov.	09·110
2/3 Nov.	12·502
2/3 Nov.	09·664
2/3 Nov.	10·581
2/3 Nov.	10·360
2/3 Nov.	07·835
2/3 Nov.	11·602
2/3 Nov.	11·096
2/3 Nov.	10·014
2/3 Nov.	08·837
2/3 Nov.	12·270
2/3 Nov.	10·496
2/3 Nov.	12·261
2/3 Nov.	10·088

Mean = 86° 07′ 09″858
Correction to Mean Pole = −0·320

86 07 09·538

9.2 *continued*

Observing Station = Fairlight Down Referring Object (R) = Herstmonceux

Date 1953 (night of)	*Astronomic Azimuth of R*
6/7 Nov.	266° 19′ 56″·617
6/7 Nov.	58·437
6/7 Nov.	57·016
6/7 Nov.	53·361
6/7 Nov.	54·625
6/7 Nov.	53·143
6/7 Nov.	54·718
6/7 Nov.	59·096
6/7 Nov.	56·827
6/7 Nov.	56·507
6/7 Nov.	56·017
6/7 Nov.	55·896
6/7 Nov.	58·001
6/7 Nov.	55·330
6/7 Nov.	57·246
6/7 Nov.	57·883
11/12 Nov.	59·110
11/12 Nov.	55·891
11/12 Nov.	52·511
11/12 Nov.	58·177
11/12 Nov.	53·842
11/12 Nov.	54·890
11/12 Nov.	53·427
11/12 Nov.	54·318
11/12 Nov.	58·643
11/12 Nov.	57·570
11/12 Nov.	57·509
11/12 Nov.	56·845
11/12 Nov.	56·363
11/12 Nov.	54·612
11/12 Nov.	54·074
11/12 Nov.	55·265

Mean = 266° 19′ 56″·056
Correction to Mean Pole = −0·231

266 19 55·825

9.3 Data for Position Lines

Observing Station = Cairn Pat $\varphi_0 = 54°\ 51'\ 48''\!.797$

Date 1953 (night of)	*Star*	*Approximate Mean A_S from Separate Faces*	*Mean of λ' from Separate Faces*
20/21 July	δ Cassiopeiae	48° 04′ 33″	$-00^{h}\ 20^{m}\ 11^{s}\!.642$
20/21 July	ε Cassiopeiae	41 55 24	11·700
20/21 July	109 Herculis	225 43 18	11·558
20/21 July	110 Herculis	222 30 51	11·564
20/21 July	α Pegasi	138 50 42	10·988
20/21 July	ι Draconis	315 28 23	11·222
21/22 July	δ Ursae Majoris	310 28 40	10·988
21/22 July	γ Delphini	133 27 51	11·216

Least squares value = φ 54° 51′ 46″·60 λ −05° 02′ 50″·34
Correction to Mean Pole = +0·01 —

φ 54 51 46·61 λ −05 02 50·34

Observing Station = Fairlight Down $\varphi_0 = 50°\ 52'\ 36''\!.505$

Date 1953 (night of)	*Star*	*Approximate Mean A_S from Separate Faces*	*Mean of λ' from Separate Faces*
9/10 Nov.	β Cygni	273° 23′ 25″	$+00^{h}\ 02^{m}\ 28^{s}\!.376$
9/10 Nov.	ζ Persei	94 05 56	28·518
9/10 Nov.	β Tauri	89 43 58	28·516
9/10 Nov.	ι Aurigae	92 14 18	28·498
9/10 Nov.	ζ Cygni	272 29 02	28·382
9/10 Nov.	κ Aurigae	86 15 45	28·610
9/10 Nov.	α Arietis	269 12 43	28·404
9/10 Nov.	α Trianguli	278 37 43	28·656
9/10 Nov.	21 Arietis	274 14 45	28·429
10/11 Nov.	β Cygni	270 14 48	28·529
10/11 Nov.	ζ Persei	90 27 48	28·690
10/11 Nov.	ξ Persei	87 02 39	28·696
10/11 Nov.	15 Vulpeculae	268 56 24	28·514
10/11 Nov.	ε Cygni	270 21 08	28·664
10/11 Nov.	41 Cygni	271 18 38	28·442
10/11 Nov.	ι Aurigae	88 01 06	28·627
10/11 Nov.	β Tauri	88 49 24	28·741
10/11 Nov.	136 Tauri	86 17 04	28·642

Only stars near the prime vertical were observed at this station, consequently the arithmetic mean was accepted, namely:

$+00^{h}\ 02^{m}\ 28^{s}\!.5519 \quad = \lambda = \ +00°\ 37'\ 08''\!.28$

9.3 *continued*

Observing Station = Fetlar $\varphi_0 = 60^\circ\ 37'\ 14''933$

Date 1953 (night of)	*Star*	*Approximate Mean As from Separate Faces*	*Mean of λ′ from Separate Faces*
28/29 Sept.	α Pegasi	140° 30′ 17″	$-00^{h}\ 03^{m}\ 25^{s}918$
28/29 Sept.	111 Herculis	228 23 02	27·804
28/29 Sept.	υ Pegasi	137 53 02	25·874
28/29 Sept.	σ Ursae Majoris	46 18 47	27·518
28/29 Sept.	α Andromedae	229 45 37	27·612
28/29 Sept.	θ Cephei	317 46 54	25·823
28/29 Sept.	η Cephei	315 37 30	26·018
28/29 Sept.	ζ Tauri	136 40 40	25·707
28/29 Sept.	α Cephei	315 38 58	25·975
28/29 Sept.	β Ursae Majoris	43 34 56	27·348
1/2 Oct.	β Camelopardi	45 20 13	27·734
1/2 Oct.	θ Draconis	313 42 12	25·998
1/2 Oct.	γ Delphini	221 21 15	27·434
1/2 Oct.	η Piscium	133 18 32	26·193
1/2 Oct.	2 Lyncis	47 11 50	27·451
1/2 Oct.	ι Pegasi	223 14 40	27·659

Least squares value =	φ 60° 37′ 09″08	λ −00° 51′ 41″74
Correction to Mean Pole =	−0·32	—
	φ 60 37 08·76	λ −00 51 41·74

9.3 *continued*

Observing Station = Greenwich Observatory (Auxiliary) φ_0 = 51° 28′ 38″·000

Date 1953 (night of)	*Star*	*Approximate Mean As from Separate Faces*	*Mean of λ′ from Separate Faces*
17/18 Aug.	β Pegasi	99° 38′ 39″	$+00^{\mathrm{h}}$ 00^{m} 00^{s}·010
17/18 Aug.	θ Coronae Borealis	273 45 34	−00 00 00·167
17/18 Aug.	α Pegasi	131 51 07	+00 00 00·057
17/18 Aug.	ι Draconis	316 05 11	+00 00 00·176
18/19 Aug.	β Cassiopeiae	47 24 37	+00 00 00·147
18/19 Aug.	δ Bootis	271 07 50	+00 00 00·024
18/19 Aug.	β Pegasi	98 02 02	+00 00 00·280
18/19 Aug.	γ Cassiopeiae	45 10 28	+00 00 00·217
18/19 Aug.	θ Coronae Borealis	271 35 49	−00 00 00·080
18/19 Aug.	α Ophiuchi	230 12 04	+00 00 00·061
18/19 Aug.	α Andromedae	94 30 36	+00 00 00·196
18/19 Aug.	ζ Pegasi	139 53 00	+00 00 00·148
18/19 Aug.	ε Aquilae	223 08 09	+00 00 00·080
18/19 Aug.	θ Draconis	311 32 44	−00 00 00·044
21/22 Aug.	β Trianguli	92 07 52	−00 00 00·054
21/22 Aug.	α Lyrae	278 32 50	+00 00 00·112
22/23 Aug.	γ Bootis	278 11 56	+00 00 00·100
22/23 Aug.	β Cassiopeiae	47 39 32	00 00 00·000
22/23 Aug.	ω Herculis	233 15 54	+00 00 00·148
22/23 Aug.	γ Delphini	142 52 06	+00 00 00·370
22/23 Aug.	ζ Ursae Majoris	311 38 07	−00 00 00·192
22/23 Aug.	γ Cassiopeiae	45 04 33	+00 00 00·024
22/23 Aug.	α Ophiuchi	227 02 49	+00 00 00·027
22/23 Aug.	α Pegasi	139 53 17	+00 00 00·146
22/23 Aug.	π Herculis	276 45 28	+00 00 00·061
22/23 Aug.	β Andromedae	91 09 10	+00 00 00·124
22/23 Aug.	θ Draconis	314 23 40	+00 00 00·194
22/23 Aug.	γ Aquilae	224 56 00	+00 00 00·170
22/23 Aug.	γ Trianguli	94 13 09	+00 00 00·022
22/23 Aug.	β Camelopardi	42 21 50	+00 00 00·088
23/24 Aug.	β Andromedae	89 23 00	+00 00 00·040
23/24 Aug.	θ Herculis	278 11 08	+00 00 00·014
24/25 Aug.	η Piscium	135 39 06	+00 00 00·038
24/25 Aug.	ξ Draconis	312 46 35	−00 00 00·114
24/25 Aug.	* Cygni	268 54 28	−00 00 00·030
24/25 Aug.	ι Aurigae	96 25 32	+00 00 00·124

Least squares value = φ 51° 28′ 38″·01 λ +00° 00′ 01″·04
Correction to Mean Pole = −0·13 —
φ 51 28 37·88 λ +00 00 01·04

* 41 Cygni on F.L.
ε Cygni on F.R.

9.3 *continued*

Observing Station = Herstmonceux $\varphi_0 = 50^\circ\ 51'\ 55''271$

Date 1953 (night of)	*Star*	*Approximate Mean As from Separate Faces*	*Mean of λ′ from Separate Faces*
26/27 Aug.	θ Coronae Borealis	271° 53′ 28″	$+00^{h}\ 01^{m}\ 22^{s}862$
26/27 Aug.	α Ophiuchi	228 40 09	22·864
26/27 Aug.	α Draconis	322 57 37	22·988
26/27 Aug.	ε Cassiopeiae	39 55 56	22·689
26/27 Aug.	ζ Pegasi	138 01 46	22·952
26/27 Aug.	η Persei	50 47 19	22·623
26/27 Aug.	β Andromedae	91 45 06	22·799
26/27 Aug.	γ Draconis	304 04 04	23·010
26/27 Aug.	η Piscium	134 27 12	22·909
26/27 Aug.	ε Pegasi	222 49 12	22·910
26/27 Aug.	η Cygni	273 41 28	22·874
26/27 Aug.	ζ Persei	92 14 10	22·798
27/28 Aug.	α Ophiuchi	227 35 23	22·962
27/28 Aug.	ε Cassiopeiae	41 03 18	22·636
27/28 Aug.	α Pegasi	142 22 34	23·088
27/28 Aug.	ι Draconis	317 25 06	22·924
27/28 Aug.	γ Trianguli	93 48 16	22·910
27/28 Aug.	γ Lyrae	271 38 37	22·934
30/31 Aug.	κ Ophiuchi	225 19 30	22·750
30/31 Aug.	γ Cassiopeiae	44 08 26	22·436
30/31 Aug.	δ Bootis	274 40 34	22·868
30/31 Aug.	ε Pegasi	141 38 55	22·884
30/31 Aug.	ζ Pegasi	134 43 28	22·732
30/31 Aug.	δ Andromedae	91 37 29	22·780
30/31 Aug.	ι Draconis	314 19 38	22·918
30/31 Aug.	η Persei	51 22 08	22·524
30/31 Aug.	70 Pegasi	139 55 30	22·753
30/31 Aug.	θ Draconis	314 37 19	22·776
30/31 Aug.	γ Aquilae	224 52 51	22·662
30/31 Aug.	θ Herculis	277 23 00	22·736
30/31 Aug.	β Trianguli	87 33 45	22·712
30/31 Aug.	κ Lyrae	274 52 40	22·824
30/31 Aug.	γ Trianguli	90 14 36	22·736
30/31 Aug.	ε Delphini	223 36 28	22·720
30/31 Aug.	β Camelopardi	42 02 15	22·542
30/31 Aug.	ξ Draconis	310 36 54	22·876
31 Aug./1 Sept.	β Cassiopeiae	46 18 59	22·716
31 Aug./1 Sept.	γ Delphini	139 23 48	23·046
31 Aug./1 Sept.	ι Ophiuchi	226 14 59	22·694
31 Aug./1 Sept.	ζ Ursae Majoris	311 19 46	22·746
31 Aug./1 Sept.	δ Bootis	273 10 06	22·858
31 Aug./1 Sept.	γ Cassiopeiae	44 35 14	22·677
31 Aug./1 Sept.	θ Coronae Borealis	270 50 40	22·837
31 Aug./1 Sept.	α Ophiuchi	230 31 32	22·969
31 Aug./1 Sept.	α Draconis	323 07 20	23·116
31 Aug./1 Sept.	ζ Pegasi	129 53 53	23·053

9.3 *continued*

Observing Station = Herstmonceux φ_0 = 50° 51′ 55″271 (*continued*)

