The Course, Learning Resources & Your Responsibility

Introduction to Continuum Mechanics

Connecting Many Courses

Continuum Mechanics, and now meets you at 300 Level.

Second Set at Unilag to be taught this way

Continuum Mechanics is a unifying theory

Many courses are special cases of the general framework of continuum mechanics.

The balance laws of mass, momentum and energy applicable to all matter;

This connection is not trivial.

Separating the natural laws from constitutive functional relationships creates a connection several courses.

Allows a synthetic approach to engineering design.

Analysis, Design, Optimization & Prototyping

Mechanics of Fluids

- Hydrodynamics, Aerodynamics, Hypersonic Flow, Rheology, Flows in arteries and other biological pipes, etc.

Elasticity

- Hyperelesticity, Hypolelasticity, Rubber Technology, Soft materials, Tissue Mechanics, Cadiovascular Solid Mechanics, Viscoelasticity, etc.

Plasticity

- Elasto-plastic loading, viscoplasticity, incremental theories, etc.

BioMechanics

- Human and Animal Tissues, Flows in blood vessels, Prosthetics, Sports Enhancement products, etc.

Heat Transfer

- Visco-Thermal Flows, Thermal stresses, etc.

Thermal Mechanics

- Thermodynamics of Materials, Material Behavior, etc.

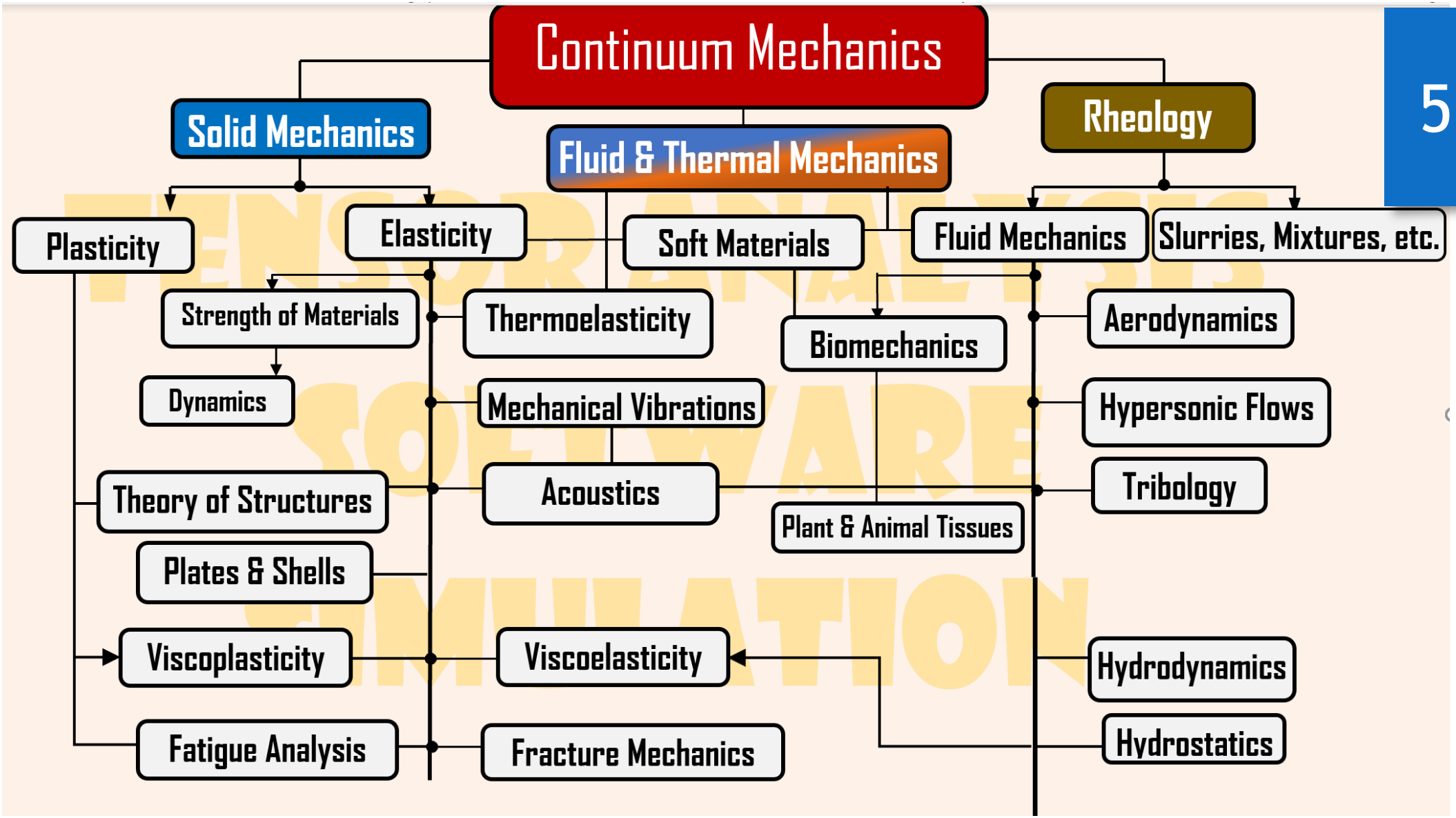
The Synthetic Approach

Design software is built on a mutiphysical foundation.

Stresses, for example, may be induced by applied heat in addition to the ones caused by mechanical loads.	Cannot afford to treat the heat transfer problem independently of the stress analysis problem.	The physical causes of these are dissimilar; engineers encounter them simultaneously

Mutliphysical approach is imperative

Already implemented in design software	Bringing instructional strategy up to that same level: Continuum Mechanics


Continuum Mechanics
Solid Mechanics
Fluid & Thermal Mechanics
Rheology
Plasticity
Elasticity
Soft Materials
Fluid Mechanics
Slurries, Mixtures, etc.
Strength of Materials
Thermoelasticity
Biomechanics
Aerodynamics
Dynamics
Mechanical Vibrations
Hypersonic Flows
Theory of Structures
Acoustics
Tribology
Plant & Animal Tissues
Plates & Shells
Viscoplasticity
Viscoelasticity
Hydrodynamics
Fatigue Analysis
Fracture Mechanics
Hydrostatics

What is Engineering?

