Chapter 2
The History of Mathematics Education: Developing a Research Methodology

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The aim of this chapter is to discuss answers to two basic questions: what does the history of mathematics education investigate, and how does it carry out its investigation? It is not enough to say, following and paraphrasing Leopold von Ranke, that the goal is to determine how mathematics education really happened: first, one must determine what is meant by the history of mathematics education (in other words, one must determine what pertains to the history of mathematics education) and, most importantly, what it means to determine something and how exactly this can be done.

Schoenfeld (2007) formulated the following three questions, which may be asked of any study:

- Why should one believe what the author says? (the issue of trustworthiness)
- What situations or contexts does the research really apply to? (the issue of generality; or scope)
- Why should one care? (the issue of importance) (p. 81)

These questions must be answered for historical studies, too. Possible answers to these questions (along with possible criticism of them) will be addressed in the discussion below.

The history of mathematics education is a branch of research that is still only taking shape, and consequently its methodology, too, is still only in its formative stage. Studies directly focused on the methodology of the history of mathematics education are comparatively few, and below they will be discussed in sections that pertain to their subject matter. At the same time, this comparatively new field inevitably inherits the techniques and methods developed both in mathematics education and in history (including the history of science).

In history, methodological discussions have been going on for centuries if not millennia, and at first glance they may seem quite distant from anything that one might argue about in the history of mathematics education, a field in which everything appears more modest and concrete. On the other hand, research methodologies in mathematics education have developed to a very considerable extent under the influence of psychological research and very frequently have been connected with quantitative methods, whose use in historical studies is often problematic, if only because much information has simply not survived. The use of qualitative research methodologies, which have become more popular in recent decades, in mathematics education likewise needs to be made more precise when research begins to address periods that lie hundreds of years in the past. Nonetheless, one cannot speak about the methodology of the history of mathematics education without touching on the two fields just mentioned, if only because it is from these fields that future researchers in the history of mathematics education usually emerge.

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Let us say at once that we see the history of mathematics education as a part of social history, which becomes meaningful only when it includes social analysis and examines what happened in mathematics education in connection with the processes that were taking place in society around it. The development of mathematics teaching is an element of the social subsystem of schools, which, in turn, is a part of the broader subsystem of education, which interacts with other social subsystems such as the labor market (Schubring 2006). Mathematics education has appeared and developed not in isolation, but in response to various social needs.

These needs cannot be understood in a primitive manner, for example, as needs of a purely economic nature. During certain periods, it was in fact the need to prepare a sufficient number of engineers, technicians, and scientists capable of developing new technological applications and organizing their production that was the main stimulus for the development of mathematics education. During other periods, by contrast, the connection with economic development was less simple – it is easy to cite instances when attention to mathematics education was predicated on the need to prepare users of already existing technologies or simply administrators. The development of mathematics education has been influenced by political views as well as philosophical and religious ones. Identifying and elucidating connections with the general course of the development of society is a crucial problem for the historian of mathematics education.

At the same time, historical analysis is often based on the analysis of texts, and since the texts that must be analyzed in this field are most often of a mathematical-pedagogical nature, it is impossible to analyze them without a knowledge and understanding of the relevant mathematical-pedagogical issues (Karp 2011). This two-sided nature of the field under discussion is precisely what makes it interesting, we would argue, although also difficult for research.

1 Historical Research: Who Cares?

The history of mathematics education is sometimes conceived of as a kind of preamble to current research. Demonstrating in two or three paragraphs that the problems that are of interest to them have occupied people for a long time and in this way showing the importance of their own research, some authors seem to say: "well, that was all history, and now to business" – and then proceed to that which really interests them, which is to say their own lucubrations. The thoughtful reader, however, might pause to wonder why a given problem was recognized when it was recognized rather than earlier or later and what happened with previous attempts to solve this problem and even why no one had previously thought of proposing what is being proposed now by this or that author. In general, the relationship between history and "business" – that is, contemporary problems – turns out to be very complicated.

It would be naïve, of course, to rely on the old maxim "*historia est magistra vitae"* and expect that answers to questions arising today can always be taken directly from history (contrary to those who say, as it were, "let's take a textbook from the last century, or even better, from the century before last, and everything will work out" – see, e.g., Kostenko n.d.). Such a search for direct guidance in the past is akin to the widespread faith in direct borrowings from abroad, a faith that virtually everyone appears to denounce, justly pointing to cultural and social differences, but which now here, now there, leads educators to use textbooks from Singapore or proudly announce that in one or another American school, everything is done as it is, say, in China.

History, however, knows numerous instances in which foreign textbooks were used to teach students – and quite successfully – for example, British and French textbooks in the United States. Why, then, did this work in some cases and in others seem laughable and naïve? Attempts to formulate general and permanent rules concerning what can and what cannot be done in mathematics education come to naught: everything depends on the specific nature of the case at hand, on the specific process of which the phenomenon under investigation constitutes a part.

Schubring (1987) justly wrote about the deficiencies of the widespread methodology in which one or another textbook is considered in isolation from everything else or is only nominally compared with certain other textbooks. Textbooks are written and used in the context of a specific system – the textbook's author consciously or unconsciously expects that teachers and students will behave in a certain manner, consciously or unconsciously relies on certain habits that influence how educational texts are read; on certain traditions and tastes that influence how assignments are used; on certain life plans on the part of the students and a certain educational background on the part of the teachers; on certain roles that will be played by parents, the school administration, and the higher educational authorities; on a certain system of testing both teachers and students; and so on and so forth. All of these habits and traditions accumulate and change (as they undoubtedly do change) gradually. Attempts to introduce something extraneous and thus to implement instantaneous transformations are, of course, different from using materials that are close to one another in spirit and style or even materials that are new for a country but used in a context in which not textbooks alone but the whole educational system is brought over from elsewhere, sometimes with teachers included. Steps that superficially seem to resemble each other turn out to be completely different when their historical context is analyzed.

