

# Preface

Relay feedback has attracted considerable research attention for more than a century. The classical work of Typikin (1984) on analysis summarizes progress up to the 1960s. Early applications of relay systems ranged from stationary control of industrial processes to control of mobile objects. In the 1950s, relays were mainly used as amplifiers but such applications are now obsolete, owing to the development of electronic technology. In the 1960s, relay feedback was applied to adaptive control. One prominent example of such an application is the self-oscillating adaptive controller developed by Minneapolis Honeywell, which uses relay feedback to attain a desired amplitude margin. It was in the 1980s that Åström and Hägglund successfully applied the relay feedback method to auto-tune PID controllers for process control, and triggered a resurgence of interest in relay methods, including extensions of the method to more complex systems. Since then, new tools and powerful results have emerged.

The present monograph presents, in a single volume, a fairly comprehensive, up-to-date and detailed treatment of relay feedback theory, use of relay feedback for process identification, and use of identified models for control system design. The materials included here are based on research results of the authors and their co-workers in the domain. Both single-variable and multivariable systems are addressed. For presentation, we have made the technical development of the results as self-contained as possible. Only knowledge of linear system theory is assumed for readers. Illustrative examples of different degrees of complexity are given to facilitate understanding. Therefore, it is believed that the book can be accessed by graduate students, researchers and practising engineers.

The table of contents gives an idea of what is contained in the book. The book is organized into three parts: Part I Analysis of Relay Feedback Systems, Part II Process Identification Using Relay Feedback, and Part III Controller Design. The three parts are related but can be read independently. *Those who are only interested in relay applications can skip Part I.* A chapter-by-chapter preview of our materials is given as follows.

Part I deals with analysis of relay feedback systems and consists of four chapters. Chapter 1 considers a SISO linear system with general relay feedback and studies the existence of solutions to the system. It is shown that existence and uniqueness of solutions are always guaranteed for the system with nonzero time delay; for the delay-free case, a necessary and sufficient condition is given for the existence of solutions. Chapter 2 deals with problems of the existence of limit cycles of linear systems with relay feedback, and presents some sufficient conditions. Moreover, if a limit cycle exists, necessary and sufficient conditions are given for determining particular limit cycles. The main tool used is Brouwer's fixed point theorem and some techniques related to system theory. Chapter 3 studies the local stability problem of limit cycles for time-delay relay feedback systems with relays containing asymmetric hysteresis. It is shown that if a certain constructed matrix is Schur stable, then the local stability of the limit cycle considered is guaranteed. Chapter 4 investigates the global stability of limit cycles in relay feedback systems. A unified framework and results are presented for global convergence. The key idea is to reduce the global stability problem to the asymptotic stability of a discrete time system.

Part II studies process identification from relay feedback and consists of four chapters. Chapter 5 deals with identifying process models from steady-state responses, or limit cycles, of relay feedback systems. We show that use of the FFT in place of the describing function approximation can give more accurate frequency response estimation. More cycles of oscillations can be employed to enhance estimation robustness. Modified relays can enable estimation of more points on the process frequency response. Chapter 6 deals with identifying process frequency response by using transient responses of relay feedback systems. Chapter 7 considers the problem of converting the frequency response identified from relay feedback to a transfer function with possible dead time. Chapter 8 develops a general identification procedure applicable to various test scenarios, covering step/relay and open-loop/closed-loop types.

Part III is concerned with control system design and consists of four chapters. Chapter 9 considers control design for SISO stable processes. Internal model control design is reviewed. Its equivalent single-loop controller is derived and usually of high complexity. The model reduction technique is employed to find its approximation. Users have the option to choose between PID and high-order controllers to better suit the applications. It turns out that high-order controllers may be necessary to achieve high performance for essentially high-order processes. Chapter 10 extends the SISO methodology in Chapter 9 to the multivariable case. For internal model control, the objective closed-loop trans-

fer functions are characterized in terms of process unavoidable non-minimum-phase elements. A multivariable controller for best achievable performance is obtained using block diagram analysis and model reduction. Chapter 11 considers control of unstable processes. The relay feedback test is used to identify an unstable process. An IMC-based single-loop controller design method is given to find the feedback controller in either PID or high-order form. Chapter 12 presents a new scheme called the Partial Internal Model Control (PIMC), which is capable of controlling both stable and unstable processes. In the PIMC, a process model is expressed as the sum of stable and anti-stable parts and only the stable part of the process model is used as the internal model. The process stable part is cancelled by the internal model and the remaining anti-stable part is stabilized and controlled using a primary controller. Chapter 13 addresses decentralized control of multivariable processes. A simple independent design method for multi-loop controllers is proposed which exploits process interactions for the improvement of loop performance.

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*Qing-Guo Wang*

*Tong Heng Lee*

*Chong Lin*

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