

---

## Contents

<b>Introduction</b> .....	1
<b>1 Preliminaries</b> .....	5
1.1 Terminology .....	5
1.2 Convex Sets in Normed Vector Spaces .....	6
1.3 Convex Functionals: Definitions and Examples .....	8
1.4 Continuity of Convex Functionals .....	11
1.5 Sandwich and Separation Theorems .....	13
1.6 Dual Pairs of Vector Spaces .....	19
1.7 Lower Semicontinuous Functionals .....	22
1.8 Bibliographical Notes and Exercises .....	24
<b>2 The Conjugate of Convex Functionals</b> .....	27
2.1 The Gamma Regularization .....	27
2.2 Conjugate Functionals .....	29
2.3 A Theorem of Hörmander and the Bipolar Theorem .....	34
2.4 The Generalized Farkas Lemma .....	36
2.5 Bibliographical Notes and Exercises .....	38
<b>3 Classical Derivatives</b> .....	39
3.1 Directional Derivatives .....	39
3.2 First-Order Derivatives .....	41
3.3 Mean Value Theorems .....	44
3.4 Relationship between Differentiability Properties .....	46
3.5 Higher-Order Derivatives .....	48
3.6 Some Examples .....	49
3.7 Implicit Function Theorems and Related Results .....	51
3.8 Bibliographical Notes and Exercises .....	57

<b>4</b>	<b>The Subdifferential of Convex Functionals</b> . . . . .	59
4.1	Definition and First Properties . . . . .	59
4.2	Multifunctions: First Properties . . . . .	63
4.3	Subdifferentials, Fréchet Derivatives, and Asplund Spaces . . . .	64
4.4	Subdifferentials and Conjugate Functionals . . . . .	73
4.5	Further Calculus Rules . . . . .	76
4.6	The Subdifferential of the Norm . . . . .	78
4.7	Differentiable Norms . . . . .	83
4.8	Bibliographical Notes and Exercises . . . . .	89
<b>5</b>	<b>Optimality Conditions for Convex Problems</b> . . . . .	91
5.1	Basic Optimality Conditions . . . . .	91
5.2	Optimality Under Functional Constraints . . . . .	92
5.3	Application to Approximation Theory . . . . .	96
5.4	Existence of Minimum Points and the Ritz Method . . . . .	99
5.5	Application to Boundary Value Problems . . . . .	105
5.6	Bibliographical Notes and Exercises . . . . .	110
<b>6</b>	<b>Duality of Convex Problems</b> . . . . .	111
6.1	Duality in Terms of a Lagrange Function . . . . .	111
6.2	Lagrange Duality and Gâteaux Differentiable Functionals . . . .	116
6.3	Duality of Boundary Value Problems . . . . .	118
6.4	Duality in Terms of Conjugate Functions . . . . .	122
6.5	Bibliographical Notes and Exercises . . . . .	129
<b>7</b>	<b>Derivatives and Subdifferentials of Lipschitz Functionals</b> . . .	131
7.1	Preview: Derivatives and Approximating Cones . . . . .	131
7.2	Upper Convex Approximations and Locally Convex Functionals . . . . .	135
7.3	The Subdifferentials of Clarke and Michel–Penot . . . . .	139
7.4	Subdifferential Calculus . . . . .	146
7.5	Bibliographical Notes and Exercises . . . . .	153
<b>8</b>	<b>Variational Principles</b> . . . . .	155
8.1	Introduction . . . . .	155
8.2	The Loewen–Wang Variational Principle . . . . .	156
8.3	The Borwein–Preiss Variational Principle . . . . .	161
8.4	The Deville–Godefroy–Zizler Variational Principle . . . . .	162
8.5	Bibliographical Notes and Exercises . . . . .	166
<b>9</b>	<b>Subdifferentials of Lower Semicontinuous Functionals</b> . . . . .	167
9.1	Fréchet Subdifferentials: First Properties . . . . .	167
9.2	Approximate Sum and Chain Rules . . . . .	172
9.3	Application to Hamilton–Jacobi Equations . . . . .	181
9.4	An Approximate Mean Value Theorem . . . . .	182
9.5	Fréchet Subdifferential vs. Clarke Subdifferential . . . . .	184

9.6	Multidirectional Mean Value Theorems .....	185
9.7	The Fréchet Subdifferential of Marginal Functions.....	190
9.8	Bibliographical Notes and Exercises .....	193
<b>10</b>	<b>Multifunctions .....</b>	<b>195</b>
10.1	The Generalized Open Mapping Theorem .....	195
10.2	Systems of Convex Inequalities .....	197
10.3	Metric Regularity and Linear Openness .....	200
10.4	Openness Bounds of Multifunctions .....	209
10.5	Weak Metric Regularity and Pseudo-Lipschitz Continuity ...	211
10.6	Linear Semiopenness and Related Properties .....	213
10.7	Linearly Semiopen Processes .....	217
10.8	Maximal Monotone Multifunctions .....	219
10.9	Convergence of Sets .....	225
10.10	Bibliographical Notes and Exercises .....	227
<b>11</b>	<b>Tangent and Normal Cones .....</b>	<b>231</b>
11.1	Tangent Cones: First Properties .....	231
11.2	Normal Cones: First Properties .....	237
11.3	Tangent and Normal Cones to Epigraphs .....	241
11.4	Representation of Tangent Cones .....	245
11.5	Contingent Derivatives and a Lyusternik Type Theorem ....	252
11.6	Representation of Normal Cones .....	255
11.7	Bibliographical Notes and Exercises .....	261
<b>12</b>	<b>Optimality Conditions for Nonconvex Problems .....</b>	<b>265</b>
12.1	Basic Optimality Conditions.....	265
12.2	Application to the Calculus of Variations .....	267
12.3	Multiplier Rules Involving Upper Convex Approximations ...	272
12.4	Clarke's Multiplier Rule .....	278
12.5	Approximate Multiplier Rules .....	280
12.6	Bibliographical Notes and Exercises .....	283
<b>13</b>	<b>Extremal Principles and More Normals and Subdifferentials .....</b>	<b>285</b>
13.1	Mordukhovich Normals and Subdifferentials .....	285
13.2	Coderivatives .....	294
13.3	Extremal Principles Involving Translations .....	301
13.4	Sequentially Normally Compact Sets .....	309
13.5	Calculus for Mordukhovich Subdifferentials.....	315
13.6	Calculus for Mordukhovich Normals .....	320
13.7	Optimality Conditions .....	323
13.8	The Mordukhovich Subdifferential of Marginal Functions.....	327
13.9	A Nonsmooth Implicit Function Theorem .....	330
13.10	An Implicit Multifunction Theorem .....	334

XII Contents

13.11 An Extremal Principle Involving Deformations.....	337
13.12 Application to Multiobjective Optimization .....	340
13.13 Bibliographical Notes and Exercises .....	343
<b>Appendix: Further Topics .....</b>	<b>347</b>
<b>References .....</b>	<b>351</b>
<b>Notation .....</b>	<b>363</b>
<b>Index .....</b>	<b>366</b>



<http://www.springer.com/978-3-540-71332-6>

Nonsmooth Analysis

Schiotzek, W.

2007, XII, 378 p. 31 illus., Softcover

ISBN: 978-3-540-71332-6