Date 1953 (night of)	*Star*	*Approximate Mean As from Separate Faces*	*Mean of λ′ from Separate Faces*
31 Aug./1 Sept.	π Andromedae	85° 13′ 47″	+00^{h} 01^{m} 22^{s}866
31 Aug./1 Sept.	δ Andromedae	88 36 27	22·838
31 Aug./1 Sept.	ε Cassiopeiae	41 20 04	22·704
31 Aug./1 Sept.	ε Herculis	268 19 12	23·010
31 Aug./1 Sept.	α Pegasi	134 01 33	22·942
31 Aug./1 Sept.	ε Aquilae	227 22 31	22·886
31 Aug./1 Sept.	β Andromedae	91 34 48	22·850
31 Aug./1 Sept.	θ Draconis	314 22 58	22·978

Least squares value = φ 50° 51′ 54″29 λ +00° 20′ 42″39
Correction to Mean Pole = −0·16 —

φ 50 51 54·13 λ +00 20 42·39

Observing Station = St. Agnes Beacon φ_0 = 50° 18′ 24″241

Date 1953 (night of)	*Star*	*Approximate As*	*Face*	*λ′*
17/18 June	β Ursae Majoris	311° 12′ 41″	R	−00^{h} 20^{m} 52^{s}054
17/18 June	δ Cephei	45 47 02	L	52·060
17/18 June	δ Cephei	46 09 40	R	52·043
17/18 June	β Ursae Majoris	313 56 17	L	52·211
17/18 June	ι Cephei	35 52 21	L	52·745
17/18 June	γ Aquilae	139 24 47	R	52·060
17/18 June	γ Aquilae	141 51 15	L	52·133
17/18 June	α Delphini	124 19 23	L	52·181
17/18 June	α Delphini	125 27 16	R	52·229
17/18 June	β Cassiopeiae	43 59 46	R	52·042
17/18 June	δ Ursae Majoris	315 33 45	R	52·290
17/18 June	δ Ursae Majoris	314 01 31	L	52·491
17/18 June	κ Ophiuchi	217 41 52	L	51·748
17/18 June	κ Ophiuchi	219 20 01	R	51·532
17/18 June	ω Herculis	233 32 56	R	52·030
17/18 June	ι Ophiuchi	229 33 04	L	51·859

Least squares value = φ 50° 18′ 25″07 λ −05° 13′ 01″65
Correction to Mean Pole = +0·19 —

φ 50 18 25·26 λ −05 13 01·65

9.3 *continued*

Observing Station = Spital Hill φ_0 = 58° 28′ 53″026

Date 1953 (night of)	*Star*	*Approximate Mean As from Separate Faces*	*Mean of λ′ from Separate Faces*
11/12 Sept.	ε Tauri	137° 35′ 11″	$-00^{h}\ 13^{m}\ 41^{s}187$
11/12 Sept.	33 Cygni	311 55 20	41·207
11/12 Sept.	23 Ursae Majoris	40 28 41	42·230
11/12 Sept.	β Arietis	219 50 07	41·975
11/12 Sept.	α Ursae Majoris	37 51 32	42·056
12/13 Sept.	γ Sagittarii	228 28 41	42·231
12/13 Sept.	α Arietis	137 20 21	41·062
12/13 Sept.	ξ Draconis	316 29 01	41·428

Least squares value = φ 58° 28′ 49″83 λ −03° 25′ 24″79
Correction to Mean Pole = −0·27 —
φ 58 28 49·56 λ −03 25 24·79

Observing Station = Warth Hill φ_0 = 58° 36′ 45″089

Date 1953 (night of)	*Star*	*Approximate Mean As from Separate Faces*	*Mean of λ′ from Separate Faces*
14/15 Sept.	ι Pegasi	230° 24′ 12″	$-00^{h}\ 12^{m}\ 19^{s}414$
14/15 Sept.	τ Pegasi	223 46 34	19·480
14/15 Sept.	σ Ursae Majoris	45 31 30	19·804
14/15 Sept.	23 Ursae Majoris	37 17 17	19·970
15/16 Sept.	η Persei	48 29 03	19·388
15/16 Sept.	111 Herculis	224 46 02	19·612
18/19 Sept.	η Cephei	315 55 40	18·300
18/19 Sept.	α Cephei	314 05 16	18·173
18/19 Sept.	ζ Tauri	134 28 41	18·111
18/19 Sept.	μ Gemini	134 36 51	18·210
18/19 Sept.	γ Gemini	136 32 22	18·173
18/19 Sept.	ζ Cephei	312 45 33	18·309
18/19 Oct.	γ Delphini	224 31 15	19·805
18/19 Oct.	β Camelopardi	48 27 02	19·610
18/19 Oct.	θ Draconis	319 54 50	17·938
18/19 Oct.	η Piscium	138 12 32	17·884

Least squares value = φ 58° 36′ 39″28 λ −03° 04′ 43″50
Correction to Mean Pole = −0·30 —
φ 58 36 38·98 λ −03 04 43·50

9.3 *continued*

Observing Station = White Horse Hill $\varphi_0 = 51^\circ\ 34'\ 29''\!.872$

Date 1953 (night of)	*Star*	*Approximate As*	*Face*	λ'
27/28 May	α Ursae Majoris	321° 19′ 26″	R	$-00^h\ 06^m\ 15^s\!.995$
27/28 May	α Ursae Majoris	322 42 57	L	15·787
28/29 May	α Cephei	42 55 34	R	14·727
28/29 May	ζ Cephei	47 03 42	L	15·796
28/29 May	ε Aquilae	135 31 37	L	16·138
28/29 May	β Ursae Majoris	315 30 13	L	15·263
28/29 May	γ Aquilae	135 50 55	L	16·174
28/29 May	γ Aquilae	139 18 57	R	15·183
28/29 May	δ Ursae Majoris	313 23 04	R	16·182
28/29 May	κ Delphini	132 38 36	R	15·029
30/31 May	β Serpentis	223 04 12	L	15·289
30/31 May	β Serpentis	224 28 49	R	15·814

Least squares value = φ 51° 34′ 31″·30 λ −01° 33′ 53″·32

Correction to Mean Pole = +0·29 —

φ 51 34 31·59 λ −01 33 53·32

APPENDIX 10

COMPLETE LIST OF FINALLY ACCEPTED GEOGRAPHICAL AND NATIONAL GRID RECTANGULAR CO-ORDINATES FOR PRIMARY STATIONS

FIGURE 1

Station	*Station Number*	*National Grid Rectangular Co-ordinates*		*Geographical Co-ordinates derived from National Grid Rectangular Co-ordinates*	
		Easting (Metres)	*Northing (Metres)*	*Latitude*	*Longitude*
Bardon Hill	58	445 992·420	313 193·662	52° 42′ 50″7713	01° 19′ 08″7381 W
Beacon Hill	15	419 499·949	142 749·481	51 10 59·2399	01 43 15·5134 W
Bradley Knoll	14	378 598·164	137 648·963	51 08 13·8684	02 18 21·3630 W
Broadway Tower	91	411 368·235	236 216·548	52 01 25·7764	01 50 03·4812 W
Bulbarrow	12	377 769·726	105 575·629	50 50 55·3697	02 18 56·9252 W
Butser	9	471 683·240	120 320·824	50 58 38·2338	00 58 43·7455 W
Charwelton	78	451 345·032	256 126·357	52 12 02·0583	01 14 55·1090 W
Cleeve Hill	69	399 694·076	224 592·756	51 55 09·9285	02 00 16·0153 W
Cold Ashby	76	464 422·157	276 588·870	52 22 59·3680	01 03 12·1993 W
Coringdon	11	400 833·086	81 185·858	50 37 47·2322	01 59 17·5916 W
Dunnose	10	456 784·439	80 149·652	50 37 03·7288	01 11 50·1015 W
Gwynydd Bach	72	321 539·301	229 973·252	51 57 44·1610	03 08 31·4443 W
Inkpen	33	437 345·969	161 624·794	51 21 07·0932	01 27 49·1572 W
Malvern	79	376 882·307	245 218·179	52 06 15·8383	02 20 15·2274 W
Martinsell	68	417 836·746	163 867·544	51 22 23·1102	01 44 37·3915 W
Mynydd Maen	73	325 995·025	197 810·600	51 40 25·3981	03 04 13·2574 W
Peglers Tump	88	378 959·629	200 029·541	51 41 53·3650	02 18 16·0969 W
Pen Hill	77	356 442·739	148 778·962	51 14 09·6525	02 37 26·3128 W
Titterstone Clee	62	359 138·473	277 950·259	52 23 51·5865	02 36 02·1852 W
Walton Hill	61	394 260·225	279 795·293	52 24 56·6903	02 05 03·8427 W
Westbury Down	13	390 112·169	151 134·726	51 15 31·5975	02 08 30·1799 W
White Horse Hill	34	430 083·684	186 375·194	51 34 29·8723	01 33 57·0308 W
Wingreen	17	392 505·628	120 645·011	50 59 04·6225	02 06 24·4031 W
Liddington Castle	35	For Final Values see Reco-ordination of Liddington Castle.			
Castle Ring	60	For Final Values see Figure 2.			
Wrekin	63	For Final Values see Figure 2.			
Radnor Forest	71	For Final Values see Figure 4.			
Gore Hill	37	For Final Values see Figure 4.			

FIGURE 2

Station	*Station Number*	*National Grid Rectangular Co-ordinates*		*Geographical Co-ordinates derived from National Grid Rectangular Co-ordinates*	
		Easting (Metres)	*Northing (Metres)*	*Latitude*	*Longitude*
Acre	132	512 111·868	396 465·657	53° 27′ 09″8936	00° 18′ 41″3758 W
Alport Heights	56	430 559·500	351 582·517	53 03 37·1333	01 32 38·2460 W
Blake Mere	29	404 133·648	360 998·845	53 08 44·9737	01 56 17·4870 W
Botton Head	28	459 431·636	501 594·101	54 24 22·2763	01 05 03·4622 W
Boulsworth	16	392 972·615	435 632·351	53 49 00·1622	02 06 24·2972 W
Castle Ring	60	404 312·038	312 881·803	52 42 47·6534	01 56 10·1869 W
Cave Wold	131	494 954·888	432 079·657	53 46 34·1903	00 33 32·1719 W
Clifton	53	451 893·415	395 942·307	53 27 26·7079	01 13 06·1607 W
Collier Law	23	401 621·591	541 790·234	54 46 15·3566	01 58 29·2481 W
Cross Fell	19	368 734·339	534 321·705	54 42 10·2226	02 29 06·8492 W
Delamere	30	354 321·268	369 626·297	53 13 17·1781	02 41 03·2168 W
Great Whernside	7	400 201·528	473 904·461	54 09 38·7962	01 59 48·8881 W
Hambleton Down	65	451 084·780	483 682·170	54 14 46·0072	01 12 57·4353 W
Hanchurch Wtr Twr	97	383 978·760	339 745·616	52 57 16·3197	02 14 18·6087 W
Harland South	52	430 088·382	368 157·063	53 12 33·6324	01 32 57·9530 W
High Street	5	344 075·439	511 048·123	54 29 29·5760	02 51 48·4628 W
Holme Moss	26	407 816·420	404 684·179	53 32 18·6069	01 52 55·3612 W
Lincoln Minster	80	497 795·732	371 807·114	53 14 02·4017	00 32 04·7133 W
Little Whernside	6	373 849·091	481 414·211	54 13 39·3955	02 24 04·2478 W
Loath Hill	54	463 497·796	353 716·606	53 04 35·7565	01 03 07·3753 W
Mallowdale Pike	4	359 251·532	458 728·544	54 01 21·9285	02 37 19·3689 W
Margery	24	418 911·406	395 695·127	53 27 26·6898	01 42 54·5667 W
Normanby Gasholder	83	488 910·035	413 720·672	53 36 44·0817	00 39 21·2865 W
Rivington	20	365 981·668	414 933·379	53 37 46·5172	02 30 52·0816 W
Royal Oak	25	420 424·963	524 960·432	54 37 09·3399	01 41 01·1760 W
Rombalds Moor	70	411 467·861	445 223·446	53 54 10·2453	01 49 31·5830 W
The Edge	27	407 698·169	389 370·440	53 24 03·0275	01 53 03·1364 W
Thoresby Wtr Twr	154	463 765·038	367 685·949	53 12 07·6833	01 02 43·0062 W
Upton Beacon	152	447 442·527	413 945·846	53 37 10·8012	01 16 57·6524 W
Water Crag	18	392 853·214	504 618·760	54 26 12·4609	02 06 36·7061 W
Weaver Hill	51	409 454·311	346 388·631	53 00 51·8808	01 51 32·6247 W
Wrekin	63	362 806·296	308 093·120	52 40 08·1275	02 33 00·2665 W
York Minster	22	460 322·345	452 181·240	53 57 43·1879	01 04 49·7302 W
Black Combe	2	For Final Values see Figure 3.			
Cold Fell Pike	99	For Final Values see Figure 3.			
Leavening Brow	8	For Final Values see Figure 3.			
Skiddaw	3	For Final Values see Figure 3.			
Warden Law	142	For Final Values see Figure 3.			
Weeton Reservoir	164	For Final Values see Figure 3.			
Cader Berwyn	87	For Final Values see Figure 4.			
Moel Fammau	86	For Final Values see Figure 4.			
Stiperstones	64	For Final Values see Figure 4.			
Belvoir Castle	81	For Final Values see Figure 5.			