- A lot of the things that make you an engineer related to continuum mechanics
  - Language: Tensor Analysis
  - Software: Catalysts to the learning Process
  - Simulation: deploy the knowledge gained to design visually and virtually in order to save prototyping costs
- Engineering is the *application of Science* to create technology **products and services**.
  - Rooted in theory. If you do not organize the learning of theory very well, you end up with full heads and no products as we have been doing.
  - If you leave theory and simply do “practicals”, you end up with half-baked crafts trade – again, no serious products.
  - We are offering you an approach to avoid both extremes and learn, in order to do engineering correctly.

Dean's Welcome Address 2016

"I want you to be ambitious. One of the biggest problems of the African mind is the absence of serious ambition. Once a Naija man can be a little better than his neighbor, he appears satisfied! If there is no electricity, and you get a small noise maker that helps you to watch Manchester United, you are already in heaven! You seem to forget that the same electricity is available to young people in Singapore 24 hours a day! And that there is absolutely no reason why Enyimba, the people's Elephant, cannot be more popular than Manchester United! What do they have? Football grass fields, one ball, 22 men and hundreds of thousands of passionate fans! With some clever marketing, this nets them more money than crude oil in its most comfortable price regimes, can get Nigeria. More depressing is that the City State of Singapore, smaller in population and size than Lagos, can actually consume over 60% of Nigeria's oil! That is the meaning of industrialization! It is lack of ambition that will cause a Minister of Aviation to steal two jeeps! Two jeeps! Even for all their rapacity, our thieves are not sufficiently ambitious! Why, for example, cannot the Minister of Aviation ensure than Nigeria can buy 100 of the latest wide body jets such as A380 or B777 and then steal two of them at the cost of nearly 1 billion dollars each! But once they can drive two jeeps in a convoy and use sirens to chase others from the road, even if they cannot comfortably get to where they are going, they are already satisfied!"

First Course in 3 Chapters

Vector Analysis

- Vector Analysis it is designed to make you deepen your understanding of this most basic of tools in learning continuum mechanics and prepare the way for tensor theory.

Tensor Algebra

- Additional Structure to extend what you already know in Vector Algebra
- Eigenvalues and Characteristic Directions

Tensor Calculus

- The extension of the well-known limiting process of scalar calculus to larger objects you deal with in engineering design such as:
  Stress and Strain, Temperature & Velocity Gradient, Elasticity Tensor, Invariants, etc.

Scope of the Course @ 300 Level

1. Vectors & Transformations
   - Elementary concepts, Index notation & Summation Convention, Coordinate Transformations, Dyads & Rotations, 3D Geometry & General Curvilinear Coordinates
2. Tensor Algebra
   - Tensors & Transformations, Components & Invariants, Decompositions, Eigensystems & Spectra Forms
3. Tensor Calculus
   - Calculus of Large Objects, Gateaux & Fréchet Derivatives, Scalar, Vector & Tensor Arguments and Return Values, The Euclidean Space and Fields. Demonstration of Field Theorems

- Ordinarily, this chapter should not be necessary
  - If we are to assume you already know the things you were taught in your lower-level courses. Our experience compels us to start here because vector algebra and calculus are key to the development of tensor theory. If you understand vector algebra deeply, armed with calculus and some geometry, tensors should be easy to master.
  - Your difficulty often arises from the cramming of things you did not fully grasp in lower-level courses
  - The vector analysis introduction will be brief, well directed and designed to lead you gently into tensor theory.
  - If you struggled with this subject previously, here is your opportunity to remedy that deficiency.
- If you got good grades in it, do not therefore be complacent.
  - You may be in for a surprise that there are still matters in vector analysis that you have not yet mastered.
  - If you are hardworking and patient, you will not only advance your vector analysis, you will also find that your knowledge makes learning tensors quite enjoyable and logical. Do yourselves a favor: pay attention to the material on vector analysis.

Chapter One

Chapter Two

Tensor Algebra

- Tensors as Linear Transformations

Types and Properties of Tensors

The eigenvalue Problem & Spectral Transformations

Chapter Three

- Calculus of Large Objects
  - Integral Theorems
  - Differentiation of Tensor Fields
  - Derivatives of Tensor Functions
- Applications to Kinematics, and Natural Laws

Future Topics

- These prepare you to learn and understand the topics to be introduced at 400 Levels including:
  - Kinematics: The Geometry of Deformation & Motion
  - The Theory of Stress & Heat Flow
  - The Balance Laws of Mass & Momentum
  - Energy and the Thermodynamic Laws
- Our goal is that at 500 Level, We need to do only two things:
  - Constitutive Laws. Here where the mechanical descriptions of fluids, solids and other engineering materials are laid bare for design and analysis.
  - Simulation, Modeling & Design with 3D Solid Models & Multiphysics

Learning Resources

- **Your Lecturers**.
  - We have prepared for you. You will quickly see that this is not “just another course”. We will listen to you; please ask us questions because we will answer you.
- **The Book**.
  - There are ten chapters. You are only covering chapters 1-3 in 300 level. If you do it well, things get easier as you go along. If not, it will keep getting more difficult. Please start early and do not fall behind. This is a marathon!
- Q&A with Self-Testing Access.
  - There are usually at least sixty problems in each chapter. Some have more than one hundred. ALL problems are solved. They are to give you practice and further elucidate the theory.
  - Some of the questions are to be programmed. Our language and environment of choice are *Python* and $Mathematica^{©}$.
  - Do not fear, we will be gentle with you and you WILL see that programming is achievable and that you can thrive in it! We teach these in the Course on Data structures and Algorithms.

Learning Resources

- Your classmates
  - That is the first line because she is the most easily accessible help for you.
- Internet & Interactive materials.
  - The book, the slides, Q&A, video, audio, and other things that we shall deploy to teach this course are all available to you online. You can post questions and we will respond as appropriate. Please use this resource thoughtfully. You can reach your lecturers at their posted email addresses. You can also ask me questions lms.s2pafrica.com specifying the issue that is problematic: page number or question number. If a matter is a general problem, we will do an addendum to explain it better.
- Mathematica.
  - You can get a lot of help on Mathematica from the installation itself. You can also join the Mathematica Stack Exchange Group on the Internet to post questions and read answers to other questions.

Your Responsibility

The first responsibility you have is to receive and act on instructions for this course.

We do not like to say things more than once. They will be written, and you can go and be looking as many times as you like. Dates and times for assignments are given. Materials to cover before coming to class are specified. Please, make things easy for everybody, read and act on instructions.

Every Problem in this course is worked out in full for you

It is your duty to practice and test yourself ahead of time.

No exam will come with new questions. There is nothing stopping you from getting full marks here!