And here we come to the main point, which is that one can achieve an understanding of what is happening only on the basis of studying and understanding the processes of which it is a part, which is precisely the concern of history. The answer to Schoenfeld's third question – "Why should one care?" – can be briefly formulated for historical research as a whole in the following way: "In order to understand what is going on." And this answer pertains by no means only to the use of foreign textbooks, which was mentioned above. The complexity of historical research, however, consists in the fact that the same event forms part of different processes and may itself be considered from different angles.

In our view, it is not only mathematics educators who would do well to care about the history of mathematics education. Contemporary historians try to understand how people lived in the past. Attempts to reconstruct the psychology of the people of another time (Huizinga 1996) or to study their everyday life (Braudel 1974) have long ago become classics of historical research. Education in general and education in mathematics in particular are a part of this daily life, and it was not for nothing that already Weber (2003) drew connections between the development of mathematics (and thus also mathematics education) and the general processes of "disenchantment" taking place during a certain period. If we try to imagine a group of seventeenth-century French schoolchildren sitting on the floor with their teacher and discussing how to fill in a table of expenditures incurred on a journey from one city to another and how to construct a pie chart based on this table, we will immediately sense the impossibility of such a thing ever having taken place – it will seem as impossible to us, perhaps, as picturing the same group of students using iPads, although by contrast with iPads lessons in arithmetic already existed in the seventeenth century, of course, and people did have to travel from city to city. The goals of teaching, the place of mathematics education in general education, anything that may be meant by the words "style of mathematics education," reveal a great deal not only about mathematics education in and of itself but also about the period under investigation as a whole.

2 The Subject of Study

Schubring (1988) once noted that

> [t]raditional historiography used to focus on administrative policy and its operationalisation by decrees, time tables and the weekly portion of mathematics instruction and by the syllabus but did not bother much about real school life and about epistemological dimensions of school knowledge. (p. 1)

Indeed, today when we speak about curricula, we distinguish between professed, intended, and enacted or implemented curricula (Stein et al. 2007); we discuss students' beliefs, with the understanding that this is one of the key components of the learning process (Leder et al. 2002); we conduct detailed analyses of various models of teacher behavior and compositions of lesson plans; and so on. In historical studies, such approaches are used, at least at present, to a far more modest degree. Justly noting that the administrative side of education is much easier to study, which is one of the reasons for its dominance in the history of mathematics education, Schubring (1988) pointed to yet another aspect of the problem: the lack of worked-out theoretical categories for analysis, in other words, the absence of clear questions that one must try to answer. It may be argued that the limitations of historical studies in mathematics education also stem from the limitations of research in mathematics education in general: many of today's problems are understood too statically and one-sidedly.

We will limit ourselves to one example. As has already been said, the role of beliefs and attitudes in mathematics education is today universally recognized. Quite often one hears it is said that, for example, the characteristic trait of Western education and specifically British and American education is the striving to make learning pleasurable (Leung 2001; Leung et al. 2006). It is natural to ask, even if this is the case, whether it has always been the case. Could "making learning pleasurable" have possibly been the goal of the sadistic teachers whose likenesses have been preserved for us in classic British literature? The reality is far more complex: in Britain itself, several different traditions in education have existed and continued to exist in parallel – traditions that have developed in different ways and that have at different times been more or less influential for different reasons. The tendency toward simplification prevalent in studies devoted to the present day – the extraction of some single aspect or feature from its whole framework and context – influences historical studies also: a great deal of research begins to seem like useless digging around in details of a kind to which no one gives any thought even when analyzing contemporary or recent times, for which the collection of the relevant data is far easier than it is for long-gone epochs.

By contrast, by transferring into the past today's conception of mathematics education as a complex phenomenon, one that is by no means reducible to a list of topics found in textbooks or even to a description of lesson formats recommended by someone or others, we see the multifacetedness of our objects of study in the past also. Effectively any topic of contemporary research (for instance, any topic mentioned in the name of a topic study group at international congresses on mathematics education) may be studied from a historical point of view. The history of the development and formation of any school mathematics subject or of teacher education or, on the other hand, the history of the development and change of beliefs and attitudes toward mathematics and its teaching, and many other topics can and should be subjects of study. Moreover, since the processes through which education developed were by no means necessarily identical in different countries, specific national characteristics in teaching (beginning from the time when such specific national characteristics first emerged) are of considerable interest, as are interactions between different countries and transmission between different cultures, about which not a little has already been said above.

Following mainly Schubring (1988), let us list several types of crosscutting studies that might be relevant for different times and different countries:

- *The state of mathematics within general education for all and within professional education.* The role of mathematics education in different societies has varied quite widely. Today, on the wall of virtually every mathematics classroom in Russia, one can read the famous words of Lomonosov (1711–1765): "Mathematics must be studied if only because it puts the mind in order." However, in Lomonosov's time, mathematics in Russia was studied not at all in order to put the mind in order, but first and foremost for practical needs. On the other hand, the Western European tradition of liberal arts allocated a place for mathematics in a system constructed – at least in principle – for the school, not for life. The development and interaction of different traditions is quite relevant for an understanding of different processes taking place in mathematics education.

