Chapter 2
The Retail Model and Its Applications

2.1 The Retail Model

The next step in the argument is through another example: a model of a retail system. This serves as a valuable demonstrator of a number of model design principles. It is both simple and easy to understand but can also be developed in a way that is rich and realistic. As usual, we have a discrete spatial system: zones for residential areas and what are taken as points for retail centres—so in this case origins, i, and destinations, j, represent different spatial systems. London is shown as an example in Fig. 2.1: there are 623 wards with centroids shown as dots, and 220 retail centres shown in blocks.

Thus if S_{ij} is the flow of money (say) spent on retail goods and services by residents of zone i in retail centre j and we denote the whole array by $\{S_{ij}\}$, in the London case, we have a 623 × 220 matrix. The power of modelling is demonstrated by the fact that we can write down one equation to represent the flow from i to j (S_{ij}) and the computer can simply repeat the calculation 623 × 220 times—which is 137,060 possible flows. We define e_i as the per capita expenditure by each of the P_i residents of zone i and W_j as a as the size of a centre, and by raising it to a power, α, it can be taken as a measure of the attractiveness of retail centre j. If $\alpha > 1$, this will represent positive returns to scale for retail centres. c_{ij} is a measure of travel cost as usual. The main variables are shown diagrammatically in Fig. 2.2.

The core spatial interaction model is then

$$S_{ij} = A_i e_i P_i W_j^{\alpha} \exp(-\beta c_{ij}) \tag{2.1}$$

where

$$A_i = \sum_k W_k^{\alpha} \exp(-\beta c_{ik}) \tag{2.2}$$

to ensure that

A. Wilson, *The Science of Cities and Regions*, SpringerBriefs in Geography,
DOI: 10.1007/978-94-007-2266-8_2, © The Author(s) 2012

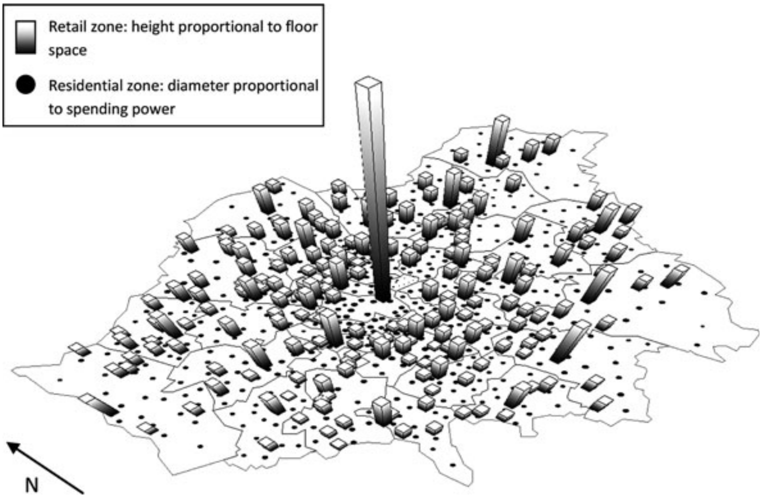
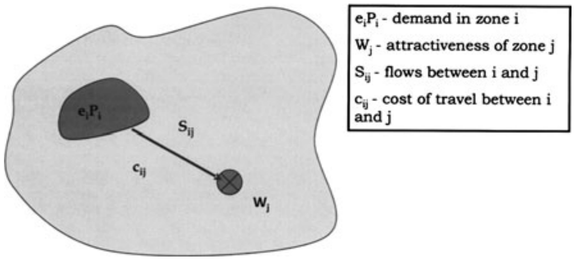


Fig. 2.1 Wards and retail centres in London

Fig. 2.2 The main variables of an aggregate retail model



$$\sum_j S_{ij} = e_iP_i \tag{2.3}$$

That is, we build in the constraint that all the money available in i is spent somewhere. Note that in this case, we have chosen a particular declining function of c_{ij}—the negative exponential function. The reason for this, and a derivation, will be given in Chap. 6. Meanwhile, it can be taken comfortably on trust. (This is not a restrictive assumption: the exponential can be easily replaced if appropriate as we will see later.)

The constraint is on all the flows leaving a residential zone. There is no such constraint on flows entering a retail zone, and so we can use the model to calculate the total revenue attracted to a particular j. If we call this D_j, then

$$D_j = \sum_k S_{ij} \tag{2.4}$$

which is, substituting from (2.1) and (2.2)

$$D_j = \sum_i \left[e_i P_i (W_j)^{\alpha} \exp(-\beta c_{ij}) \Big/ \sum_k (W_k)^{\alpha} \exp(-\beta c_{ik}) \right] \tag{2.5}$$

This is a very important example because it shows how, in appropriate cases, the spatial interaction model also functions as a location model. We have already seen examples of this, of course, in the Lowry model though with more primitive interaction models.