There is also nothing stopping you from failing woefully.

The Choice is yours to make!

SSG 321 Introduction to Continuum Mechanics

Instructors: OA Fakinlede & O Adewumi

Website: www.s2pAfrica.org, www.oafak.com

Vectors & Linear Independence

Remind you much of what you already know; Quick review and an opportunity to overcome difficulties up to this point.

Move quickly to a more advanced & actionable understanding: Be able to visualize, animate & simulate 3D geometry.

- Geometry: Important Math for Mechanical Prototyping, Design & Analysis.

Index Notation & the Summation Convention: Important thing to master at this stage of your preparation for Tensor Analysis.

- **Not knowing it will haunt you through the next three years.** Better come to terms with it now. You will have the help you need if you make the effort.

Vectors & Transformations: The Goals

Tensor Algebra

Tensors & Transformations

- What tensors are: Virtually every analysis you do in Engineering is about manipulating them.

Components & Invariants

- Tensors: Not usually handled directly;
- Components and invariants of tensors:
  - Von Mises stress, Moments of inertia, Dilatation

Decompositions, Eigensystems & Spectra Forms

- Specific Cases: Principal stresses, Natural frequencies, Mode shapes and many engineering concepts electrical, mechanical, chemical, etc.

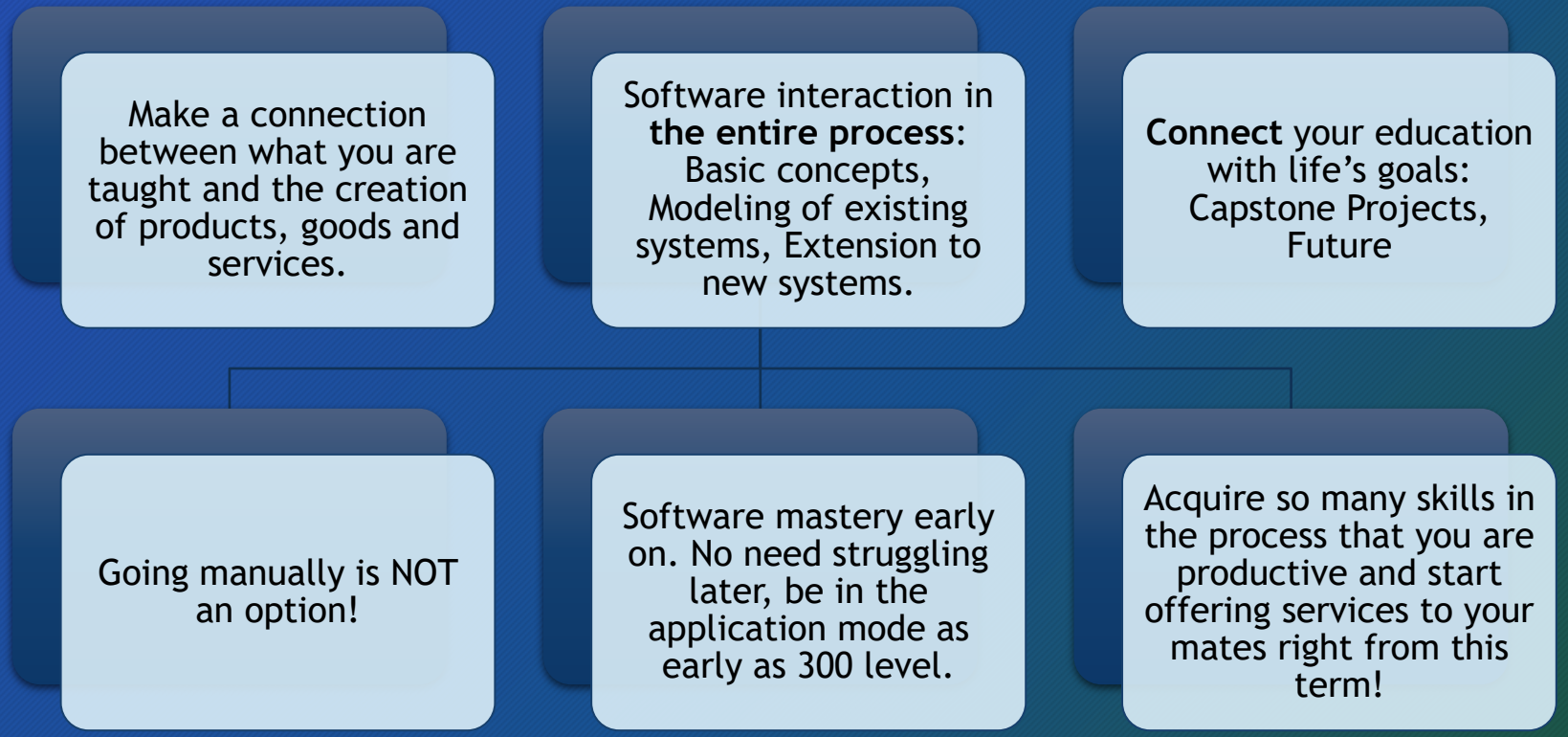
Tensor Calculus

Calculus of Large Objects

- Gateaux Differentials & Fréchet Derivatives
- Move from your present calculus of scalars to more realistic engineering objects. Your scalar calculus will be key. You will need to revise that to cope here!

The Euclidean Space and Fields.

- Demonstration of Field Theorems
- Many results (called theorems to frighten you) will not be proved in the usual way. Our method is to make you use software to demonstrate them in a way you can never forget!
- That requires your quick mastery of Mathematica. I am willing to give you a special tutorial if you want it and are ready for it.



The Goal: Eye on the Ball!

Main Course Text

Continuum Mechanics for Modeling, Simulation & Design

- Ten Chapters, 700 pages
- 300 Level covers the first three chapters
- 500 Solved problems
- Hundreds of objective questions in the teacher specific database.

Testing & Exams

- Computer based with personalized feedback
- Immersive learning: Improve by practice
- Testing and evaluation can be self-paced

Mathematica - Wolfram Research

Install it, Learn it. Documentation inside your installation

Answers to Homework, etc. will be by programming.

Main Course Text: Designed for you, Free for you

You will gain access by registering at www.S2PAfrica.org or directly from my personal webpage www.oafak.com

You will receive feedbacks on how you are doing by email. Your performance throughout the term will be measured.

