

---

## Contents

<b>1</b>	<b>Introduction .....</b>	<b>1</b>
<b>2</b>	<b>Introduction to Contact Mechanics .....</b>	<b>11</b>
2.1	Contact in a Mass Spring System .....	11
2.1.1	General formulation .....	11
2.1.2	Lagrange multiplier method .....	15
2.1.3	Penalty method .....	17
2.2	Finite Element Analysis of the Contact of Two Bars .....	18
2.3	Thermo-mechanical Contact .....	21
2.4	Impact .....	25
<b>3</b>	<b>Continuum Solid Mechanics and Weak Forms.....</b>	<b>31</b>
3.1	Kinematics .....	31
3.1.1	Motion and deformation gradient .....	31
3.1.2	Strain measures .....	34
3.1.3	Transformation of vectors and tensors .....	36
3.1.4	Time derivatives .....	36
3.2	Balance Laws .....	38
3.2.1	Balance of mass .....	38
3.2.2	Local balance of momentum and moments of momentum	38
3.2.3	First law of thermodynamics .....	39
3.2.4	Transformation to the initial configuration, different stress tensors .....	39
3.3	Weak Form of Balance of Momentum, Variational Principles ..	41
3.3.1	Weak form of balance of momentum in the initial configuration .....	41
3.3.2	Spatial form of the weak formulation .....	42
3.3.3	Minimum of total potential energy .....	43
3.4	Constitutive Equations .....	44
3.4.1	Hyperelastic response function .....	44
3.4.2	Incremental constitutive tensor .....	46

## VIII    Contents

3.5	Linearizations . . . . .	49
3.5.1	Linearization of kinematical quantities . . . . .	51
3.5.2	Linearization of constitutive equations . . . . .	52
3.5.3	Linearization of the weak form . . . . .	53
3.5.4	Linearization of a deformation dependent load . . . . .	55
4	Contact Kinematics . . . . .	57
4.1	Normal Contact of Three-dimensional Bodies . . . . .	58
4.2	Tangential Contact of Three-dimensional Bodies . . . . .	62
4.2.1	Stick condition . . . . .	62
4.2.2	Slip condition . . . . .	63
4.3	Variation of the Normal and Tangential Gap . . . . .	66
4.3.1	Variation of normal gap . . . . .	66
4.3.2	Variation of tangential gap . . . . .	67
5	Constitutive Equations for Contact Interfaces . . . . .	69
5.1	Normal Contact . . . . .	69
5.1.1	Constraint formulation . . . . .	70
5.1.2	Constitutive equations for normal contact . . . . .	72
5.2	Tangential Contact . . . . .	76
5.2.1	Stick as a constraint . . . . .	77
5.2.2	Coulomb law . . . . .	77
5.2.3	Regularization of the Coulomb law . . . . .	79
5.2.4	Elasto-plastic analogy for friction . . . . .	80
5.2.5	Friction laws for metal forming . . . . .	86
5.2.6	Friction laws for rubber and polymers . . . . .	88
5.2.7	Friction laws for concrete structures on soil . . . . .	90
5.2.8	Friction laws from computational homogenization procedures . . . . .	93
5.3	Lubrication . . . . .	98
5.4	Adhesion . . . . .	100
5.5	Decohesion . . . . .	102
5.6	Wear . . . . .	103
5.7	Fractal Contact Interfaces . . . . .	106
6	Contact Boundary Value Problem and Weak Form . . . . .	109
6.1	Frictionless Contact in Linear Elasticity . . . . .	109
6.2	Frictionless Contact in Finite Deformations Problems . . . . .	113
6.3	Treatment of Contact Constraints . . . . .	115
6.3.1	Lagrange multiplier method . . . . .	117
6.3.2	Penalty method . . . . .	118
6.3.3	Direct constraint elimination . . . . .	120
6.3.4	Constitutive equation in the interface . . . . .	121
6.3.5	Nitsche method . . . . .	122
6.3.6	Perturbed Lagrange formulation . . . . .	124