In this presentation, we have used the customary definitions of variables, but it is a special case of the X-Y-Z-W notation of the previous chapter. e_iP_i is an X-variable, S_{ij} is a Y-flow, D_j, a Z-variable and W_j is a structural variable. Indeed, the S_{ij} variables can be seen as accounts which mirror the inter-sectoral-inter-zonal table of the previous chapter—though in this simple demonstrator, there is only one sector—with e_iP_i as the row sums and D_j as the column sums.

This example of a model has the advantage that it has a straightforward intuitive interpretation. If we substitute for A_i in (2.1) from (2.2), the flow model can be written in the form

$$S_{ij} = e_i P_i W_j^{\alpha} \exp(-\beta c_{ij}) \Big/ \sum_k W_k^{\alpha} \exp(-\beta c_{ik}) \tag{2.6}$$

This shows that $W_j^{\alpha}\exp(-\beta c_{ij}) / \sum_k W_k^{\alpha}\exp(-\beta c_{ik})$ is the share of e_iP_i that goes to j. This will be large if $Wj^{\alpha}\exp(-\beta c_{ij})$ is large compared to $\Sigma_k W_k^{\alpha}\exp(-\beta c_{ik})$ and the make up of the terms in the sum represent the *competition* of all other centres. $W_j^{\alpha}\exp(-\beta c_{ij})$ is a combination of pulling power (W_j^{α}) and the opposite effect of greater distance [$\exp(-\beta c_{ij})$].

2.2 Disaggregation

It is possible to build these models for very fine levels of detail and this is necessary to make them realistic. This has been done by consumer type, by store/retail centre type and for types of goods, so the science is well known and extensively tested. It is systematically employed—at both centre and store level—by major retailers and its value in this context is proven. Some of this experience will be described in Chap. 6. There are more challenging questions, which we will address in Chap. 9 about whether it could be applied in public sector areas.

The aggregate variables used to introduce the model could be disaggregated as follows: population by type (m)—P_i^m; type of good (g); expenditure by person type and type of good—e_i^{mg}; shopping centres by type (n). There could be different elements of 'attractiveness' by person type and type of good and so W_j could be disaggregated to become:

$$W_j^{mng} = W_j^{(1)mng} W_j^{(2)mng} \ldots\ldots \tag{2.7}$$

and the cost of travel could be broken down into different elements—different kinds of time, money cost and so on to become a generalised cost:

$$c_{ij}^{m} = t_{ij}^{m} + m_{ij}^{m} + \ldots\ldots \tag{2.8}$$

The parameters such as α and β would also be disaggregated. For example, β^{mg} would be lower for higher value goods (g)—i.,e. generating longer trips—than for those of lower value.

2.3 Structural Dynamics

In the two examples presented so far—the Lowry model and the retail model—the model cores have been concerned with spatial interaction and the structural variables have been specified exogenously. A new challenge is to model the evolution of these structural variables. We can illustrate this with the retail example. In this case, a hypothesis for the structural dynamics can be presented as

$$\Delta W_j(t,\ t+1) = \varepsilon\left[D_j(t) - C_j(t)\right]W_j(t) \tag{2.9}$$

We can replace $C_j(t)$ with an assumption that costs are proportional to size—say KW_j—and then we will see in the next chapter that this is a form of Lotka-Volterra equation. The expression $[D_j(t) - C_j(t)]$, or $[D_j(t) - KW_j(t)]$ using the linearity assumption for centre costs, can be seen as a measure of 'profit' (or 'loss'). So Eq. 2.9 is representing a hypothesis that if a centre is profitable, it will grow; otherwise, it will decline. The parameter ε measures the strength of response to this signal.

At equilibrium, $\Delta W_j(t, t + 1)$ is zero, so

$$D_j = C_j = KW_j \tag{2.10}$$

That is

$$\sum_i \{e_i P_i W_j^{\alpha} \exp(-\beta c_{ij}) / \sum_k W_k^{\alpha} \exp(-\beta c_{ik})\} = k_j W_j \tag{2.11}$$

These are rather fierce nonlinear equations in $\{W_j\}$ and we will explore them further in Chap. 7. Considerable progress can be made.