Equipping for the Course

Lai, WM, Rubin, D & Krempl, E “Introduction to Continuum Mechanics” 4th Ed, Butterworth-Heinemann 2010

Gurtin, et. al., “Mechanics and Thermodynamics of Continua”, Cambridge University Press, 2010

Wolfram, S, “An Elementary Introduction to the Wolfram Language”, Wolfram Media, 2017 ($20 @Amazon.com)

Other Recommended Textbooks

Class Discipline

These lectures will be delivered on time. The materials are on the web. If you don't go on strike from facebook, there is no need to go on strike from learning.

Q&A at the end of the chapter are your practice arena. Objective questions for tests and exams are randomly picked from them.

Required reading before coming to class are REQUIRED before you come. You are thereby enabled to ask good questions that help everyone to learn.

You are better off staying away from class than disturbing with latecoming.

Teaching & Practice Schedule

Main Topics	Read Pages	Practice Q&A
Vectors, Linear Independence, Products	1-25	1-6, 14-21,24, 30
Index Notation, Summation Convention, Kronecker Delta, Alternating Symbol, Dyads	25-39	2,3,7,8,13,23,25,26,29-35,39,50-55
Vector Space, Point Space, Transformations, Position Vector, Coordinate curves & surfaces	39-57	56,57,70-74
Programming Practicum, Vector Geometry	57-71	56-73
May be omitted until we cover Chapter Two		9-12,17,27,28 31,41,43,54,89

Teaching Strategy

A deliberate principle is to allow you to practice the questions and answers in the sections on Solved Problems

The previous slides show the approximate groupings of these problems for your convenience.

We recommend that the questions marked red be omitted at first pass through the book unless you have extra background.

Tests after two topics are based on the text pages shown and the questions listed.

Note that every idea in the book is tested.

If you want to do well in the test and final exams, practice these questions

Practicing for Tests & Exams

The text for this course solves all problems it poses. Usually, each chapter will have as many as 100 solved problems. Don't let that overwhelm you. They are NOT to hurt you; they are to help those who want to learn.

These are for your practice. Real tests will be computer based.

Computer questions are paraphrased from the same questions that have been solved for you.	Want to get 100% ? Then work through ALL the solved problems; there will be NO surprises!	Want to get 30% ? Then work 30% of the problems, you will achieve your goal!

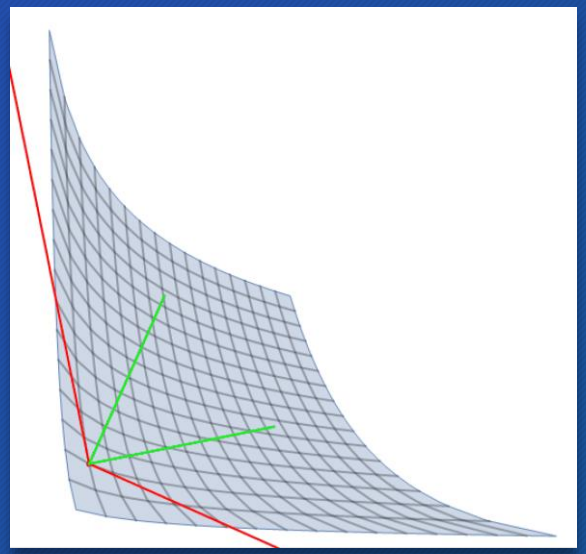
The Best Student

Is NOT the one that scores 100%. There is a silly attachment to getting a good class of degree with nearly empty heads when all you really have done is know how to cram and return crammed matter to the lecturer!

The best student is one that can do something with what is learned here and be successful to change his lot, change his environment and lead Nigeria out of its mess.

Several things you learn are not easily testable. They will be tested by the fires of real life. That is where you earn useful grades!

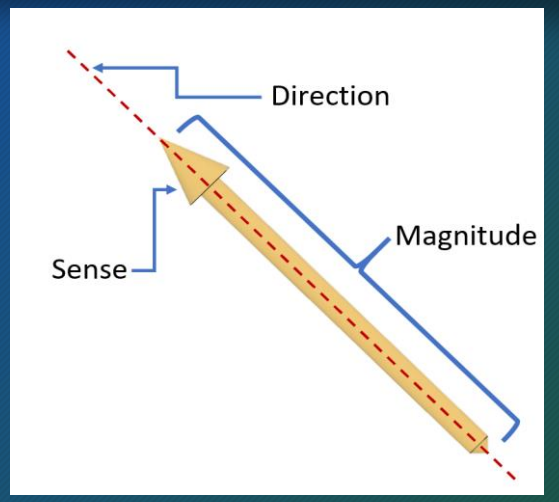
An Example

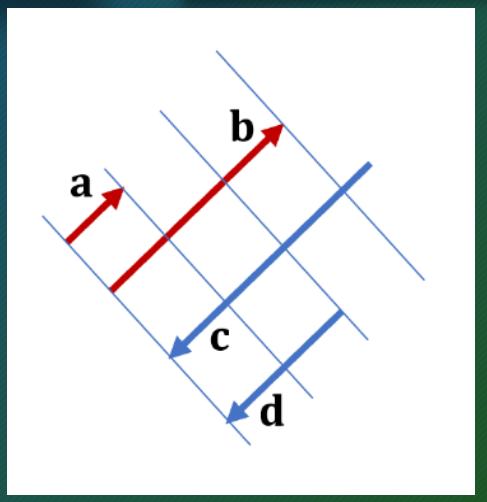


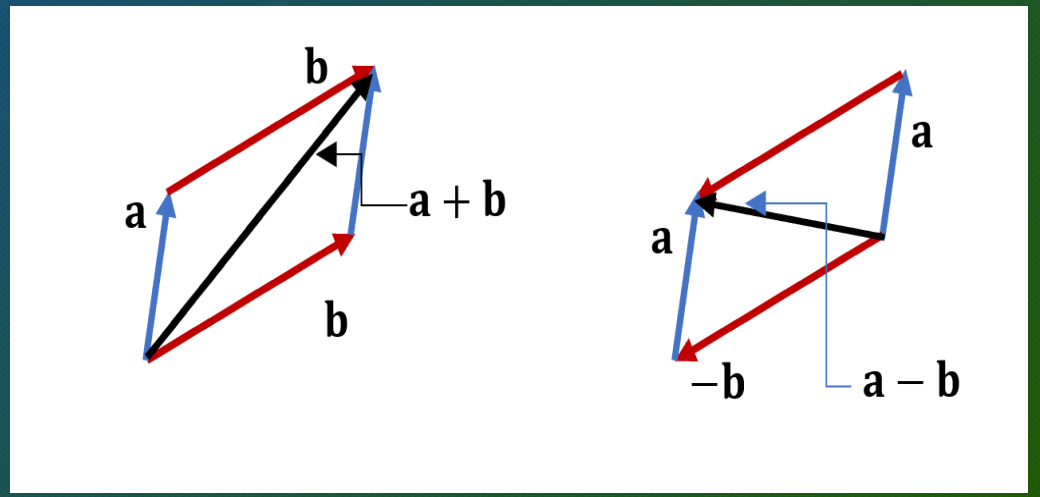
- The picture teaches the concept of covariance & contravariance
- Not only can the concept the explained, you will learn to write the computer program that created the explanation!
- Let the “cram and pass” people make first class. Hire them in your company when they are jobless and looking for employment!
- Then you are the best student in my class! If you also make a first class in that process, great!

