Preface

This research monograph concerns the design and analysis of discrete-time approximations for stochastic differential equations (SDEs) driven by Wiener processes and Poisson processes or Poisson jump measures. In financial and actuarial modeling and other areas of application, such jump diffusions are often used to describe the dynamics of various state variables. In finance these may represent, for instance, asset prices, credit ratings, stock indices, interest rates, exchange rates or commodity prices. The jump component can capture event-driven uncertainties, such as corporate defaults, operational failures or insured events. The book focuses on efficient and numerically stable strong and weak discrete-time approximations of solutions of SDEs. Strong approximations provide efficient tools for simulation problems such as those arising in filtering, scenario analysis and hedge simulation. Weak approximations, on the other hand, are useful for handling problems via Monte Carlo simulation such as the evaluation of moments, derivative pricing, and the computation of risk measures and expected utilities. The discrete-time approximations considered are divided into regular and jump-adapted schemes. Regular schemes employ time discretizations that do not include the jump times of the Poisson jump measure. Jump-adapted time discretizations, on the other hand, include these jump times.

The first part of the book provides a theoretical basis for working with SDEs and stochastic processes with jumps motivated by applications in finance. This part also introduces stochastic expansions for jump diffusions. It further proves powerful results on moment estimates of multiple stochastic integrals. The second part presents strong discrete-time approximations of SDEs with given strong order of convergence, including derivative-free and predictor-corrector schemes. The strong convergence of higher order schemes for pure jump SDEs is established under conditions weaker than those required for jump diffusions. Estimation and filtering methods are discussed. The third part of the book introduces a range of weak approximations with jumps. These weak approximations include derivative-free, predictor-corrector, and simplified schemes. The final part of the research monograph raises questions on numerical stability and discusses powerful martingale representations and variance reduction techniques in the context of derivative pricing.

The book does not claim to be a complete account of the state of the art of the subject. Rather it attempts to provide a systematic framework for an understanding of the basic concepts and tools needed when implementing simulation methods for the numerical solution of SDEs. In doing so the book aims to follow up on the presentation of the topic in Kloeden & Platen (1999) where no jumps were considered and no particular field of application motivated the numerical methods. The book goes significantly beyond Kloeden & Platen (1999). It is covering many new results for the approximation of continuous solutions of SDEs. The discrete time approximation of SDEs with jumps represents the focus of the monograph. The reader learns about powerful numerical methods for the solution of SDEs with jumps. These need to be implemented with care. It is directed at readers from different fields and backgrounds.

The area of finance has been chosen to motivate the methods. It has been also a focus of research by the first author for many years that culminated in the development of the benchmark approach, see Platen & Heath (2006), which provides a general framework for modeling risk in finance, insurance and other areas and may be new to most readers. The book is written at a level that is appropriate for a reader with an engineer's or similar undergraduate training in mathematical methods. It is readily accessible to many who only require numerical recipes.

Together with Nicola Bruti-Liberati we had for several years planned a book to follow on the book with Peter Kloeden on the "Numerical Solution of Stochastic Differential Equations", which first appeared in 1992 at Springer Verlag and helped to develop the theory and practice of this field. Nicola's PhD thesis was written to provide proofs for parts of such a book. It is very sad that Nicola died tragically in a traffic accident on 28 August 2007. This was an enormous loss for his family and friends, his colleagues and the area of quantitative methods in finance.

The writing of such a book was not yet started at the time of Nicola's tragic death. I wish to express my deep gratitude to Katrin Platen, who then agreed to typeset an even more comprehensive book than was originally envisaged. She carefully and patiently wrote and revised several versions of the manuscript under difficult circumstances. The book now contains not only results that we obtained with Nicola on the numerical solution of SDEs with jumps, but also presents methods for exact simulation, parameter estimation, filtering and efficient variance reduction, as well as the simulation of hedge ratios and the construction of martingale representations.

I would like to thank several colleagues for their collaboration in related research and valuable suggestions on the manuscript, including Kevin Burrage, Leunglung Chan, Kristoffer Glover, David Heath, Des Higham, Hardy Hulley, Constantinos Kardaras, Peter Kloeden, Uwe Küchler, Herman Lukito, Remigius Mikulevicius, Renata Rendek, Wolfgang Runggaldier, Lei Shi and Anthony Tooman. Particular thanks go to Rob Lynch, the former Dean of the Faculty of Business at the University of Technology Sydney, who made the writing of the book possible through his direct support. Finally, I like to thank the Editor, Catriona Byrne, at Springer for her excellent work and her encouragement to write this book as a sequel of the previous book with Peter Kloeden.

It is greatly appreciated if readers could forward any errors, misprints or suggested improvements to: eckhard.platen@uts.edu.au. The interested reader is likely to find updated information about the numerical solution of stochastic differential equations on the webpage of the first author under "Numerical_Methods":

http://www.business.uts.edu.au/
finance/staff/Eckhard/Numerical_Methods.html

Sydney, January 2010 *Eckhard Platen*

Springer

http://www.springer.com/978-3-642-12057-2

Numerical Solution of Stochastic Differential Equations
with Jumps in Finance

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2010, XXVIII, 856 p., Hardcover
ISBN: 978-3-642-12057-2