Chapter 2
Encounters with Reinhard Selten: An Office Mate's Report

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Background Story

In winter term 1960/1961 about 100 students applied for the economic seminar held by Professor Heinz Sauermann. As an economics student at the University of Frankfurt, I took part in that seminar. Among the students it had the reputation of being the most challenging course in the field of economic theory.

After passing a test, about 20 of the best students were allowed to participate. Sauermann always organized this seminar in economic theory together with all his research assistants, Reinhard Selten being one of them, and with some guests (former assistants, future colleagues).

At the end of the term I decided to talk to Sauermann during his office hour and ask for a position as a student assistant. To take that step was not easy for me and I was all the more surprised and happy by his answer: "You can start tomorrow".

During the seminar I did make some critical remarks but more with the intention that I, with a major in Business Administration, could forget about economics later on and would not have any more contact with it at my already planned and partly started second study program in mathematics after finishing my degree. But it should all turn out differently.

First Encounters with Reinhard Selten

A very important step for me was that Professor Sauermann, who then already had more than ten assistants, had Reinhard Selten and me to share an office. It stayed like that for many years and looking back I can say that nothing better could have happened to me.

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The Sauermann-assistants soon recognized that we both used to discuss all different kinds of problems for hours and soon the saying spread: “Even if you do not see the two of them, you will hear them”.

We never attended the same lectures but shared interests for the same research fields, such as psychology. From my almost 10 years of professional practice before studying (6 years as insurance broker, 4 years as certified financial accountant), I was able to share all different aspects of my practical experience. On the other side, Reinhard Selten as mathematician and especially as game theorist was an excellent teacher. We did not only concern ourselves with economic questions; we also enjoyed solving all kinds of riddles. Reinhard Selten was a very successful problem solver.

Common Fundamental Beliefs

Regarding economics as an academic discipline we shared a number of common fundamental beliefs from a methodological point of view. For us it seemed very doubtful and unrealistic that human behavior (regardless of the decision-maker being a household or a firm) could be explained by the rationality postulate. It appeared to be very far from reality for a number of reasons. To justify this opinion is easy. One need only think of the large set of goods desired by the household and of the quantities of producible goods by the firms to have reasonable doubts about the practical solvability of the household maximizing utility and the firms maximizing profits. In addition, the rationality principle requires a temporal interdependency of economic decisions (always meaning long-term planning). With the complexity of the decision tasks alone it seemed impossible, even with a short-term time horizon.

We both agreed that for a introductory lecture it is sufficient to demonstrate the fundamental tasks of economic decisions in a two-goods-world and to illustrate rational behavior in this two dimensional cosmos; and why not, it all seems very reasonable at first sight.

Certainly this is not a realistic consideration of the real economic decisions tasks. This can be proven normatively and exploratory. If one is not content with an “as-if-explanation,” something new had to be conceived. In this connection Reinhard Selten, like Herbert A. Simon, whom he knew in person, thought about developing an own concept in the direction of “limited rationality,” that has to be applicable and that must be able to provide experimentally verifiable results. This laid the foundation for the later by Reinhard Selten developed “Aspiration Adaptation Theory”. But one life task persisted: the experimental consideration (i.e. verification) of this theory, which Reinhard Selten is still preoccupied with until today.

Naturally Reinhard Selten’s academic interests were not only focused on this topic. A lot of our discussions aimed at topics that were not represented when building up the departments at the Frankfurt Faculty of Economics and Social

Sciences. Apart from the field of game theory, this concerns: operations research, business informatics including computer applications and finally econometrics.

Led by Reinhard Selten, soon a group was formed that according to their interests dealt with questions from the just mentioned areas. An accompanying study of literature for our increase of knowledge played an important role.

Within the group there was more than one person capable of programming so there were no significant problems with the numerical analysis. There was only seldom the need to buy software, the more so as for numerical analysis Selten could fall back on me (FORTRAN) and on Reinhard Tietz (ALGOL).

A New Seminar Develops at the Sauermann Chair

Thanks to Reinhard Selten Heinz Sauermann could be convinced to offer another seminar at his chair, especially because of the current and future planned DFG projects and to give the research in this area an institutional frame. It was his seminar for "Mathematical Economic Research and Econometrics".