What is a Vector?

- We retain the elementary understanding we took from high school:
- A directed line: Magnitude, Direction, Sense
- An abstract description of many quantities of interest and importance.
- Properties:
  - Can be added (parallelogram law), Subtraction is the addition of the opposite.
  - Can be scaled (Multiplication by a scalar) This is the first idea of multiplication: One scalar acts on one vector, produces a vector

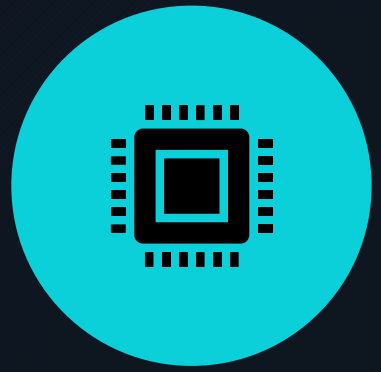






The Scalar Product

THIS IS A PRODUCT BETWEEN TWO VECTORS RESULTING IN A SCALAR.



WHY IS IT ALSO CALLED A DOT PRODUCT?

THE NEED FOR DISAMBIGUATION

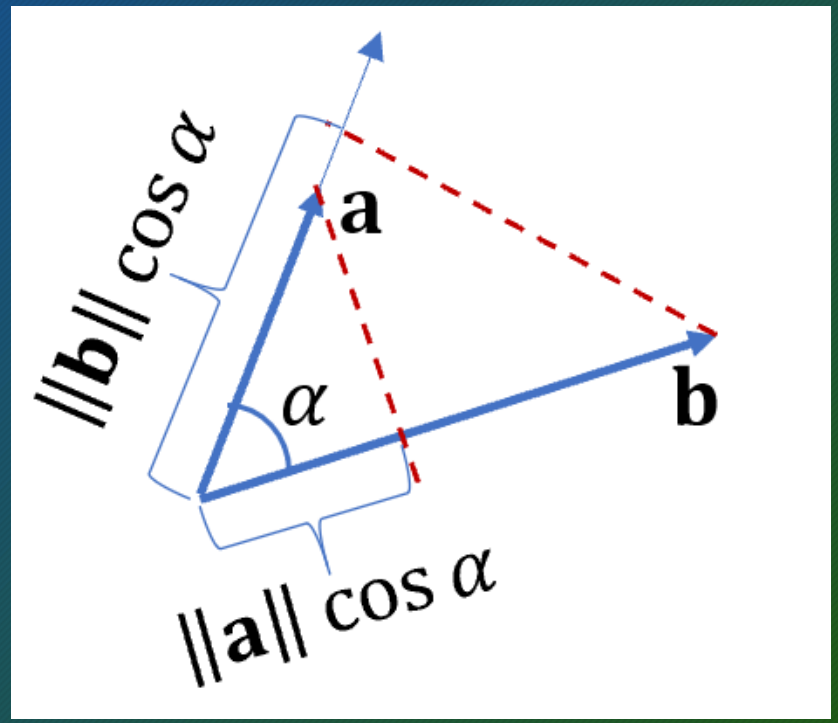
The Scalar Product: Projections

- We examine the product of the projection of

$$\|\mathbf{a}\| \cos \alpha \times \|\mathbf{b}\|.$$

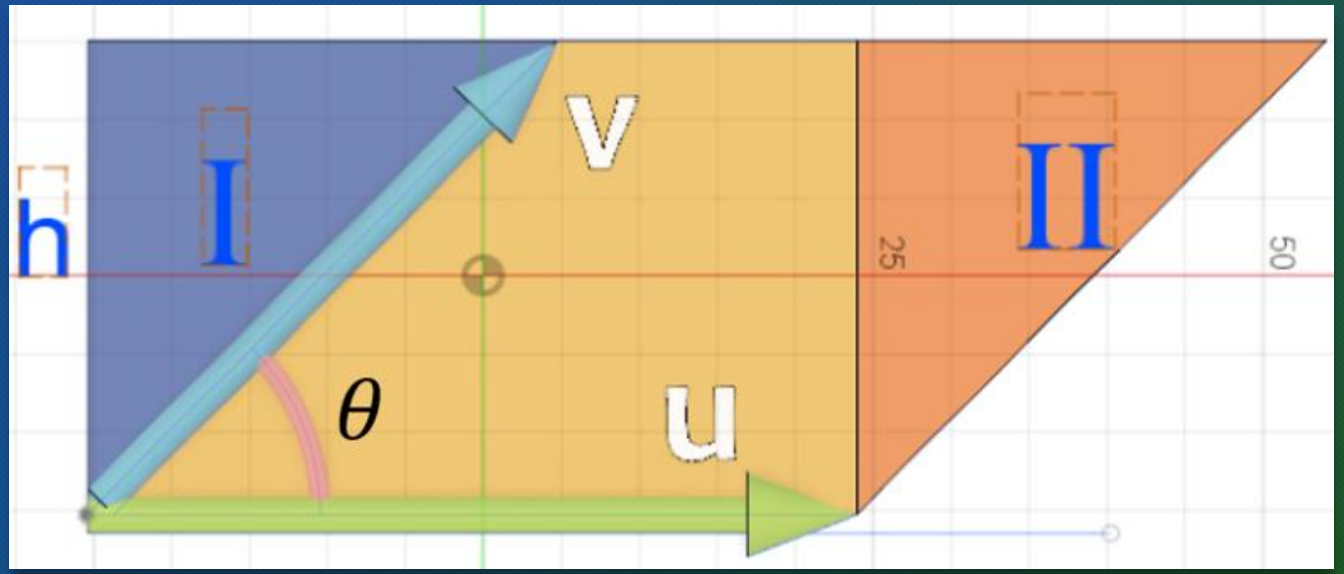
- Comparing this with the product of the projection of $\mathbf{b}$ on $\mathbf{a}$ and the magnitude of vector; we find they are equal:

$$\mathbf{a} \cdot \mathbf{b} \equiv \|\mathbf{a}\| \cos \alpha \times \|\mathbf{b}\| = \|\mathbf{b}\| \cos \alpha \times \|\mathbf{a}\|$$



The Vector Product

- Is this connected to the area of a parallelogram?
- The proof of equality is in the picture since the two triangles marked I and II are congruent
  $\mathbf{u} \times \mathbf{v} \equiv \mathbf{A} = A_p \mathbf{e} = \|\mathbf{u}\|\|\mathbf{v}\| \sin\theta\, \mathbf{e}$
- Our first hint at the fact that area is a vector directed along its normal.