- *Divisions, interactions, and influences between different stages in education, including secondary and elementary and secondary and higher education.* Today's system, in which a university student, before enrolling in the university, has gone through secondary and elementary schools along with others who did not go on to a university, is the product of a comparatively recent past. Differences between secondary and higher education have not always been as clear as they are today (and even today, they are often easier to trace organizationally, than in terms of curricula). Elementary schools, on the other hand, often made no provisions at all for the continuation of education beyond them. A certain degree of interaction, balance, and even mutual influence nonetheless existed among different institutions.
- *The organization of the instructional process, teaching practices, the role of the teacher, and the function of textbooks.* It would not be wrong to say that historical studies in these topics in many countries are only beginning. The identification of the specific character of the development of mathematics as an educational subject against the background of the development of education as a whole needs to continue.
- *The professional role of the mathematics teacher, teacher education, and its influence on the instructional process.* The teacher of mathematics in a school, a familiar figure today, also appeared historically not that long ago, and the appearance of this figure and the changes in the professional life of the teacher, including the appearance of a special mathematics teacher education, are all important topics for research.
- *The relation between scientific knowledge and school knowledge.* The content of mathematics education, including the mathematical form in which it is presented, despite widespread views to the contrary, has by no means been absolutely stable. How it changed and what roles have been played by conceptions of mathematics as a science and by conceptions of its role as a science are all topics that require further study.
- *Local, national, and international in mathematics education*. The development of mathematics education, while reflecting the local context in which it takes place, is at the same time subject to powerful influences from international processes. Such influences are evident and regularly discussed today, but they existed in the past as well, as has already been pointed out. Meanwhile, the manner in which various national and regional systems of mathematics education interacted with one another in the past has not yet been studied in the majority of cases (Schubring 2009).

The topics listed above, which pertain to the place of mathematics education, interactions among its various parts, the contents of education, the nature of instruction, and those who implement this instruction, are important for characterizing the state and development of mathematics education. They do not exhaust all possible research topics, of course, and each of them furthermore contains numerous subtopics while being in itself connected to broader topics, such as *mathematics education and religion; the cultural determination of school knowledge; the contribution of mathematics education, and specifically textbooks, to the social history of ideas; mathematics education and political movements;* and so on.

3 The Theoretical and the Descriptive

Studies in topics listed above and others may be carried out in different ways. Going back to Schubring's (1988) proposition concerning the lack of worked-out theoretical categories, we would argue that theorization in general is sometimes seen as being opposed to dispassionate research and historical generalization and even the use of general concepts – whether Marxist socioeconomic formations or the *mentalité* of the Annales school – as a distortion of reality. It is, indeed, not difficult to cite instances in which historical theory did not grow out of facts, but preceded them, and the expert scientist was invited only in order to find facts that could confirm one or another set of propositions from, say, the famous Soviet *Short Course in the History of the Communist Party (Bolsheviks)* or some other collection of wise remarks by a political leader (the history of mathematics education, naturally, does not contain as many examples of this kind as does political history, but they also exist and will be addressed later). In opposition to this, there has been a tendency to avoid theory and to present "just the facts."

Without question, the collection and publication of existing evidence, that is, of surviving documents, is undoubtedly useful, and one can only welcome, say, such collections of documents on administrative history as D'Enfert et al. (2003). At the same time, as was justly remarked by Andrey Kiselev, Russian author of mathematics textbooks who was himself involved in politics, "not everything by which the common man lives is based only on laws, not everything is prescribed in them" (cited in Karp 2002, p. 15). By limiting ourselves to the decrees of the central government or even if we include in our studies the directives of local governments, we automatically paint an incomplete picture.

In most cases, however, it is impossible to present all of the facts: sometimes a great quantity of materials has survived (official memoranda from a large school district alone can fill up many volumes), and sometimes, conversely, the most important information has not reached us. Conscientious researchers take into account the greatest possible number of the materials that are known to them and seem relevant to them, but for purposes of publication, they almost always make a selection from them (this is the case in all, not just historical, research). Writing about the methodology of all research in mathematics education, Schoenfeld notes: "These acts of selection/rejection are consequential for the subsequent representation and analysis of those data" (p. 71).

Below, we will have an opportunity to discuss which incomplete presentations of data seem possible in historical research and which do not seem possible. Here, we will merely state once again that, in any case, it is still impossible to make do without general theoretical positions. They are expressed both in the selection of materials and in their presentation. "Theoretical" studies, which represent new conceptions and new approaches toward describing and understanding the changes that have taken place, are accordingly no worse (although not necessarily any better) than "descriptive" ones, whose authors do not explicitly state what it was that led them to give some particular description and what conclusions they reached on the basis of this description or have not yet found an adequate form in which to express their understanding.

At this point, however, it must be said that the very notion of process and evolution is understood in historical literature in various different ways. Bayly (2011) writes:

> Late twentieth century postmodernist scholars rejected the whole idea of the historical evolution of forms, stressing instead the fragment, the unique experience, and arguing that far from an objective, evolving process, history was constituted by the discourses of the present. World history therefore became, following Foucault, the history of discourses of global powers. (p. 13)

As part of the ongoing process of change in the interests of historians (a process whose existence no one appears to deny, although one might also speak about it in terms of changing discourses), the notion of causation itself is rejected (Wong 2011) and not just some primitive conception of this notion, but the very existence of cause-effect connections between historical events. Indeed, the very notion of a fact becomes open to doubt, because historians allegedly see not facts themselves, but their reflections in the perceptions of various people, who inevitably perceive reality in different ways.[1]

Russian researcher Andrey Zaliznyak (2010) objects quite sharply to such views, writing that the "paradigm of postmodernism, initially enthusiastically perceived as a sign of new freedom, in reality now leads to many destructive consequences," causing people "gradually to forget how to draw rigid distinctions between the true and the false, the accurate and the inaccurate."

