Preface

This book grew out from the Lecture Notes of the course in General Relativity which I gave for more than 15 years at the University of Torino. That course has a long tradition since it was attached to the Chair of Relativity created at the beginning of the 1960s for prof. Tullio Regge. In the years 1990–1996, while prof. Regge was Member of the European Parliament the course was given by my long time excellent friend and collaborator prof. Riccardo D'Auria. In 1996 I had the honor to be appointed on Regge's chair[1] and I left SISSA of Trieste to take this momentous and challenging legacy. Feeling the burden of the task laid on my shoulders I humbly tried to do my best and create a new course which might keep alive the tradition established by my so much distinguished predecessors. In my efforts to cope with the expected standards, I obviously introduced my own choices, view-points and opinions that are widely reflected in the present book. The length of the original course was of about 120 hours (without exercises). In the new $3 + 2$ system introduced by the Bologna agreements it was split in two courses but, apart from minor readjustments, I continued to consider them just as part one and part two of a unique entity. This was not a random choice but it sprang from the views that inspired my teaching and the present book. I always held the opinion that University courses should be long, complex and articulated in many aspects. They should not aim at a quick transmission of calculating abilities and of ready to use information, rather they should be as much formative as informative. They should offer a general overview of a subject as seen by the professor, in this way giving the students the opportunity of developing their own opinions through the critical absorption of those of the teacher.

One aspect that I always considered essential is the historical one, concerning on one side the facts, the life and the personalities of the scientists who shaped our present understanding, on the other hand concerning the usually intricate development of fundamental ideas.

The second aspect to which I paid a lot of attention is the use of an updated and as much as possible rigorous mathematical formalism. Moreover I always tried to

[1] At that time Regge had shifted from the University to the Politecnico of Torino.

convey the view that Mathematics should not be regarded as a technical tool for the solution of Physical Problems or simply as a language for the formulation of Physical Laws, rather as an essential integral part of the whole game.

The third aspect taken not only into account but also into prominence, is the emphasis on important physical applications of the theory: not just exercises, from which I completely refrained, but the full-fledged *ab initio* development of relevant applications in Astrophysics, Cosmology or Particle Theory. The aim was that of showing, from A to Z, as one goes from the first principles to the actual prediction of experimentally verifiable numbers. For the reader's or student's convenience I included the listing of some computer codes, written in MATHEMATICA, that solve some of the posed problems or parts thereof. The aim was, once again formative. In the course of their theoretical studies the students should develop the ability to implement formal calculations on a machine, freeing themselves from the slavery to accidental errors and focusing instead all their mental energies on conceptual points. Furthermore implementation of formulae in a computer code is the real test of their comprehension by the learners, more efficient in its task than any *ad-hoc prepared* exercise.

As for the actual choice of the included and developed material, I was inspired by the following view on the role of the course I used to gave, which I extended as a mission to the present book. General Relativity, Quantum Mechanics, Gauge Theories and Statistical Mechanics are the four pillars of the Physical Thought developed in the XX century. That century laid also the foundations for new theories, whose actual relations with the experimental truth and with observations will be clarified only in the present millennium, but whose profound influence on the current thought is so profound that no-one approaching theoretical studies can ignore them: I refer to supersymmetry, supergravity, strings and branes. The role of the course in *General Relativity* which I assumed as given, was not only that of presenting Einstein Theory, in its formulation, historical development and applications, but also that of comparing the special structure of Gravity in relation with the structure of the Gauge-Theories describing the other fundamental interactions. This was specially aimed at the development of critical thinking in the student and as a tool of formative education, preparatory to the study of unified theories.

The present one is a Graduate Text Book but it is also meant to be a self-contained account of Gravitational Theory attractive for the person with a basic scientific education and a curiosity for the topic who would like to learn it from scratch, being his/her own instructor.

Just as the original course given in Torino after the implementation of the Bologna agreements, this book is divided in two volumes:

1. **Volume 1**: *Development of the Theory and Basic Physical Applications.*
2. **Volume 2**: *Black Holes, Cosmology and Introduction to Supergravity.*