FIGURE 3

		National Grid Rectangular Co-ordinates		*Geographical Co-ordinates derived from National Grid Rectangular Co-ordinates*	
Station	*Station Number*	*Easting (Metres)*	*Northing (Metres)*	*Latitude*	*Longitude*
Ben Aigan	348	330 993·834	848 190·887	57° 31′ 07″8658	03° 09′ 07″8063 W
Ben Cleugh	307	290 272·313	700 639·126	56 11 08·8438	03 46 05·2765 W
Beneraird	363	213 543·393	578 515·803	55 03 57·5164	04 55 13·1792 W
Ben Lawers	315	263 550·891	741 423·350	56 32 42·2486	04 13 10·4872 W
Ben Lomond	336	236 702·688	702 865·729	56 11 25·5460	04 37 54·6765 W
Ben Macdhui	302	298 898·340	798 942·909	57 04 14·0746	03 40 03·7161 W
Bennachie	341	368 227·156	822 391·402	57 17 28·4271	02 31 37·9453 W
Bin of Cullen	349	347 997·621	864 263·442	57 39 55·7778	02 52 18·3123 W
Black Combe	2	313 549·149	485 488·093	54 15 27·3224	03 19 38·0451 W
Black Mount	352	307 992·802	645 968·469	55 41 54·3979	03 27 50·6061 W
Brimmond	316	385 668·888	809 079·897	57 10 21·0753	02 14 13·3168 W
Brown Carrick	346	228 355·249	615 952·466	55 24 26·5714	04 42 41·1291 W
Cairnsmore of Deugh	328	259 441·830	597 991·304	55 15 21·5495	04 12 42·5135 W
Cairnsmore of Fleet	343	250 156·832	567 074·685	54 58 32·2936	04 20 29·3332 W
Cairn Pat	360	204 427·704	556 351·096	54 51 48·7971	05 02 51·9665 W
Cairn Table	311	272 427·387	624 226·186	55 29 42·4841	04 01 10·5217 W
Carn Gower	332	297 045·574	773 195·457	56 50 20·1932	03 41 15·9394 W
Cheviot	308	390 905·205	620 525·190	55 28 42·3993	02 08 38·0664 W
Cold Fell Pike	99	360 571·997	555 634·401	54 53 37·6931	02 36 53·3032 W
Corryhabbie	342	328 091·698	828 867·646	57 20 41·4394	03 11 41·7924 W
Corse Hill	329	259 837·014	646 468·388	55 41 29·0838	04 13 48·1755 W
Craigowl	353	337 696·097	739 992·082	56 32 52·4204	03 00 48·5178 W
Criffell	96	295 725·247	561 867·652	54 56 25·7292	03 37 40·5037 W
Cutties Hillock	347	318 033·483	863 596·409	57 39 18·1675	03 22 25·3292 W
Cutties Hillock East	357	318 515·297	863 306·371	57 39 09·1050	03 21 55·9168 W
Dunrig	313	325 369·612	631 587·300	55 34 20·0032	03 11 01·3992 W
Earls Seat	327	256 984·742	683 800·305	56 01 32·7934	04 17 42·2661 W
Easington	55	474 994·927	519 474·011	54 33 53·2930	00 50 24·0350 W
Findlays Seat	340	325 812·203	854 913·779	57 34 42·2883	03 14 26·5655 W
Glas Maol	322	316 699·182	776 576·201	56 52 23·6882	03 22 00·4685 W
Greensheen Hill	344	405 630·645	635 757·983	55 36 55·3837	01 54 38·1450 W
Hart Fell	320	311 364·035	613 573·883	55 24 28·9742	03 24 00·1634 W
Hill of Stake	319	227 353·567	663 003·747	55 49 45·7041	04 45 24·2126 W
Kellie Law	321	351 744·190	706 457·567	56 14 53·7440	02 46 43·7282 W
Kings Seat	310	323 060·940	733 004·531	56 28 58·6488	03 14 57·9028 W
Knock	339	353 721·511	855 166·425	57 35 03·8537	02 46 26·6534 W
Leavening Brow	8	479 544·652	462 302·497	54 03 01·2740	00 47 05·5806 W
Lossiemouth Base East Terminal	350	329 623·645	866 419·940	57 40 56·5164	03 10 49·2023 W
Lossiemouth Base West Terminal	351	323 333·117	869 857·434	57 42 43·9568	03 17 12·8483 W
Loose Howe	44	470 179·695	501 190·364	54 24 04·2868	00 55 07·7440 W
Lumsdaine	324	387 233·345	668 335·135	55 54 28·7606	02 12 15·2440 W
Meall Dearg	305	288 659·874	741 494·702	56 33 08·3877	03 48 41·1351 W

Figure 3 *continued*

Station	*Station Number*	*National Grid Rectangular Co-ordinates*		*Geographical Co-ordinates derived from National Grid Rectangular Co-ordinates*	
		Easting (Metres)	*Northing (Metres)*	*Latitude*	*Longitude*
Merrick	301	242 757·019	585 548·688	55° 08′ 21″2362	04° 28′ 01″8542 W
Mormond	338	398 126·671	856 956·244	57 36 10·2581	02 01 52·8579 W
Mount Battock	304	354 963·812	784 468·916	56 56 57·9951	02 44 25·5800 W
Sayers Law	333	358 121·098	661 739·435	55 50 49·4591	02 40 08·0915 W
Sca Fell	92	321 540·019	507 216·651	54 27 14·8831	03 12 37·1259 W
Scald Law	345	319 166·826	661 084·169	55 50 10·4323	03 17 26·8257 W
Skiddaw	3	326 040·563	529 085·979	54 39 04·7853	03 08 47·0477 W
Tinto	318	295 278·907	634 371·876	55 35 30·1196	03 39 42·6905 W
Tosson Hill	95	400 482·355	598 246·383	55 16 41·9394	01 59 32·6620 W
Trusta	306	378 163·788	786 836·945	56 58 20·6189	02 21 33·2147 W
Warden Law	142	436 991·116	550 619·965	54 50 56·1316	01 25 25·7992 W
Weeton Reservoir	164	339 734·229	434 372·275	53 48 06·9875	02 54 54·5567 W
West Hills	312	353 521·785	744 737·407	56 35 32·4930	02 45 24·9358 W
West Lomond	334	319 730·494	706 638·971	56 14 44·0265	03 17 43·5363 W
Whitelyne Common	93	360 139·434	580 923·461	55 07 15·7221	02 37 30·2710 W
Wisp Hill	317	338 644·899	599 347·695	55 17 03·8139	02 57 57·9567 W
Wuddy Law	354	362 994·832	752 337·682	56 39 41·3481	02 36 13·5081 W
Ailsa Craig Lighthouse	364	Not Co-ordinated as a Primary.			
Ben Alder	335	For Final Values see Figure 6.			
Beinn Bhreac Mhor	356	For Final Values see Figure 6.			
Carn nan-tri-tighearnan	325	For Final Values see Figure 6.			
Goat Fell	309	For Final Values see Figure 6.			
Cnoc Moy	365	For Final Values see Figure 6.			
Carleton Fell	362	For Final Values see Figure 7.			
Inshanks	361	For Final Values see Figure 7.			
Rottington	1	For Final Values see Figure 7.			

FIGURE 4

Station	*Station Number*	*National Grid Rectangular Co-ordinates*		*Geographical Co-ordinates derived from National Grid Rectangular Co-ordinates*	
		Easting (Metres)	*Northing (Metres)*	*Latitude*	*Longitude*
Aberystwyth	108	259 253·827	283 340·656	52° 25′ 46″3041	04° 04′ 13″3249 W
Aran Fawddwy	102	286 268·582	322 382·286	52 47 12·0558	03 41 11·8312 W
Arenig	118	282 703·102	336 947·453	52 55 00·5007	03 44 40·9585 W
Bagborough	49	316 531·966	135 154·749	51 06 32·6698	03 11 32·8153 W
Bartinney	180	139 447·901	29 328·373	50 06 22·1413	05 38 39·8472 W
Bin Down	198	227 540·084	57 644·703	50 23 33·8807	04 25 35·7371 W
Blackdown	46	361 298·259	87 601·382	50 41 10·3186	02 32 52·4856 W
Blagdon	41	348 460·913	157 264·510	51 18 41·9479	02 44 22·3199 W
Brown Willy	172	215 868·983	79 998·088	50 35 24·1660	04 36 05·9474 W
Cader Berwyn	87	307 220·094	332 716·100	52 53 00·8521	03 22 44·2556 W
Cader Idris	105	271 111·033	313 035·172	52 41 57·4287	03 54 27·3728 W
Capel Cynon	114	237 301·772	249 391·738	52 07 06·4423	04 22 35·8347 W
Carnmenellis	177	169 558·198	36 439·405	50 10 56·8491	05 13 41·8651 W
Carn Galver	179	142 061·815	35 715·675	50 09 52·6624	05 36 44·0562 W
Cefn Bryn	119	251 803·423	188 965·348	51 34 46·4357	04 08 20·5788 W
Cyrn-y-Brain	103	321 367·280	349 648·492	53 02 16·8715	03 10 22·2906 W
Dodman	176	200 187·356	39 389·531	50 13 12·3188	04 48 04·8393 W
Drygarn	90	286 226·484	258 375·742	52 12 41·1779	03 39 55·2950 W
Dumpdon	202	317 601·024	104 008·549	50 49 45·0503	03 10 12·4350 W
Dunkery	74	289 142·501	141 587·355	51 09 44·1280	03 35 08·0949 W
Eastacott Hill	165	246 812·709	142 885·954	51 09 50·9712	04 11 28·1912 W
Furland	169	289 468·968	53 185·357	50 22 03·2270	03 33 15·6605 W
Garnedd Ugain	111	261 078·255	355 157·867	53 04 30·9280	04 04 26·3343 W
Garn Fawr	106	189 584·658	238 869·932	52 00 28·3942	05 03 58·3481 W
Goonhilly Down	182	172 541·925	21 109·795	50 02 45·3834	05 10 38·7555 W
Gore Hill	37	363 738·169	103 860·562	50 49 57·2891	02 30 53·9063 W
Great Ormes Head	115	276 750·792	383 334·130	53 19 56·1898	03 51 03·6917 W
Hendon Moor	166	226 052·084	118 248·820	50 56 12·7091	04 28 33·6043 W
Hensbarrow	174	199 679·431	57 544·809	50 22 58·8532	04 49 05·1028 W
Little Haldon	168	291 673·058	75 222·046	50 33 57·9703	03 31 47·0898 W
Llaneilian	116	247 283·709	391 720·144	53 23 59·7101	04 17 50·1994 W
Llangeinor	89	291 274·930	194 769·866	51 38 26·6680	03 34 17·0265 W
Llyn Du	148	277 375·198	260 576·716	52 13 45·5505	03 47 44·3466 W
Lundy Island Lighthouse	Int. 1	213 192·966	144 282·945	51 10 00·1845	04 40 20·1747 W
Lundy Island NW Point Lighth'se	Int. 2	213 059·702	148 130·899	51 12 04·4685	04 40 34·2381 W
Marros Beacon	104	220 373·282	208 020·297	51 44 29·7489	04 36 07·3533 W
Moel Fammau	86	316 164·416	362 653·461	53 09 14·8220	03 15 13·8135 W
Moelfre Isaf	163	295 140·803	373 372·149	53 14 48·1806	03 34 18·0067 W
Mynydd Margam	150	281 914·132	188 856·489	51 35 08·5263	03 42 16·6771 W
Mynydd Rhos-Wen	113	247 921·191	233 472·263	51 58 42·3999	04 12 52·3515 W
Parracombe	140	270 002·483	143 472·650	51 10 30·6250	03 51 35·5605 W
Pendine	149	223 425·482	209 803·365	51 45 30·9045	04 33 31·6165 W

Figure 4 *continued*

Station	*Station Number*	*National Grid Rectangular Co-ordinates*		*Geographical Co-ordinates derived from National Grid Rectangular Co-ordinates*	
		Easting (Metres)	*Northing (Metres)*	*Latitude*	*Longitude*
Pilsdon	36	341 353·646	101 142·066	50° 48′ 22″6735	02° 49′ 56″6580 W
Plynlimon	101	278 968·403	286 940·435	52 27 59·7588	03 46 54·6156 W
Portlemouth	208	275 155·232	38 154·346	50 13 46·4470	03 45 02·0729 W
Prescelly	107	209 406·256	231 155·377	51 56 44·7426	04 46 24·4730 W
Radnor Forest	71	318 217·188	263 896·678	52 16 00·2872	03 11 54·8334 W
Rat Island Lighthouse	Int. 3	214 399·311	143 665·426	51 09 41·6278	04 39 16·9749 W
Ryders Hill	167	265 977·444	69 062·547	50 30 19·4577	03 53 24·7364 W
St Agnes Beacon	175	171 011·559	50 215·333	50 18 24·2412	05 12 58·6547 W
St Anns Hill	112	181 128·082	204 169·926	51 41 35·0756	05 10 02·1727 W
Stiperstones	64	336 751·856	298 644·562	52 34 53·6565	02 56 00·7838 W
Talsarn	151	254 228·310	259 915·014	52 13 03·8395	04 08 02·6976 W
Three Barrows	209	265 326·617	62 574·220	50 26 48·9517	03 53 49·3500 W
Trecastle	85	282 533·161	221 791·834	51 52 54·6960	03 42 24·5478 W
Tregonning Hill	181	159 923·998	30 040·278	50 07 16·2242	05 21 32·3621 W
Trendrine Hill	178	147 876·296	38 758·154	50 11 39·9982	05 31 58·7734 W
Trevose Head	173	185 235·096	76 474·727	50 32 52·5755	05 01 54·6806 W
Wembury	210	252 532·060	51 441·452	50 20 37·6431	04 04 22·0038 W
Wirswall	21	354 998·447	343 843·644	52 59 23·0085	02 40 13·6918 W
Yes Tor	203	258 089·728	90 145·156	50 41 34·9608	04 00 33·9678 W
Yr Eifl	109	236 493·345	344 746·592	52 58 29·2231	04 26 07·4452 W
Holyhead	117	For Final Values see Figure 7.			
Rhiw	110	For Final Values see Figure 7.			