6.3.7	Barrier method . . . . .	124
6.3.8	Augmented Lagrange methods . . . . .	126
6.3.9	Cross-constraint method . . . . .	128
6.4	Comparison of Different Methods . . . . .	131
6.4.1	Normal Contact . . . . .	131
6.4.2	Frictional Contact . . . . .	136
6.5	Linearization of the Contact Contributions . . . . .	140
6.5.1	Normal contact . . . . .	141
6.5.2	Tangential contact . . . . .	144
6.5.3	Special case of stick . . . . .	147
6.6	Rolling Contact . . . . .	147
6.6.1	Special reference frames for rolling contact . . . . .	148
6.6.2	Strain measures . . . . .	150
6.6.3	Weak Form . . . . .	151
6.6.4	Constitutive equation . . . . .	152
6.6.5	Contact kinematics . . . . .	153
<b>7</b>	<b>Discretization of the Continuum . . . . .</b>	<b>157</b>
7.1	Isoparametric Concept . . . . .	158
7.1.1	Isoparametric interpolation functions . . . . .	161
7.1.2	One-dimensional shape functions . . . . .	161
7.1.3	Two-dimensional shape functions . . . . .	163
7.1.4	Three-dimensional shape functions . . . . .	165
7.2	Discretization of the Weak Forms . . . . .	167
7.2.1	FE formulation of the weak form with regard to the initial configuration . . . . .	168
7.2.2	Linearization of the weak form in the initial configuration . . . . .	172
7.2.3	FE formulation of the weak form in the current configuration . . . . .	178
7.2.4	Linearization of the weak form in the current configuration . . . . .	180
<b>8</b>	<b>Discretization, Small Deformation Contact . . . . .</b>	<b>183</b>
8.1	General Approach for Contact Discretization . . . . .	184
8.1.1	Lagrange multiplier method . . . . .	184
8.1.2	Penalty method . . . . .	187
8.2	Node-to-Node Contact Element . . . . .	188
8.2.1	Frictionless contact . . . . .	188
8.2.2	Contact with friction . . . . .	192
8.3	Isoparametric Discretization of the Contact Contribution . . . . .	194
8.3.1	Examples for isoparametric contact elements . . . . .	199
8.4	Discretization for Non-matching Meshes . . . . .	205
8.4.1	Discretization with contact segments . . . . .	206
8.4.2	Mortar method . . . . .	211
8.4.3	Nitsche method . . . . .	217

<b>9</b>	<b>Discretization, Large Deformation Contact</b>	225
9.1	Two-dimensional Node-to-Segment Contact Discretization	226
9.2	Alternative Discretization for the Two-dimensional NTS-Contact	235
9.3	Three-dimensional Contact Discretization	241
9.3.1	Node-to-surface contact element	242
9.4	Three-Node Master Segment for Frictionless Contact	246
9.4.1	Matrices for Node-To-Edge (NTE) elements	249
9.4.2	Matrices for Node-To-Node (NTV) elements	251
9.5	Mortar Discretization for Large Deformations	253
9.5.1	Introduction	253
9.5.2	Mortar discretization for finite deformations	254
9.5.3	Linear approximation for the frictionless case	256
9.5.4	Quadratic approximation for the frictionless case	262
9.5.5	Numerical examples for frictionless contact	265
9.5.6	Quadratic Approximation for the Frictional Case	270
9.5.7	Numerical examples for frictional contact	273
9.6	Smooth Contact Discretization	279
9.6.1	Hermite interpolation for frictionless contact	280
9.6.2	Bezier interpolation for frictionless contact	286
9.6.3	Bezier interpolation for frictional contact	291
9.6.4	Three-dimensional contact discretization	299
9.7	Numerical Examples	303
9.7.1	The sheet/plate rolling simulation	303
9.7.2	Simulation of a sliding and rolling wheel	305
<b>10</b>	<b>Solution Algorithms</b>	309
10.1	Contact Search	311
10.1.1	Spatial search, phase (I)	314
10.1.2	Contact detection, phase (II)	316
10.2	Solution Methods for Unconstrained Nonlinear Problems	321
10.2.1	Algorithms for time-independent problems	321
10.2.2	Algorithms for time-dependent problems	323
10.3	Global Solution Algorithms for Contact	328
10.3.1	Basic notation	329
10.3.2	Dual formulation	333
10.3.3	Penalty method	335
10.3.4	Lagrange multiplier method	336
10.3.5	Augmented Lagrange method, Uzawa algorithm	338
10.3.6	Partitioning method	340
10.3.7	SQP method	344
10.3.8	Active set method for quadratic program	346
10.3.9	Linear complementary problem	347
10.3.10	Contact algorithm of Dirichlet–Neumann type	348
10.3.11	Contact algorithm based on projected gradients	349