[1] In one respected American pedagogical journal, I have had occasion recently to read a teacher's disquisition about how she tried to open the eyes of 6-year-old children to the fact that one should not say that the three little pigs from the well-known story are good and the wolf is bad. In reality, they just have different perspectives. (In support of this proposition, it was pointed out how pleasant it can be to eat some good ham.)

What is undoubtedly true and what is false is more difficult to establish in history than it is in mathematics. Schoenfeld (2007) quotes Henry Pollak: "there are no theorems in mathematics education" (p. 92) – although even in mathematics the level of a proof's strictness has not always been the same and consequently could be criticized (Grabiner 1974). It is noteworthy that an orientation toward examining the "unique experience" is sometimes used to deny connections and dependencies and indeed all attempts at generalization in the spirit of mathematical reasoning, in which, as is known, one counterexample suffices to disprove a proposition.

Mathematics education by its very nature has to do with specific individuals and individual lives. The specific characteristics of individuals can sometimes contradict any observable tendency, which undoubtedly implies that no such tendency can be represented as an iron law that is always in effect. At the same time, neither can the existence of such a tendency and the possibility of a theoretical generalization be denied on the basis of one or even several counterexamples. Analyzing the biographies of outstanding Russian mathematicians who have received Fields Medals (Karp 2011), one can observe that very many among them were winners of high-level mathematics Olympiads and attended famous mathematical schools. This is not true for all of them, however. It would therefore be as wrong to claim that a Russian who did not attend such a school cannot become a major mathematician as it would be to claim that whether or not a person attended such a school has no importance.

In those cases in which insufficient information has survived, it will be more difficult to identify a tendency than in cases for which we have all the data. Indeed, even when we have all the data, establishing such important characteristics of education as the existence of restrictions for various categories of citizens, for example, often meets with objections: what kind of restrictions on education can there be, it is asked, when so many people have obtained it? Such facts are even more difficult to demonstrate when the discussion concerns the past. For example, even the relatively obvious fact that individuals of non-noble origins in Russia in the 1830s were restricted in obtaining an education in general and a mathematics education in particular can be argued with by citing examples of people who did obtain it. The lives of such individuals are undoubtedly worthy of study, and references to this or that set of laws are insufficient, if only because laws, generally speaking, are not necessarily obeyed,[2] but the fact that someone managed to overcome the restrictions does not mean that they did not exist. In general, identifying tendencies and generalizing them is quite a demanding task – which does not mean, however, that it ought not to be undertaken.

Generalization and theorization, however, must always take into account the complex nature of the object being studied – mathematics education – which is, on the one hand, truly international, but on the other hand, not even national, but regional and local. American researchers of contemporary mathematics education usually understand that observations made at a suburban school cannot be generalized for all cases. In historical studies, it is not easy to answer the question posed above – "What situations or contexts does the research really apply to?" – if only because first one must determine what factors constitute the situations and what defines the contexts, and this requires a deep immersion into the history and culture of the period in question; the difference between suburban and urban districts is today known to practically everyone, but certain differences between regions, obvious to contemporaries, may get lost, particularly if they are not economic in nature, but based on some other parameter that many people today are not familiar with.[3]

[2] In reality, analysis of the biographies of persons who did obtain an education shows that the authorities monitored adherence to the established order quite closely: when, for example, a graduate from St. Petersburg's Third Gymnasium was discovered to have been originally admitted to the school from among the low-level social strata without the requisite forms and permissions, the director was quite severely reprimanded (No author 1835).

[3] The Russian writer Alexander Herzen wrote: "The power of the governor in general grows in direct proportion to the distance from St. Petersburg, but it grows geometrically in gubernias where there is no gentry" (Herzen 1956, p. 237). The researcher of education in Novgorod and Vyatka during the 1830s must take these considerations into account.

Robson (2002) justly noted, when discussing the history of Babylonian mathematics (which is, as a matter of fact, practically inseparable for that time from the history of mathematics education), that the popular metaphor of Sherlock Holmes traveling to the ancient world and understanding what is going on around him – by using his wits and powers of observation – is completely wrong. One cannot get by on acumen alone; one has to understand the conditions in which one or another text was written, and only by relying on such an understanding can one formulate one's hypotheses. (Conan Doyle's Holmes, incidentally, kept an abundance of manuals and never denied the importance of specialized knowledge.)

Undoubtedly, one can imagine studies in which tables of contents from various textbooks are ingenuously analyzed and, for example, chapters and sections that were missing at a certain stage but appeared at later stages are identified. Moreover, such a study, unencumbered by considerations about the era in which these textbooks were used, can possess a certain value, but only, unfortunately, as a preliminary step. Subsequently, the researcher will need to try to understand why these changes took place, and at this point he or she will have no choice but to go beyond the perusal of tables of contents.