FIGURE 5

Station	*Station Number*	*National Grid Rectangular Co-ordinates*		*Geographical Co-ordinates derived from National Grid Rectangular Co-ordinates*	
		Easting (Metres)	*Northing (Metres)*	*Latitude*	*Longitude*
Abberton Wtr Twr	230	600 402·773	219 010·057	51° 49′ 59″6877	00° 54′ 32″2282 E
Beachy Head	194	559 038·377	95 789·995	50 44 21·6320	00 15 15·1626 E
Belvoir Castle	81	481 981·442	333 712·946	52 53 39·4249	00 46 52·4673 W
Benfleet Wtr Twr	219	579 052·312	186 711·974	51 33 01·3175	00 34 58·1663 E
Bethersden Air Beacon	Int. 4	593 124·558	140 583·787	51 07 53·0102	00 45 37·9712 E
Bignor Beacon	39	496 596·968	113 116·276	50 54 31·9379	00 37 33·2748 W
Bolnhurst	433	505 879·616	259 778·365	52 13 32·2280	00 26 58·9225 W
Boston Tower	264	532 655·784	344 179·092	52 58 41·5509	00 01 26·4580 W
Brenchley Air Beacon	Int. 5	567 965·573	142 236·203	51 09 15·0190	00 24 07·4033 E
Buckminster Wtr Twr	153	488 170·167	322 950·908	52 47 47·6699	00 41 31·8235 W
Bunwell Ch Twr	255	612 544·918	292 769·257	52 29 27·5566	01 07 51·9884 E
Burrough Green Wtr Twr	241	563 214·328	256 400·115	52 10 52·5550	00 23 15·0717 E
Caister Wtr Twr	293	651 409·903	313 177·271	52 39 27·2531	01 43 04·5177 E
Charnwood	57	450 936·053	314 808·474	52 43 41·4381	01 14 44·3790 W
Chedburgh	236	578 690·613	255 857·133	52 10 17·7322	00 36 48·1640 E
Chipping Barnet Ch Twr	185	524 538·488	196 463·188	51 39 09·6501	00 11 58·4315 W
Church Farm Wtr Twr	279	654 028·088	294 349·659	52 29 14·7717	01 44 31·6407 E
Cold Harbour	266	526 592·538	381 214·084	53 18 44·7752	00 05 58·7010 W
Collyweston	431	500 078·974	303 199·045	52 37 01·1074	00 31 17·7734 W
Coombe Hill	204	489 068·197	209 997·074	51 46 51·9231	00 42 31·4082 W
Crimplesham	424	564 839·965	304 270·330	52 36 38·8956	00 26 05·4375 E
Crowborough	196	551 169·211	130 761·184	51 03 20·6416	00 09 26·0605 E
Crown Corner	260	625 514·095	270 170·219	52 16 58·5507	01 18 23·8233 E
Dexthorpe	265	540 661·255	373 017·751	53 14 06·9152	00 06 28·0060 E
Ditchling	32	533 162·816	113 063·037	50 54 04·0149	00 06 21·7531 W
Docking Ch Twr	284	576 508·010	336 971·658	52 54 03·0094	00 37 28·5924 E
Dunmow	437	564 886·697	222 350·406	51 52 29·5423	00 23 43·9793 E
Dunstable Down	94	500 879·723	219 418·096	51 51 49·5757	00 32 05·2699 W
East Grinstead Ch Twr	170	539 631·179	138 001·964	51 07 25·4414	00 00 16·2073 W
Ely Cathedral	430	554 048·139	280 275·770	52 23 54·2672	00 15 52·0876 E
Epping Wtr Twr	188	546 705·441	202 764·897	51 42 14·2563	00 07 23·9855 E
Fairlight Down	193	584 340·282	111 923·339	50 52 36·5055	00 37 14·0592 E
Faxton	443	480 589·538	275 413·442	52 22 13·6226	00 48 58·1736 W
Fayway	432	506 679·108	278 492·703	52 23 37·2025	00 25 55·4384 W
Felixstowe Wtr Twr	233	628 697·969	236 384·287	51 58 42·0416	01 19 49·8006 E
Firle Beacon	199	548 557·172	105 922·255	50 49 59·4568	00 06 35·4910 E
Framingham	261	626 238·249	302 646·415	52 34 26·8916	01 20 21·1080 E
Fransham	426	592 507·706	310 418·048	52 39 24·8745	00 50 47·7514 E
Frog Hill	262	587 200·013	291 090·083	52 29 06·7012	00 45 26·2692 E
Harrowby	429	494 620·788	335 766·548	52 54 38·3965	00 35 34·0821 W
Helion Bumpstead	248	562 492·844	241 622·942	52 02 55·4009	00 22 11·7177 E
Hindhead	31	489 984·580	135 909·726	51 06 53·5555	00 42 51·4434 W
Hingham Ch Twr	287	602 154·586	302 126·358	52 34 44·2274	00 59 02·2867 E

Figure 5 *continued*

Station	*Station Number*	*National Grid Rectangular Co-ordinates*		*Geographical Co-ordinates derived from National Grid Rectangular Co-ordinates*	
		Easting (Metres)	*Northing (Metres)*	*Latitude*	*Longitude*
Hockley Wtr Twr	220	582 441·469	192 208·498	51° 35′ 55″1576	00° 38′ 04″2441 E
Icomb Tower	67	420 179·690	222 880·938	51 54 13·2000	01 42 23·9498 W
Ilketshall St Andrews Ch Twr	290	637 904·114	287 239·454	52 25 51·2262	01 30 00·0965 E
Kessingland Ch Twr	278	652 765·807	286 264·886	52 24 55·7474	01 43 02·8171 E
Leith Hill Tower	50	513 949·281	143 161·713	51 10 32·8895	00 22 10·9797 W
Lenham Wtr Twr	205	592 574·666	152 842·751	51 14 30·0984	00 45 33·3702 E
Linch Ball	38	484 804·616	117 371·734	50 56 56·3553	00 47 33·4423 W
Mablethorpe Wtr Twr	269	550 554·384	384 164·116	53 19 57·6025	00 15 40·2698 E
Manningtree	245	608 327·302	229 541·330	51 55 29·6818	01 01 48·6029 E
Maplestead	235	583 017·341	234 470·897	51 58 41·1107	00 39 54·4080 E
Massingham	272	579 482·933	320 139·055	52 44 55·2529	00 39 34·3452 E
Metfield	258	631 245·969	280 009·244	52 22 07·8936	01 23 50·1691 E
Muswell Hill	100	464 129·083	215 295·540	51 49 55·7121	01 04 09·2634 W
Nedging Tye	240	601 971·974	249 713·799	52 06 30·2362	00 56 59·1418 E
North Walsham Wtr Twr	283	627 846·134	329 200·505	52 48 42·4483	01 22 52·5632 E
Orford Castle	254	641 944·445	249 878·456	52 05 37·8303	01 31 57·2412 E
Paddlesworth	190	For Final Values see Co-ordination of Frittenfield and Paddlesworth.			
Peterborough Cathedral	447	519 426·546	298 646·136	52 34 19·7356	00 14 15·2665 W
Piggs Grave	263	602 653·381	332 998·559	52 51 21·4075	01 00 37·2937 E
Puttocks Hill	246	589 820·253	269 583·164	52 17 28·1294	00 47 01·1614 E
Rollright	66	427 877·959	229 860·053	51 57 57·9171	01 35 39·0541 W
Rumfields Wtr Twr	201	637 754·156	167 767·191	51 21 31·0839	01 24 55·5915 E
Salle	259	635 859·534	266 256·810	52 14 36·4810	01 27 19·0123 E
Selsey	47	486 827·633	95 745·731	50 45 15·2563	00 46 08·2476 W
Severndroog Castle	189	543 186·143	176 199·773	51 27 58·1086	00 03 41·7321 E
Shirburn Hill	207	472 344·363	195 240·742	51 39 02·9992	00 57 15·1064 W
Shurland	191	600 157·970	171 679·966	51 24 29·7701	00 52 42·1025 E
Sibleys Wtr Twr	434	556 480·193	229 994·444	51 56 45·5105	00 16 37·1841 E
Skegness Wtr Twr	267	555 782·792	364 408·125	53 09 13·3137	00 19 47·9204 E
South Lopham Ch Twr	237	603 959·176	281 755·406	52 23 43·3503	00 59 53·1338 E
Southwold Ch Twr	280	650 734·308	276 388·787	52 19 40·1019	01 40 48·9230 E
Stoke by Nayland Ch Twr	249	598 596·655	236 273·899	51 59 20·1432	00 53 33·8069 E
Swaffham	425	583 912·644	309 253·005	52 38 57·9476	00 43 08·4463 E
Swilland	244	618 239·615	253 813·747	52 08 20·5012	01 11 22·5365 E
Therfield	441	533 184·175	237 242·048	52 01 01·8225	00 03 32·2205 W
Tilton Pile	75	476 739·963	305 904·124	52 38 42·3267	00 51 56·3801 W
Topcroft Ch Twr	296	626 575·127	292 894·813	52 29 11·2902	01 20 15·0369 E
Uppingham	442	485 119·971	298 887·193	52 34 50·7640	00 44 37·0646 W
Walpole St Peters	427	550 202·519	316 622·003	52 43 33·6594	00 13 28·0226 E
Walton on the Naze Twr	227	626 486·512	223 538·866	51 51 50·1745	01 17 23·6522 E
Warley Wtr Twr	224	559 102·888	191 527·159	51 35 58·5974	00 17 50·9995 E
Woolpit	247	599 634·652	262 291·884	52 13 19·9066	00 55 23·0576 E
Wyck Beacon	144	420 190·077	220 792·752	51 53 05·6032	01 42 23·8465 W
Wyton Wtr Twr	444	528 152·267	273 816·575	52 20 49·3452	00 07 06·3027 W
Wrotham	192	559 322·787	160 004·988	51 18 58·7165	00 17 11·2898 E

FIGURE 6

Station	*Station Number*	*National Grid Rectangular Co-ordinates*		*Geographical Co-ordinates derived from National Grid Rectangular Co-ordinates*	
		Easting (Metres)	*Northing (Metres)*	*Latitude*	*Longitude*
Ailsa Craig	479	201 910·512	599 828·519	55° 15′ 09″8254	05° 07′ 01″6638 W
An Cuaidh	373	176 499·789	889 126·543	57 50 09·4789	05 45 55·7422 W
Anteallach	389	206 901·074	884 369·814	57 48 26·8544	05 15 01·6657 W
Askival	374	139 308·622	795 222·601	56 58 28·8102	06 17 25·7919 W
Bad Mor	376	299 850·143	955 057·401	58 28 21·3051	03 43 02·2180 W
Balta	455	466 245·928	1 208 187·146	60 45 06·5926	00 47 04·1071 W
Beinn a' Bha' ach Ard	380	236 058·776	843 484·964	57 27 07·9912	04 43 57·5025 W
Beinn Bhan	382	180 359·833	845 038·764	57 26 33·4954	05 39 38·0131 W
Beinn Bheula	330	215 477·938	698 325·832	56 08 30·9955	04 58 13·0627 W
Beinn Bhreac Mhor	356	267 805·214	819 860·118	57 15 01·7628	04 11 28·6688 W
Beinn Mhor	476	80 853·907	831 095·293	57 15 32·3519	07 17 38·6406 W
Beinn na Caillich	375	160 145·514	823 306·788	57 14 15·5563	05 58 31·5568 W
Beinn Tart a' Mhill	383	121 057·806	656 985·044	55 43 32·8317	06 26 35·5205 W
Ben Alder	335	249 616·189	771 856·070	56 48 50·3889	04 27 49·7064 W
Ben Cruachan	314	206 965·150	730 470·587	56 25 37·1255	05 07 50·1769 W
Ben Hogh	369	118 105·557	758 073·201	56 37 46·3799	06 35 50·0541 W
Ben Hutig	378	253 859·479	965 288·245	58 33 05·1134	04 30 42·1633 W
Ben Hynish	368	96 790·881	740 114·129	56 27 19·7524	06 55 20·9386 W
Ben Klibreck	381	258 526·114	929 916·002	58 14 07·8780	04 24 35·3627 W
Ben More (Mull)	377	152 575·348	733 078·281	56 25 29·9909	06 00 46·7258 W
Ben Nevis	323	216 674·794	771 283·665	56 47 49·3810	05 00 08·3726 W
Ben Wyvis	379	246 296·340	868 378·006	57 40 45·1270	04 34 40·6114 W
Brassa	456	450 286·503	1 138 721·891	60 07 49·7332	01 05 41·1851 W
Carn an Fhreiceadain	331	272 559·865	807 132·201	57 08 15·2461	04 06 21·7017 W
Carn Eige	386	212 355·415	826 197·472	57 17 16·6881	05 06 50·1541 W
Carn nan-tri-tighearnan	325	282 311·489	839 035·458	57 25 35·8256	03 57 36·5710 W
Carra Duagh	385	189 276·577	710 272·556	56 14 17·7146	05 24 02·7279 W
Clisham	472	115 484·873	907 304·028	57 57 50·6850	06 48 41·4333 W
Cnoc an t' Sabhail	359	272 162·266	881 714·793	57 48 25·2558	04 09 05·7763 W
Cnoc Moy	365	161 140·590	615 230·712	55 22 22·0748	05 46 13·3280 W
Col Bheinn	358	288 445·662	911 003·172	58 04 27·5556	03 53 29·4486 W
Conival	384	230 331·884	919 937·948	58 08 09·7903	04 52 55·8222 W
Creach Bheinn	372	187 059·883	757 648·947	56 39 43·8242	05 28 30·2653 W
Creag Riabhach	387	227 887·886	963 803·050	58 31 42·9731	04 57 22·7847 W
Deerness	457	356 890·748	1 007 387·946	58 57 06·9595	02 44 57·7948 W
Dunnet Head	388	320 525·449	976 512·017	58 40 10·1680	03 22 13·4366 W
Fair Isle	458	420 837·885	1 073 402·221	59 32 47·2758	01 37 53·0665 W
Fetlar	459	462 228·597	1 193 520·946	60 37 14·9328	00 51 46·1735 W
Fitty Hill	460	342 976·664	1 044 871·177	59 17 12·8550	03 00 03·5790 W
Foula	461	394 780·042	1 139 507·651	60 08 26·1513	02 05 38·3760 W
Goat Fell	309	199 135·405	641 538·659	55 37 33·1044	05 11 26·8940 W
Healaval Beg	390	122 493·201	842 215·270	57 23 08·5840	06 37 06·8907 W
Heaval	475	67 808·774	799 413·705	56 57 58·9716	07 28 02·4096 W