First Forecast Experiments with the Time Course HISTO

The first experimental collaboration with Reinhard Selten were preceded by discussions on so called "technical stock price analysis" that was also popular in the 1960s in Germany. We asked ourselves a just as obvious as provoking question: What is it like, if someone looks at a price trend graph of the recent past of a share and knows neither anything about the company, nor anything about the price movement of other corporations? Are there concrete general rules about price movements of shares that forecasts could be based on them?

A well-directed experimental approach was easy to realize. After a preliminary study with some empirical data, I attended the task as follows: I created a long random series of realizations of a linear second order difference equation with damped oscillations and a "white noise term," so that graphically a picture of a more or less irregular sine wave was produced.

From a longer series of more than 200 periods, 42 periods were chosen for the actual experiment. The test subjects had no information on where the data came from, nor on what it meant. For us this seemed to be best to compare it to a newspaper reader, who finds a time course whose meaning he does not know, but thinks about how this time course would continue if it was a stock price.

In individual tests, the test subject had the task to predict the next number in the row. Thereupon they were told the actual number, which they had to write in the chart, to go on predicting the next number.

Our first aim was simply to explain the mean predictive value of the test subjects.

In the Gestalt Psychology there have been similar experiments a lot earlier (already in the 1930s). The test subjects were showed a pile of dots at the beginning and had to guess, what object they would form. The number of dots was intentionally that small, so the test subjects were not able to guess right at the first try. The relevant object was a cup e.g., of which only a few dots or rather fragments were shown. Then, successively more dots were added until the test subjects were able to identify the object. It took some test subjects longer to identify the object than others. Our experiment is of course a lot more complicated but there are some analogies. By adding more dots, the sine wave could not be recognized wholly but the local extrema should have been considered for their prognosis as "dots of special value" (Selten) by the test subjects. On this a first explanatory model for the mean projection was built.

The time course HISTO, as we named it, is still today subject of academic research. Already in the 1960s the number of participants was extensively increased. In cooperation with Ulrike Leopold-Wildburger, a "Bounds and Likelihood-Theory" was developed that explained the mean projections comparatively well. The difference to the first, predominantly wholistic explanatory model lays mainly in the reversal probabilities of the time course being estimated from the reversed cases observed up to then. In later versions of the experiment, additional information in terms of leading series with a lead of a period and more or less high deviations compared to the base course were added. In the next step experiments were conducted in which structural breaks are built in the base course.

Some of these experiments were conducted during my time at the Karl Franzens-University in Graz. Ulrike Leopold-Wildburger, who under Selten already habilitated several years ago, played an important role by carrying on the experiments initiated by me.

The current state of the experimental research of the HISTO-data in Graz is that the test subjects are attached to an electronic indicator, the so called "eye tracker", with which in nearly all of the 42 periods all movements of the eye in viewing direction, location and time period can be recorded. All in all it has to be noted that meanwhile there are several publications on the HISTO experiments. Further publications are in preparation.

OR-Problems, Ultima Ratio and Good Heuristics

The field of operations research is another chapter of the collaboration with Reinhard Selten. Though from mathematical view seemingly trivial, yet n-factorial sequences have to be compared and the value n by no means is small, otherwise this problem would not exist. At this class of problems there is always the question if all possible cases can be calculated (the ultima ratio), or if only the search for a preferably efficient heuristic is promising.

Firstly, one of these n-factorial complete problems is described, for which a certain practical meaning cannot be denied. It is the *travelling salesman problem*:

A travelling salesman has to visit n people (or stations) as fast as possible. All information is restricted to a type (n,n) matrix, from which emanates what time it takes to get from station i to station j. For small n this task is trivial, as the solution is only to compare all n! periods and to look for a period with the shortest time requirement.

First of All a Side Note on Available Computer Technology Now and Then

Is n, the number of stations, notably smaller than 20, already in the 1960s this task could be solved in adequate time with the main frame computers (qua ultima ratio) available. But despite all technical progress, for $n > 200$ the ultima ratio would still take too much computing time. Besides, this entire problem with deterministic running times is an exercise at the most, because realistically, these running times often have quasi random character.

From the view of a practitioner it is solely about finding a good heuristic that has a tolerably acceptable efficiency relation (running time of the optimal solution in relation to the heuristic).