4 Sources: Their Identification, Analysis, and Interpretation

Contemporary readers who for one reason or another have never before given any thought to the question "What is the historical method?" will most likely look on the Internet and learn that the historical method consists first and foremost in working with primary sources, analyzing them, criticizing them, and interpreting them. This is true of studies in the history of mathematics education as well. It is another matter that both in history in general and in the history of mathematics education, the difficulty consists precisely in determining which sources may be used and how to use them.

Without even attempting to enter into questions of historiography here, let us recall that attitudes toward evidence from the past have varied from almost total trust to almost total distrust, and the change in these attitudes has not by any means always been steady and in the same direction: quite recently, studies that took everything in the sources they used at face value have appeared alongside of hypercritical works (probably the most vivid examples of the latter, actually lying beyond the bounds of scientific literature, are the works of the mathematician Anatoly Fomenko and his collaborators, which prove that Classical Antiquity never existed, that Pope Gregory VII and Jesus Christ were really the same person, etc., and that at one time people simply fabricated a great deal of counterfeit evidence about life in the past). The specific character of the history of mathematics education as a scholarly discipline is such that, due to a certain incompleteness in its development, already mentioned above, the question of what could be considered a source has also often been understood in a narrow way, so that the issue of critiquing and comparing sources often did not arise at all. Therefore, we must begin by discussing possible primary sources.

Again, textbooks and the Internet report that primary sources are usually divided into "relics" and "narratives." In our field, relics consist of what are also called "tools of mathematics education" (Kidwell et al. 2008). This includes manipulatives, blackboards, computers and calculators, models, and even textbooks. The appearance of such tools and their technological development are significant for the development of mathematics education (one might compare it to the influence of the development of musical instruments on the development of music, which was studied by Weber 1958). The technical parameters of the instruments used in schools themselves contain quite a bit of information; still, in discussing blackboards or models (textbooks will have to be discussed separately – their role is more complicated), we cannot limit ourselves to such parameters. Naturally, the fact that blackboards replaced slates in classrooms and the ways in which they came to be improved demonstrates that the nature of work in mathematics classes changed over time, but we will be able to get serious insights into the changes that occurred only if we make use of people's testimony – only if we make use of narratives.

This category of sources encompasses a great deal. Effectively any text that concerns mathematics education and, even more broadly, that concerns the life of a person involved in the development of mathematics education can under certain circumstances serve as a source in the history of mathematics education. Naturally, this includes official documents pertaining to mathematics education, for example, testing documents, complaints, or internal reviews of textbooks; but useful information can also be derived from strictly personal diaries and memoirs, newspapers and journals in which advertisements or announcements of various materials were published, novels that depict classes and teachers, poems and songs composed by students about their education, and much else besides (some examples are given in Karp 2008).

The notion that sources in the history of mathematics education can be listed once and for all is erroneous. Marc Bloch once wrote: "even those texts and archeological finds which seem the clearest and the most accommodating will speak only when they are properly questioned" (Bloch 2004, p. 53). The authors of various diaries would probably be surprised to learn that they have provided evidence about mathematics education, since they wrote about something completely different. Nonetheless, sometimes indirect evidence turns out to be substantive and indispensable. Therefore, a text may be useful within the framework of one approach and useless within the framework of another. Consequently, texts that had not been considered sources and had not been used in research previously may turn out to be useful in the future.

Researchers studying the work of the already mentioned Russian author of mathematics textbooks Andrey Kiselev may, generally speaking, limit themselves to analyzing how his famous geometry textbook is constructed, how it differs from the previously most popular Russian textbooks (e.g., the textbook by Davidov), and how it is connected with textbooks from other countries, above all French textbooks. Further research might include an analysis of the changes that took place from one edition to the next (in fact, memoir accounts have survived about Kiselev attending teachers' meetings and writing down the suggestions and comments of working teachers) and above all the changes introduced into the Soviet editions of the textbook, edited by N.A. Glagolev and published in enormous numbers. Finally, another further step might be connected with interpreting what happened with Kiselev's textbooks in the context of the changes taking place in mathematics education as a whole, which reflected the radical changes taking place in the country during the late 1920s and early 1930s.

For the first of these studies, researchers could limit themselves to Kiselev's textbook and several others, although, of course, it would be useful to examine reviews or transcripts of teachers' meetings, as well as surviving normative documents. The second study would necessarily have to include such materials. The third study would necessarily have to rely on a very broad range of materials, among which, for example, it would be very sensible to include prerevolutionary newspaper publications portraying Kiselev as a conservative political figure, consequently very far from the revolutionary ideology of 1917–1918, but fitting in much more among those who gained ascendancy under Stalin.

Analyzing and interpreting relevant documents means being able to read a cultural code. The cultural historian Yuri Lotman (1992) wrote about this notion, analyzing the comments of a French traveler who met Russians from different circles, accurately reported what happened and what was said, but drew absolutely mistaken conclusions, precisely due to a lack of understanding of the attendant circumstances. The French traveler saw and noted that various individuals chuckled at the czar but drew the conclusion that they were freethinkers, while they were on the contrary the czar's closest collaborators and precisely as such permitted themselves to smirk and to grumble. Historians of mathematics education (like all cultural historians) must connect their observations with a general culturological and social analysis of the era; a simple reproduction of the judgments expressed by various mathematics educators, various mission statements, or even official statistical data that does not take into account the circumstances of the time in which they were made can be misleading.[4]

This is all the more true because the meanings of terms used in the past, even if they continue to be employed, usually change. Today's mathematics educators who wish to promote problem solving do not necessarily mean by this term the same thing that was meant by their colleagues 30 years ago (and even today "problem solving" often means different things to different people), while differences between what is meant by "equality in education" today and what was meant by this phrase during the French Revolution are altogether glaring. The genuine meanings of words emerge in the course of analysis that combines the special and the general, sociohistorical.