Figure 6 *continued*

Station	*Station Number*	*National Grid Rectangular Co-ordinates*		*Geographical Co-ordinates derived from National Grid Rectangular Co-ordinates*	
		Easting (Metres)	*Northing (Metres)*	*Latitude*	*Longitude*
Hill of Yarrows	391	329 613·844	942 797·979	58° 22′ 05″·9043	03° 12′ 11″·9424 W
Jura	392	149 805·314	674 947·126	55 54 08·9738	06 00 11·2608 W
Marrival	477	80 860·737	870 031·789	57 36 26·1974	07 20 40·6550 W
Mealisval	473	102 197·596	927 040·563	58 07 55·1903	07 03 36·7242 W
Meall nan Con	393	150 385·990	768 137·490	56 44 17·0014	06 04 55·7880 W
Muirnag	474	147 962·048	948 935·448	58 21 23·6197	06 18 32·6845 W
Point of Stoer	394	201 751·010	934 601·128	58 15 20·3548	05 22 45·3271 W
Ronas Hill	462	430 529·336	1 183 485·633	60 32 03·7603	01 26 36·9780 W
Saxavord	463	463 120·624	1 216 622·752	60 49 41·0388	00 50 20·6454 W
Scaraben	397	306 607·242	926 839·378	58 13 14·5786	03 35 24·1559 W
Sgurr na Ciche	371	190 216·679	796 683·279	57 00 49·2193	05 27 20·8781 W
Sliabh Gaoil	303	181 876·266	674 227·873	55 54 41·9012	05 29 26·0156 W
South Ronaldsay	464	345 540·929	988 649·500	58 46 56·5008	02 56 31·4682 W
Spital Hill	398	316 776·904	955 642·702	58 28 53·0264	03 25 38·5129 W
Storr	396	149 537·793	854 046·299	57 30 26·6501	06 10 55·2348 W
Stronsay	465	368 879·938	1 023 145·919	59 05 40·1814	02 32 35·5685 W
Ward Hill	466	322 875·861	1 002 248·690	58 54 03·6503	03 20 19·5006 W
Warth Hill	399	337 118·216	969 864·913	58 36 45·0887	03 04 57·0132 W
Yell	467	450 087·205	1 185 095·902	60 32 48·7496	01 05 12·4789 W

FIGURE 7

Station	*Station Number*	*National Grid Rectangular Co-ordinates*		*Geographical Co-ordinates derived from National Grid Rectangular Co-ordinates*	
		Easting (Metres)	*Northing (Metres)*	*Latitude*	*Longitude*
Carleton Fell	362	240 231·725	537 897·307	54° 42′ 38″·0565	04° 28′ 49″·0297 W
Holyhead	117	221 853·814	382 945·188	53 18 47·3540	04 40 28·0111 W
Inshanks	361	211 399·990	535 524·486	54 40 45·4652	04 55 32·6799 W
Rhiw	110	222 845·687	329 388·836	52 49 57·0813	04 37 48·3016 W
Rottington	1	295 216·792	513 370·813	54 30 16·8407	03 37 06·2272 W
Snaefell	468	239 770·040	488 085·911	54 15 47·2972	04 27 37·5384 W
South Barrule	469	225 767·870	475 919·641	54 08 57·5699	04 40 05·2960 W

ADDITIONAL PRIMARY WORK

Station	*Station Number*	*National Grid Rectangular Co-ordinates*		*Geographical Co-ordinates derived from National Grid Rectangular Co-ordinates*	
		Easting (Metres)	*Northing (Metres)*	*Latitude*	*Longitude*
Liddington Castle	35	420 981·982	179 752·992	51° 30′ 56″·9808	01° 41′ 51″·3145 W
Dimlington	452	539 597·773	420 678·716	53 39 49·3065	00 06 46·8276 E
Stone Creek	450	524 842·475	418 824·177	53 39 02·7580	00 06 39·3779 W
Tunstall	451	529 824·625	433 762·698	53 47 01·5306	00 01 45·5987 W
Frittenfield	480	598 116·020	148 970·754	51 12 18·0670	00 50 11·1289 E
Paddlesworth	190	619 999·270	139 527·200	51 06 43·9659	01 08 36·4987 E
Hillhead Farm	478	327 801·461	963 351·281	58 33 09·2991	03 14 26·8009 W
Herstmonceux	481	565 074·215	110 000·390	50 51 55·2713	00 20 45·8817 E
Epping	483	546 700·127	202 780·870	51 42 14·7780	00 07 23·7330 E
Greenwich Observatory	482	538 882·663	177 328·999	51 28 38·5045	00 00 00·4173 E
North Tolsta	484	152 866·777	947 353·395	58 20 42·6477	06 13 25·6532 W
St Kilda	486	9 969·656	900 033·903	57 49 10·0357	08 34 20·4621 W

APPENDIX 11

THEODOLITE TESTS

The method of test adopted was originated by Rannie and Dennis and described by them in their excellent paper *Axis Strain in Theodolites, Its Effects and One Method of Removal*, published by the National Research Council of Canada. The reader is referred to this paper for fuller details and proof of the method, although it is desirable to summarise here the method of test in order that the results in this case may be fully understood.

The theodolite under test is used in various positions to measure an angle of approximately 60° between two collimators A and B. As in the case of field observations the procedure is to swing right on Face Left, bisect A, then B, and close on A; change face; swing left on Face Right, bisect A, then B, and again close on A. The mean of such double-face readings reduced to a zero initial reading on A is entered in each row of the last three columns of the double Table I; the last column showing the mean closure on A. Similar sets of readings are taken on each of six symmetrical positions of the circle as shown in the third column. The mean of these six sets corrected for closure appears in the eighth line of Table I: it is considered to be free from circle graduation and observational error, but affected by any axis strain associated with the particular position of the theodolite base used for this series of readings. Six similar means are obtained in each of six symmetrical positions of the theodolite base as shown in the first column of Table I and are entered in the second column of Table II. In order to minimise the effect of any uniform movement of the apparatus during the test, the observations are taken in the order shown in the second column of Table I.

The grand mean of the six entries in the second column of Table II is 10·93 seconds and is the accepted value of the measured angle AB. Departures from it are entered with the correct sign in the third column of residuals headed p; the mean of these residuals regardless of sign being an indication of the amount of axis strain present in the instrument. In order to determine the amount of strain occurring in each diameter of the axis associated with each of the six positions of the theodolite base, Rannie and Dennis complete the table as follows:

Entries in the fourth column are:

$$r_1 = p_1, \qquad r_2 = p_2 + r_1, \ldots, r_5 = p_5 + r_4$$

Denote the algebraic mean of these values of r by C (= +0·01 seconds). Then the diametral strain errors shown in the last column are respectively $-C$, $-C+r_1$, $-C+r_2$, etc. The mean regardless of sign of the six entries in this column (0·08 seconds) is called the 'average strain error' and is a criterion of the performance of the theodolite axis.

TABLE I

GEODETIC TAVISTOCK NO. 35209: AXIS TEST

Theodolite Base	*Sequence of Readings*	*Circle Reading on Collimator A*	*Observer*	*Coll. A*	*Coll. B*	*Coll. A*	*Theodolite Base*	*Sequence of Readings*	*Circle Reading on Collimator A*	*Observer*	*Coll. A*	*Coll. B*	*Coll. A*
°		°		″	″	″	°		°		″	″	″
0	1	0	J	00·0	10·6	60·05	−180	4	0	J	00·0	10·55	60·1
		60			10·8	60·0			60			11·1	60·05
		120			10·8	60·0			120			10·95	59·95
	12	180	F		10·75	60·3		9	180	F		11·05	59·8
		240			11·2	60·1			240			11·15	59·8
		300			10·9	60·0			300			10·8	60·0
					*10·84 †10·80	60·07						10·93 10·96	59·95
−60	2	0	J	00·0	10·85	59·8	−240	5	0	J	00·0	11·0	59·9
		60			11·3	60·2			60			10·9	59·85
		120			11·2	60·1			120			10·9	59·95
	11	180	F		10·9	60·1		8	180	F		10·95	60·05
		240			10·75	60·3			240			11·05	59·75
		300			11·2	59·95			300			10·8	60·05
					11·03 10·99	60·08						10·93 10·97	59·92
−120	3	0	J	00·0	11·0	60·05	−300	6	0	J	00·0	10·85	59·8
		60			11·15	60·05			60			10·85	60·15
		120			11·2	60·15			120			10·75	60·15
	10	180	F		10·85	60·1		7	180	F		10·8	59·95
		240			11·0	60·15			240			10·65	59·9
		300			11·35	60·1			300			10·75	59·9
					11·09 11·05	60·10						10·78 10·79	59·98

* Observed mean. † Mean corrected for closure.

The same observations are also used in Tables III and IV to determine the graduation errors of the three principal circle diameters used in the test. The observations in Table I are transferred to the double entry Table III in a manner which will be readily apparent; each observation being corrected separately for closure on Collimator *A*. The mean of each line in Table III is considered free from strain error, but affected by relative graduation error between the graduations used for sighting *A* and *B* on this particular circle setting. The means for circle settings 0° and 180° (and 60° and 240°, etc.) are meaned in order to provide an indication of the graduation errors associated with each of the three diameters and entered in the second column of Table IV. This table is completed by the same process as was used for Table II to provide the diametral graduation error and the 'average' graduation error.

TABLE II

GEODETIC TAVISTOCK NO. 35209: CALCULATION OF DIAMETRAL STRAIN ERRORS

Theodolite Base	*Seconds of Angle*	*Residual* p	r	*Diametral Strain Error*
0°	10″80	$-0''13 = p_1$	$-0''13 = r_1$	−0″01
60°	10·99	$+0{\cdot}06 = p_2$	$-0{\cdot}07 = r_2$	−0·14
120°	11·05	$+0{\cdot}12 = p_3$	$+0{\cdot}05 = r_3$	−0·08
180°	10·96	$+0{\cdot}03 = p_4$	$+0{\cdot}08 = r_4$	+0·04
240°	10·97	$+0{\cdot}04 = p_5$	$+0{\cdot}13 = r_5$	+0·07
300°	10·79	$-0{\cdot}14 = p_6$	0	+0·12
	Mean 10·93	Arith. Mean 0·09	$\Sigma r + 0{\cdot}06$ $\frac{\Sigma r}{n} + 0{\cdot}01 = C$	Average Strain Error 0·08″

TABLE III

OBSERVATIONS IN TABLE I ARRANGED FOR CALCULATION OF GRADUATION ERRORS

Circle Setting	*Theodolite Base*						
	0°	60°	120°	180°	240°	300°	*Mean*
0°	10·6*	10·9	11·0	10·5	11·0	10·9	10·82
60°	10·8	11·2	11·1	11·1	11·0	10·8	11·00
120°	10·8	11·1	11·1	11·0	10·9	10·7	10·93
180°	10·6	10·9	10·8	11·1	10·9	10·8	10·85
240°	11·2	10·6	10·9	11·2	11·2	10·7	10·97
300°	10·9	11·2	11·3	10·8	10·8	10·8	10·97

* Readings in Table I are corrected for closure before inclusion in Table III.

