Contents

1 Doubly Periodic Functions 1
1.1 Definition and First Properties 1
1.2 The $\wp$-Function of Weierstrass 4
1.3 The Differential Equation of the $\wp$-Function 7
1.4 Eisenstein Series 8
1.5 Bernoulli Numbers and Values of the Zeta Function 9
1.6 Exercises and Remarks 11

2 Modular Forms for $\mathbf{SL_2}(\mathbb{Z})$ 15
2.1 The Modular Group 15
2.2 Modular Forms 20
2.3 Estimating Fourier Coefficients 30
2.4 L-Functions 31
2.5 Hecke Operators 37
2.6 Congruence Subgroups 51
2.7 Non-holomorphic Eisenstein Series 53
2.8 Maaß Wave Forms 63
2.9 Exercises and Remarks 73

3 Representations of $\mathbf{SL_2}(\mathbb{R})$ 79
3.1 Haar Measures and Decompositions 79
3.1.1 The Modular Function 84
3.2 Representations 87
3.3 Modular Forms as Representation Vectors 92
3.4 The Exponential Map 97
3.5 Exercises and Remarks 100

4 p-Adic Numbers 105
4.1 Absolute Values 105
4.2 $\mathbb{Q}_p$ as Completion of $\mathbb{Q}$ 107
4.3 Power Series 109
4.4 Haar Measures 112

4.5 Direct and Projective Limits . . . 115
4.6 Exercises . . . 120

5 Adeles and Ideles . . . 123
5.1 Restricted Products . . . 123
5.2 Adeles . . . 125
5.3 Ideles . . . 128
5.4 Fourier Analysis on $\mathbb{A}$ . . . 131
5.4.1 Local Fourier Analysis . . . 134
5.4.2 Global Fourier Analysis . . . 136
5.5 Exercises . . . 139

6 Tate's Thesis . . . 143
6.1 Poisson Summation Formula and the Riemann Zeta Function . . . 143
6.2 Zeta Functions in the Adelic Setting . . . 145
6.3 Dirichlet L-Functions . . . 150
6.4 Galois Representations and L-Functions . . . 156
6.5 Exercises . . . 160

7 Automorphic Representations of $\mathbf{GL_2}(\mathbb{A})$ . . . 163
7.1 Principal Series Representations . . . 163
7.2 From Real to Adelic . . . 168
7.3 Bochner Integral, Compact Operators and Arzela–Ascoli . . . 172
7.3.1 The Arzela–Ascoli Theorem . . . 180
7.4 Cusp Forms . . . 182
7.5 The Tensor Product Theorem . . . 185
7.5.1 Synthesis . . . 185
7.5.2 Analysis . . . 193
7.5.3 Admissibility of Automorphic Representations . . . 205
7.6 Exercises and Remarks . . . 206

8 Automorphic L-Functions . . . 211
8.1 The Lattice $M_2(\mathbb{Q})$ . . . 211
8.2 Local Factors . . . 212
8.3 Global L-Functions . . . 224
8.4 The Example of Classical Cusp Forms . . . 235
8.5 Exercises and Remarks . . . 238

Appendix Measure and Integration . . . 241
A.1 Measurable Functions and Integration . . . 241
A.2 Fubini's Theorem . . . 243
A.3 L^p-Spaces . . . 245

References . . . 247

Index . . . 249

http://www.springer.com/978-1-4471-4434-2

Automorphic Forms
Deitmar, A.
2012, IX, 252 p. 2 illus., Softcover
ISBN: 978-1-4471-4434-2