L7 Index of Refraction

Erick Oduniyi (erickoduniyi@ku.edu)
Department of Electrical Engineering & Computer Science, University of Kansas
1450 Jayhawk Blvd, Lawrence, KS 66045, USA

Objective

The objective of this lab was to understand the phenomenon of refraction. In route to learning about this phenomena, experiments using a prism spectrometer, color prism, and vapor lamp were used. Accordingly, the quantities measured during this lab were the *apex angle A*, and *minimum deviation D*. Using these measured quantities the *index of refraction n* of the hydrogen lines series, and their *experimental wavelength* λ_{exp} was calculated.

Figure 1: Album art for Pink Floyd's *The Dark Side of the Moon*

Setup and Diagram

Initially, the *prism spectrometer*, *prism*, and *vapor lamp* were carefully calibrated according to the PHSX 316 L1 Manual

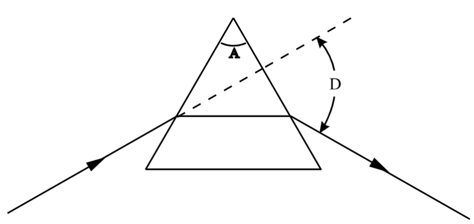

Figure 2: The minimum deviation angle D shown as light passes through glass prism

Index of Refraction Part A

The illuminated slit was set to be narrow and vertical, aligning it to the cross hairs of the telescope. Then, one of the prism's vertices was directed towards the *collimator*. From there, the apex angle A was measured by successively setting the telescope T_1 and T_2 position on the reflected images and at each at each position recording the readings from the spectrometer's A and B verniers.

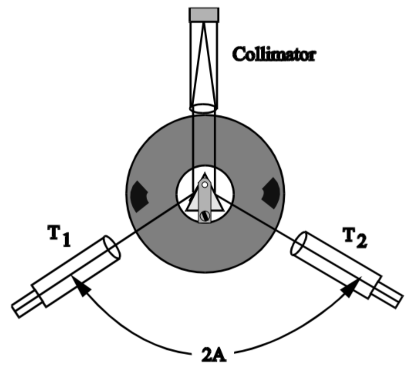

Figure 3: Configurations of the spectrometer for measuring the apex angle A

Index of Refraction Part B

One of the prism's vertices was chosen, and spectrum lines were found at position T_2. Then, the prism table was rotated in order to find the position at which the image of the line reverses the direction of its motion, again measuring both verniers. Then, the prism table was rotated such that the deviated light was visible at position T_1 and accordingly measuring the position of minimum deviation. This was process was repeated for each mercury series.

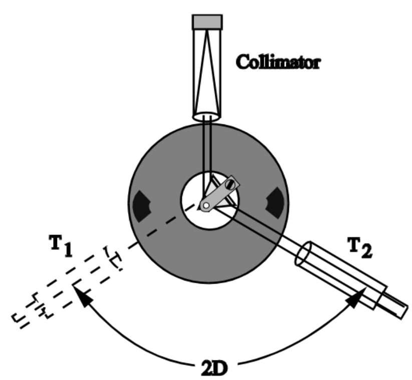

Figure 4: Configurations of the spectrometer for measuring the minimum deviation angle D

Index of Refraction Part C

In the final part of the lab the index of refraction n was calculated using the apex angle A and the minimum deviation angle D for the hydrogen series. Finally, the experimental wavelength λ_{exp} was calculated using Microsoft Excel's LINEST function.

Sample Calculations

The equation for calculating the *apex angle A* and *minimum deviation angle D* is given by equation (1-2)

$$A = \frac{\frac{1}{2}(T_1A - T_2A) + \frac{1}{2}(T_1B - T_2B)}{2} \tag{1}$$

$$D = \frac{\frac{1}{2}(T_1A - T_2A) + \frac{1}{2}(T_1B - T_2B)}{2} \tag{2}$$

Where T_1A,T_2A,T_1B,T_2B are the respective A and B vernier measurements for the spectrometer telescope at position T_1 and T_2:

$$n(\lambda) = m\frac{1}{\lambda^3} + b$$

$$\lambda_{exp} = \sqrt{\frac{m}{n-b}} \tag{3}$$

and where m and b are the respective slope and y-intercept[1]