2.4 An Urban Systems Example

It is interesting at this stage to present a third example of a model at a different scale but which can use modified versions of the retail model equations of the previous section. We now interpret 'retail centres' as towns or cities and the flows as some composite measure of trade and migration. The variables now become:

P_i = the population of the *i*th city
e_i = the level of economic activity per capita—so we can distinguish in principle between 'poor' and'rich' cities
W_i = a measure of the level of economic activity
S_{ij} = trade/migration flows
K_i = cost of maintaining the level of economic activity per unit—relates to 'rent'
c_{ij} = cost of interaction

The model to be developed—for a full description, see Wilson and Dearden (2011)—will be applied to the evolution of the North American urban system from 1790 to 1870—the period chosen because of the availability of good Census data on populations and an excellent account of the development of the railway system—in Cronon (1992). In this case, therefore, we need to add population dynamics to the system. A suitable equation, assuming that it is driven (via migration) by the economic activity—W_j-dynamics, is

$$P_i(t+1) = \mu(t)\{P_i(t)[1+\phi_{1i}] + \phi_2 \Delta W_j(t, t+1)\} \tag{2.12}$$

where ϕ_{1i} represents 'noise' and is a random variable less than 1; ϕ_2 measures the response of population to changes in economic activity and $\mu(t)$ is a normalising factor so that the total matches that in the available Census data.

The model is run through successive time periods—annually from 1790 to 1870. The aggregate population is increased in proportion to the known Census total and the W_js are constrained to this. We make a number of assumptions about the way the exogenous variables, represented in the vector $[\{e_i\}, \{P_i\}, \{c_{ij}\}, \{K_i\}, \alpha, \beta, \lambda_t]$, change over time and focus on introducing the railways exogenously to explore that effect on the urban system of cities

The system of interest is the area covering the East coast to the Midwest of the United States and we focus on the development of Chicago as the major city in the Midwest. The zone system is shown in Fig. 2.3. (This was originally seen as Fig. 1.2b in Chap. 1 but is repeated here for convenience.)

We had available population data at county level from the historical census (1790–1870)—source: NHGIS (www.nhgis.org). County boundaries change each decade and so they were aggregated to a regular grid of 434 cells. The "settlements" are then located at the grid square centroids. The transport system is represented by a spider network. This is a reasonably good approximation to a real network and is constructed by linking nearby zone centroids. We have separate links for land, water and rail. The travel cost from settlement i to settlement j is then the cost of shortest path through the spider network. When railway construction occurs the link costs change and the shortest paths are recalculated. The detail of the spider network for 1870 with land, water and railroads, is shown in Fig. 2.4.

Some results for a sample of years are shown in Fig. 2.5. The impact of the railways on the development of the Midwest is obviously of major significance.

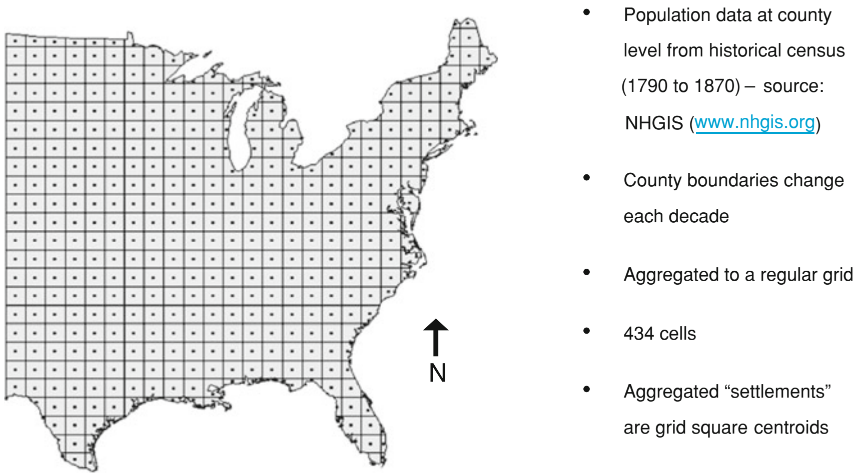


Fig. 2.3 A grid zoning system for the North East and mid-West USA

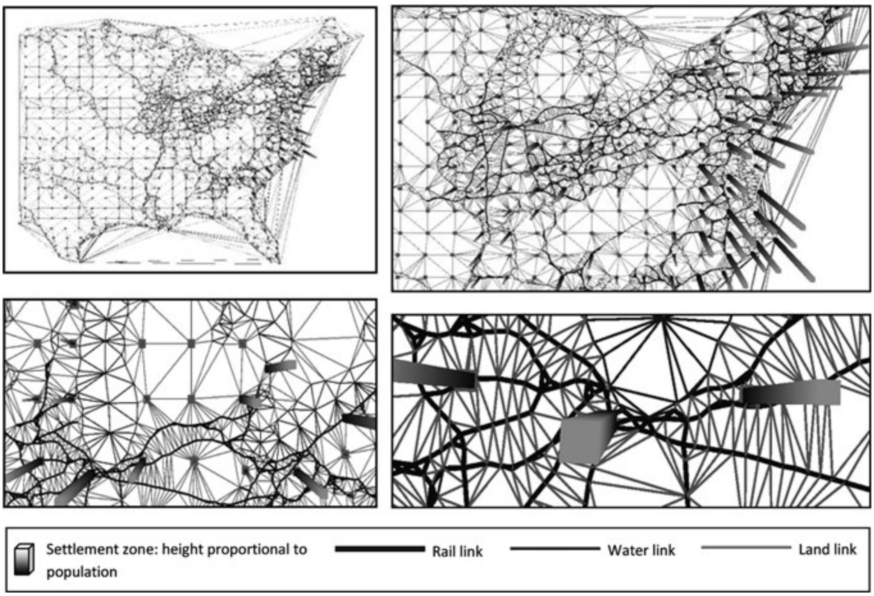


Fig. 2.4 A spider network representation of the North American transport system in 1870