10.3.12 Algorithm for dynamic contact . . . . .	352
10.4 Global Algorithms for Friction . . . . .	355
10.5 Local Integration of Constitutive Equations in the Contact Area . . . . .	359
10.5.1 Evolution of adhesion . . . . .	360
10.5.2 Friction laws . . . . .	360
<b>11 Thermo-mechanical Contact . . . . .</b>	<b>365</b>
11.1 Equations for the Continuum . . . . .	366
11.1.1 Kinematical relations, multiplicative split . . . . .	366
11.1.2 Thermoelastic constitutive law . . . . .	367
11.2 Constitutive Equations for Thermo-mechanical Contact . . . . .	368
11.2.1 Heat conductance through spots . . . . .	370
11.2.2 Heat conductance through gas . . . . .	371
11.2.3 Heat conductance by radiation . . . . .	373
11.3 Initial Value Problem for Thermo-mechanical Contact . . . . .	374
11.4 Weak Forms in Thermo-mechanical Analysis . . . . .	376
11.5 Algorithmic Treatment . . . . .	377
11.6 Discretization Techniques . . . . .	378
11.6.1 Node-to-node contact element . . . . .	379
11.6.2 Node-to-segment contact element . . . . .	382
11.7 Examples . . . . .	384
11.7.1 Heat transfer at finite deformations . . . . .	385
11.7.2 Frictional heating at finite deformations . . . . .	386
<b>12 Beam Contact . . . . .</b>	<b>391</b>
12.1 Kinematics . . . . .	391
12.1.1 Normal contact . . . . .	392
12.1.2 Tangential contact . . . . .	395
12.2 Variation of the Gap in Normal and Tangential Directions . . . . .	396
12.3 Contact Contribution to Weak Form . . . . .	399
12.4 Finite Element Formulation . . . . .	400
12.5 Contact Search for Beams . . . . .	402
12.6 Examples . . . . .	404
12.6.1 Three beams in frictionless contact . . . . .	404
12.6.2 Two beams in contact with friction . . . . .	406
<b>13 Computation of Critical Points with Contact Constraints . . . . .</b>	<b>411</b>
13.1 Inequality Constraints for Contact . . . . .	412
13.2 Calculation of Stability Points . . . . .	413
13.3 Extended System with Contact Constraints . . . . .	414
13.4 Examples . . . . .	415
13.4.1 Block pressing on arch . . . . .	416
13.4.2 Two arches . . . . .	419

<b>14 Adaptive Finite Element Methods for Contact Problems</b>	423
14.1 Contact problem and discretization	425
14.2 Residual Based Error Estimator for Frictionless Contact	426
14.3 Error Indicator for Contact Based on Projection	430
14.4 Error Estimators based on Dual Principles	434
14.4.1 Displacement error control	434
14.4.2 Stress error control	436
14.5 Adaptive Mesh Refinement Strategy	437
14.6 Numerical Examples	441
14.6.1 Hertzian contact problem	441
14.6.2 Crossing tubes	443
14.6.3 Fractal interface	444
14.7 Error Indicator for Frictional Problems	448
14.7.1 Adaptive strategies	450
14.7.2 Transfer of history variables	452
14.7.3 Numerical examples	455
14.8 Adaptive methods for thermo-mechanical contact	461
14.8.1 Staggered solution scheme with independent spatial grids	461
14.8.2 Error measures for the coupled problem	464
14.8.3 Adaptive method for the coupled problem	466
14.8.4 Example	467
<b>A Gauss integration rules</b>	473
A.1 One-dimensional Integration	473
A.2 Two-dimensional Integration	474
<b>B Convective Coordinates</b>	477
<b>C Parameter Identification for Friction Materials</b>	483
<b>References</b>	487
<b>Index</b>	513



<http://www.springer.com/978-3-540-32608-3>

Computational Contact Mechanics

Wriggers, P.

2006, XII, 518 p., Hardcover

ISBN: 978-3-540-32608-3