TABLE IV

CALCULATION OF DIAMETRAL GRADUATION ERRORS

Circle Reading	*Seconds of Angle*	*Residual p*	*r*	*Diametral Graduation Error*
	″	″	″	″
0°	10·84	−0·08	−0·08	+0·03
60°	10·98	+0·06	−0·02	−0·05
120°	10·95	+0·03	+0·01	+0·01
	10·92		−0·03	Average 0·03

The results of complete tests for three instruments, together with a test on a small Tavistock theodolite selected at random for the sake of comparison, are shown in the following Table:

TABLE V

	Geodetic Tavistock No. 35209	35210	36039	*Small Tavistock No.* 35203
	″	″	″	″
Average Axis Strain	0·08	0·13	0·06	0·09
Average Diametral Graduation Error	0·03	0·33	0·05	0·09

No. 35210 was taken early in the series of tests, immediately after the initial practice afforded to the observers, and the test on it is probably affected by observational error to an abnormal extent. An abbreviated axis test on this instrument undertaken later shows, for instance, an average axis strain of no more than 0·06 second; and two more Geodetic Tavistock instruments (Nos. 36037 and 36038) subjected to the same abbreviated test show comparable average strain errors of no more than 0·05 second and 0·04 second respectively. Rannie and Dennis, after testing a large number of instruments, conclude that an average axis strain of 0·12 second is a satisfactory performance for a first-order theodolite. It must be concluded that the Tavistock family (including 'little brother', whose performance for one so young is nothing short of amazing) comes very well indeed out of these searching tests.

It was considered possible that the phenomenally low graduation errors revealed by the above method of test might arise from exceptionally careful setting of the dividing machine for the principal 60° graduations, and that the remaining graduations might be subject to greater errors. A complete test of the circle graduation errors was accordingly undertaken in one position of the theodolite base in order to ensure a constant strain error throughout. A set of observations consisted of double-face measures of the collimator angle, corrected for closure, on three positions of the circle 60° apart. Three micrometer readings were taken for each pointing and the mean of the set is considered free from observational error. Eighteen similar sets were measured on circle positions 10° apart and the 18 means are analysed to provide a probable error of graduation on the assumption that residuals from the grand mean are due to accidental graduation error. The probable graduation error thus obtained is only 0·09 second for No. 35209, 0·08 second for No. 36039, and 0·21 second for the small Tavistock; No. 35210 not being tested in this manner. Here, again, it must be concluded that the graduation of these instruments attains a higher standard than has hitherto been realised, and the makers are to be congratulated on achieving a remarkable advance in precise theodolite construction.

APPENDIX 12

INSTRUCTIONS TO OBSERVERS

1. The Geodetic Tavistock Theodolite is to be rigidly emplaced. On pillar stations it is to be anchored down by cords or web straps passing over the loose footscrew clamping plate, but not so excessively as to strain the tribrach. On earth stations the tripod feet are to be cemented to specially prepared rock or concrete footings, in which small dents have first been cut to receive the points of the tripod shoes. The tops of the legs are to be clamped firmly as soon as the tripod has been correctly centred.

2. The instrument is to be carefully levelled and the levelling checked occasionally between rounds. Particular care should be taken before observing to focus the diaphragm against the sky (or diaphragm illumination) with both eyes open, and then to eliminate parallax with the internal focusing ring; these adjustments then being left undisturbed throughout the observations at that set-up. No field adjustments for horizontal or vertical collimation are to be made.

3. The circles are to be illuminated electrically for all observations, whether by day or by night. Care should be taken before observing to equalise the illumination on both limbs and to set the light-gap for easy and accurate micrometer setting; these adjustments then being left undisturbed at least during a particular round on one face. A general guide to setting the light-gap is to make the white spaces, on either side of the central bar, about the width of a graduation; but, since this is to a large extent a personal matter, each observer should determine by practice which setting gives him the most confidence and the least range in his readings. Intense concentration is necessary for accurate readings, and the fingers should be taken off the micrometer after each small movement while the setting is being examined. It is desirable to make the final movements of the micrometer always in the same direction.

4. On Face Left the instrument is always to be swung right, that is rotated clockwise viewed from above; and on Face Right the instrument is always to be swung left. The same rule applies to the horizontal slow motions, whether this results in a movement with or against the retaining spring; and this implies always turning the slow-motion screw over and away from the observer on both faces. Before commencing a round the instrument should always be rotated at least one complete turn in the direction above for the face on which it is set; and should be rotated several times in this direction before first commencing observations, in order to take up any slack in the axis, etc. If the beacon is overshot, whether on the slow-motion screw or not, the instrument should be rotated completely in the correct direction and the overshot beacon re-intersected. If the instrument is swung accidentally in the wrong direction, whether on the slow-motion screw or not, the instrument is to be rotated in the correct direction and a fresh round started on any previously intersected beacon in the first half of the round—preferably the R.O.—the readings for this second half round being booked in a fresh column. Observers must practise coming straight on to the mark in the correct direction with the large slow-motion screw in diminishing steps, the fingers being

taken off the screw after each step while the intersection is being examined. They must avoid 'dithering about' on both sides of the mark. Intense concentration is required during final intersection.

5. The steadiest and most reliable light should be chosen as R.O. A rolling woolly light or one which is likely to be frequently interrupted should not be chosen, although it will often be necessary to strike a balance between these conflicting requirements.

6. Observation will normally be by continuous rounds (or directions), commencing on the R.O. on Face Left, changing face after intersecting the last beacon, intersecting the latter first on Face Right, and finishing on the R.O. Single-face rounds need not be closed on the R.O. unless the observer is uncertain whether the instrument has been displaced during the round. If a single-face round fails to close on the R.O. within two seconds, it is to be rejected and repeated entirely.

7. The procedure for 'broken rounds' when all lights are not showing is as follows. A light which is temporarily out may be filled in at any time (and booked in the same column) during a single-face round, provided that the direction of rotation of the instrument is not, and has not been, changed during that round, and provided the instrument has not otherwise been disturbed. After completion of a single-face round on all available lights, wait not more than one minute to see if any missing lights show up for inclusion in the same round. If not, change face and intersect all available lights on this other face, again not waiting more than one minute to see if any other lights show up. Now concentrate on lights which have been observed on one face but not on the other. If any such show up, tie them in on the missing face to the R.O. (or other light which has been well observed on that face in the main round). If a light which has not been observed on either face shows up, it should be tied in on both faces to the R.O. (or other light which has been well observed on both faces in the main round). After not more than five minutes without observing, change zero, and repeat the whole process. Any lights which have not been observed on the first zero, or have been observed on one face only, must be tied in subsequently on both faces on that zero to the R.O. (or other light which has itself been well tied to the R.O. on that zero). The general principles are as follows:

(*a*) Observations on a particular zero are not complete until it is possible to derive from them double-face directions of all lights from the R.O.

(*b*) If the instrument has been disturbed in any way during a single-face round (whether by reversing the direction of rotation, accidental displacement, or too long an interval of time since the last observation), then that round is to be terminated and any directions subsequent to the disturbance must include a fresh intersection on the R.O. (or other beacon which has previously been well tied to the R.O. on that face in that zero).

(*c*) Too long an interval of time—or a change of zero—must not occur between the balanced observations on the two faces. The light gap or illumination should preferably not be changed between faces, but this is not essential.

8. Double-face directions are to be measured once to all lights on each of eight 'zeros', or circle positions, given by the following Face Left readings on the R.O.:

Zero 1	00°	01′	05″
2	90	08	55
3	45	02	10
4	135	07	50
5	22	33	20
6	112	36	40
7	67	34	30
8	157	35	30

These circle settings, in which the odd minutes and seconds are required to eliminate errors of run of the micrometer, are to be set within 3–4 seconds. As soon as a complete round on any zero has been measured, the circle readings for all lights on all zeros should be tabulated, so as to facilitate picking up lights, and to facilitate setting the circle on any other light should the R.O. be out temporarily.

9. The above number of observations is sufficient in the case of observation on the short rays to and from secondary substitute stations, unless the observer is himself dissatisfied with them, and is sufficient also for observations to secondary up-stations. They may be considered sufficient at primary stations where considerable difficulty and delay are experienced; in such cases the observer should send in to Headquarters a complete copy of his observations with a report and a request to move on, when he will be notified by telegram whether to move or not. In all other cases, a second set of observations should be added on the following zeros:

Zero 1 ×	11°	16′	05″
2 ×	101	23	55
3 ×	56	17	10
4 ×	146	22	50
5 ×	33	48	20
6 ×	123	51	40
7 ×	78	49	30
8 ×	168	50	30

10. Three micrometer readings are to be taken at each intersection of a beacon, and observers should practise micrometer reading until the range on three such readings does not usually exceed one second.

11. Clamping screws are to be tightened only enough to make the slow-motion screw work. A very light pressure is sufficient for this purpose, and under no circumstances should the clamp be 'savaged'. So far as possible the vertical circle should be left clamped throughout a single-face round, and differences in elevation of the beacons taken up on the vertical slow motion. Care should of course be taken to set slow-motion screws in the middle of their runs before commencing a round.

12. The pointing on the beacon should be checked after reading the micrometer. If it has moved, unclamp, rotate in the correct direction and re-intersect.

13. In addition to cases given in para. 7, a single-face round should be terminated (and the remaining lights intersected in a fresh round off the R.O., or other beacon which has been well tied to the R.O. in that round) in the following cases, which are to be avoided as far as possible:

(*a*) When the micrometer is bumped against its stop.
(*b*) When the circle illumination has been changed, possibly by knocking the illuminating bulb.
(*c*) When the light-gap has been changed, possibly by bearing on the plate underneath the horizontal micrometer.
(*d*) Whenever the slow-motion screw runs out.
(*e*) After any accidental disturbance of the instrument.

14. The instrument must generally be carefully handled. Any rough usage, particularly when changing face, may result in strain which is released gradually during the succeeding round, with resulting inaccuracy. Remember to change over the micrometer eye-piece gently after changing face, before commencing the round. A reversing eye-piece prism is provided to eliminate constant error

due to observations on beacons of different magnitude. This eye-piece should also be reversed on changing face. Rotate the instrument by grasping the bottom plate lightly with the thumbs and first fingers fully extended round the plate; never by grasping the standards or telescope. Reverse the telescope by grasping it lightly with the thumb and first finger as close to the transit axis as possible. These precautions are unnecessary for rough tertiary work, but the ruling triangulation of Great Britain is not rough tertiary work.

15. All observations are to be booked in ink on the squared paper provided. Mistakes in booking are to be lightly crossed through but not erased. Legible figuring is essential, and under no circumstances is a figure to be corrected by superimposing another.

16. Immediately on completion of the observations at a station the original observation sheets are to be sewn together and forwarded to Headquarters by registered post. Receipt will be acknowledged.

17. Each observer will be supplied with a sheet of specimen bookings and will adhere rigidly to the system shown on this specimen and amplified by the following notes:

(*a*) On a title-page are to be shown the following:

Name of Station (as given on the triangulation diagram).

Name of Observer (including a statement as to which zeros were observed by each observer, when two are employed).

Number of Instrument.

Names of lightkeepers at each surrounding station.

(*b*) On the following pages:

A brief dated diary of events at the station, including hours of observing; weather conditions; visibility, roughly, in miles at which opaque objects can just be seen with the naked eye at sunset; which lights were not showing and for what periods; and the quality of the lights showing.

(*c*) Double pages as necessary for main horizontal observations. The names of stations are entered clockwise commencing with the R.O.; and finishing with the R.O. in case it is desired to close a round; two squares for each station. If the number of lights exceeds 11, the paper should be turned round 90°, thus making room for 15 stations.

Degrees and minutes are booked for the first two zeros only and are to be checked by mental abstract by the observer's assistant; thereafter micrometer seconds only.

Each column contains readings in a single-face round only. If for any reason, the round has to be terminated (see paras. 7 and 13), the fresh round must be entered in a fresh column. All three micrometer readings are to be booked in the two-square deep space. If more are taken, or if mistakes are made in booking, extra figures may be added in the next column (which is not in that case to be used for any part of the next round), and bracketed to the column to which they refer.

At the head of each column is entered, without explanation and in this order, the zero number (e.g. 2×), the Face (L or R), and the date (day figures only with M, A or N for Morning, Afternoon or Night observations, the 'Night' referring to the previous date. For instance, observations between 1.0 a.m. and 2.0 a.m. on 28th June would be entered 27N).

Except for the observer's own information, no means or abstracts need be made in the field and none will be entered on the observation sheets.

(*d*) Double pages as necessary for horizontal observations to secondary up-stations.

SPECIMEN OBSERVATIONS

	1: L 4N	1: R 4N	2: L 4N	2: R 4N	3: L 4N	3: R 4N
	° ′ ″	′ ″	° ′ ″	″	″	″
Llangeinor	00 01 04·9	00 59·0	90 08 56·4	48·4	09·0	02·6
	04·7	59·0	56·2	48·9	09·3	02·8
	04·7	58·4	56·4	48·6	09·8	02·6
Trecastle	33 54 41·6	34·8	Faded out			
	41·9	34·3				
	41·6	34·6				
Malvern	142 02 54·7	49·6	232 10 39·6	35·0	Faded out	
	54·2	49·3	39·4	34·7		
	54·4	49·2	39·8	35·2		
Peglers Tump	182 37 27·9	20·4	272 45 17·1	09·4	34·0	26·8
	27·6	20·1	17·3	09·3	34·3	26·3
	27·4	20·7	17·2	09·1	33·9	26·3
Pen Hill	243 11 06·6	01·5	333 19 03·3	18 55·5	17·7	09·0
	06·8	01·5	02·5	55·1	17·3	09·9
	07·0	02·1	02·9	55·4	17·2	09·9
Blagdon	246 02 05·0	01 58·5	336 09 56·3	49·2	11·1	04·6
	04·4	58·9	56·6	49·3	10·5	04·2
	05·1	58·8	57·1	49·8	10·9	04·6

APPENDIX 13

DIARY OF THE FIELDWORK OF THE PRIMARY RETRIANGULATION

1935	Reconnaissance	*April–October*	Verification of the 'paper scheme' for Figures 1 and 2 (central chain south and north).
1936	Reconnaissance	*April–November*	Verification of 'paper scheme' for Figure 3 (South and Central Scotland) and Figure 4 (West England and Wales). A start made on the southern portion of Figure 5 (South-East England).
	Station Marking	*March–November*	All stations in the central chain of Figures 1, 2 and 3 completed. A start made on Figure 4.
	Observing	*April–October*	Figures 1 and 2 completed by three observing parties.
1937	Reconnaissance	*January–July*	Reconnaissance of Figure 5 and additional stations in Figure 3 completed.
		June	Preliminary reconnaissance of Ridgeway Base.
	Station Marking	*February–May*	Remaining stations in Figure 4 completed.
		June–August	Southern portion of Figure 5 completed.
		July–August	Reconstruction of a number of pillars in Wales which had been found to be unsound due to the work of an unreliable pillar constructor.
	Observations	*April–July*	The main chain was extended to the Lossiemouth Base by the observation of Figure 3.
		July–September	Figure 4 completed.
	Base Measurement	*November–December*	Ridgeway Base measurement.