Reinhard Selten suggested an approach for the production of all periods, which he recalled from a combinatorics lecture and that was easy to program for any n. This was the trick: To every number from 1 to n a running direction (left or right) was assigned. At the beginning e.g. all numbers run to the right. All permutations only result from exchanging one number with its neighboring number. There is an exactly defined running rule. If no number can be exchanged according to this rule any more, the procedure stops automatically and all permutations have been played. There is one basic rule: Every number can only be exchanged in its momentarily running direction and only by a higher number.

Step 1: You start with the elemental period $+1, +2, +3,\ldots,+n$, giving all numbers the algebraic sign $+$. It means their exchange direction is to the right.

Step 2: Every try to exchange starts with $K = 1$, wherever this number is located, followed by testing the basic rule:

(a) If allowed, the exchange takes place and you go back to step 2.
(b) If not allowed, a change of direction of the number 1 takes place and a skip to step 3 with $K = 2$.

Step 3: The try to exchange the number K in its momentary direction is tested.

(a) If allowed by the basic rule, the exchange takes place and you go back to step 2.
(b) If not allowed, only a change of direction of the number K takes place and $K + 1$ continues with step 3 as long as $K < n$.

In case of $K = n$ all permutations are produced.

The simple example for n = 3 produces the six permutations in the following row:

+1+2+3	+2+1+3	+2+3+1	+3+2−1	+3−1+2	−1+3+2

In the last position, no number can be exchanged.

There is no need to test whether all n! periods are checked using this procedure, because at the end in the case of n = 6, for example, the last position is: −1−2−3−4+6+5. The first four numbers cannot be exchanged to the left, the number 6 can never be exchanged and the number 5 cannot be exchanged to the right.

The mathematical proof for the possibility of producing all n! sequences is waived here. A draft of the proof in the direction of the above mentioned procedure should suffice: Imagine for the numbers of 1 to (n−1) any arrangement of the (n−1)! permutations. In every one of these (n−1) factorial permutations you put the number n in front and pull this number diagonally through all of the (n−1)!-permutations until the end. Therewith you produce exactly n(n−1)! = n! permutations.

In a program it is recommendable to fill the positions 0 and n + 1 with the number 0 to avoid exceeding the boundary. Regarding the target function, the new value can be calculated by exchanging three numbers from the old value of the objective function as one passage is reversed and both the new tie-in sections have to be replaced, too. Anyway, this “elegant” solution of Selten with n as input parameter of the program is better than all other programming solutions that work with an n-times nested Do-nest.

Now what about a feasible heuristic for the concrete task of combinatory optimization? It is known for a long time as “nearest neighbor”. The name is strictly speaking actually self explaining. To determine the efficiency of this heuristic you can, for example, produce a higher number of purely random topologies for a yet acceptable n. For the time needed from i to j, you take a value proportional to the Euclidean distance of the random target points. For the constructed time matrices you generally find a mean efficiency of more than 90%. Admittedly, to be correct, the efficiency can drop to nearly 50% occasionally. But these cleverly constructed extreme counter examples are based on neighborhood relations that you can get under control by using easy to observe modifications.

Another Example: Helmstädter’s Linearity Measure

The sector alignment of an input–output-matrix in the broader sense follows the typical alignment of a production process, starting with the primary industry, over the manufacturing industry in different sectors, to the consumer oriented output stages. The allocation of the production sums above the main diagonal indicates on how the production process develops straight (in the sense of “linear”) towards the output stages. Helmstädter's question is: At which allocation of the sectors is the sum of the delivery chains above the main diagonal at a maximum? If all industries would only deliver downstream, or technically speaking, if all sectors were aligned in way that below the main diagonal of a input–output table no regressive delivery chains would exist, than maximum linearity exists according to Helmstädter. As I/O tables often consist of more than 30 sectors, even here it is not possible to calculate all possible n! rows in an adequate computing time.

A reasonable heuristic to use is a ratio criterion (sum of all delivery chains above/sum of all delivery chains below the main diagonal) to stepwise decide whether a sector can be integrated in the still available free spot ahead (maximum ratio) or back (minimum ratio). The deliveries of the already assigned sectors play no role anymore at the calculation of the ratio. A control calculation shows that a reassignment of the middle sectors often results in slightly improved solutions but this reassignment is often accompanied by small robustness. The conclusion is: for smaller n an exact efficiency value is calculable and the described heuristic reaches a relatively high efficiency.