Volume 1, starting from a summary of Special Relativity and a sketchy historical introduction of its birth, given in Chap. 1, develops the general current description of the physical world in terms of gauge connections and sections of the bundles on which such connections are constructed. The special role of Gravity as the gauge theory of the tangent bundle to the base manifold of all other bundles is emphasized. The mathematical foundations of the theory are contained in Chaps. 2 and 3. Chapter 2 introduces the basic notions of differential geometry, the definition of manifolds and fibre-bundles, differential forms, vector fields, homology and cohomology. Chapter 3 introduces the theory of connections and metrics. It includes an extensive historical account of the development of mathematical and physical ideas which eventually lead to both general relativity and modern gauge theories of the non-gravitational interactions. The notion of geodesics is introduced and exemplified with the detailed presentation of a pair of examples in two dimensions, one with Lorentzian signature, the other with Euclidian signature. Chapter 4 is devoted to the Schwarzschild metric. It is shown how geodesics of the Schwarzschild metric retrieve the whole building of Newtonian Physics plus corrections that can be very tiny in weak gravitational fields, like that of the Solar System, or gigantic in strong fields, where they lead to qualitatively new physics. The classical tests of General Relativity are hereby discussed, perihelion advance and the bending of light rays, in particular. Chapter 5 introduces the Cartan approach to differential geometry, the vielbein and the spin connection, discusses Bianchi identities and their relation with gauge invariances and eventually introduces Einstein field equations. The dynamical equations of gravity and their derivation from an action principle are developed in a parallel way to their analogues for electrodynamics and non-Abelian gauge theories whose structure and features are constantly compared to those of gravity. The linearization of Einstein field equations and the spin of the graviton are then discussed. After that the bottom-up approach to gravity is discussed, namely, following Feynman's ideas, it is shown how a special relativistic linear theory of the graviton field could be uniquely inferred from the conservation of the stress-energy tensor and its non-linear upgrading follows, once the stress-energy tensor of the gravitational field itself is taken into account. The last section of Chap. 5 contains the derivation of the Schwarzschild metric from Einstein equations. Chapter 6 addresses the issue of stellar equilibrium in General Relativity, derives the Tolman Oppenheimer Volkhoff equation and the corresponding mass limits. Next, considering the role of quantum mechanics the Chandrasekhar mass limits for white dwarfs and neutron stars are derived. Chapter 7 is devoted to the emission of gravitational waves and to the tests of General Relativity based on the slowing down of the period of double star systems.

Volume 2, after a short introductory chapter, the following two chapters are devoted to Black Holes. In Chap. 2 we begin with a historical account of the notion of black holes from Laplace to the present identification of stellar mass black holes in the galaxy and elsewhere. Next the Kruskal extension of the Schwarzschild solution is considered in full detail preceded by the pedagogical toy example of Rindler space-time. Basic concepts about Future, Past and Causality are introduced next. Conformal Mappings, the Causal Structure of infinity and Penrose diagrams are discussed and exemplified.

Chapter 3 deals with rotating black-holes and the Kerr-Newman metric. The usually skipped form of the spin connection and of the Riemann tensor of this metric is calculated and presented in full detail, together with the electric and magnetic field strengths associated with it in the case of a charged hole. This is followed by a careful discussion of the static limit, of locally non-rotating observers, of the horizon and of the ergosphere. In a subsequent section the geodesics of the Kerr metric are studied by using the Hamilton Jacobi method and the system is shown to be Liouville integrable with the derivation of the fourth Hamiltonian (the Carter constant) completing the needed shell of four, together with the energy, the angular momentum and the mass. The last section contains a discussion of the analogy between the Laws of Thermodynamics and those of Black Hole dynamics including the Bekenstein-Hawking entropy interpretation of the horizon area.

Chapters 4 and 5 are devoted to cosmology. Chapter 4 contains a historical outline of modern Cosmology starting from Kant's proposal that nebulae might be different island-universes (galaxies in modern parlance) to the current space missions that have measured the Cosmic Microwave Background anisotropies. The crucial historical steps in building up the modern vision of a huge expanding Universe, which is even accelerating at the present moment, are traced back in some detail. From the Olbers paradox to the discovery of the stellar parallax by Bessel, to the Great Debate of 1920 between Curtis and Shapley, how the human estimation of the Universe's size enlarged, is historically reported. The discovery of the Cepheides law by Henrietta Leavitt, the first determination of the distance to nearby galaxies by Hubble and finally the first measuring of the universal cosmic recession are the next episodes of this tale. The discovery of the CMB radiation, predicted by Gamow, the hunt for its anisotropies and the recent advent of the Inflationary Universe paradigm are the subsequent landmarks, which are reported together with biographical touches upon the life and personalities of the principal actors in this exciting adventure of the human thought.