Schubring (1987) suggested that researchers choose a basic "unit" for studies "where at least some of the relevant dimensions can already be seen in interaction." As one such unit, he proposed looking at the life of the individual mathematics educator, in which working with teachers, writing textbooks, and simply the ordinary life of a person in a given era are all intertwined. As another technique, he proposed studying different editions of the same textbook: by connecting the changes taking place in life and pedagogy as a whole with the changes introduced into different editions of a textbook, he argued, one can arrive at a better understanding of what has taken place.

The historian of mathematics education in general must as far as possible strive to achieve a kind of competition among different sides; one must always try to find sources representing different sides and presenting mathematics education in different ways. For example, complaints about lowered grades or investigations of conflicts between teachers make it possible to portray existing practices, prevailing values, and the outlines of the contradictions between them. Indeed, the comparison of sources constitutes the main technique for critiquing and analyzing them.

A source in the history of mathematics education can be as unreliable as any other historical source. The teacher of mathematics who cleans up mistakes on students' tests in order not to be scolded for poor instruction by a visiting inspector is, of course, far less ambitious than the author of the Pseudo-Isidorean Decretals. Nonetheless, in both cases historians have to deal with falsifications. Evidence about the teaching of mathematics is no more reliable than evidence about any other human activity – here, too, one can find deliberate lies, and here, too, one can find honest mistakes, confusions between what is desirable and what is real, and so on. Critical analysis is indispensable in such cases, too.

The trouble, however, consists in the fact that sources rarely come to us fully complete – for various reasons, a great deal turns out to be inaccessible for researchers. It is unlikely that one could give a universal solution for overcoming this incompleteness: everything depends on the question being examined. For example, when we examine memoirs about school years and the teaching of mathematics, we evidently have to do with authors who, of course, cannot be considered representative of all students of a given time – far from everyone who writes memoirs. It would therefore be incorrect automatically to consider the assessments expressed in surviving memoirs (concerning, say, the importance of various sections of the school curriculum or different school subjects), even if all surviving memoirs express the same views, to have been shared by all the students of the time. On the other hand, if all memoirs describe, for example, certain lesson formats or certain typical homework assignments, this can be regarded as substantial evidence of the fact that the memoirists' schools did indeed employ such lesson formats and homework assignments.

[4] The same Herzen relates how the German traveler Humboldt, who was accompanied by a Cossack, inquired of the latter about the temperature of the water in a spring that they had come to. The Cossack "put up a stony front and replied: 'whatever duty demands, Your Grace – and we are glad to do what we can,' since to himself he thought: 'No, sir, you won't put anything over on me'" (Herzen 1956, p. 126). And the question in this case concerned merely a measurement of temperature, something far more simple than any measurement of education.

In studying the history of mathematics education, just as in any other kind of historical research, one might be confronted with the question of whether it is admissible to use evidence drawn from sources that are not entirely reliable. Discussing this issue, the Russian historian Yakov Lurie (2011) wrote that only by putting together all the facts that pertain to a given source (such as its overall structure and its textual history), along with observations concerning the specific items in it, and thus developing a general preliminary picture of the source as a whole can one acquire a footing for assessing the veracity of particular facts. The historian noted that, instead of doing this, researchers quite often leap from observations contained in the source directly to assessing their likelihood (and in addition sometimes interpreting their likelihood as proof of the fact that what is described in the source actually took place). Lurie analyzes transcripts of interrogations of heretics in Western Europe and Russia, but methodologically similar situations arise in studies of far more peaceful circumstances.

For example, in a diary from 1892 that has survived in manuscript form, St. Petersburg resident Alexandra M. describes how she and her girlfriends called on Yakov Gurevich, the director of a gymnasium and the editor of an influential journal, in order to persuade him to give them copies of a mathematics examination paper before the examination was given, acquaintance with which would have naturally been useful to her brother. According to her, Gurevich heard the girls out, remaining well disposed (in particular, he led them away from some doormen who could have informed on them (M., Alexandra 1892)). Not a little surviving evidence points to the fact that indeed the pedagogues of that time did not take the secrecy of exams too seriously (e.g., Brushtein (1985) tells a story pretty similar to the one just mentioned). At the same time, an analysis of the text as a whole – which abounds in such phrases as "I coldly and somewhat derisively asked" or "we were as if drunk," as well as self-congratulatory observations by the author of the diary about her and her girlfriends' cleverness – suggests that in this particular instance the story recounted by the author nonetheless could be pure fantasy.

By casting doubt on the veracity of the story told in this diary about Gurevich, we by no means must reject the diary as a whole as a source: considered in the context of other sources, it confirms that in and of itself procuring copies of an examination paper in advance of the exam was a rather quotidian affair. An interpretation of a source that is grounded in a criticism of the source is not reducible to a paraphrase of that information in the source which criticism recognizes to be reliable but possess a far more complex character.

The analysis of a source's general characteristics, including its history, sometimes reveals significant facts in and of itself. Let us mention one more Russian example. An inspector's report from 1946, which has survived, tells of third-grade students who made a mistake in copying a text from the blackboard, omitting a word, and thereby making the copied phrase "politically harmful." Probably even more useful than the details of the inspector's report for understanding the system of education that existed at that time is the fact that this report ended up in the Leningrad City Committee of the Communist Party, which found time, while governing a city with a population of several million, for investigating the causes of the politically harmful behavior of third graders (Karp 2010).