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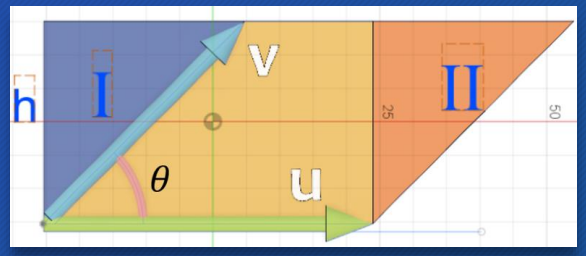

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- Our first hint at the fact that area is a vector directed along its normal.

The Area of a Rectangle

- Why is the area of rectangle length times breadth?
- Can you prove it?

Disambiguation

- It makes sense to say, “find the product of two numbers”;
- The above statement means one thing and one thing only.
- Once vectors are involved, it can mean at least four different things!
- We are looking at three of these things here. REASON why each product involving vectors has an adjective: Disambiguation!
- Each meaning is a product and behaves like a product. How does a product behave?

Product	Operation	Result	Other Names
Scalar Multiplication	$\mathbf{b} = \alpha\mathbf{a}$	Vector $\mathbf{b}$ in the same direction as $\mathbf{a}$. Scaled to the value of α. Sense depends on the sign of α	Scaling
Scalar Product	$\mathbf{a} \cdot \mathbf{b} \equiv \|\mathbf{a}\|\|\mathbf{b}\| \cos\alpha$	Result is a scalar value. Here α is the angle between the two vectors.	Dot Product, Inner Product
Vector Product	$\mathbf{u} \times \mathbf{v} \equiv \mathbf{A} = A_p\mathbf{e}$ $= \|\mathbf{u}\|\|\mathbf{v}\| \sin\alpha\, \mathbf{e}$	Result is a vector value. Its magnitude is the scalar area of the parallelogram formed by the vectors. Here α = angle between the two vectors. $\mathbf{e}$ = unit vector perpendicular to both	Cross Product

Disambiguation

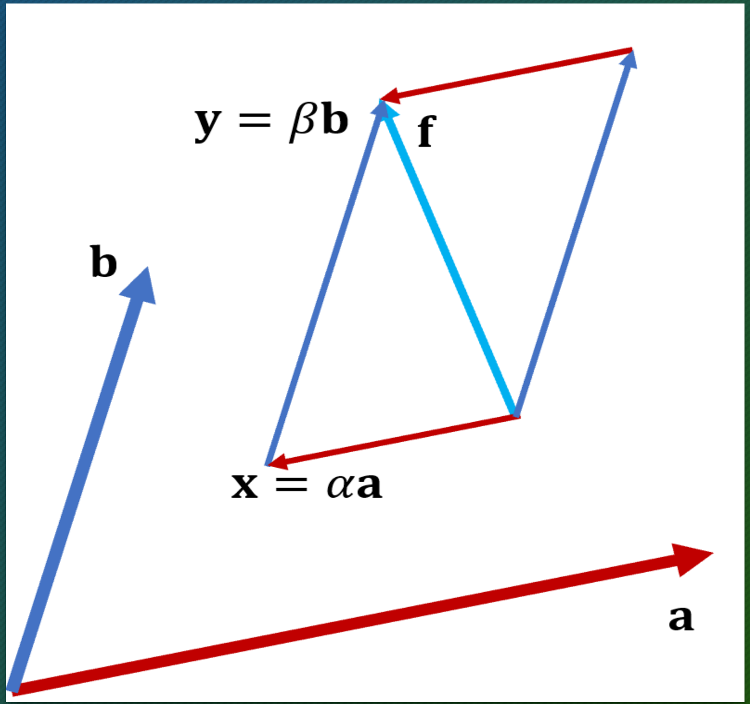
- At the tip of the vector $\mathbf{f}$, we draw a line parallel to $\mathbf{b}$. At the tail of the same vector, we draw another line parallel to $\mathbf{a}$.

$$\mathbf{x} = \alpha\mathbf{a};\ and\ \mathbf{y} = \beta\mathbf{b}$$

- where α and β are the scaling factors (real numbers that can be positive or negative). Clearly,

$$\begin{aligned}\mathbf{f} &= \mathbf{x} + \mathbf{y} \\ &= \alpha\mathbf{a} + \beta\mathbf{b}\end{aligned}$$

- Any other vector on the plane can be treated this way $\Rightarrow \mathbf{a}\ and\ \mathbf{b}$ span the plane.