Chapter 5, entitled *Cosmology and General Relativity: Mathematical Description of the Universe*, provides a full-fledged introduction to Relativistic Cosmology. The chapter begins with a long mathematical interlude on the geometry of coset manifolds. These notions are necessary for the mathematical formulation of the Cosmological Principle, stating homogeneity and isotropy, but have a much wider spectrum of applications. In particular they will be very important in the subsequent chapters about Supergravity. Having prepared the stage with this mathematical preliminaries, the next sections deal with homogeneous but not isotropic cosmologies. Bianchi classification of three dimensional Lie groups is recalled, Bianchi metrics are defined and, within Bianchi type I, the Kasner metrics are discussed with some glimpses about the cosmic billiards, realized in Supergravity. Next, as a pedagogical example of a homogeneous but not isotropic cosmology, an exact solution, with and without matter, of Bianchi type II space-time is treated in full detail. After this, we proceed to the Standard Cosmological Model, including both homogeneity and isotropy. Freedman equations, all their implications and known solutions are discussed in detail and a special attention is given to the embedding of the three type of standard cosmologies (open, flat and closed) into de Sitter space. The concept of particle and event horizons is next discussed together with the derivation of exact formulae for read-shift distances. The conceptual problems (horizon and flatness) of the Standard Cosmological Model are next discussed as an introduction to the new inflationary paradigm. The basic inflationary model based on one scalar field and the slow rolling regime are addressed in the following sections with fully detailed calculations. Perturbations, the spectrum of fluctuations up to the evaluation of the spectral index and the principles of comparison with the CMB data form the last part of this very long chapter.

The last four chapters of the book provide a conceptual, mathematical and descriptive introduction to Supergravity, namely to the Beyond GR World.

Chapter 6 starts with a historical outline that describes the birth of supersymmetry both in String Theory and in Field Theory, touching also on the biographies and personalities of the theorists who contributed to create this entire new field through a complicated and, as usual, far from straight, path. The chapter proceeds than with the conceptual foundations of Supergravity, in particular with the notion of Free Differential Algebras and with the principle of rheonomy. Sullivan's structural theorems are discussed and it is emphasized how the existence of p-forms, that close the supermultiplets of fundamental fields appearing in higher dimensional supergravities, is at the end of the day a consequence of the superPoincaré Lie algebras through their cohomologies. The structure of M-theory, the constructive principles to build supergravity Lagrangians and the fundamental role of Bianchi identities is emphasized. The last two sections of the chapter contain a thorough account of type IIA and type IIB supergravities in $D = 10$, the structure of their FDAs, the rheonomic parameterization of their curvatures and the full-fledged form of their field equations.

Chapter 7 deals with the brane/bulk dualism. The first section contains a conceptual outline where the three sided view of branes as 1) classical solitonic solutions of the bulk theory, 2) world volume gauge-theories described by suitable world-volume actions endowed with κ-supersymmetry and 3) boundary states in the superconformal field theory description of superstring vacua is spelled out. Next a New First Order Formalism, invented by the author of this book at the beginning of the XXI century and allowing for an elegant and compact construction of κ-supersymmetric Born-Infeld type world-volume actions on arbitrary supergravity backgrounds is described. It is subsequently applied to the case of the $D3$-brane, both as an illustration and for the its intrinsic relevance in the gauge/gravity correspondence. The last sections of the chapter are devoted to the presentations of branes as classical solitonic solutions of the bulk theory. General features of the solutions in terms of harmonic functions are presented including also a short review of domain walls and some sketchy description of the Randall-Sundrun mechanism.

Chapter 8 is a bestiary of Supergravity Special Geometries associated with its scalar sector. The chapter clarifies the codifying role of the scalar geometry in constructing the bosonic part of a supergravity Lagrangian. The dominant role among the scalar manifolds of homogeneous symmetric spaces is emphasized illustrating the principles that allow the determination of such U/H cosets for any supergravity theory. The mechanism of symplectic embedding that allows to extend the action of U-isometries from the scalar to the vector field sector are explained in detail within the general theory of electric/magnetic duality rotations. Next the chapter provides a self-contained summary of the most important special geometries appearing in

$D = 4$ and $D = 5$ supergravity, namely Special Kähler Geometry, Very Special Real Geometry and Quaternionic Geometry.

Chapter 9 presents a limited anthology of supergravity solutions aimed at emphasizing a few relevant new concepts. Relying on the special geometries described in Chap. 8 a first section contains an introduction to supergravity spherical Black Holes, to the attraction mechanism and to the interpretation of the horizon area in terms of a quartic symplectic invariant of the U duality group. The second and third sections deal instead with flux compactifications of both M-theory and type IIA supergravity. The main issue is that of the relation between supersymmetry preservation and the geometry of manifolds of restricted holonomy. The problem of supergauge completion and the role of orthosymplectic superalgebras is also emphasized.

Appendices contain the development of gamma matrix algebra necessary for the inclusion of spinors, details on superalgebras and the user guide to Mathematica codes for the computer aided calculation of Einstein equations.

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http://www.springer.com/978-94-007-5360-0

Gravity, a Geometrical Course
Volume 1: Development of the Theory and Basic Physical Applications
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2013, XVIII, 338 p., Hardcover
ISBN: 978-94-007-5360-0