The incompleteness of sources mentioned above often prevents researchers from carrying out quantitative analyses, which are in general so popular in mathematics education. At the same time, in certain (and even in many) cases, such analyses are in fact possible. For example, for a relatively large number of nineteenth-century secondary schools, teachers' journals have survived, and consequently, it is quite possible to determine quantitatively how grades in mathematics were given in various places, at various times, and sometimes even to follow the individual lives of good students and bad ones. Surviving examination statistics sometimes make it possible to discover typical mistakes made by students or to carry out other kinds of studies along the lines of those which are conducted using the results of today's examinations.

Quantitative methods in historical research, of course, are quite feasible and useful. Yet one ought not to contrast them with other kinds of methods and assume that quantitative methods are more reliable. Without qualitative analysis and comparisons with other facts, computations are unlikely to be meaningful (e.g., going back to the example of the analysis of school grades, it is important not only to examine their distribution but also to understand what value grades possessed in general and what value grades given for specific assignments possessed in particular).

In general, to repeat, the methods of the history of mathematics education are first and foremost historical methods. It is clear, however, that much greater mathematical preparedness is demanded of the researcher in the history of mathematics education than of the general historian: the texts studied are often mathematical texts.

Grattan-Guinness (2009) remarked that the history of mathematics is “too mathematical for historians and too historical for mathematicians.” This is at least to a certain extent true of the history of mathematics education as well, although for some reason it is sometimes assumed that school-level mathematics is something that everyone knows. A superficial knowledge of it leads researchers to confine their analysis of mathematical texts – textbooks or examination materials – to superficial approaches, such as subdividing problems only into obvious categories, for example, problems that require proofs or word problems. A deeper analysis often makes it possible to see how problems become more or less difficult over time, the changes in the principles underlying their selection, and the influence of various sources or social forces. This last point may seem like an exaggeration: such a title as “The Social History of Quadratic Equations” sounds like a parody, since we are accustomed to think that quadratic equations are in no way connected to social life and that the solving of quadratic equations, by contrast, may serve as an example of a lack of social activism. This, however, is not the case: the authors of textbooks or exam problems write in response to certain demands, defining both the form and content of their assignments in a corresponding manner (Karp 2011). The historian of mathematics education must be able to see such connections and to command both historical and mathematical-methodological methods of analysis.

Summing up, and attempting, as on an exam, to give a one-sentence response to Schoenfeld’s question: “Why should one believe what the author [of a historical study] says?” – we can say that the answer often comes down to the support (direct or indirect) found in different sources for the conclusions drawn by the author and to their general conformity with the existing understanding of what took place at a particular time. However, people by no means believe only that for which support may be found.

5 Myths in the History of Mathematics Education

The mythologies that arise on the basis of history are studied by many researchers. For example, in a recently published book, Margaret MacMillan (2010) lists different instances of the “abuse of history,” in which invented or exaggerated historical facts have been used for political purposes, including in this last category all possible group interests as well. The history of mathematics education may seem too small a field to possess its own mythology: it is a discipline, one might say, that is not big enough for such britches. This, however, is not the case. Mathematics education lies at the intersection of a large number of politically important topics, an intersection large enough for people to seek to falsify its history.

Furthermore, a mythology is not always invented deliberately in someone’s interests: sometimes it arises simply from a striving for simplification. The noble baron in Mark Twain’s *A Connecticut Yankee in King Arthur’s Court* (1889) cannot find the product of 9 and 6 on an exam, but obtains the position he desires because all of his ancestors were also noble barons. The difference between the positions of a baron and a weaver in the Middle Ages was undoubtedly enormous, but it would be wrong to project contemporary notions of discriminatory procedures into the past: barons did not take exams in mathematics (let alone together with weavers). Mark Twain deliberately modernized the past – no less in the given instance than when describing the use of a telegraph in King Arthur’s court – but sometimes the past is seriously conceived of as being not very different from the present day.

The striving for simplification likewise reinforces the most widespread myths, which may be called "the myth of the blessed past" and "the myth of inevitable progress." These two myths are in a certain sense opposed to one another. The Russian mathematics educator Yuri Kolyagin (2001) opens his book with remarks about the rapid growth and improvement of Russian education during the years of Soviet rule (particularly during the Stalinist period) and its subsequent decline due in large part to the machinations of the CIA (on p. 7, Kolyagin reproduces excerpts from the so-called Dulles Declaration, a long-discredited forgery). The book also informs us, however, that even prior to Soviet rule, Russian schools were excellent and all was well with the world – but eventually troublemakers came along and ruined everything. This book, of course, represents a kind of extreme case, but the conviction that in the past everything was perfect and that people then started changing everything for some reason can be found in milder formulations in different countries. Above, we have already mentioned the naive belief that when old textbooks were being used, everyone knew and understood everything.

Examples demonstrating successes achieved in the past are indeed sometimes not difficult to find, as are situations in which something appears to us to be better than what it subsequently turned into. The creators and propagators of the myth, however, usually take an absolutist stance toward such examples, ignoring the existence of other aspects of what took place. The Hungarian educators Halmos and Varga (1978) once described the state of affairs in a school undergoing reforms as follows: "It goes without saying that the last four years of this general school could offer less to every pupil than what the first four years of the earlier eight-grade secondary school could give to a highly selected population of the same age" (p. 225). This sentence underscores the complexity of the processes taking place – some things improved, some things got worse. A given change for the worse may be obvious, but the process is far more complex than the creators of myths like to recognize. Even leaving aside Dulles, and without going into other details, let us say that a good prerevolutionary mathematics education in Russia was accessible to very few individuals, while mass-scale Soviet education developed against the background of the difficult predicament in which other school subjects found themselves, which in itself makes it impossible to characterize those years as a happy time even for school education.