1938	Training	*January–March*	Training in the erection of Bilby steel observing towers.
	Reconnaissance	*February–June*	Remainder of Figure 5 completed. To avoid the re-erection of steel towers and the re-occupation of roof stations where special staging was used, the reconnaissance for the secondary triangulation around such primary stations was done concurrently with the primary reconnaissance.
	Station Marking	*March–October*	At steel tower stations in Figure 5, pillars were built after the steel towers had been erected.
	Observations	*April–October*	The observation of Figure 5 was completed and the secondary triangulation was also observed around steel tower stations.
	Base measurement	*July–August*	Lossiemouth Base measurement.
1939	General		During 1939 the main fieldwork effort was switched from primary to secondary and tertiary triangulation in areas where the new large scale plans were to be produced. Such areas included the main industrial centres and coalfields. Consequently, little progress was made on the primary.
	Reconnaissance	*July–August*	The reconnaissance of Figure 6 (North and West Scotland) was completed except for Orkney and Shetland. Preliminary reconnaissance for the Caithness Base completed and terminals sited.
	Station marking	*August*	Terminals for the Caithness Base built.
	Observations	*April–May*	A small extension to Figure 3, involving three steel towers, was observed.
1939/ 1945			Second World War during which no fieldwork was carried out on the primary Retriangulation.
1945/ 1946			Construction carried out of a few pillars in Northwest Scotland.
1947			The maintenance of pillars only.

1948	Reconnaissance	*July*	Reconnaissance of Orkney and Shetland (part of Figure 6) completed.
	Station Preparation	*June–October*	The marking of stations in eastern and northern parts of Figure 6 completed, except for six of the more difficult stations where only surface marks had been inserted.
1949	Training	*January–April*	Training of lightkeepers.
	Station Marking	*May–July*	Marking of stations in Figure 6 completed.
	Observations	*May–July*	The eastern and northern portions of Figure 6, including Orkney and Shetland, were completed by two observing parties.
1950	Reconnaissance	*April–July*	Reconnaissance of western part of Figure 6 (the Western Isles) and Figure 7 (Isle of Man) completed.
		July	Detailed reconnaissance of the Caithness Base completed and an intermediate station sited midway between the terminals.
	Re-observations	*July–August*	Figure 6, north of line Foula (461)–Brassa (456).
	Marking	*May–October*	The stations in Figures 6 and 7, reconnoitred in 1950, were marked. The maintenance of primary stations built before 1939 was completed.
1951	Reconnaissance	*March–April*	Reconnaissance for the connection to France completed including the strengthening of the triangulation in Kent by the inclusion of one new station.
	Station Preparation	*April*	Steel towers were erected on five stations for the England/France connection.
	Observations	*April–September*	Observations in the western part of Figure 6 and in Figure 7 completed.
		May–July	Observations for the Cross Channel connection completed.
	Base Measurement	*October*	Re-measurement of the Ridgeway Base.
1952	Observations	*April–September*	Observations for the connection to Eire and Northern Ireland completed in conjunction with the respective Survey Departments.
	Base Measurement	*April–June*	Measurement of the Caithness Base.

1953	Astronomical Observations	*May–November*	The programme of Laplace azimuths completed consisting of observations at six pairs of stations, and observations at the Royal Observatory, Greenwich to determine observer's personal equation in longitude. Astronomical latitude and longitude were found also at seven of the stations, and longitude only at an eighth.
		June–October	Assistance to the Astronomer Royal in conjunction with his determination of the azimuth of the line Greenwich to Pole Hill.
		August	Observations to connect the Royal Greenwich Observatory, Herstmonceux, to the triangulation.
1954		*April–May*	Observations to connect the Royal Observatory, Greenwich, to the triangulation.
1955		*June–July*	Observations to connect one terminal used by the United States Air Force in the Shoran connection to Iceland.
1957		*May*	Observations to co-ordinate a primary station on the island of St Kilda in connection with the Guided Missile Range.

APPENDIX 14

CO-ORDINATION BY SEMI-GRAPHIC METHODS

1. Introduction

In semi-graphic methods no assumptions are made about triangle misclosures because the computation depends fundamentally on bearings and resection angles. Triangles are not used. For the same reason it is not necessary to observe a continuous triangulation network. The main advantage of semi-graphic methods is that the plotted graph gives a general picture of the fixation at a glance and bad pointings are obvious. Under these circumstances it is not difficult to ensure local consistency among the points—an important consideration when the control is the basis for large-scale surveys.

2. Semi-graphic Intersection

The semi-graphic intersection can best be described as a method of showing graphically on a large scale the relative positions of a series of plane observed grid bearings into a station. Fig. 1 illustrates this.

In Fig. 1 observations to *P* are made from the fixed stations *A*, *B*, *C*, and *D*. In the computation the graph that is actually plotted is the portion of Fig. 1 in the immediate vicinity of the point *P*, the scale of the graph being such that the co-ordinates of *P* can be read off to an accuracy of 0·01 m. Ideally, all the pointings will go through one point and there is then no doubt about the position of *P*. The ideal is rarely found in practice, and it is necessary to assess the graph to find the most likely position for *P*. There are no hard and fast rules, experience and common sense being the best guides.

3. Semi-graphic Intersection and Resection

If observations have been made at *P* to the fixed stations, another estimate of the co-ordinates of *P* can be made by resection. By combining the intersection and resection on one graph a very strong fixation can be obtained for *P*.

The two fixations are combined by first computing the semi-graphic intersection, and then utilising the plotted intersection graph to construct the tangents due to the observations at *P*. The so-called tangents at *P* should in fact be arcs of circles. The curvatures of the latter are so small, however, that no error is introduced if they are plotted as straight lines, or tangents. (See Fig. 2.) Assume in Fig. 2 that *P* observes the two stations *A* and *B*. Then the tangent at *P* should actually be part of the circle passing through *APB*. Observation to a third point C gives two more circles, *APC* and *BPC*; three tangents are the minimum necessary to define *P*. The minimum of three tangents is not very reliable, since even with gross errors in the observations at *P*, the three tangents will always trisect. Such errors will always be detected, however, by the intersecting observations. In general, if *P* observes *n* stations there will be $n(n-1)/2$ tangents.

Although the intersection and resection are plotted on one graph, they are completely independent fixations for the same point.

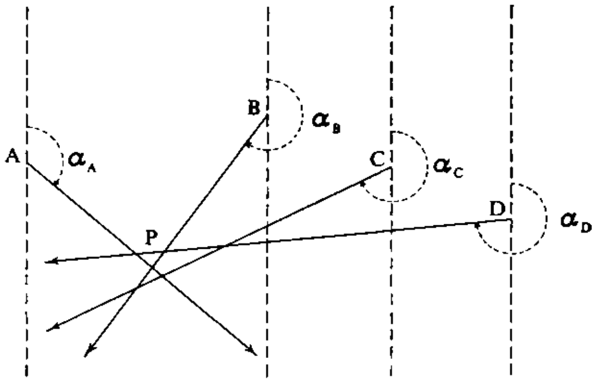

FIG. 1. The intersection

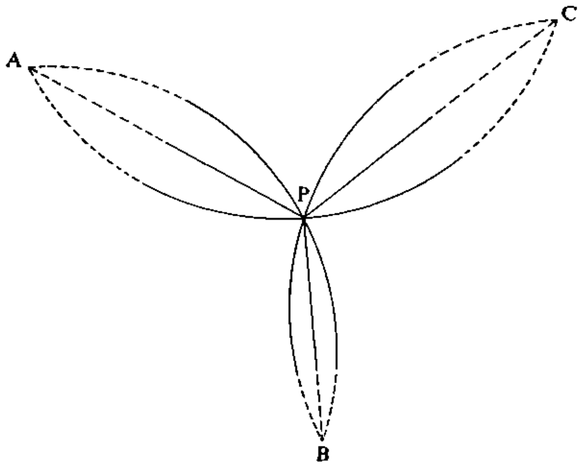

FIG. 2. The resection

4. A Worked Example (see Fig. 3)

The computation is done in two stages, the first stage being the intersection.

Data

Station	*E.*	*N.*	*Grid Bearing to P* (α)	*Function*
A	275 982·62 m.	145 917·71 m.	344° 39′ 19″·0	Tan −0·274 40819
B	270 002·48 m.	143 472·65 m.	36 16 21·0	Tan +0·733 83433
C	270 744·58 m.	148 713·98 m.	71 42 06·3	Cot +0·330 68450
D	276 500·35 m.	149 376·93 m.	292 40 51·6	Cot −0·417 91947

$$\text{Tan}\ \alpha_A - \text{Tan}\ \alpha_B = -1{\cdot}008\ 24252$$

Formulae

(*a*) $$E_P = \frac{E_A \,.\, \cot\alpha_A - E_B \,.\, \cot\alpha_B - N_A + N_B}{\cot\alpha_A - \cot\alpha_B}, \quad \text{and} \quad N_P = (E_P - E_A)\cot\alpha_A + N_A$$
$$= (E_P - E_B)\cot\alpha_B + N_B \text{ (Check)}$$

alternatively,

(*b*) $$N_P = \frac{N_A \,.\, \tan\alpha_A - N_B \,.\, \tan\alpha_B - E_A + E_B}{\tan\alpha_A - \tan\alpha_B}, \quad \text{and} \quad E_P = (N_P - N_A)\tan\alpha_A + E_A$$
$$= (N_P - N_B)\tan\alpha_B + E_B \text{ (Check)}$$

Two of the stations are selected whose bearings to *P* give a good intersection with each other, that is, whose bearings cross as near 90° as possible. These stations are nominated *A* and *B* in the formulae, and their bearings to *P* are α_A and α_B respectively. The choice of formulae (*a*) or (*b*) depends on the collective magnitude of the tangent/cotangent functions of the bearings. If the tangents of α_A and α_B are collectively smaller than the cotangents use (*b*), if the cotangents are smaller use (*a*). Generally the aim is to keep the functions as small as possible and this should be kept in mind when choosing *A* and *B*; because although 90° is the ideal intersection angle it is only an aim and not a necessary condition; keeping the magnitude of the functions down is important. In the example formulae (*b*) were used, and gave:

$$E_P = 274\ 843{\cdot}37 \text{ m.} \qquad N_P = 150\ 069{\cdot}36 \text{ m.}$$

E_P and N_P are now used to compute the cuts of the remaining bearings into *P*. Let suffix *N* indicate any station other than *A* and *B*, then:

$$E \text{ cut of station } N = (N_P - N_N)\tan\alpha_N + E_N$$

or

$$N \text{ cut of station } N = (E_P - E_N)\cot\alpha_N + N_N$$

If α_N is between 315° and 45° or 135° and 225° compute *E* cut, in other cases compute *N* cut.

Thus

$$N \text{ cut of } \alpha_C \text{ on } E_P = (E_P - E_C)\cot\alpha_C + N_C = 150\ 069{\cdot}39 \text{ m.}$$
$$N \text{ cut of } \alpha_D \text{ on } E_P = (E_P - E_D)\cot\alpha_D + N_D = 150\ 069{\cdot}41 \text{ m.}$$

On graph paper plot E_P and N_P, these are the axes of the graph. Plot *N* cuts on the E_P axis, and *E* cuts (if any) on the N_P axis. With a protractor plot all the bearings from their respective cuts, α_A and α_B being both plotted from the point E_P, N_P. Fig. 4 shows the plot for the worked example.