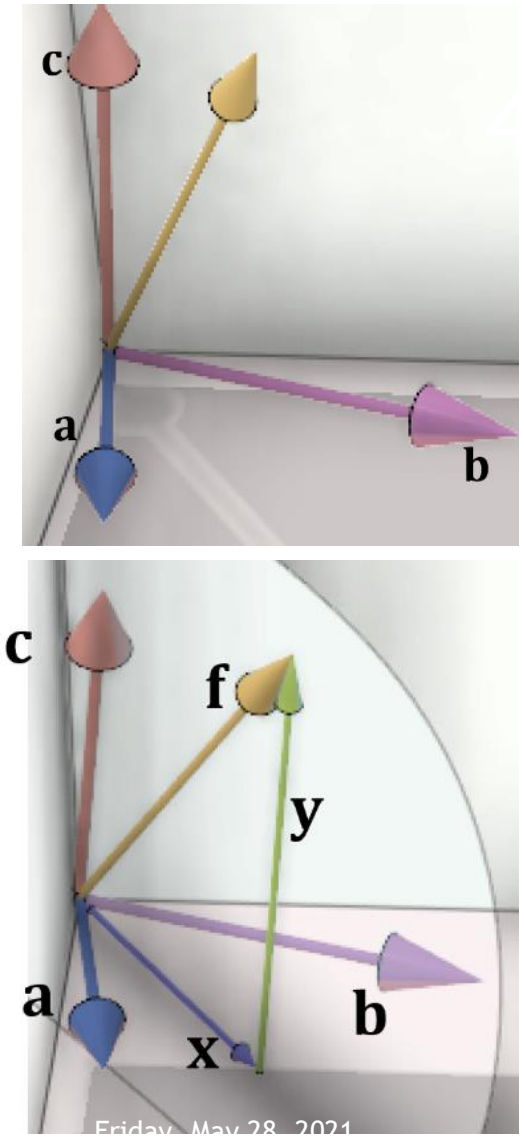
Linear Independence, 3D

- We hereby introduce the set of vectors $\{\mathbf{a}, \mathbf{b}, \mathbf{c}\}$ that are not collinear and not coplanar as shown.
- From the tip of $\mathbf{f}$ draw a line parallel to $\mathbf{c}$ to the plane of $\{\mathbf{a}, \mathbf{b}\}$, meeting it at the tip of vector $\mathbf{x}$. We can therefore write

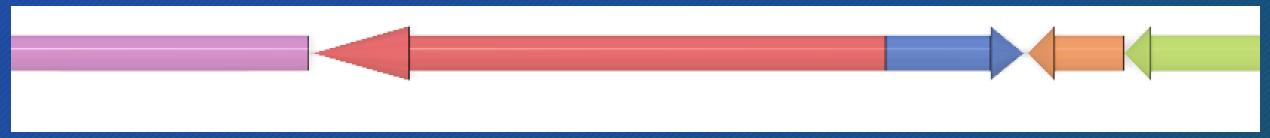
$$\mathbf{x} = \alpha\mathbf{a} + \beta\mathbf{b}$$

- And since $\mathbf{y}$ is parallel to $\mathbf{c}$, we can find γ:

$$\mathbf{f} = \mathbf{x} + \mathbf{y} = \alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c}$$



Dimensionality of Space: 1D



- For a set of vectors that are all on one line as shown, select one of them, say vector $\mathbf{a}$. Any other vector $\mathbf{b}$ among the rest satisfies the equation,

$$\alpha\mathbf{a} + \beta\mathbf{b} = \mathbf{o}$$

- can always be solved for $\mathbf{b}$ provided $\beta \neq 0$, this equation can be simplified to $\mathbf{b} = -\frac{\alpha}{\beta}\mathbf{a}$. This means every other member of that set is simply a scaling of vector $\mathbf{a}$. The space is one dimensional.

Dimensionality of Space: 2D

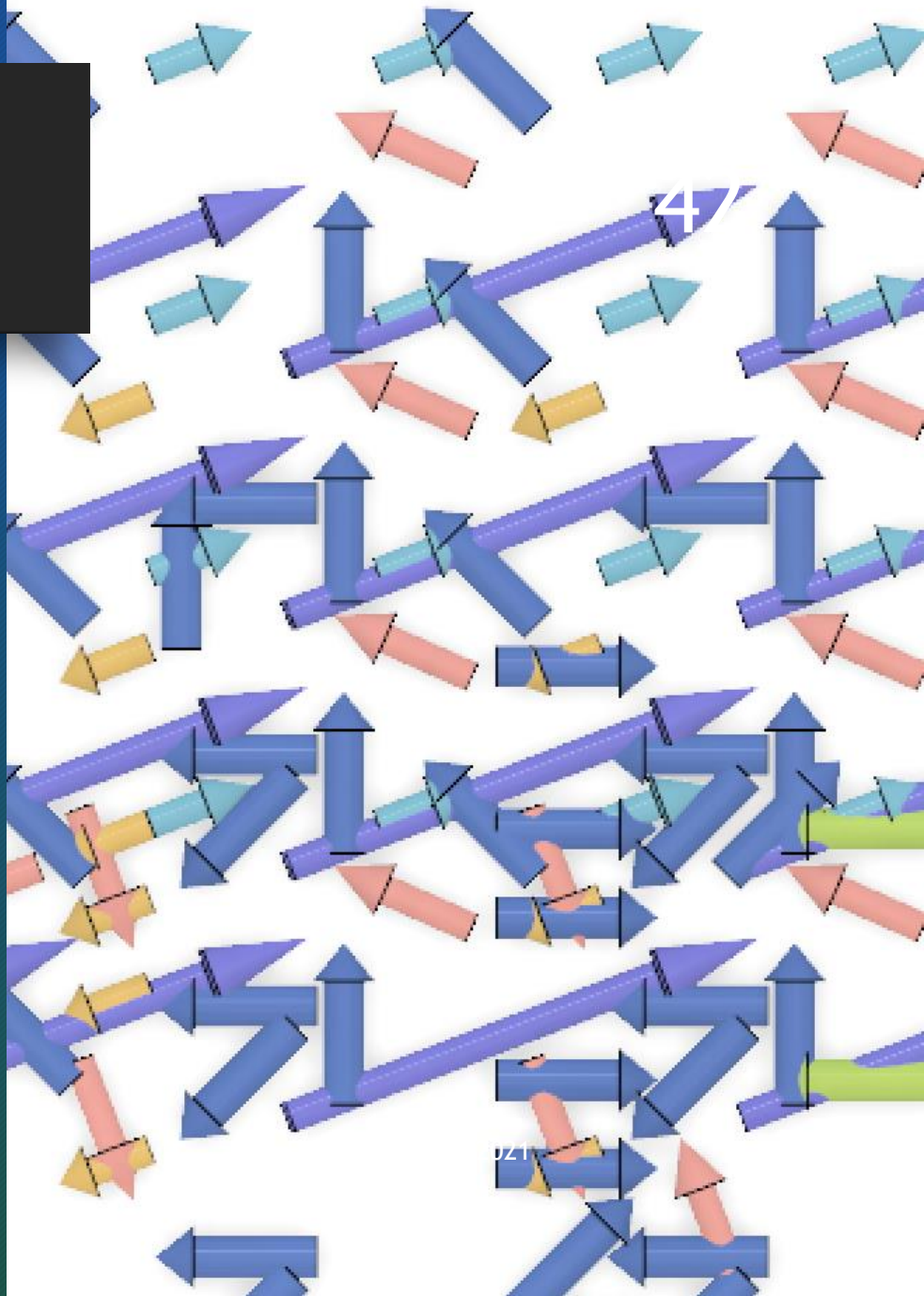
- We also showed further that once these are chosen, any other vector $\mathbf{x}$ can be expressed as a sum of scaled versions

$$\mathbf{x} = \alpha\mathbf{a} + \beta\mathbf{b}$$

- given any other vector c in the space, the equation,

$$\alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c} = 0$$

- can be solved for $\mathbf{c}$ provided γ is not zero. This means that the maximum number of linearly independent vectors in this space is two. This makes the plane a two-dimensional vector space.