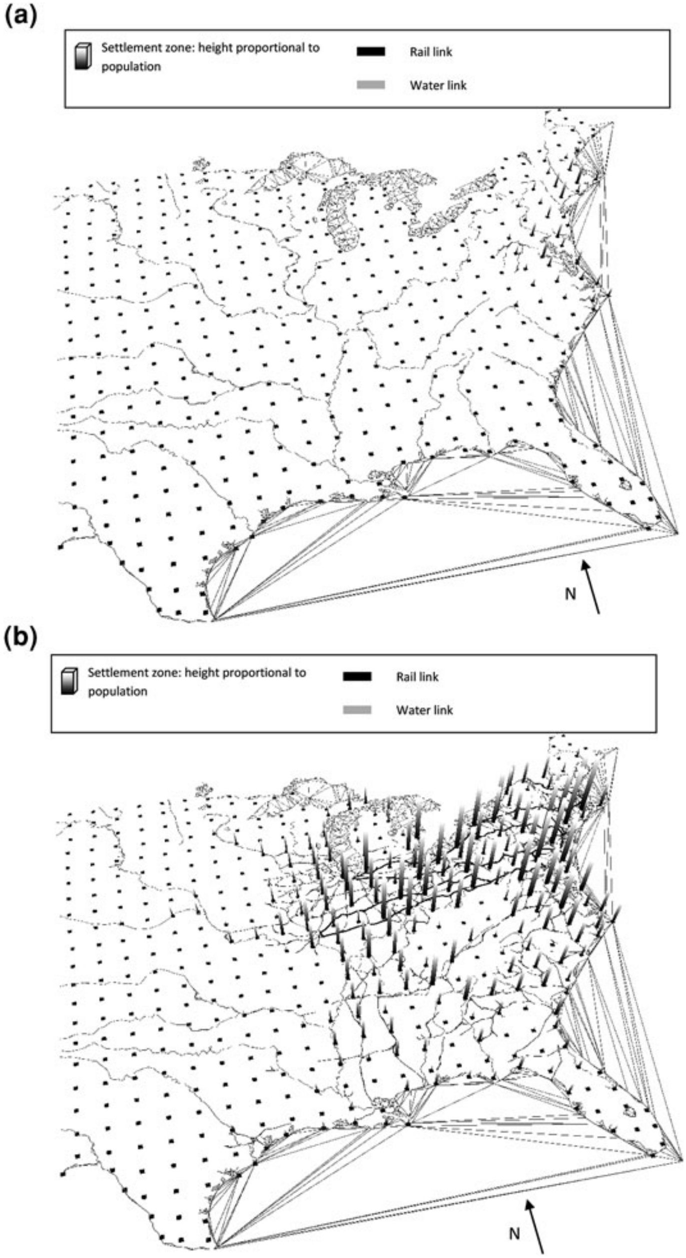


Fig. 2.5 Model-predicted growth of the North American urban system, 1790–1870

There are ongoing challenges of course. We can ask the question: what is the variety of models that can be constructed within this particular paradigm? And what are the alternative approaches to modelling this kind of system? Later, we will seek to review alternatives systematically and 'compare and contrast'.

Further Reading

Cronon W (1992) Nature's metropolis: Chicago and the great west. W. W. Norton, New York

Dearden J, Wilson AG (forthcoming-A) A framework for exploring urban retail discontinuities, Geog Anal 43:172–187

Harris B, Wilson AG (1978) Equilibrium values and dynamics of attractiveness terms in production-constrained spatial-interaction models. Env Planning A 10:371–388

Lakshmanan TR, Hansen WG (1965) A retail market potential model. J Amer Inst Planners 31:134–143

Wilson AG (2000) Complex spatial systems. Prentice Hall, Englewood Cliffs, Chapter 6

Wilson AG, Dearden J (2011) Tracking the evolution of regional DNA: the case of Chicago. In: Clarke M, Stillwell JCH (eds) Understanding population trends and processes. Springer, Berlin, pp 209–222

http://www.springer.com/978-94-007-2265-1

The Science of Cities and Regions
Lectures on Mathematical Model Design
Wilson, A.
2012, XII, 102 p. 31 illus., Softcover
ISBN: 978-94-007-2265-1