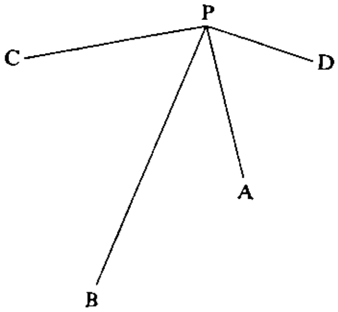

FIG. 3. Diagram of fixation

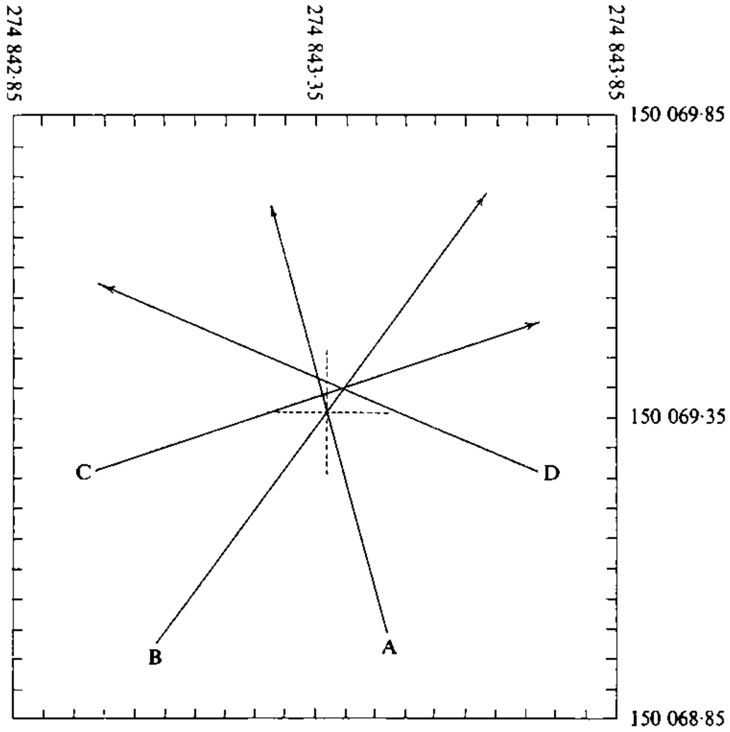

FIG. 4. Graph of intersection

The second stage consists of plotting the resection on Fig. 4.

Data

Observed Directions at P			
To *A*	272°	57′	11″8
To *B*	324	34	12·2
To *C*	359	59	59·6
To *D*	220	58	43·4

Approximate Distances in km.			
PD	1·8	*DB*	8·8
PA	4·3	*DC*	6·0
PB	8·2	*AB*	6·5
PC	4·5	*AC*	6·2
DA	3·5	*BC*	5·2

In Fig. 5, A to P and B to P represent any pair of intersecting rays in Fig. 4, and P is the point of intersection of the pair. The angle APB is given by the difference between the reversed intersecting bearings. If the observed angle at P is $AP'B$, then the observer was at some point on the circle through $AP'B$ when the observation was made. When the angle $AP'B$ is smaller than APB, P' lies on the opposite side of P from the line AB; and vice versa.

If now we plot the tangent XY on the intersection graph, raise from P a perpendicular to the tangent, and plot along the perpendicular at the scale of the graph the distance Z' (see Fig. 5), we can get the required tangent $X'Y'$ at P' by drawing a parallel from XY. The direction of XY on the graph is found by plotting from P along the rays PA, PB, their distances in reverse order, that is, along the ray PA plot distance PB, and along ray PB plot distance PA. Any convenient scale is used. It can be shown that:

$$Z' = \frac{PA \times PB}{AB} \times 0{\cdot}0048 \times d\alpha'' \qquad \text{(approximately)}$$

where the distances PA, PB, AB, are in km., Z' is in metres, and $d\alpha''$ is the difference in seconds of arc between the angles $AP'B$ and APB. The distances are only required to the nearest 0·1 km.

The above procedure is carried out for each pair of intersecting rays in turn, the final result being the resection tangents on the intersection graph. It is convenient in practice to plot the direction of XY on the side of P on which P' falls, and then carry out the construction without plotting XY itself, bearing in mind that Z' is always plotted from P.

Applying the above routine to the data for the worked example gives:

Pair	*Angle from Reversed Bearings*	*Observed Angle*	$d\alpha''$	Z'	*Direction of P′ from P*
AB	51° 37′ 02″0	51° 37′ 00″4	01·6	0·04 m.	Away from *AB*
AC	87 02 47·3	87 02 47·8	00·5	0·01 m.	Towards *AC*
AD	51 58 27·4	51 58 28·4	01·0	0·01 m.	Towards *AD*
BC	35 25 45·3	35 25 47·4	02·1	0·07 m.	Towards *BC*
BD	103 35 29·4	103 35 28·8	00·6	0	$P' = P$
CD	139 01 14·7	139 01 16·2	01·5	0·01 m.	Towards *CD*

Plotting on Fig. 4 gives the final combined graph shown in Fig. 6. In practice the intersecting rays and the tangents are drawn in contrasting coloured inks to emphasise the different plotted data. Construction lines are lightly drawn in pencil and finally erased. The selected final value for P is E 274 843·38 N 150 069·39, and is indicated in Fig. 6 by the dot inside the triangle.

For the best results a minimum of four observations in and four out, to give six tangents, is desirable. Occasionally a pointing is made from P to a fixed station from which no intersecting

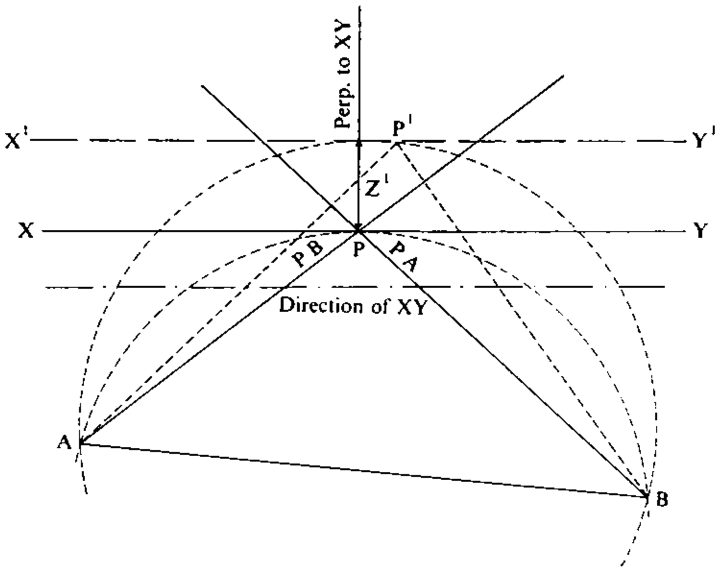

FIG. 5. Plotting a tangent

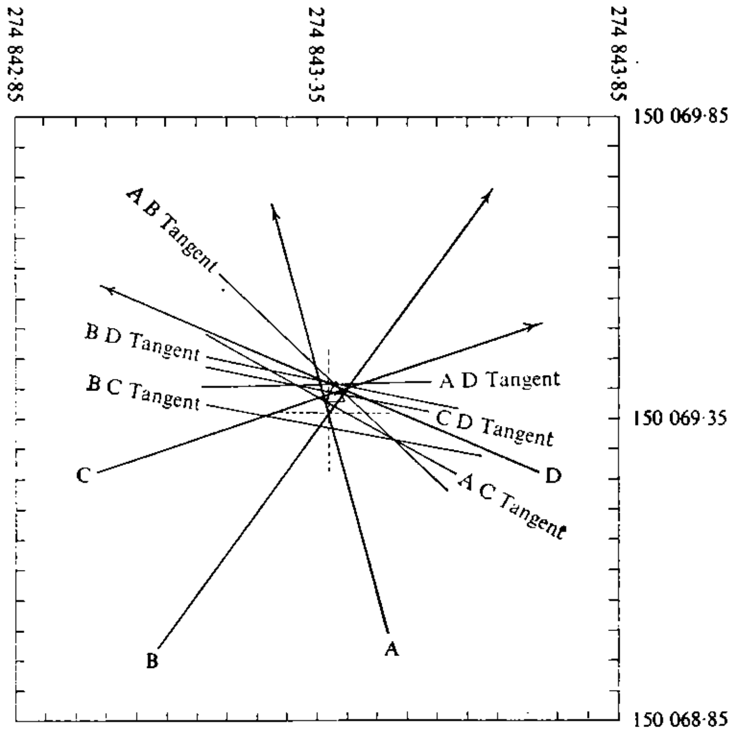

FIG. 6. Graph of intersection and resection

observation has been made to P. Observations to church spires are typical cases. Such pointings can be used in the semi-graphic fixation by computing a bearing from E_P, N_P to the observed station, plotting the bearing on the graph through E_P, N_P, and using it as already described to plot the tangents. The computed bearing is used to find the angle APB in Fig. 5. This computed ray on the graph must not be considered in any way when selecting the final position of P; it is not an intersecting ray. Only the tangents from it contribute to the fixation of P.

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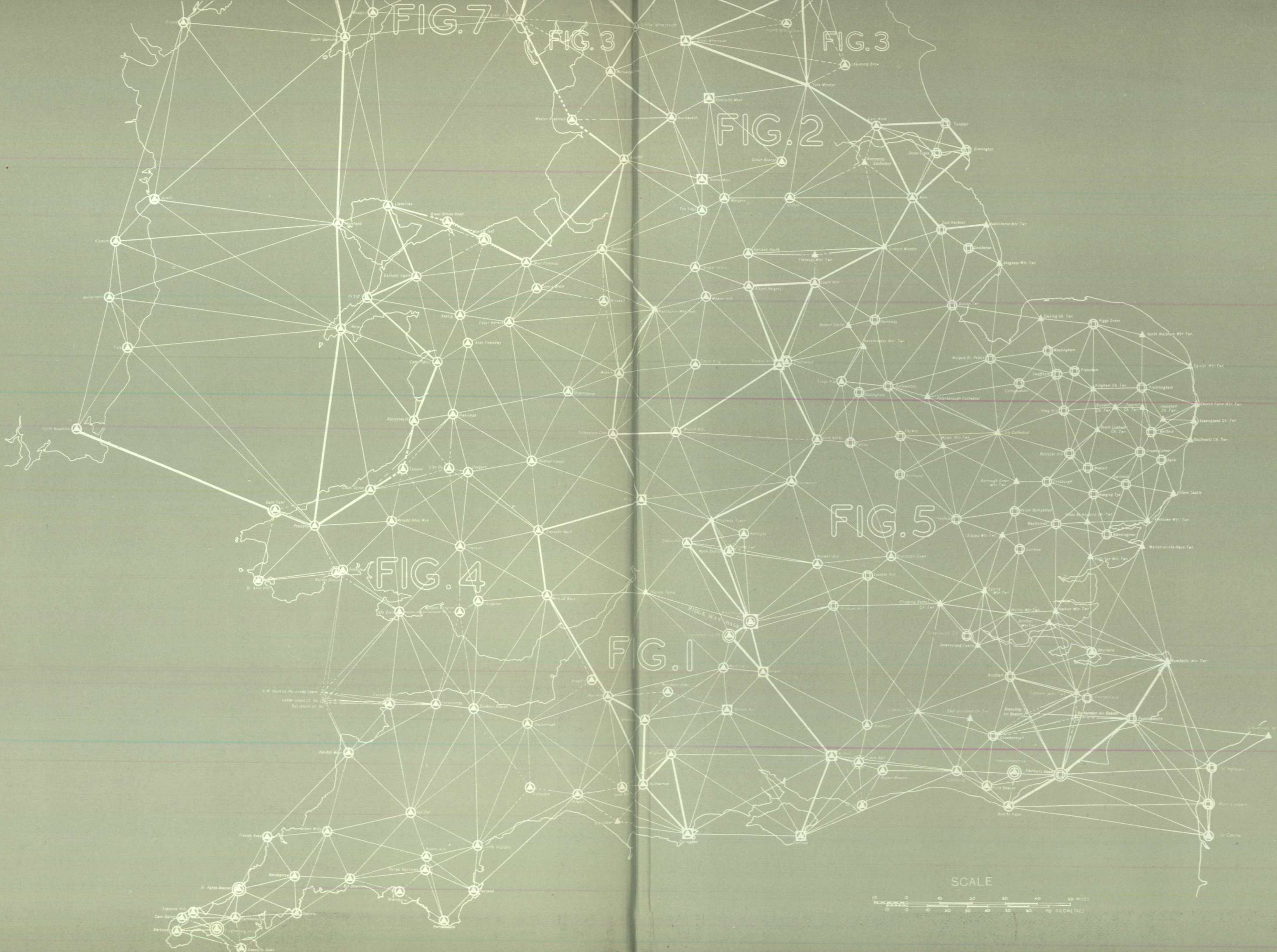
FIG. 7
FIG. 3
FIG. 3
FIG. 2
FIG. 4
FIG. 5
FIG. 1
Little Whernside
Great Whernside
Hambleton Down
Leavening Brow
York Minster
Rombalds Moor
Boulsworth
Upton Beacon
Holme Moss
Margery
The Edge
Clifton
Cave Wold
Tunstall
Stone Creek
Dimlington
Normanby Gasholder
Acre
Cold Harbour
Mablethorpe Wtr. Twr.
Harland South
Thoresby Wtr. Twr.
Lincoln Minster
Deatherage
Skegness Wtr. Twr.
Louth Hill
Alport Heights
Docking Ch. Twr.
Figgs Grave
North Walsham Wtr. Twr.
Massingham
Swaffham
Fransham
Caister Wtr. Twr.
Crimplesham
Hingham Ch. Twr.
Framlingham
Frog Hill
Church Farm Wtr. Twr.
Kessingland Ch. Twr.
Ely Cathedral
South Lopham Ch. Twr.
Southwold Ch. Twr.
Puttocks Hill
Crown Corner
Saxle
Burrough Green Wtr. Twr.
Orford Castle
Hedging Tye
Helion Bumpstead
Felixstowe Wtr. Twr.
Wennington
Walton-on-the-Naze Twr.
Dunmow
Epping Wtr. Twr.
RIDGE WAY BASE
SCALE
10
0
10
20
30
40
50 MILES
10
0
10
20
30
40
50
60
70 KILOMETRES