The SINTO Market

There exists a company game, called SINTO market, which Selten and I invented in 1967 and we played it for the first time at the Control Data Company in Frankfurt/M. This off-campus location was chosen because the University of Frankfurt/M. did not have the adequate computer equipment needed for our experiment. We needed a main frame computer system working for several hours. Nowadays you only need a mediocre computer.

The development of the game was preceded by numerous considerations about corporate strategies, e.g. oligopolistic price policy, brand products produced from a uniform raw material, possibilities of quality variation and product-related advertising with aftereffects of it in later periods. This was all supposed to be realized after the company foundation. The products to be introduced on the market are subjected to a typical product life cycle and the course of the game should reach exactly 15 periods to the maximum sale stage.

In previous years we closely examined a series of practical, managerial questions, also covering the field of quality variation, which now suggested that we should experimentally examine a practical example ourselves. It was supposed to be an example for a market, on which a certain good is offered in different quality variations and by a small number of companies. Therefore, we invented an oligopoly market in which three companies could offer and sell the same product (called SINTO) in different quality variations on one market.

Each of the companies was allowed to produce and sell up to ten qualitatively different brands of SINTO. For each brand, three quality criteria had to be guaranteed that each could be varied in 10 steps from 0 (low level) to 9 (maximum level). To avoid any previous knowledge on the product itself and its production, we invented with SINTO an albuminous “dietary sublement substance, that was not yet invented but could be at any time,” i.e. a new product to be introduced on the market.

In order not to distract the decision-makers in the three companies with financing problems and questions of profit distribution, we invented parent organizations to which a certain amount from the earned profit had to be paid. So these parent organizations were responsible for all financial duties and responsibilities of the companies.

The company’s scope of action: In each of the 15 periods they were able to cancel an already existing brand, introduce new brands and at the same time carry on the old ones and, if desired, change their quality criteria. In every period they had to decide for each brand for the production output, the offer price and advertising expense.

At the beginning of each subsequent period the companies were presented the following documents:

1. The closing balance
2. The profit and loss statement
3. A market overview on all products on the market including their prices and quality criteria
4. A breakeven analysis structured by brands

Thus, up to 30 different brands can be introduced to the market. The first two quality criteria (fine graindness and tartness) are cost neutral and only serve for differentiation of taste. To present the participants with an indicator for the preferences of the customers, they were told that there is a buyer for each criteria combination but the middle criteria stages are preferred. In contrast, the third criterion is not cost neutral but a quality criterion that causes constantly increasing additional costs.

As an ideal casting for the companies we imagined teams of up to six people. The team should decided on a division of labor, consult together on the situation of the company, and then reach a decision. So far for the short description.

Regarding the course of the game, we were interested from a theoretical point of view in various questions:

1. Which price, investment, and advertising policy is chosen?
2. How many different brands are introduced?
3. Where are the brands placed in the market landscape?
4. How do the companies organize their decisions?

The underlying mathematical model was designed by Reinhard Selten in a way that it could be aggregated regardless of the numerous decisions of the companies and it therefore allowed detailed analyses and benchmark tests with theoretical solutions.

From 1967 until today, this game has not lost any of its attractiveness. Even for experienced Business Administration Majors it is not easy to play; not to mention students that are often not even able to understand all the documents correctly.

This also explains why a lot of companies could not generate even half as much profit as with an experienced play.

For this experiment there are several result oriented questions. It allows e.g. an answer to the question if and in what ways firms with female or male staff differ. Decades later, we could convince Ulrike Leopold-Wildburger, University of Graz, and Oliver Heil, University of Mainz, to repeat the game. The data from Graz presented similar results to our former observations.

One interesting result from the Graz experiment should be pointed out. A company with solely female staff – on average – earns less profit than with mixed staff or solely male staff. The women comparatively often had too low starting prices. Male teams spend a lot on advertising. A typical mistake was obviously not to act whole-heartedly in case a brand completely sold out. In this case, one expects that the obviously too low starting price is drastically increased and the additional cost of stockpiling is accepted, especially since the sales are planned for the following period.

A Resume of Reinhard Selten Back Then Until Today

To draw a resume of the years spent together with Reinhard Selten is easy for me: He was an excellent provider of ideas, a mathematician quickly adjusting to the special problems in Economics, an all-rounder who liked talking about psychology as much as about science and eventually one of the best game theorists worldwide already at an early age.

And nothing of that has changed ever since.

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