Dimensionality of Space: 3D

The arguments above can be carried to three dimensions. A geometric interpretation can be given. With a more accurate mathematical definition of vectors, we can even go to higher dimensions. Once we are past three dimensions, however, a geometric interpretation will no longer be possible, but the concept can remain useful for analytical purposes.

The maximum number of linearly independent vectors in a three-dimensional space is three. These will, in addition to not being collinear, they MUST NOT all be coplanar. That means that once you have four or more vectors, one will be expressible in terms of the other three.

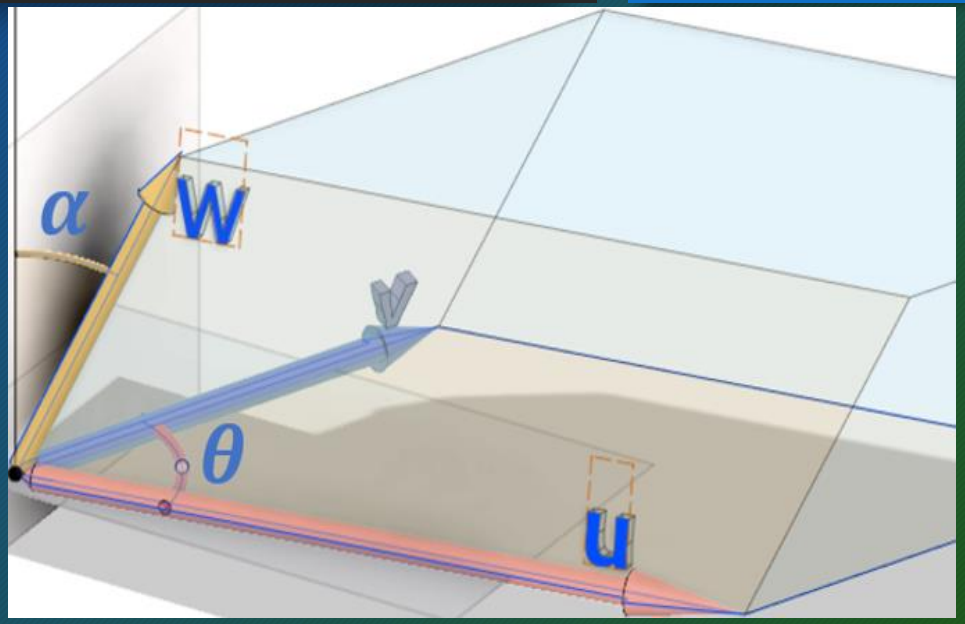
The Papallelepiped

- Consider a parallelepiped with the edge vectors $\mathbf{u}, \mathbf{v}$ and $\mathbf{w}$ with $\mathbf{u}$ subtending an angle θ on the horizontal plane while $\mathbf{w}$ is inclined at angle α to the vertical axis. The base area

$$A = \|\mathbf{u}\|\|\mathbf{v}\| \sin\theta = \|\mathbf{u} \times \mathbf{v}\|$$

- Vertical height, h, of the object is $\|\mathbf{w}\| \cos\alpha$. Volume therefore is

$$V = Ah = \|\mathbf{u} \times \mathbf{v}\|\|\mathbf{w}\| \cos\alpha$$
$$= |\mathbf{u} \times \mathbf{v} \cdot \mathbf{w}|.$$



Parallelepiped & Linear Independence

- What happens to the volume of a parallelepiped when the edge vectors are linearly dependent? $|\mathbf{u} \times \mathbf{v} \cdot \mathbf{w}|$?
- The answer to this question is deeper than it looks. Understanding it can help you go a long way in vector and tensor analysis.

Orthonormal Basis (ONB) Vectors

- Instead of writing $\{\mathbf{i}, \mathbf{j}, \mathbf{k}\}$ we write, $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$. Clearly,
$$\mathbf{e}_1 \times \mathbf{e}_2 \cdot \mathbf{e}_3 = 1$$
- The base vectors of the coordinate system is now an indexed object. We can depict them as $\mathbf{e}_1, \mathbf{e}_2$ and $\mathbf{e}_3$. We could also have written, $\mathbf{e}_i, i = 1,2,3$ or $\mathbf{e}_i, i = 1, \dots, 3$. A typical vector $\mathbf{f}$ can be written in terms of the basis vectors as,
$$\mathbf{f} = \alpha\mathbf{a} + \beta\mathbf{b} + \gamma\mathbf{c} = a_1\mathbf{e}_1 + a_2\mathbf{e}_2 + a_3\mathbf{e}_3$$
- The scalars a_1, a_2, a_3 in this case are easily found by taking the dot product of the equation with $\mathbf{e}_1$,
$$\mathbf{f} \cdot \mathbf{e}_1 = a_1\mathbf{e}_1 \cdot \mathbf{e}_1 + a_2\mathbf{e}_2 \cdot \mathbf{e}_1 + a_3\mathbf{e}_3 \cdot \mathbf{e}_1 = a_1.$$
- And we can similarly take products with $\mathbf{e}_2$ and $\mathbf{e}_3$ respectively and obtain that, $a_2 = \mathbf{f} \cdot \mathbf{e}_2$, and $a_3 = \mathbf{f} \cdot \mathbf{e}_3$.

Programming & Mathematica

- Two main software products are useful in this course. They are:
  1. Mathematica & The Wolfram Language. Recommended Textbook 3 is to help you in this aspect. It is for those who like printed books. For others, there is sufficient documentation of all features in the installation. There will often be full code of solutions for you to play with so that you can write your own.
  2. Fusion 360 by Autodesk. Systems Engineering students have taken a course in this at 200 level. It is good to continue to practice. The consummation of this course is to lead to simulation and prototyping. We will, at the end be doing a lot of Fusion 360 in later years. This is the time to master the software so you are not wasting time later.

Programming & Mathematica