Preface

Around 1980, recursions for aggregate claims distributions started receiving attention in the actuarial literature. Two common ways of modelling such distributions are as compound distributions and convolutions.

At first, one considered recursions for compound distributions. In particular, Panjer (1981) became crucial in this connection. He considered a class of counting distributions consisting of the negative binomial, Poisson, and binomial distributions, that is, the three most common types of counting distributions in the actuarial literature.

In the mid-eighties, De Pril turned his attention to recursions for convolutions and published several papers on that topic. These recursions can be rather time- and space-consuming, and he, and other authors, therefore developed approximations based on these recursions and error bounds for these approximations.

By extending the Panjer class, Sundt (1992) presented a framework that also covered De Pril's recursions for convolutions.

Originally, the recursions were deduced for the probability function of the distribution, but later they were adapted to other functions like cumulative distributions and stop loss transforms. As in this book we focus mainly on recursions for probability functions of distributions on the integers, we shall normally refer to such functions as distributions.

From the late nineties, the theory has been extended to multivariate distributions.

In the present book, we give a presentation of the theory. We restrict to classes of recursions that are somehow related to those of Panjer (1981), and we aim at giving a unified presentation. Although the theory has been developed mainly within the actuarial literature, the recursions can also be applied in other fields, and we have therefore tried to make the presentation relatively general. However, we use applications from insurance to illustrate and motivate the theory. Not all the recursions that we are going to present, are equally interesting in practice, but we wish to give a broad presentation of recursions within the framework of the book with emphasis on how they can be deduced.

We have tried to make the book self-contained for readers with a reasonable knowledge of probability theory. As an exception, we present Theorem 12.1 without proof.

Our main goal is to deduce recursions and give general ideas of how such recursions can be deduced. Although applications are used to illustrate and motivate the theory, the book does not aim at giving guidelines on what methods to apply in situations from practice. We consider the recursions primary as computational tools to be used for recursive evaluation of functions. However, we shall also use the recursions in analytical proofs of for instance characterisations of infinitely divisible distributions and normal distributions.

One area that we have not considered, is numerical stability. That is obviously an important aspect when considering recursions, but the tools and methodology would have been quite different from the rest of the book. Furthermore, if we should have kept the principle of making the book self-contained, then we would have needed to include a lot of deductions of results from numerical mathematics, an area where the present authors do not consider themselves as specialists. Some references are given at the end of Chap. 13.

The most compact way of presenting the theory would have been to start with the most general results and deduce the more specific results as special cases. We have chosen the opposite approach, that is, we start with the simple cases, and gradually develop the more general and complex results. This approach better reflects the historical development, and we find it more pedagogical. When knowing and understanding a proof in a simple model, it will be much easier to follow a generalised proof of an extended result. Furthermore, a reader who wants a recursion for a compound geometric distribution, should not first have to go through a complicated deduction of a messy recursion for a general multivariate distribution and get the desired result as a special case. To make it easier for the reader to see what changes have to be made in the proof when extending a result, we have sometimes copied the proof of the simple case and edited it where necessary. A problem with going from the simple to the complex, is how much to repeat. When the proof of the extension is straight forward, then we sometimes drop the proof completely or some details. We apologise to readers who find that we give too much or too little.

Each chapter is preceded by a summary. We do not give literature references in the main text of the chapter, but allocate that to a section *Further remarks and references* at the end of the chapter.

The book consists of two parts. Part I (Chaps. 1–13) is restricted to univariate distributions, whereas in Part II (Chaps. 14–20), we extend the theory to multivariate distributions. To some extent, we have tried to make Part II mirror Part I.

Chapter 1 gives an introduction to Part I. After having given a motivation for studying aggregate claims distributions in insurance, we introduce some notation and recapitulate some concepts from probability theory.

In Chap. 2, we restrict the class of counting distributions to distributions that satisfy a recursion of order one, that is, the mass of the distribution in an integer n is a factor multiplied by the mass in $n-1$; this factor will normally be in the form $a+b/n$ for some constants a and b. Within this framework, we characterise various classes of counting distributions and deduce recursions for compound distributions. We also deduce recursions for convolutions of a distribution.

Chapter 3 is devoted to compound mixed Poisson distributions. Such distributions are also discussed in Chap. 4 on infinitely divisible distributions. There we in particular use recursions presented in Chap. 2 to prove that an infinitely divisible distribution on the non-negative integers is infinitely divisible if and only if it can be expressed as a compound Poisson distribution.

In Chap. 5, we extend the classes of counting distributions to distributions that satisfy recursions of higher order, and we discuss properties of such distributions. We allow for infinite order of the recursions. Within that setting, the coefficients of a recursion can be expressed as a De Pril transform, a term introduced by Sundt (1995) for a function that is central in De Pril's recursions for convolutions. Many results on De Pril transforms follow as special cases of results deduced in Chap. 5 and are given in Chap. 6, which is devoted to properties and applications of De Pril transforms.

Chapter 7 is devoted to individual models, including collective approximation of such models.

In Chap. 8, we extend the theory to recursions for cumulative functions and tails, and Chap. 9 is devoted to recursions for moments of distributions.

As pointed out above, De Pril presented approximations based on his exact recursions for convolutions. These approximations consist of approximating the mass at zero and the De Pril transform of the distributions. Such approximations constitute the subject of Chap. 10. As the approximations to the distributions are not necessarily distributions themselves, we have to extend the theory to a wider class of functions.

Up to this stage, we have restricted to distributions on the non-negative integers. In Chap. 11, we extend the theory of Chap. 10 to distributions that are bounded from below, and Chap. 12 opens for negative severities.

Part I closes with Chap. 13, where we discuss how we can modify the recursions to avoid problems with numerical underflow or overflow.

In Part II, Chap. 14 mirrors Chap. 1.

In the multivariate case, we distinguish between two cases of compound distributions; those with univariate counting distribution and multivariate severity distribution and those with multivariate counting distribution and univariate severity distributions. The former case is the most straight-forward to obtain by extending the univariate case, and that is the topic of Chap. 15. From the theory presented there, we introduce the De Pril transform of a multivariate function in Chap. 16, which mirrors Chaps. 6 and 7.

Chapters 17 and 18 mirror Chaps. 9 and 10 respectively. It should be relatively straight-forward to also extend the recursions of Chap. 8 to a multivariate setting, but we have not found it worth while to pursue that in this book.

In Chap. 19, we deduce recursions for compound distributions with multivariate counting distribution and univariate severity distributions. Such distributions are also considered in Chap. 20, which mirrors Chap. 3.

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