If the myth about the good old days is often fueled by reactionary political views (while at the same time serving such views), then the myth of progress often arises from a transfer into education of what is observed in technology. Computers over the past 20 years have become thousands of times more effective; it is natural (although mistaken) to think about improvements, even if not such rapid ones, in other fields. Unfortunately, there are no grounds whatsoever for this view, just as there are no grounds to think that the teaching of mathematics over the course of centuries has invariably grown better.

Moreover, myths about improvement and deterioration usually presuppose the existence of some common, universal scale, using which one can determine, for all times, what is good and what is bad – which is obviously far from reality. But the challenge for the history of mathematics education is sometimes seen as consisting precisely in finding "good practices" and sometimes even in extolling them. This becomes especially clear when attempts are made to use the history of mathematics education for nationalist motives, by creating and sustaining a *myth about the special role of national mathematics education.*

This myth comes in different versions: one might see the system of education in one's country as the cradle of international education, claiming that it was specifically in a given country and a given place that the most important ideas were born; or one might see the system of education in one's country as a bastion of international education, contending that even if mathematics education developed later in one's country than in some other places, it nevertheless attained greater heights; or one might not even make this claim and criticize the system of education in one's country, simultaneously assuming, however, that it is the most important system of education all the same, since the country as a whole is very important.

Such mathematics education nationalism becomes more acute during periods of conflicts and wars (Karp 2007) but is by no means limited to such periods, manifesting itself during quite peaceful times as well. In reality, the processes of development are far more complex; even when it became possible to speak about national education (which happened relatively recently – certainly not before the formation of nation states), processes and ideas from different countries often echoed each other, and different ideas were first realized in different countries. This does not mean that one can never or that one should not strive to determine when exactly they first appeared, but it rules out the use of such information to fuel national pride. Of particular interest in this connection are instances when ardent patriots struggling for their country's national independence preferred foreign curricula or textbooks in mathematics, assuming that they would be of greater benefit to their homeland and not subscribing to the view that their own was invariably better (Zuccheri and Zudini 2007).

Each country has its own history of the development of mathematics and mathematics education, and this history has different pages. Already in the nineteenth century, Dmitry Tolstoy (1885), a Russian political figure and historian of education, indignantly quoted a learned German of the eighteenth century who had argued that not all nations were capable of genuinely scientific undertakings and therefore that Russians, who were incapable of reaching the true summits of learning, should devote themselves to lower concerns, namely, mathematics. In the nineteenth century, and even more so in the twentieth, mathematics ceased to be regarded as an insufficiently scientific enterprise; however, the belief that not all nations were capable of reaching the true summits of learning (now by learning mathematics) was one that people sometimes continued to espouse and to express.

The assertion that Russians or Americans are incapable of learning mathematics, made in the past, could only make us laugh today. Meanwhile, we know that in France and Germany there were both major mathematicians and a quite developed system of mathematics education at a time when nothing of the kind existed in Russia, while by the time that Russia could boast of the names of Lobachevsky or Chebyshev and of substantive courses in gymnasia and universities, the United States could only look forward to anything comparable. One might think that this alone should make highly suspect any contemporary claims about representatives of countries which today occupy less prominent positions in the world of mathematics than Russia and the United States and their alleged inability to learn mathematics. However, one often encounters as a counterweight to such claims what may be called the *myth of universal simultaneous development*. The fact that certain techniques, ideas, and organizational structures were borrowed from abroad by all countries begins to be considered invariably offensive, and people begin to claim that every place had its own indigenous system, which not only deserves to be taken into account and respected but in general requires no additions or improvements whatsoever.

The myths listed above, along with others not mentioned, are most often found in popular literature, but they exert an influence on scholarly literature as well, or more precisely, on the people who study it. Determining the truth turns out to be less important than not contradicting existing viewpoints – which calls for a completely different methodology. Historians feel obligated to seek supporting evidence for these viewpoints and are afraid of the conclusions that can be reached on the basis of the documents they study. As MacMillan (2010) writes, these are indeed "dangerous games."

6 Conclusion

The aim of this chapter has not been to list all possible research methodologies in the history of mathematics education. Oral history alone – and consequently the practice of interviewing, a powerful research tool in the history of mathematics education, in which even the relatively recent past has not been sufficiently documented – has been the subject of numerous books and articles. Considerable literature has also been devoted to other methodologies of historical research and qualitative research methodologies in mathematics education, including methodologies used in the studies discussed in this handbook.

The goal of this chapter has been to describe the twofold nature of the field: historical in terms of methodologies and mathematical-pedagogical in terms of the objects of study. Overcoming a simplistic understanding of both these objects and these methodologies, which reduces research to reprinting tables of contents from textbooks and the like, may be the most important methodological objective of all.

As has already been repeatedly said, the history of mathematics education is still in its formative stages as a scientific discipline. Over time, it will likely become enriched by vivid examples and model studies (although not so little has already been done), while its methodology will expand and acquire new resources. The principles and spirit of conscientious research based on all available information and aiming to reconstruct a realistic picture of what has taken place will, one would like to hope, remain unchanged.

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