Error Analysis

Using the total differential formula, the uncertainty of the experimental wavelength λ_{exp} can be determined as:

$$\delta\lambda_{exp} = \sqrt{\left(\frac{\partial\lambda_{exp}}{\partial m}\delta m\right)^2 + \left(\frac{\partial\lambda_{exp}}{\partial n}\delta n\right)^2 + \left(\frac{\partial\lambda_{exp}}{\partial b}\delta b\right)^2}$$

where

$$\frac{\partial\lambda_{exp}}{\partial m} = -\frac{1}{2(n-b)^2\sqrt{\frac{m}{n-b}}},$$

$$\frac{\partial\lambda_{exp}}{\partial n} = -\frac{m}{-2(n-b)^2\sqrt{\frac{m}{n-b}}},$$

$$\frac{\partial\lambda_{exp}}{\partial b} = \frac{m}{2(n-b)^2\sqrt{\frac{m}{n-b}}},$$

$$\delta m = 4.8725 * 10^{10},$$

$$\delta n = 0,$$

$$\delta b = 0.0000676173$$

Hence, the uncertainty of the experimental wavelength λ_{exp} is given by:

$$\lambda_{exp} = \sqrt{\frac{4.8725 * 10^{10}}{m(n-b)^3} + \frac{0.0000676173m}{(n-b)^3}}$$

[1]The uncertainties for m and b are obtained from Microsoft Excel's LINEST function.

The total uncertainty in the experimental wavelength can be calculated by:

$$\delta\lambda_{exp-total} = \sqrt{\delta\lambda^2_{exp-1} + \delta\lambda^2_{exp-2} + \delta\lambda^2_{exp-3}}$$

Therefore, uncertainty in the wavelength with 95% confidence is given by:

$$\delta\lambda_{95\%} = \frac{(\lambda_{exp-total})(CC_{95})}{\sqrt{(3)-1}}$$

Results

The final results and calculations for part a, b, and c are reported below. The results obtained from spectrometer apparatus are slightly confusing and did not align with any initial hypothesis[2]. For example, the dispersion curve generated from plotting n vs $1/\lambda^2$ has a negative slope. In principal, this seems to be fine but certaintly unexpected. Additionally, the only calculated hydrogen wavelength close to the actual wavelength was red. For both the purple and blue wavelengths they were very off. The total uncertainty obtained from the experimental wavelength calculation are unacceptable and are not reported as the values generated were extraneous; such a wild uncertainty could have been generated from a number of sources and requires further investigation.

Index of Refraction Part A

A [deg]
59.45
59.64
59.97

Thus, the average apex angle A_{avg} is:

$$A_{avg} = 59.68$$

Index of Refraction Part B

Color (Hg)	λ [0.1*nm]	$\frac{1}{\lambda^2}$	D [deg]	n
Strong Violet	4358	5.26E-08	36.63	1.50
Faint Blue	4916	4.14E-08	36.44	1.50
Strongest Red	6908	2.09E-08	38.63	1.52
Strong Green	5460	3.35E-08	37.25	1.50
Strong Yellow	5790	2.98E-08	37.92	1.51

Index of Refraction Part C

Color (H)	λ_{exp} [0.1*nm]	λ_{actual} [0.1*nm]	D [deg]	n
Red	7107	6562	36.63	1.52
Blue	10916	4861	39.25	1.53
Purple	16543	4340	39.58	1.53

[2]hypothesis specified in original lab manual

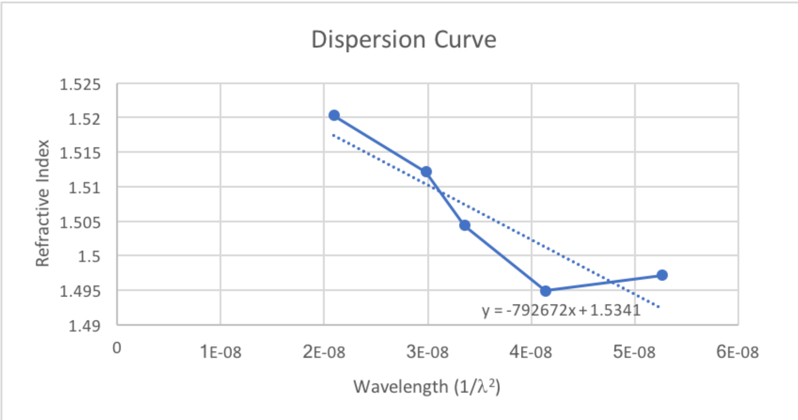

Figure 5: Dispersion curve generated from n vs. $\frac{1}{\lambda^2}$